Theoretical analyses of $\bar{B} \to \chi_c K_S$

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We use the perturbative QCD methods of Lepage and Brodsky to calculate the rate for $\bar{B} \to \chi_c K_S$. We find agreement with recent ARGUS data. We compare our results with those of Kaplan, Kuhn, Nussinov, and Ruckl—there is no disagreement between our work and that of these authors.

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I. INTRODUCTION

The possibility of probing *CP* violation in the proposed SLAC-LBL-LNL Asymmetric *B* Factory [1] requires a very aggressive luminosity for the device: allowing for the use of the $B \to \psi/JK_{+}^{*}$, one needs $\approx 10^{33}/\text{cm}^2\,\text{sec}$ as a minimal luminosity parameter $\mathcal{L}$, as we and others have explained in detail elsewhere [1]. Hence, it is imperative to continue to look for other *B* decay modes to soften further this requirement on $\mathcal{L}$.

Recently, an extremely interesting possibility for such a mode has been suggested by results from ARGUS [2], who have found the preliminary results $B(B \to \chi_c X) = (1.05 \pm 0.35 \pm 0.25)\%$ and $B(B^+ \to \chi_c K^+) = (0.19 \pm 0.13 \pm 0.06)\%$. These results should be compared to the gold-plated mode [3] $B \to \psi/JK_S$: $B(B \to \psi/JK_S) = (4.0 \pm 1.4) \times 10^{-4}$; it is this mode which sets the value 3×10^{33} for $\mathcal{L}$. There is the clear suggestion that $B \to \chi_c K_S$, $\chi_c K_{+}^{*}$ decays may provide another set of useful modes for *CP*-violation studies at the SLAC-LBL-LNL Asymmetric *B* Factory. In this paper, we will analyze this $\chi_c K_S$ possibility from a theoretical standpoint to get an assessment of its ultimate utility in reducing the required value of $\mathcal{L}$ for the *CP* violation objectives of the respective colliding beam device. The $\chi_c K_{+}^{*}$ mode will be analyzed elsewhere [4].

Our work is organized as follows. In Sec. II, we analyze $\bar{B} \to \chi_c K_S$ using the methods of Lepage and Brodsky [5], and we also compare our results with the work of Kaplan, Kuhn, Nussinov, and Ruckl (KKNR) [6]. Section III contains our summary remarks. The Appendix contains relevant details of our work discussed in Sec. II.

II. $\bar{B} \to \chi_c K_S$

In this section, we use the method of Lepage and Brodsky [5] to analyze the decay $\bar{B} \to \chi_c K_S$. We begin by setting the notation.

Specifically, our kinematics is illustrated in Fig. 1, where we show the basic process of interest to us at the level of QCD Feynman diagrams. We note that χ_c in Fig. 1 actually corresponds to the states χ'_{c1}, χ_{c0}, χ_{c1}, and χ_{c2} of spins $J = 1$, 0, 1, and 2 respectively. In our work, we will use the average rest mass 3.50 GeV for the χ_c's, since the attendant approximation is within the accuracy of approximations in our amplitudes. We further note that the parity of our χ_c's is positive and that the charge conjugation eigenvalue C is $+1$ for χ_{cj}, $j = 0,1,2$, and is -1 for χ'_{c1}. Thus, the net contribution of $\chi_{c1} K_S$ to the *CP* violating asymmetry in $\bar{B} \to \chi_c K_S$ is of the opposite sign to that of $\chi'_{c1} K_S$, $\chi_{c0} K_S$ and $\chi_{c2} K_S$, for example.

In Fig. 1, the external wave functions are all Lepage-Brodsky distribution amplitudes as we have illustrated in [7], for example, in our analysis of $B \to \psi/JK_{+}^{*}$. Here, we will always compute the rates for the $\bar{B} \to \chi_c K_S$ processes in units of the gold-plated mode $\bar{B} \to \chi_c K_S$. This will facilitate our assessment of the utility of the $\chi_c K_S$ modes in *CP*-violation studies. Correspondingly, the somewhat uncertain normalization of the *B* meson distribution amplitude drops out of our discussion. The process $\bar{B} \to \psi/JK_S$ is shown in Fig. 2 for definiteness. With these preliminary remarks, we now proceed with our analysis.

