Testing the standard model and schemes for quark mass matrices with CP asymmetries in B decays

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(Received 31 July 1992)

The values of $\sin 2\alpha$ and $\sin 2\beta$, where α and β are angles of the unitarity triangle, will be readily measured in a B factory (and maybe also in hadron colliders). We study the standard model constraints in the $\sin 2\alpha$-$\sin 2\beta$ plane. We use the results from recent analyses of f_B and $\tau_b |V_{cb}|^2$ which take into account heavy-quark symmetry considerations. We find $\sin 2\beta \geq 0.15$ and most likely $\sin 2\beta \gtrsim 0.6$, and emphasize the strong correlations between $\sin 2\alpha$ and $\sin 2\beta$. Various schemes for quark mass matrices allow much smaller areas in the $\sin 2\alpha$-$\sin 2\beta$ plane. We study the schemes of Fritzsch, of Dimopoulos, Hall, and Raby, and of Giudice, as well as the "symmetric Cabibbo-Kobayashi-Maskawa" idea, and show how CP asymmetries in B decays will crucially test each of these schemes.

PACS number(s): 12.15.Ff, 11.30.Er, 13.25.+m, 14.40.Jz

CP asymmetries in neutral B decays will provide a unique way to measure the Cabibbo-Kobayashi-Maskawa (CKM) parameters. In a high-luminosity e^+e^- collider running at the energy of the $\Upsilon(4S)$ resonance (a "B factory"), two of the three angles of the unitarity triangle (see Fig. 1) will be readily measured [1]: the CP asymmetry in, e.g., $B \to \pi^+\pi^-$ will determine $\sin 2\alpha$, while that in, e.g., $B \to \psi K_S$ will determine $\sin 2\beta$. It may also be possible to measure $\sin 2\beta$ in a hadron collider, but $\sin 2\alpha$ would be difficult due to the large background (see, e.g., [2]). The experimental measurements are expected to be highly accurate and the theoretical calculations are, to a large extent, free of hadronic uncertainties. Furthermore, CP asymmetries in neutral B decays are a powerful probe into possible sources of CP violation beyond the standard model (SM). The richness of available B decay modes would allow one to determine detailed features of the new sources of CP violation if the SM predictions are not borne out. In this work, we refer to both aspects of CP asymmetries in B decays, namely, the determination of the CKM parameters within the SM, and the testing of extensions of the SM, with a special emphasis on the information that can be extracted by measuring two angles of the unitarity triangle rather than, say, $\sin 2\beta$ alone.

In the first part of this work, we investigate in detail the SM predictions for $\sin 2\alpha$ and $\sin 2\beta$. In particular, we study the correlation between the two quantities and present our results in the $\sin 2\alpha$-$\sin 2\beta$ plane. We update previous analyses with emphasis on recent theoretical developments which involve the heavy-quark symmetry.

In the second part of this work, we show how various schemes for quark mass matrices can be tested through their predictions for $\sin 2\alpha$ and $\sin 2\beta$. We analyze the Dimopoulos-Hall-Raby (DHR) scheme [3], the Giudice scheme [4], the Fritzsch scheme [5], and the idea that the CKM matrix is symmetric in the absolute values of its entries [6] (including the two-angle parametrization of Kielanowski [7]). Each of these schemes allows a range for the asymmetries which is much smaller than in the SM and thus may be clearly excluded when the asymmetries are measured.

Various bounds on the CKM parameters are usually presented as constraints on the form of the unitarity triangle (for a review see [8,9,2] and references therein). However, the quantities directly measurable via CP violation in a B factory are $\sin 2\alpha$ and $\sin 2\beta$, so we will present our constraints in terms of these observables. The time-dependent CP asymmetry in the decay of a B or $\bar{B}$ into some final CP eigenstate f is given by

