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Radiative and hadronic transitions of the charmonium $^1P_1$ state

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Hadronic transitions of the charmonium $^1P_1$ ($=h_c$) state into J/ψ are considered in the QCD multipole expansion. Using the decay rate of $\chi_{cJ}\to J/\psi+\gamma$ and a quantum-mechanical sum rule, we estimate $\Gamma(h_c(1P)\to J/\psi\pi^0)>1.6$ keV, which is close to the recent measurement by E760, $\Gamma(h_c(1P)\to J/\psi\pi^0)\sim 1$ keV. Other transitions of h_c are also considered in the same context.

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The spin-singlet P-wave quarkonium state $[n\,^1P_1(Q\bar{Q})\equiv h_Q(nP)]$ is an interesting object for both theoretical [1,2] and experimental investigations. Theoretically, one can test the ideas of potential models for heavy quarkonia, QED multipole expansion for radiative decays, and QCD multipole expansion for hadronic decays into lower level quarkonia states, as well as perturbative QCD for $Q\bar{Q}$. Especially, it can be a useful source to reach $n\,^1S_0$ (e.g., η_b) through the radiative decay, $h(1P)\to n\,^1S_0+\gamma$. However, it is difficult to produce the $^1P_1$ state in e^+e^- annihilation, since the parity of $^1P_1$ is even. Further, the $^1P_1$ state cannot be produced from the radiative decay of a higher $^3S_1$ state because of its negative charge-conjugation parity. Therefore, it would be best to look for the $^1P_1$ state in $p\bar{p}$ annihilation. Recently, the charmonium $^1P_1$ state $h_c(1P)$ has been found in the $h_c(1P)\to J/\psi\pi^0$ channel by E760 Collaboration at Fermilab [3]:

$$\frac{\Gamma(h_c(1P)\to J/\psi\pi\pi)}{\Gamma(h_c(1P)\to J/\psi\pi^0)}<0.18\ (90\%\ \text{C.L.})\ . \tag{1}$$

Also, the data suggest that

$$\Gamma_{\text{tot}}(h_c(1P))\sim 700\ \text{keV}\ , \tag{2}$$

$$\Gamma(h_c(1P)\to J/\psi\pi^0)\sim 1\ \text{keV}\ . \tag{3}$$

Two theretical approaches discussed in the literature [1,2] lead to rather different results on Eqs. (3) and (1) as well as on the counterparts in the bottomonium system. Since the E760 result (1) agrees only with the prediction of Voloshin [2], we consider absolute decay rates of the above decays in the framework of Ref. [2]. We try to minimize dependence of our results on specific models on the quarkonium potential. Therefore, a specific form of the potential will not be needed in this work except for the size of the $\Upsilon(1S)$ state.

First, the matrix element for $^1P_1\to {}^3S_1+\pi^0$ in the QCD multipole expansion is discussed. Then, they are related to the matrix element for the electric dipole radiative transitions, $^{2S+1}P_1\to {}^{2S+1}S_1+\gamma$. To get information on the Green's function G_8 of color-octet $Q\bar{Q}$ states, we consider another class of hadronic transitions between quarkonia: $n\,^3S_1\to m\,^3S_1+\pi\pi$ and $n\,^3S_1 m\,^3S_1+\eta$ (or π^0). Using a quantum-mechanical sum rule, one can then derive a lower bound on G_8, and predict the absolute branching ratio of $h_c(1P)\to J/\psi\pi^0$ to be greater than ~ 1.6 keV. Similar decays in the bottomonium system, $h_b\to\Upsilon(1S)\pi^0$ and $\Upsilon(3S)\to h_b\pi^0$, will be discussed elsewhere [4].