More specifically, entirely standard manipulations allow us to write the amplitudes of interest to us as

$$\mathcal{M}(\bar{B} \to \psi/JK_S)|_{J_z=0} = \frac{-8iG_F a_2 V_{cb} V_{cs}^{*} m_{\psi/J} f_{\psi/J} \sqrt{3} f_K a_B g_s^2 C_F g_V(-1/\sqrt{2})}{(\sqrt{2})^3 (2E_{\psi/J})^{1/2} (2E_K)^{1/2} (2m_B)^{1/2} (2\pi)^{9/2}} \times \int dy_1 \frac{y_1 (p m_B / m_{\psi/J})}{x_2 (m_B^2 + m_K^2 - m_{\psi/J}^2)} \times \left\{ \frac{m_{\psi/J}^2 - 2m_B m_b + m_K m_b + y_1 (m_B^2 - m_{\psi/J}^2 - 2m_B m_K)}{\bar{m}_b^2 - m_b^2 + 2x_1 x_2 m_B^2 - y_2 (m_B^2 + m_K^2 - m_{\psi/J}^2) + i\epsilon} + \frac{x_2 m_{\psi/J}^2 + x_1 m_K^2 + m_s m_B - 2m_K m_B x_1 - 2m_K m_s}{\bar{m}_b^2 - m_s^2 + x_2 m_{\psi/J}^2 - x_1 (m_B^2 - m_K^2) + i\epsilon} \right\}, \tag{1a}$$

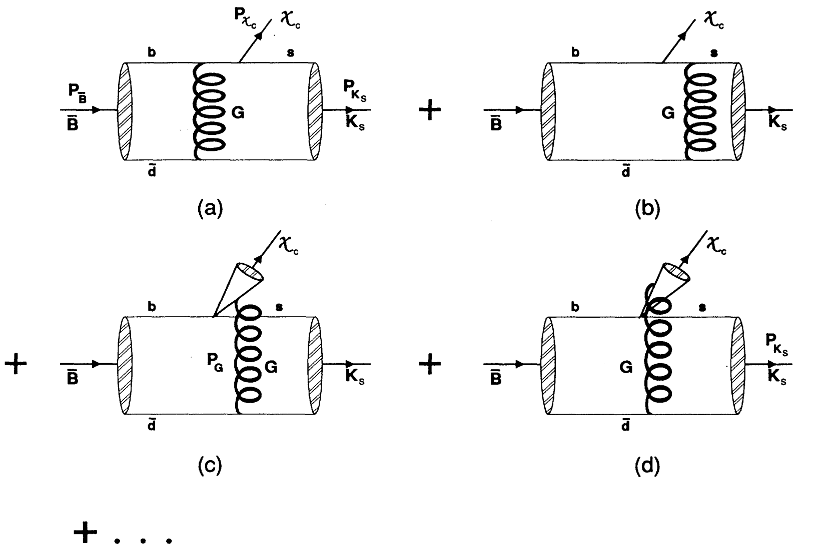

FIG. 1. Feynman diagrams for $\bar{B}\to\chi_c K_S$ in the Lepage-Brodsky theory. Here, P_A is the four-momentum of A and G is a QCD gluon. Note that $-P_G^2\simeq 2x_2 y_2 P_{\bar{B}}\cdot P_{K_S}\simeq 0.56 y_2$ GeV^2; thus, since the Lepage-Brodsky distribution amplitudes contain all G with momentum transfer squared $-P_G^2\le 1/r_H^2\simeq(0.2\text{ GeV})^2$, we see that y_2 should be kept above 0.079 here, where r_H is the typical hadron radius. This shows that the Lepage-Brodsky theory applies here at the $\sim 10\%$ level insofar as the size of $-P_G^2$ is concerned.