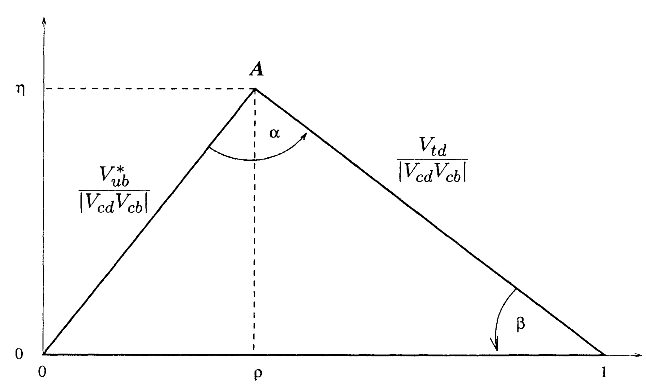

FIG. 1. The unitarity condition $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$ represented as a triangle in the complex plane. The sides have been divided by $|V_{cb} V_{cd}|$ so that the vertices may be placed at (0,0), (1,0), and (ρ,η). The angles α and β are measured counterclockwise as shown.

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$$\frac{\Gamma(B^0(t)\to f)-\Gamma(\bar{B}^0(t)\to f)}{\Gamma(B^0(t)\to f)+\Gamma(\bar{B}^0(t)\to f)}=-\mathrm{Im}\lambda(f)\sin\Delta M t\ , \quad (1)$$

where $\Delta M \equiv M(B_{\mathrm{heavy}})-M(B_{\mathrm{light}})$, $B^0(t)$ $[\bar{B}^0(t)]$ is a state which starts out as the flavor eigenstate B^0 $[\bar{B}^0]$ at a time $t=0$, and $\lambda(f)$ is a complex number with (almost exactly) unit magnitude. Then, within the SM (and in all schemes considered in this work),

$$\mathrm{Im}\lambda(\pi^+\pi^-)=\sin 2\alpha\ ,\quad \mathrm{Im}\lambda(\psi K_S)=\sin 2\beta \quad (2)$$

(where we took into account the fact that ψK_S is a CP-odd state). Thus, our figures in the $\sin 2\alpha$-$\sin 2\beta$ plane simply present the allowed range in the $\mathrm{Im}\lambda(\pi^+\pi^-)$-$\mathrm{Im}\lambda(\psi K_S)$ plane. This gives an important advantage to our method: the presentation in the $\mathrm{Im}\lambda(\pi^+\pi^-)$-$\mathrm{Im}\lambda(\psi K_S)$ plane allows a direct comparison of the SM predictions (or the experimental results) with models of new physics where the asymmetries are not related to angles of the unitarity triangle.

We use the following relations to transform from the (ρ,η) coordinates of the free vertex A of the unitarity triangle to $(\sin 2\alpha, \sin 2\beta)$

$$\sin 2\alpha=\frac{2\eta[\eta^2+\rho(\rho-1)]}{[\eta^2+(1-\rho)^2][\eta^2+\rho^2]}\ ,$$
$$\sin 2\beta=\frac{2\eta(1-\rho)}{\eta^2+(1-\rho)^2}\ . \quad (3)$$

Note that these coordinate transformations are highly nonlinear; hence the predictions in the $\sin 2\alpha$-$\sin 2\beta$ plane will be very different from the more familiar constraints in the ρ-η plane. Furthermore, since (3) are not (uniquely) invertible, we may not simply map the regions in the ρ-η plane allowed by each of the various constraints into corresponding regions in the $\sin 2\alpha$-$\sin 2\beta$ plane, and then assume that the overlap in the latter is allowed. To see this, note that a single point in the overlap region in the $\sin 2\alpha$-$\sin 2\beta$ plane may correspond to two different points in the ρ-η plane. If each of these two points is allowed by one constraint but forbidden by the other, then the original point in the $\sin 2\alpha$-$\sin 2\beta$ plane is in fact forbidden though it is in the overlap of two regions allowed by the individual constraints. We therefore form the overlap in the ρ-η plane first, and then map this overall-allowed region into $\sin 2\alpha$-$\sin 2\beta$ coordinates. Finally, even in the ρ-η plane the overlap of two allowed regions may not entirely be allowed: a given point in the overlap may meet the various constraints only by using different values of some parameter which enters into both constraints. But this correlation is unimportant in practice, since the uncertainties in the parameters which enter into more than one constraint never dominate both constraints.