In the QCD multipole expansion, spin-flip transitions, $1\,^1P_1\to 1\,^3S_1+\pi^0$ and $n\,^3S_1\to 1\,^1P_1+\pi^0$ occur through the $E1-M1$ transition [1,2]:

$$\begin{aligned}\mathcal{M}(1\,^1P_1\to 1\,^3S_1+\pi^0)&\\ &=\mathcal{M}(n\,^3S_1\to 1\,^1P_1+\pi^0)\\ &=\langle\pi^0|\pi\alpha_s E_i^a(0)H_k^a(0)|0\rangle I_{\text{SP}}\hat{\epsilon}_k^{(s)}\hat{\epsilon}_i^{(P)}\ ,\end{aligned} \tag{4}$$

where $\hat{\epsilon}_k^{(S)}$ and $\epsilon_i^{(P)}$ are spin vectors of $^3S_1$ and $^1P_1$ states, respectively. The gluonic matrix element can be reduced to $G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$; and then evaluated using the low-energy theorem for a pion [5]:

$$\begin{aligned}\langle\pi^0|\pi\alpha_s E_i^a(0)H_k^a(0)|0\rangle&\\ &\equiv A_0\delta_{ik}\\ &=\tfrac{1}{12}\delta_{ik}\langle\pi^0|\pi\alpha_s G^a_{\mu\nu}\tilde{G}^{a\mu\nu}|0\rangle \qquad (5)\\ &=\tfrac{1}{12}\delta_{ik}\frac{4\pi^2}{\sqrt{2}}\left[\frac{m_d-m_u}{m_d+m_u}\right]f_\pi m_\pi^2\ . \qquad (6)\end{aligned}$$

Using $f_\pi=132$ MeV and $(m_d-m_u)/(m_d+m_u)=0.3$, we get $A_0=1.7\times10^{-3}$ GeV^3. The matrix element between quarkonia I_{SP} is defined as

$$I_{SP}=-\frac{2\sqrt{3}}{9m_Q}\langle nS|G_{8,S}(E)r+rG_{8,P}(E)|1P\rangle\ , \tag{7}$$

where $G_{8,S}(E)$ and $G_{8,P}(E)$ are the Green's functions for the color-octet $Q\bar{Q}$ intermediate states in the S and P waves. Finally, the decay rate from Eq. (4) is

$$\begin{aligned}\Gamma(1\,^1P_1\to 1\,^3S_1+\pi^0)&=\Gamma(n\,^3S_1\to 1\,^1P_1+\pi^0)\\ &=\frac{1}{2\pi}(A_0 I_{SP})^2|\mathbf{p}_\pi|\ .\end{aligned} \tag{8}$$

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To get the absolute decay rate for hadronic transitions between quarkonia, it is imperative to know more about the "I_{SP}" defined in Eq. (7). Because of our ignorance of the confinement in QCD, it lies beyond our ability to calculate I_{SP} from the first principle in QCD. We have to make reasonable approximations. There are two sources of uncertainties in the matrix element I_{SP}.

The first is the Green's function for the color-octet $Q\bar{Q}$ intermediate states, $G_8(E)$:

$$G_8(E)=\sum_k \frac{|k\rangle\langle k|}{E_k-E} . \tag{9}$$

Here, k runs over color octet $Q\bar{Q}$ states only, E and E_k are energies of the initial and the intermediate states. This quantity is unknown due to our ignorance of quark confinement in QCD, even in the potential model description of quarkonia. It can be assumed to be a constant with dimension of inverse mass, since this assumption leads to nice agreements of theretical predictions and the measured $\pi\pi$ spectra in $\psi'\to J/\psi\pi\pi$ and $\Upsilon(2S)\to\Upsilon(1S)\pi\pi$:

$$G_{8,S}(E)=G_{8,P}(E)=G_8 . \tag{10}$$

The second uncertainty comes from the matrix element $\langle nS|r|1P\rangle$. In previous estimations, this quantity was replaced by the radius of the initial or the final quarkonium. However, such estimates can be improved, since this matrix element is also relevant to electric dipole radiative transitions between quarkonia. Electric dipole radiative transitions are described by

$$\Gamma(E1)=\frac{4}{27}\alpha e_q^2 k_\gamma^3 S_{if}(2J_f+1)|\langle 1S|r|2P\rangle|^2 , \tag{11}$$

where J_f is the spin of the final quarkonium and k_γ is the energy of the emitted photon. $S_{if}=3$ for ${}^1P_1\to\eta_c+\gamma$ and $S_{if}=1$ for other transitions. Comparing Eq. (11) and the measured decay rate of $\chi_{cJ}\to J/\psi+\gamma$, we get

$$|\langle 1S|r|1P\rangle_\psi|\approx 1.57\ \text{GeV}^{-1}=0.3\ \text{fm} , \tag{12}$$