$$\mathcal{M}(\bar{B}\to\chi_{c0}K_S)=\frac{4\left[\dfrac{G_F}{\sqrt{2}}\right]C_1 V_{cb}V_{cs}^{*}g_s^2 C_F a_B i_{\chi_{c0}} f_K\left[\dfrac{-1}{\sqrt{2}}\right]\tilde{f}_{\chi_{c0}}m_\chi\left[\dfrac{1}{\sqrt{3}}\right]m_\chi\left[\dfrac{p^2 m_B^2}{m_\chi^2}+\frac{3}{2}m_B m_K\right]}{x_2(m_B^2-m_\chi^2+m_K^2)\sqrt{N_c}(2E_K)^{1/2}(2E_\chi)^{1/2}\sqrt{2m_B}(2\pi)^{9/2}}\,, \tag{1b}$$

with entirely analogous results for the χ_{c1}, χ'_{c1}, and χ_{c2} states (these are given in the Appendix for completeness), where V_{UD} is the Cabibbo-Kabayashi-Maskawa (CKM) matrix, g_s is the QCD coupling constant, $a_B=f_B/\sqrt{12}$ where f_A is the constant of meson A,

$$\tilde{f}_\chi=\sqrt{3/4\pi}\frac{4\sqrt{3}}{\sqrt{m_{c,\text{eff}}}}\phi'(0)\,,$$

where $m_{c,\text{eff}}$ is the Cornell-type [8] constituent value of m_c and is taken from Ref. [8] as 1.84 GeV, where $\phi'(0)$ is the derivative of the χ_c wave function at the origin [we take $\phi'(0)$ from the Cornell-type [8] potential model so that it [9] is 0.363 GeV$^{5/2}$], $i_{\chi_{c0}}$ is the analogue of the integral over y_1 of the denominator of the first term in curly brackets in Eq. (1a) and is given in (A15), $C_F=\frac{4}{3}$ is the eigenvalue of the quadratic Casimir invariant of the quark color representation and $p=(E_K^2-m_K^2)^{1/2}$ is the magnitude of the K_S three-momentum in the B rest frame. The QCD correction parameters a_2 and C_1 are given [10] by $a_2\simeq-0.24$ and $C_1\simeq 1.1$. Here, $\bar{m}_b\equiv(1-x_2)m_B$, $m_b=4.5$ GeV, and $m_c=1.3$ GeV. The vector coupling $g_V(Q^2)$ is given by the simple pole form $1/[1-Q^2/(5.43\text{ GeV})^2]$, where the pole 1^- mass is taken from Ref. [10] and $Q^2=m_{\psi/J}^2$. The results (1), and their analogues for the other χ_c states as given in the Appendix, are the basic results of our analysis.

From (1) and the results (A7), (A13), and (A14), we get the total $\bar{B}\to\chi_c K_S$ rate in ratio to that for $\bar{B}\to\psi/JK_S$ as

$$\sum_{\chi_c}\Gamma(\bar{B}\to\chi_c K_S)/\Gamma(\bar{B}\to\psi/JK_S)=1.10 \tag{2}$$

and the CP odd fraction of the $\bar{B}\to\chi_c K_S$ rate as

$$r_{CP}^{\chi}=\frac{\Gamma(\bar{B}\to\chi_{c1}K_S)}{\sum_{\chi_c}\Gamma(\bar{B}\to\chi_c K_S)}=0.365\,. \tag{3}$$

Here we use the notation

$$\sum_{\chi_c}\Gamma(\bar{B}\to\chi_c K_S)\equiv\Gamma(\bar{B}\to\chi'_{c1}K_S)+\sum_{j=0,1,2}\Gamma(\bar{B}\to\chi_{cj}K_S)\,. \tag{4}$$

We see that indeed $\bar{B}\to\chi_c K_S$ is a significant mode for possible CP-violation exploration. Our results are in

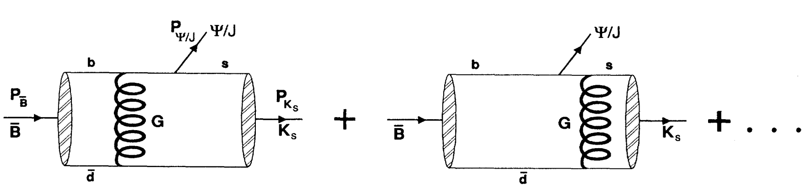

FIG. 2. Reference process $\bar{B}\to\psi/JK_S$. P_A is the 4-momentum of A and G is a QCD gluon.

agreement with the preliminary findings of ARGUS. Furthermore, we also are consistent with the work of Refs. [6]. The latter authors worked in the limit of the factorization assumption, by which (a) and (b) in Fig. 1 are the only allowed graphs. In this limit, our formulas (1) and the analogues for χ'_{c1}, χ_{c1}, and χ_{c2} reproduce their result that only χ_{c1} is produced in $B \to \chi_c K_S$ and, further, we are also consistent with their result that $\Gamma(B\to\chi_c X)/\Gamma(B\to\psi/JX)\simeq 0.27$, when one allows for the 20% errors on our methods. Hence, we find no theoretical contradiction between our work and that of Refs. [6].