We now analyze the SM predictions for $\sin 2\alpha$ and $\sin 2\beta$, updating previous analyses of constraints on the CKM parameters. The most significant update is in the constraint from B-$\bar{B}$ mixing, which determines the length of one side of the unitarity triangle:

$$(1-\rho)^2+\eta^2=\frac{(1.3\times 10^7\ \mathrm{GeV})x_d}{(B_B f_B^2)y_t f_2(y_t)(\tau_B|V_{cb}|^2)|V_{cd}|^2\eta_B}\ , \quad (4)$$

where $\eta_B=0.85$ is a QCD correction, $y_t=(m_t/M_W)^2$ and

$$f_2(x)=1-\tfrac{3}{4}x(1+x)(1-x)^{-2}[1+2x(1-x^2)^{-1}\ln(x)]\ .$$

Recently, both lattice and QCD sum-rule calculations of the f_B decay constant were made which rely on heavy-quark symmetry considerations. Results from the two techniques now converge to a consistent range and, we believe, should be preferred over previous, more model-dependent, calculations. We use the result of Ref. [10] from QCD sum rules, which is consistent with lattice calculations (see [11] and references therein):

$$f_B=190\pm 50\ \mathrm{MeV}\ . \quad (5)$$

Since the B_B factor is expected to be close to unity, we simply take $B_B=1$ and neglect the uncertainty in B_B relative to that in f_B [or, equivalently, absorb it into the uncertainty in (5)]. Heavy-quark symmetry considerations have also been applied to find the combination $|V_{cb}|^2\tau_B$. We again believe that the new results, in which only the corrections to the heavy quark limit are model dependent, should replace previous calculations which were completely model dependent. We take the analysis of Ref. [12] with updated input data [13]:

$$|V_{cb}|(\tau_b/1.3\ \mathrm{ps})^{1/2}=0.040\pm 0.005\ . \quad (6)$$

For the mixing parameter x_d, we use [14]

$$x_d=0.67\pm 0.11\ . \quad (7)$$

Finally, we use $|V_{cd}|=|V_{us}|=0.221\pm 0.002$.

Our second constraint comes from the end point of the lepton spectrum in charmless semileptonic B decays. We adopt the range quoted by the Particle Data Group [15]:

$$|V_{ub}/V_{cb}|=0.10\pm 0.03\ . \quad (8)$$

This determines the length of the other side of the unitarity triangle:

$$\rho^2+\eta^2=\left|\frac{V_{ub}}{V_{cb}V_{cd}}\right|^2 . \quad (9)$$

The third constraint comes from the CP-violating ϵ parameter in the K^0 system:

$$\rho=\left[1+\frac{[\eta_3 f_3(y_c,y_t)-\eta_1]y_c}{\eta_2 y_t f_2(y_t)|V_{cb}|^2}\right] -\frac{1}{\eta}\left[\frac{2.5\times 10^{-5}|\epsilon|}{\eta_2 y_t f_2(y_t)|V_{cb}|^4 B_K |V_{cd}|^2}\right]\ , \quad (10)$$

where $\eta_1=0.7$, $\eta_2=0.6$, and $\eta_3=0.4$ are QCD corrections [16], $y_c=(m_c/M_W)^2$ and

$$f_3(x,y)=\ln(y/x)-\tfrac{3}{4}y(1-y)^{-1}[1+y(1-y)^{-1}\ln(y)]\ .$$

The uncertainties here lie in the value of the B_K parameter, estimated to be

$$B_K=2/3\pm 1/3\ , \quad (11)$$

and in the range for $|V_{cb}|$. Using [15] $\tau_B=1.29\pm 0.05$ ps, we deduce, from (6),

$$|V_{cb}|=0.040\pm0.007 \ . \tag{12}$$

We further use $|\epsilon|=(2.26\pm0.02)\times10^{-3}$ and [17] $m_c(m_c)=1.27\pm0.05$ GeV.