and thus,

$$\Gamma(h_c(1P)\to\eta_c\gamma)\approx 400\ \text{keV} . \tag{13}$$

Since $|\langle 1S|r|1P\rangle_\psi|$ is determined from the experimental data on $\Gamma(\chi_{cJ}\to J/\psi+\gamma)$, our prediction (13) is independent of specific potentials. Taking $m_c=1.65$ GeV and $G(E)=1\ \text{GeV}^{-1}$ in Eq. (7), we get $|I_{SP}|\approx 0.73\ \text{GeV}^{-3}$ and

$$\Gamma(h_c(1P)\to J/\psi\pi^0) =0.09\left[\frac{1.65}{m_c\ (\text{GeV})}\right]^2 |G_8(\text{GeV}^{-1})|^2\ \text{keV} . \tag{14}$$

This is smaller than the measured value by an order of magnitude if $G_8\sim 1\ \text{GeV}^{-1}$.

To get a better idea on the numerical value of G_8, we consider another class of decays: $n\,{}^3S_1\to m\,{}^3S_1+\pi\pi$ and $n\,{}^3S_1\to m\,{}^3S_1+\eta$ (or π^0). These decays occur through $E1$-$E1$ and $E1$-$M2$ transitions in the QCD multipole expansions [1,5]. For simplicity, we give an explicit expression of the amplitude only for the latter decay, referring to original papers [1,2,5] for more detailed discussion:

$$\mathcal{M}(n\,{}^3S_1\to m\,{}^3S_1+\eta) =i(\partial_k\langle\eta|\pi\alpha_s E_k^a H_j^a|0\rangle)m_Q^{-1}\epsilon_{ijl}\epsilon_i\epsilon_l' I_{SS} =\frac{\pi^2}{9}\left[\frac{3}{2}\right]^{1/2} f_\pi m_\eta^2 m_Q^{-1}[\epsilon_{ijk}\epsilon_i\epsilon_j'(\mathbf{p})_k]I_{SS} , \tag{15}$$

where ϵ and ϵ' are the spin vectors of the initial and final quarkonia, and

$$I_{SS}=\frac{2}{9}\langle mS|r_i G(E) r_i|nS\rangle . \tag{16}$$

Under the same assumptions on $G(E)$, I_{SS} is simplified to

$$I_{SS}=\tfrac{2}{9}G_8\langle mS|r^2|nS\rangle . \tag{17}$$

Therefore, once $\langle 2S|r^2|1S\rangle$ is known, the absolute decay rates for these decays can be readily obtained. However, potential model calculation of this matrix element is not available on the contrary to dipole matrix elements $\langle f|r|i\rangle$ and the mean square radius of a quarkonium, $\langle i|r^2|i\rangle$.

Instead, we use a quantum-mechanical sum rule to get an upper bound on this matrix element. It turns out that the following sum rule is useful to study this matrix element:

$$m_Q\sum_n (E_n-E_i)|\langle i|O(\mathbf{r})|n\rangle|^2=\langle i\|\nabla O(\mathbf{r})|^2|i\rangle . \tag{18}$$

This sum rule can be derived by considering the identities [6]

$$\langle i|[[H,O(\mathbf{r})],O^\dagger(\mathbf{r})]|i\rangle=2\sum_n (E_i-E_n)|\langle i|O(\mathbf{r})|i\rangle|^2 \tag{19}$$

and

$$[[H,O(\mathbf{r})],O^\dagger(\mathbf{r})]=-\frac{2}{m_Q}|\nabla O(\mathbf{r})|^2 , \tag{20}$$

with $H=\mathbf{p}^2/m_Q+V(\mathbf{r})$. This reproduces the optical sum rule for $O(\mathbf{r})=\mathbf{r}$.

We choose $O(\mathbf{r})\equiv r^2$ and $|i\rangle=|1S\rangle$, and get the following bound on $|\langle 1S|r^2|nS\rangle|$:

$$|\langle 1S|r^2|nS\rangle|^2<\frac{4}{m_Q}\frac{|\langle 1S|r^2|1S\rangle|}{E_{nS}-E_{1S}} . \tag{21}$$

An upper bound on $|\langle 1S|r^2|nS\rangle|^2$ can be obtained from Eq. (21), once $|\langle 1S|r^2|1S\rangle|$ is reliably known. We use the value of $|\langle 1S|r^2|1S\rangle|$ obtained in Ref. [7]. This is the only place where our results depend on a specific potential. However, this quantity is almost universal in various potential models, and can be regarded reliable.