III. CONCLUSION

In this paper, we have used perturbative QCD to analyze the decay $B\to\chi_c K_S$, following the methods of Lepage and Brodsky [5]. We find that the respective decay rate is consistent with recent preliminary observations by ARGUS. As a consequence, this decay mode provides another avenue to *CP*-violation experimentation in the proposed SLAC-LBL-LNL Asymmetric *B*-Factory device. This will reduce the required luminosity of the device by a significant amount [11]. We should also note that, in the course of our analysis, we have found results which are consistent with those of KKNR in Refs. [6] and with the recent work on factorization in Refs. [12]. We continue to encourage the respective proponents of the SLAC-LBL-LNL Asymmetric *B* Factory to pursue its construction vigorously.

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APPENDIX

In this appendix we derive the expressions (1) and their analogues for the amplitudes for $\bar{B}\to\chi'_{c1}$, $\chi_{c1}K_S$, and $\chi_{c2}K_S$. We use the methods of Lepage and Brodsky in Ref. [5].

Specifically, considering first the process $\bar{B}\to\psi/JK_S$, we follow the prescription in Ref. [5] to obtain the following from Fig. 2. (Here, we presume the factorization ansatz of Bauer *et al.* in Ref. [10]; it means that the QCD correction coefficients $C_{1,2}$ in the notation of Ref. [10] are mapped into the parameters $a_{1,2}$ which multiply effective hadron operators in the full weak Hamiltonian; the latter operators represent physical hadron states via the current field identity, for example. The relevant weak interaction is the usual

$$\mathcal{L}_{\rm int}=\frac{G_F}{\sqrt{2}}\{C_1\bar{c}\gamma_\mu(1-\gamma_5)s'\bar{s}'\gamma^\mu(1-\gamma_5)c + C_2\bar{s}'\gamma_\mu(1-\gamma_5)s'\bar{c}\gamma^\mu(1-\gamma_5)c\}\ ,$$

where as usual d', s', and b' are the CKM rotations of d, s, and b, respectively. After the Bauer-Stech-Wirbel (BSW) [10] map, we get in $\mathcal{L}_{\rm int}$ the replacement $C_i\to a_i$ and the subscript H appended to each operator therein to denote its effective hadron interpretation as prescribed in [10]. We use one such operator $(\bar{c}\gamma_\mu(1-\gamma_5)c)_H$ to interpolate the ψ/J state and we use perturbative QCD via the methods of Lepage and Brodsky to compute the B to K_S transition via the operator $(\bar{s}\gamma^\mu(1-\gamma_5)b)_H$ at momentum transfer $Q=m_{\psi/J}$, which is well into the perturbative QCD regime. The deviation of $a_{1,2}$ from their naive QCD expectations then already takes into account the nonfactorizable effects represented by Figs. 1(c) and 1(d). If we use factorization and the empirical values of $a_{1,2}$, we have already included the entire set of graphs in Fig. 1 in the framework used by Bauer *et al.* [10]. This is also consistent with recent arguments by Dugan and Grinstein and by Blok and Shifman [12].)