Since the x_d and ϵ constraints depend on m_t, we have carried out our analysis for various m_t values within the range 90 GeV $\le m_t \le$ 185 GeV. We present our results in Fig. 2 in two ways. First, the thin black curves encompass all values of (sin2α,sin2β) which satisfy all three constraints using values of the input parameters within their 1σ ranges (or within the theoretically favored ranges for the parameters B_K and f_B). That is, the SM can accommodate a B-factory result anywhere within these curves without stretching any input parameter beyond its 1σ range. We will refer to these regions as the "allowed" areas of the SM. (A somewhat similar plot of sin2α versus ρ appears in [11].) Second (and similarly to [18]), in order to get a sense of the expected value of (sin2α,sin2β) given our current knowledge of the various input parameters, we generated numerous sample values for these parameters based on a Gaussian distribution for $|V_{cd}|$, $\tau_B|V_{cb}|^2$, $|V_{ub}/V_{cb}|$, τ_B, x_d, m_c, and $|\epsilon|$, and a uniform distribution (=0 outside of the "1σ" range) for f_B. For each sample set we used the constraints (4) and (9) to determine ρ and η, and then rejected those sets which did not meet the constraint (10) for $\frac{1}{3}\le B_K \le 1$. We binned the sets which passed in the sin2α-sin2β plane, and thus obtained their probability distribution. We show in Fig. 2 the resulting 68% and 90% probability contours in dark gray and light gray, respectively. Since we do not know the true origin of the CKM parameters and thus do not know the true probability distribution from which the experimental inputs result, and since the theoretical restrictions on f_B and B_K cannot be posed statistically, we can only interpret these probability contours as an indication of likely outcomes for B-factory results based on the SM. For example, the "tail" of the allowed areas which extends towards small values of (sin2α,sin2β) requires many of the parameters to be stretched to their 1σ bounds and so seems unlikely and lies outside both probability contours.

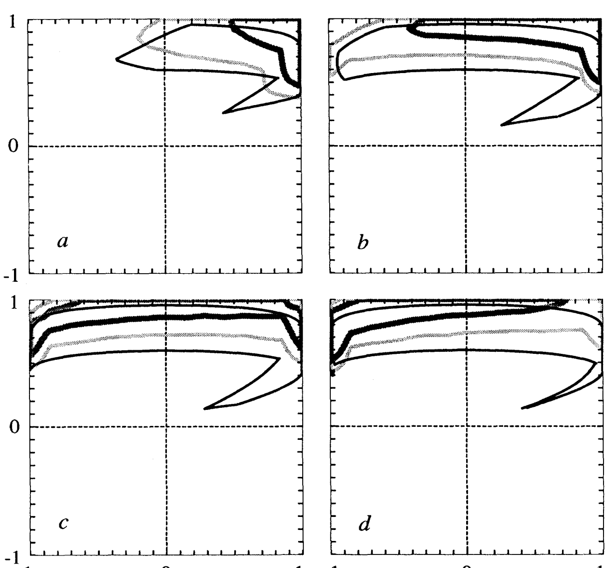

FIG. 2. The SM predictions in the sin2α(horizontal)-sin2β(vertical) plane, for four different top-quark masses: (a) 90 GeV, (b) 130 GeV, (c) 160 GeV, and (d) 185 GeV. The regions allowed by the 1σ ranges for all parameters described in the text are outlined by the thin black lines. The 68% probability contours generated as described in the text are shown as thick dark-gray lines, while the 90% contours are indicated by thinner light-gray lines.

Similarly to previous analyses (see, e.g., [2,11,19–23]), we find that sin2α can have any value in the full range from -1 to 1, while sin2β is always positive and has a lower bound

$$\sin2\beta\ge0.15 \ . \tag{13}$$

Furthermore, sin2α is likely to be positive if the top-quark mass is near its present lower bound, and most importantly *the favored values for* sin2β *are above 0.5.* We also find that the bounds on the two quantities are correlated (as also noted in [21]). In particular, we note that the magnitude of at least one of the two asymmetries is always larger than 0.2, and probably larger than 0.6 and if sin2$\beta\le$0.4, then sin2α must be positive—in fact, above 0.2. Once the top-quark mass is measured firmer predictions will of course be possible, based on one of the graphs in Fig. 2.