The transition matrix element $|\langle 1S|r^2|nS\rangle|^2$ can be smaller than the typical size of the initial quarkonium $|\langle nS|r^2|nS\rangle|^2$ by an order of magnitude. For example, the above sum rule gives a bound

$$|\langle 1S|r^2|2S\rangle_\Upsilon|^2<1.6\ \text{GeV}^{-4} \tag{22}$$

for $m_b = 4.8$ GeV and $|\langle 1S|r^2|1S\rangle_\Upsilon| = 1.1$ GeV [7]. This is much smaller than

$$|\langle 2S|r^2|2S\rangle_\Upsilon|^2 = (2.6\ \text{GeV}^{-1})^4 = 45.7\ \text{GeV}^{-4}\ ,$$

which has been frequently used along with $G_8 \sim 1$ GeV^{-1} in order to estimate the absolute decay rate for $n\,^3S_1 \to m\,^3S_1\pi\pi$ and $n\,^3S_1 \to m\,^3S_1\eta$.

By considering $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$ with the correct upper bound used for $|\langle 1S|r^2|2S\rangle_\Upsilon|^2$, one obtains a lower bound on G_8:

$$|G_8|^2 > 18\ \text{GeV}^{-2}\ . \tag{23}$$

If one assumes that G_8 is independent of the hidden flavor of quarkonia, we get the following lower bound on $h_c(1P) \to J/\psi\pi^0$ from Eq. (14):

$$\Gamma(h_c(1P) \to J/\psi\pi^0) > 1.6\ \text{keV}\ . \tag{24}$$

This is consistent with E760 data (3), and moreover, very close to it. Of course, this would be uncertain by a factor of ~ 2, depending on the choice of m_c. [If $\psi' \to J/\psi\eta$ is considered instead, then the lower bound on $|G_8|^2$ becomes looser, $|G_8|^2 > 10$ GeV^{-2} and $\Gamma(h_c(1P) \to J/\psi\pi^0) > 0.9$ keV. If the E760 data (2) become smaller, G_8 would depend on the hidden flavor of quarkonia.]

Another hadronic decay mode, $1\,^1P_1 \to 1\,^3S_1 + \pi\pi$, does not receive any contribution from the trace of the energy-momentum tensor in QCD, and is not enhanced over $1\,^1P_1 \to 1\,^3S_1 + \pi^0$ [2]. Using the result of Ref. [2], we predict

$$\frac{\Gamma(h_c(1P) \to J/\psi\pi\pi)}{\Gamma(h_c(1P) \to J/\psi\pi^0)} \approx \frac{\lambda^2}{30}\ . \tag{25}$$

Here, a new parameter $\lambda \equiv \pi\alpha_s\rho_G$ measures the glounic contribution to the energy-momentum of a pion [8]. One can extract λ from the $\pi\pi$ spectrum in $\psi' \to J/\psi\pi\pi$: $\lambda \approx 2.2$. Then, the ratio in Eq. (25) is 0.16%, which is consistent with the E760 data (3). It is actually very close to the current upper limit for $\lambda \sim 2$. Therefore, $h_c(1P) \to J/\psi\pi\pi$ may be observed in the near future.

In conclusion, hadronic decays of h_c are considered in the framework of QCD multipole expansion. Assuming the Green's function for the color-octet states is independent of E, and using a quantum-mechanical sum rule for the matrix elements of r^2, we could predict $\Gamma(h_c(1P) \to J/\psi\pi^0) > 1.6$ keV, $\Gamma(h_c(1P) \to J/\psi\pi\pi) \approx 0.16\Gamma(h_c(1P) \to J/\psi\pi^0)$, and $\Gamma(h_c(1P) \to \eta_c\gamma) \approx 400$ keV. The first two results are consistent with the recent data from E760. Clearly, one has to await improved numbers of these decays to test our predictions.

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