$$\mathcal{M}(\bar{B}\to\psi/JK_S)=\frac{-i\frac{G_F}{\sqrt{2}}a_2V_{cb}V_{cs}^*m_{\psi/J}f_{\psi/J}f_K(-1/\sqrt{2})a_Bg_s^2C_FN_c}{\sqrt{2E_{\psi/J}}(2\pi)^{9/2}\sqrt{2E_K2m_BN_c}(\sqrt{2})^2}$$
$$\times\frac{\int_0^1dy_1\,y_1y_2}{P_G^2}\mathrm{tr}\left[\frac{\gamma_\alpha\gamma_5(\not{P}_K+m_K)\not{\epsilon}^*_{\psi/J}(g_V-g_A\gamma_5)(x_1\not{P}_B-\not{P}_G+m_b)\gamma^\alpha\gamma_5(\not{P}_B-m_B)}{(\bar{m}_b^2-m_b^2+P_G^2-2x_1P_{\bar{B}}\cdot P_G+i\epsilon)}\right.$$
$$\left.+\frac{\gamma_\alpha\gamma_5(\not{P}_K+m_K)\gamma^\alpha(x_1\not{P}_B-\not{P}_{\psi/J}+m_s)\not{\epsilon}^*_{\psi/J}(g_V-g_A\gamma_5)\gamma_5(\not{P}_B-m_B)}{\bar{m}_b^2+m_{\psi/J}^2-2x_1P_{\bar{B}}\cdot P_{\psi/J}-m_s^2+i\epsilon}\right]\ , \tag{A1}$$

where $g_{V,A}$ are the respective vector and axial-vector couplings [10]. On effecting the traces in (A1), we arrive at the result in (1) for the $\bar{B}\to\psi/JK_S$ amplitude. Here, we record the required integrals over $y_1=1-y_2$ as

$$i_{1;2}=\frac{\int_0^1dy_1\{y_1;y_1^2\}}{y_1-y_1^0+i\epsilon}$$
$$=\{[-i\pi y_1^0+1+y_1^0\ln|(1-y_1^0)/y_1^0|];[-i\pi y_1^{0^2}+\tfrac{1}{2}+y_1^0+y_1^{0^2}\ln|(1-y_1^0)/y_1^0|]\}\ , \tag{A2}$$

where we define

$$(m_B^2+m_K^2-m_{\psi/J}^2)(y_1-y_1^0)=\bar{m}_b^2-m_b^2-2x_1P_B\cdot P_G$$
$$=\bar{m}_b^2-m_b^2+2x_1x_2m_B^2-y_2(m_B^2+m_K^2-m_{\psi/J}^2) \quad \text{(A3)}$$

so that

$$y_1^0=1-(\bar{m}_b^2-m_b^2+2x_1x_2m_B^2)/(m_B^2+m_K^2-m_{\psi/J}^2)\simeq 0.591 \ . \quad \text{(A4)}$$

This means that our final result for $\mathcal{M}(\bar{B}\to\psi/JK_S)$ may be represented by

$$\mathcal{M}=\frac{-4i(G_F/\sqrt{2})a_2V_{cb}V_{cs}^*m_{\psi/J}f_{\psi/J}\sqrt{3}f_Ka_Bg_s^2C_Fg_V(-1/\sqrt{2})pm_B/m_{\psi/J}}{x_2[2E_{\psi/J}2E_K2m_B(2\pi)^9]^{1/2}(m_B^2+m_K^2-m_{\psi/J}^2)}$$
$$\times\left[i_1(m_{\psi/J}^2-2m_Bm_b+m_Km_b)/d_1+i_2(m_B^2-m_{\psi/J}^2-2m_Bm_K)/d_1\right.$$
$$\left.+\frac{\frac{1}{2}(x_2m_{\psi/J}^2+x_1m_K^2+m_sm_B-2m_Km_Bx_1-2m_Km_s)}{\bar{m}_b^2-m_s^2+x_2m_{\psi/J}^2-x_1(m_B^2-m_K^2)+i\epsilon}\right] , \quad \text{(A5)}$$

where we have defined

$$d_1=m_B^2+m_K^2-m_{\psi/J}^2 \ . \quad \text{(A6)}$$

Similarly, we use again the methods in Ref. [5] to evaluate $\bar{B}\to\chi_cK_S$ via the diagrams in Fig. 1. Here, an important difference emerges in the attempt to use the simple factorization procedure in Ref. [10]. Specifically, if one uses the factorization procedure in Ref. [11], one finds [6] that only the $\bar{B}\to\chi_{c1}K_S$ amplitude would be nonzero. For this latter amplitude, we may proceed in complete analogy with our analysis of $\bar{B}\to\psi/JK_S$ in (A1)–(A6), for example. In this way, we get