Various estimates may be made of the allowed ranges for the input parameters. In particular, there is no single obvious way to evaluate theoretical uncertainties. Furthermore, future improvement in both experimental measurements and theoretical analyses would certainly strengthen the constraints. Thus, it is useful to understand the sensitivity of our analysis to the various uncertainties. To this end we have displayed in Fig. 3 how the allowed regions of the SM depend on the choice of input parameters, for a representative top-quark mass of 130 GeV. For Figs. 3(a), 3(b), and 3(c) we have allowed somewhat larger ranges for $0.05\le|V_{ub}/V_{cb}|\le0.15$ and 100 MeV $\le f_B \le$ 300 MeV. All other ranges are kept as before. The five solid lines of Fig. 3(a) correspond, from bottom to top, to the constraint (9) when $|V_{ub}/V_{cb}|$ increases from 0.05 to 0.15. The twelve solid lines of Fig. 3(b) correspond, from left to right, to the constraint (4) when the values of f_B and $\tau_B|V_{cb}|^2$ decrease within their respective ranges. The six solid lines of Fig. 3(c) correspond, from left to right, to the constraint (10) when $|V_{cb}|^2$ and B_K decrease within their respective ranges. [Note that each solid line in these figures must meet all three constraints. For Fig. 3(b) this disallows the lower end of the range for f_B and $\tau_B|V_{cb}|^2$, while for Fig. 3(c) it is the lower end of the range for $|V_{cb}|^2$ and B_K that is not allowed.] One can then read off the approximate allowed region for a more restricted choice of input parameter ranges. For completeness we have also plotted in Fig. 3(d) the allowed region obtained by accepting the range $0.15\le|V_{ub}/V_{cb}|\le0.20$ suggested by Isgur *et al.* [24],

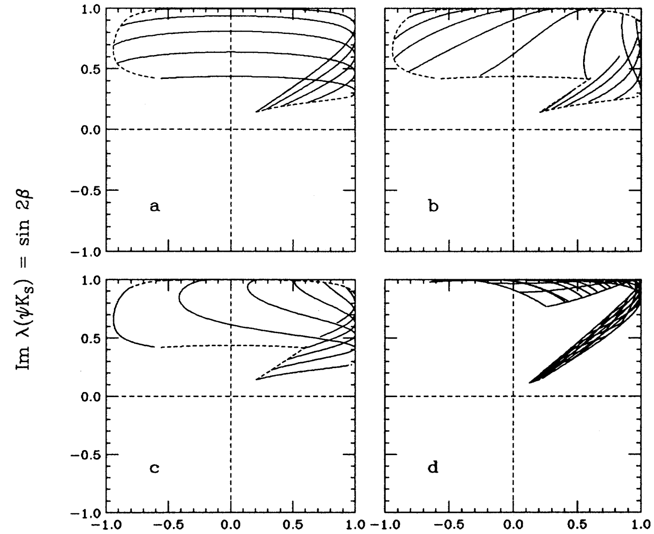

FIG. 3. The dependence of the allowed regions in the sin2α-sin2β plane on the input parameter ranges, for a representative value of $m_t=130$ GeV. The largest region allowed by all three constraints is outlined by the dashed lines. For this figure we have allowed the wider range 100 MeV $\le f_B \le$ 300 MeV but kept all other ranges as before, with the exception of $|V_{ub}/V_{cb}|$: in (a), (b), and (c) we allow the wider range $0.05 \le |V_{ub}/V_{cb}| \le 0.15$, while in (d) we adopt the higher range of Ref. [24], $0.15 \le |V_{ub}/V_{cb}| \le 0.20$. The five solid lines in (a) correspond, from bottom to top, to the constraint (9) when $|V_{ub}/V_{cb}|$ increases from 0.05 to 0.15. The twelve solid lines of (b) correspond, from left to right, to the constraint (4) when its right-hand side increases from 0.29 to 2.71. The six solid lines of (c) correspond, from left to right, to constraint (10) when its first bracketed expression increases from 1.34 to 1.40 and the second increases from 0.24 to 0.96.

while keeping all other parameters as in the rest of Fig. 3. In this case it is likely that sin2β is very close to unity, or else (and this is unlikely) sin$2\alpha \sim$ sin2β and they can both be as small as roughly 0.1 if $|V_{ub}/V_{cb}|$ and B_K are as large as possible and f_B is as small as possible.

We next turn to the testing of various schemes for quark mass matrices. We use the following ranges for quark masses at 1 GeV [17]:

$$m_c=1.36\pm0.05 \text{ GeV}, \quad m_b=5.6\pm0.4 \text{ GeV} , \tag{14}$$

and, for mass ratios,

$$\frac{m_d}{m_s}=0.051\pm0.004 , \quad \frac{m_u}{m_c}=0.0038\pm0.0012, \quad \frac{m_s}{m_b}=0.030\pm0.011 . \tag{15}$$

In the remainder of our analysis *we allow only* 1σ *ranges for all inputs,* since we believe that if any of these schemes need to be stretched beyond their 1σ predictions then their motivation is largely lost. These 1σ ranges should only be viewed as the *favored* values within the schemes; one should not rule out any scheme simply on the basis that the experimental results do not quite fall within the 1σ predictions we obtain. In Fig. 4 we display these predictions of the four schemes for the same sample values of m_t as in Fig. 2. Only the symmetric CKM *Ansatz* admits a sufficiently large range of m_t to be included in more than one graph. For reference we have also indicated, in gray, the 1σ allowed areas of the SM.

We first discuss the Fritzsch scheme [5]:

$$M_u=\begin{bmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & b_u & c_u \end{bmatrix} ,$$
$$M_d=\begin{bmatrix} 0 & a_d e^{i\phi_1} & 0 \\ a_d e^{-i\phi_1} & 0 & b_d e^{i\phi_2} \\ 0 & b_d e^{-i\phi_2} & c_d \end{bmatrix} . \tag{16}$$

It fits ten parameters (six masses, three mixing angles, and a CP-violating phase) with eight parameters and therefore makes two predictions. It is now nearly exclud-

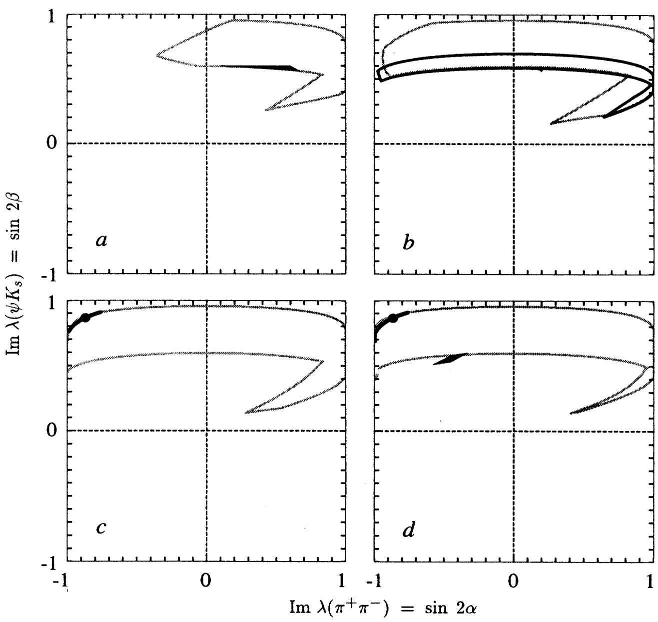

FIG. 4. The 1σ allowed regions predicted by various mass matrix schemes in the sin2α-sin2β plane, for the same sample values of m_t as in Fig. 2. The allowed regions within the SM are outlined for reference in light gray. In (a) the value of $m_t=90$ GeV is consistent only with the Fritzsch *Ansatz,* which predicts the values within the thin black wedge. A top mass of $m_t=130$ GeV in (b) is compatible only with the scheme of Giudice, which allows the region within the band outlined in black. A symmetric CKM matrix is consistent with a top-quark mass of 160 GeV (c) and 185 GeV (d); its predictions lie along the short black curve, while the special case of Kielanowski is shown as the small filled circle in each of these figures. The DHR scheme predicts the heavy top-quark mass $m_t=185$ GeV of (d), and allows only the tiny region shown in black.