$$\mathcal{M}(\bar{B}\to\chi_{c1}K_S)|_{J_z=0}=\mathcal{M}(\bar{B}\to\psi/JK_S)|_{m_{\psi/J}\to m_{\chi_{c1}},f_{\psi/J}\to -i\tilde{f}_{\chi_{c1}},P_{\psi/J}\to P_{\chi_{c1}},J_z=0} , \quad \text{(A7)}$$

where the axial-vector decay constant $\tilde{f}_{\chi_{c1}}$ is given by our Cornell-type [8] potential model used in Ref. [9] and is readily identified as

$$\tilde{f}_{\chi_{c1}}=\frac{4\sqrt{3}}{\sqrt{m_{c,\text{eff}}}}\sqrt{3/4\pi}\phi'(0)/m_{\chi_{c1}}\approx 0.259 \text{ GeV} , \quad \text{(A8)}$$

where [8] $m_{c,\text{eff}}=1.84$ GeV and $\phi'(0)$ is the respective wave function at the origin and is computed in Ref. [9] using standard finite difference methods.

Whenever a quantum amplitude vanishes as a result of an approximation, it is systematic to check the validity of the approximation in that case by computing the first corrections thereto, at a minimum. Accordingly, for χ_{c1}', χ_{c0}, and χ_{c2}, we explore the validity of the factorization approach by computing the graphs (c) and (d) in Fig. 1. Always using the standard manipulations, we get

$$\mathcal{M}(\bar{B}\to\chi_cK_S)=2C_1(N_c/2)C_Fg_s^2(-iG_F/\sqrt{2})V_{cb}V_{cs}^*\sqrt{3}f_K(-1/\sqrt{2})$$
$$\times\frac{a_B\int_0^1 dy_1\,y_1y_2}{\sqrt{2^2}\sqrt{2E_K}\sqrt{N_c^3}\sqrt{2m_B}\sqrt{2E_\chi}(2\pi)^{9/2}}$$
$$\times\int d^3k\frac{\psi_\chi(\mathbf{k})}{(2\pi)^{3/2}2\sqrt{m_\chi}}\operatorname{tr}\gamma_\alpha\gamma_5(\boldsymbol{P}_K+m_K)\gamma^\mu(1-\gamma_5)\gamma_5(\boldsymbol{P}_{\bar{B}}-m_B)$$
$$\times\operatorname{tr}\{[1+\not{k}/(2m_c)]\not{\epsilon}_\chi^*(\boldsymbol{P}_\chi+m_\chi)[1+\not{k}/(2m_c)]\gamma^\alpha(\tfrac{1}{2}\boldsymbol{P}_\chi+\not{k}+\boldsymbol{P}_G+m_c)$$
$$\times\gamma_\mu(1-\gamma_5)/[(P_\chi/2+k+P_G)^2-m_c^2+i\epsilon]$$
$$+(-\boldsymbol{P}_\chi/2+\not{k}-\boldsymbol{P}_G+m_c)\gamma^\alpha[1+\not{k}/(2m_c)]\not{\epsilon}_\chi^*(\boldsymbol{P}_\chi+m_\chi)$$
$$\times[1+\not{k}/(2m_c)]\gamma_\mu(1-\gamma_5)/[(P_\chi/2-k+P_G)^2-m_c^2+i\epsilon]\} , \quad \text{(A9)}$$

where $\psi_\chi(\mathbf{k})$ is the bound-state wave function of the χ_c in the relative momentum $\mathbf{k}$ space and we have only retained the terms up to first order in $\mathbf{k}$ in the bound-state spinor part of the χ wave function, where ϵ_χ^μ is the appropriate polarization vector when the $c\bar{c}$ spin state is that of spin 1 in the χ_c. When the $c\bar{c}$ spin state is that of spin 0, one simply introduces the substitution

$$\not\epsilon_\chi^* \to \gamma_5 \tag{A10}$$

in (A9). Evidently, ψ_χ carries the orbital polarization of the $c\bar{c}$ motion so that there is an implied Clebsch-Gordan sum in the product $\psi_\chi \not\epsilon_\chi^*$ which we do not exhibit explicitly, but which is obvious for $\chi_c = \chi_{cj}$, $j=0,1,2$.