ed [25]. The main difficulty lies in the relation

$$|V_{cb}| = \left| \left[\frac{m_s}{m_b} \right]^{1/2} - e^{-i\phi_2} \left[\frac{m_c}{m_t} \right]^{1/2} \right| , \tag{17}$$

which can only be satisfied if the top quark is close to the experimental lower bound:

$$m_t \sim 90 \text{ GeV} . \tag{18}$$

If the top quark is indeed this light, then the next crucial test for the Fritzsch scheme would be its predictions for CP asymmetries in B^0 decays. The allowed range for $(\sin 2\alpha, \sin 2\beta)$ is shown as the black wedge in Fig. 4(a). We find

$$0.10 \le \sin 2\alpha \le 0.67, \quad 0.56 \le \sin 2\beta \le 0.60 . \tag{19}$$

We turn next to the scheme of Giudice [4], which requires the charged fermion mass matrices to have the following form at the grand unified theory (GUT) scale:

$$M_u = \begin{bmatrix} 0 & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{bmatrix} ,$$

$$M_d = \begin{bmatrix} 0 & fe^{i\phi} & 0 \\ fe^{-i\phi} & d & 2d \\ 0 & 2d & c \end{bmatrix} , \tag{20}$$

$$M_l = \begin{bmatrix} 0 & f & 0 \\ f & -3d & 2d \\ 0 & 2d & c \end{bmatrix} .$$

This scheme fits the quark *and* lepton mass matrices with six parameters and therefore makes seven predictions. Among them we find

$$m_t \sim 125-155 \text{ GeV}, \quad |V_{cb}| \sim 0.048 ,$$
$$0.07 \le |V_{ub}/V_{cb}| \le 0.084 \left[\frac{130 \text{ GeV}}{m_t} \right] . \tag{21}$$

Note that our allowed range for m_t is smaller than in Ref. [4], due to our stronger bounds on $|V_{ub}/V_{cb}|$. (This range is very sensitive to the bottom-quark mass, and thus could be enlarged by adopting more conservative estimates of the uncertainty in m_b.) It is not unlikely that this scheme would survive the various measurements until a B factory starts running. Then it allows only a narrow band in the $\sin 2\alpha$-$\sin 2\beta$ plane, as shown in Fig. 4(b). The overall constraint is

$$-0.98 \le \sin 2\alpha \le +1.0, \quad 0.2 \le \sin 2\beta \le 0.7 . \tag{22}$$

However, for low $\sin 2\beta$ values, there is a strong correlation between the two asymmetries. In particular, if $\sin 2\beta \lesssim 0.45$, then $\sin 2\alpha \ge 0.65$.

The scheme of Dimopoulous, Hall, and Raby (DHR) [3] requires that, at the GUT scale, charged fermion mass matrices are of the form

$$M_u = \begin{bmatrix} 0 & c & 0 \\ c & 0 & b \\ 0 & b & a \end{bmatrix} ,$$

$$M_d = \begin{bmatrix} 0 & fe^{i\phi} & 0 \\ fe^{-i\phi} & e & 0 \\ 0 & 0 & d \end{bmatrix} , \tag{23}$$

$$M_l = \begin{bmatrix} 0 & f & 0 \\ f & -3e & 0 \\ 0 & 0 & d \end{bmatrix} .$$

It has seven parameters and therefore six predictions, among which we find (cf. [26])

$$m_t \sim 185 \text{ GeV}, \quad |V_{cb}| \sim 0.047, \quad |V_{ub}/V_{cb}| \sim 0.065 . \tag{24}$$

(Note that the latter prediction, which is at the top of the 1σ range for this scheme, is just below our allowed range. We therefore predict a very narrow range of the DHR parameter χ which accounts for much of the uncertainty in this scheme: $\chi^2 \simeq \frac{4}{3}$.) Thus, future measurements of m_t, or theoretical improvement in determining $|V_{cb}|$ or $|V_{ub}/V_{cb}|$, may easily exclude the DHR scheme. If it survives these tests, then it would provide very powerful predictions for CP asymmetries in B^0 decays. Only a very narrow range in the $\sin 2\alpha$-$\sin 2\beta$ plane is allowed, as shown in 4(d). The overall constraint is

$$-0.58 \le \sin 2\alpha \le -0.33, \quad 0.51 \le \sin 2\beta \le 0.60 . \tag{25}$$

Once again the values of the two asymmetries are correlated, providing an even stronger test than implied by (25).

Our last example is the symmetric *Ansatz* [6] for the CKM matrix:

$$|V_{ij}| = |V_{ji}| . \tag{26}$$

The theoretical motivation for this *Ansatz* is more obscure than for the previous *Ansätze.* In particular, it is still to be demonstrated that the constraints (26) can result from some symmetry of the Lagrangian [27]. This *Ansatz* leads to (cf. [28–31])

$$m_t \gtrsim 160 \text{ GeV}, \quad |V_{ub}/V_{cb}| \ge |V_{cd}|/2 \simeq 0.11 . \tag{27}$$

(This bound on m_t is lower than in some previous analyses due to our higher allowed range of f_B, as already remarked in [18].) CP asymmetries in B^0 decays would be extremely powerful in testing (26). The correlation between $\sin 2\alpha$ and $\sin 2\beta$ is strongest here, as (26) leads to

$$\rho = \tfrac{1}{2} \Longrightarrow \sin 2\alpha = -2 \sin 2\beta \cos 2\beta . \tag{28}$$

For a fixed m_t value, (28) leads to an allowed *curve* in the $\sin 2\alpha$-$\sin 2\beta$ plane, as shown in Figs. 4(c) and 4(d). For the overall bounds we find

$$-1.0 \le \sin 2\alpha \le -0.76, \quad 0.68 \le \sin 2\beta \le 0.91 . \tag{29}$$

The two-angle parametrization of the CKM matrix proposed by Kielanowski [7] is a special case of this *Ansatz,* in which $\eta \simeq 1/(2\sqrt{3})$ (to within a few percent). Consequently (cf. [29,30]) $\sin 2\alpha = -\sqrt{3}/2 = -\sin 2\beta$, as indicated by the small filled circle in Figs. 4(c) and 4(d).

Before concluding, let us mention a discussion of the structure of quark mass matrices by Bjorken [32]. His assumptions lead to a prediction for the angle γ of the unitarity triangle, $\gamma \approx \pi/2$. For the asymmetries discussed here, this implies $\sin 2\alpha = \sin 2\beta$, opposite to the prediction of the superweak scenario in which $\sin 2\alpha = -\sin 2\beta$. A discussion of the experimental prospects of excluding the latter relation can be found in Refs. [33,34].

To summarize, we have examined the predictions of the SM and of various quark mass matrix schemes for $\sin 2\alpha$ and $\sin 2\beta$ or, equivalently, for the *CP* asymmetries in $B \to \pi\pi$ and $B \to \psi K_S$. Our main results are presented in Figs. 2 and 4. We have displayed them in the $\sin 2\alpha$-$\sin 2\beta$ plane to facilitate direct comparison with future experiments or nonstandard models, and to show the importance of the correlation between the predictions for $\sin 2\alpha$ and for $\sin 2\beta$. (This correlation was also used in [21,35].) The predictions are quite encouraging for experimenters.

(i) Recent improvements in theoretical calculations lead to a lower bound on the asymmetry in $B \to \psi K_S$ of order 0.15, somewhat higher than previous analyses.

(ii) If the asymmetry in $B \to \psi K_S$ is close to its lower bound, then it is highly correlated with the asymmetry in $B \to \pi\pi$ and at least one of the two is larger than 0.2.

(iii) For the asymmetries to both be small, many parameters have to assume values close to their 1σ bounds, which is improbable. It is more likely that at least one of the asymmetries is larger than 0.6.

(iv) Various schemes for quark mass matrices allow a much smaller range for the two asymmetries than does the SM. Therefore, they would be stringently tested when the asymmetries are measured.

Note added. After completing this work, we learned of a very interesting related paper [36] by Soares and Wolfenstein, who also consider in the $\sin 2\alpha$-$\sin 2\beta$ plane the correlations predicted by the standard model and by an extension thereof.

U.S. acknowledges partial support from the Albert Einstein Center for Theoretical Physics at the Weizmann Institute, and he thanks the members of the particle theory group at the Weizmann Institute for their kind hospitality and Lawrence Hall for illuminating discussions. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098. Y.N. was supported in part by the Israel Commission for Basic Research, United States–Israel Binational Science Foundation and the Minerva Foundation.

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