For definiteness, let us note the elementary connection

$$\int \frac{d^3k}{(2\pi)^{3/2}} \psi_\chi(\mathbf{k}) k_\mu = \begin{cases} i\epsilon_\mu^*(1)\sqrt{3/4\pi}\phi'(0), & L_z = 1 , \\ i\epsilon_\mu^*(0)\sqrt{3/4\pi}\phi'(0), & L_z = 0 , \\ i\epsilon_\mu^*(-1)\sqrt{3/4\pi}\phi'(0), & L_z = -1 , \end{cases} \tag{A11}$$

where $\epsilon_\mu^*(L_z)$ is the respective polarization vector for spin 1 with z component of angular momentum equal to L_z, and L_z is the z component of the $c\bar{c}$ orbital angular momentum in the χ.

Using (A10) and working to leading nontrivial order in $\mathbf{k}$ in (A9), we get, then, from entirely standard methods, the result given in (1) in the text for $\bar{B} \to \chi_{c0} K_s$, namely,

$$\mathcal{M}(\bar{B} \to \chi_{c0} K_S) = \frac{4C_1 g_s^2 C_F G_F V_{cb} V_{cs}^* a_B i_{\chi_{c0}} f_K \tilde{f}_{\chi_{c0}} m_\chi}{x_2\sqrt{2}(m_B^2 - m_\chi^2 + m_K^2)\sqrt{N_c}\sqrt{2E_K 2E_\chi 2m_B}(2\pi)^{9/2}} \sqrt{1/3}\, m_\chi (p^2 m_B^2/m_\chi^2 + \tfrac{3}{2} m_B m_K) \tag{A12}$$

as well as

$$\mathcal{M}(\bar{B} \to \chi'_{c1} K_S) = \frac{-p_{\chi'_{c1}}(m_B^2 - m_K^2)}{m_{\chi'_{c1}} \left[\frac{2}{\sqrt{3}} p_{\chi_{c0}}^2 m_B / m_{\chi_{c0}} + \sqrt{3} m_K m_{\chi_{c0}} \right]} [\tilde{f}_{\chi'_{c1}}/\tilde{f}_{\chi_{c0}}][m_{\chi'_{c1}}/m_{\chi_{c0}}] \frac{i_{\chi'_{c1}}}{i_{\chi_{c0}}} \mathcal{M}(\bar{B} \to \chi_{c0} K_S) , \tag{A13}$$

$$\mathcal{M}(\bar{B} \to \chi_{c2} K_S) = \{\sqrt{2} p_{\chi_{c2}}^2 m_B / [m_{\chi_{c2}}^2 (m_B p_{\chi_{c0}}^2 / m_{\chi_{c0}}^2 + \tfrac{3}{2} m_K)]\} [\tilde{f}_{\chi_{c2}}/\tilde{f}_{\chi_{c0}}] \frac{m_{\chi_{c2}}}{m_{\chi_{c0}}} \frac{i_{\chi_{c2}}}{i_{\chi_{c0}}} \mathcal{M}(\bar{B} \to \chi_{c0} K_S) , \tag{A14}$$

where

$$\begin{aligned} i_\chi &= \int_0^1 dy_1 \frac{y_1}{\left\{ \frac{m_\chi^2}{4} - x_2(m_B^2 + m_\chi^2 - m_K^2) + y_2 \left[\frac{(m_B^2 - m_\chi^2 - m_K^2)}{2} - x_2(m_B^2 + m_K^2 - m_\chi^2) \right] - m_c^2 + i\epsilon \right\}} \\ &= -1/T + (1/T + A/T^2)\ln|1 + T/A| , \end{aligned} \tag{A15}$$

where

$$T = (m_B^2 - m_\chi^2 - m_K^2)/2 - x_2(m_B^2 + m_K^2 - m_\chi^2) ,$$

$$A = m_\chi^2/4 - x_2(m_B^2 + m_\chi^2 - m_K^2)/2 - m_c^2 .$$

Here, we have set $m_{\chi_{cj}} = m_{\chi'_{c1}} \equiv m_\chi = 3.50$ GeV and $\tilde{f}_{\chi_{cj}} = \tilde{f}_{\chi'_{c1}} \equiv \tilde{f}_\chi$ so that $\tilde{f}_\chi$ is given by (A8), for example.

This completes our Appendix.

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