Angular distribution and *CP* effects in rare decays of vector mesons

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We calculate the differential decay rate for the semileptonic decay $V \rightarrow Rl^+l^-$ when R is a scalar, a pseudoscalar, and a vector as a function of the invariant mass of the l^+l^- pair and the scattering angle in the l^+l^- center-of-mass frame. We obtain the helicity dependence of the decays and calculate the asymmetry factor $\alpha(s)$ and the ratio of the contributions from the *CP*-odd and *CP*-even channels. We repeat the calculations for the nonleptonic decay $V \rightarrow RR'$. In the calculations we use the heavy-quark and factorization approximations.

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I. INTRODUCTION

In this paper our aim is to calculate the differential decay rate for the semileptonic decay $V \rightarrow Rl^+l^-$, where the relevant operator is $b \rightarrow ql^+l^-$, and the nonleptonic decay $V \rightarrow RR'$, where the relevant operator is $b \rightarrow qq_1\bar{q}_2$. R represents the bound state of the $\bar{b}q$ pair, and R' represents the bound state of the $q_1\bar{q}_2$ pair. We do the calculations for a scalar, a pseudoscalar, and a vector resonance. In the calculations we use the heavy-quark and factorization approximations; therefore, we expect our results to be more reliable when the quarks involved are heavy. These decay channels are important because of their dependence on the top-quark mass as well as their providing an opportunity to study *CP* violations. Even though we do the calculations for b-quark decay, the results can be applied to rare decays of other heavy-quark mesons.

The main purpose of our work is to obtain the angular distribution of these decays for different helicity and *CP* channels in the l^+l^- rest frame. After obtaining these contributions, we calculate the asymmetry factor $\alpha(s)$, which we define to be the ratio of the $\cos^2\vartheta$ term to the flat term where ϑ is the scattering angle in the l^+l^- rest frame, the configuration for which is shown in Fig. 1. The information on the angular distribution is important because of the fact that one can use Monte Carlo simulation for the experimental setup to obtain the efficiency and to eliminate the background effects.[1]

In Sec. IV we repeat the calculations for $V \rightarrow RR'$.

In doing the calculations mentioned above, we use the heavy-quark and factorization approximations [2–4]. These works should provide a reference mark for experiments and, as a result, should provide information on how well these approximations work for these decays.

II. KINEMATICS

The three-body differential decay rate for an unstable particle is given by

$$d\Gamma = \frac{1}{2M_V}\prod_{i=1}^{3}\left[\frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i}\right](2\pi)^4\delta^{(4)}\left[P-\sum_{i=1}^{3}p_i\right]|\mathcal{M}|^2 . \tag{1}$$

Here P is the four-momentum of the decaying particle and p_i are the four-momenta of the final-state particles; we choose p_1 and p_2 to be the four-momenta of l^+ and l^- and $p_3 \equiv P_R$ to be the four-momentum of the remaining particle. $\mathcal{M}$ is the matrix element for the decay. By introducing the invariant mass $s=(p_1+p_2)^2$ of the l^+l^- pair via

$$\begin{aligned}1 &= \int ds\,\delta(s-p^2)d^4p\,\delta^{(4)}(p-p_1-p_2)\\ &= \int ds \frac{d^3\mathbf{p}}{2E_{\mathbf{p}}}\delta^{(4)}(p-p_1-p_2)\,,\end{aligned} \tag{2}$$

we obtain

$$\begin{aligned}\frac{d\Gamma}{ds\,d\cos\vartheta} = {}& \frac{1}{2^9\pi^3 s M_V^3}[\Lambda(s,m_1^2,m_2^2)]^{1/2}\\ &\times[\Lambda(M_V^2,M_R^2,s)]^{1/2}|\mathcal{M}|^2\,,\end{aligned} \tag{3}$$

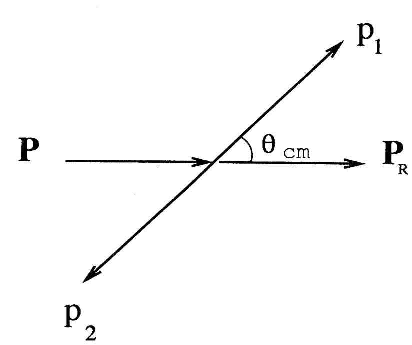

FIG. 1. Momentum configuration for the V decay in the l^+l^- center-of-mass frame.

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[1]See, for example, Ref. [1].

where

$$\Lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc \ . \tag{4}$$

For the two-body decay, we have

$$\Gamma=\frac{1}{8\pi M_V^3}[\Lambda(M_V^2,M_R^2,M_R'^2)]^{1/2}|\mathcal{M}|^2 \ . \tag{5}$$

We define the ratio of contribution to the differential decay rate from the different helicity states by

$$\mathcal{R}_{\lambda,\lambda'}\equiv\frac{1}{d\Gamma/ds}\left[\frac{d\Gamma}{dsd\cos\vartheta}\right]_{\lambda,\lambda'} \tag{6}$$

and the ratio of the contribution to the differential decay rate from the *CP*-odd channel by

$$\mathcal{R}_{\rm odd}\equiv\frac{1}{d\Gamma/ds}\left[\frac{d\Gamma}{dsd\cos\vartheta}\right]_{CP\ \rm odd} \ . \tag{7}$$

In the l^+l^- center-of-mass frame $\mathbf{p}_1=-\mathbf{p}_2$, $\mathbf{P}_V=\mathbf{P}_R=\mathbf{P}$ (the configuration is as shown in Fig. 1),

$$\begin{aligned} &P_V=(E_V,0,0,P),\quad P_R=(E_R,0,0,P)\ ,\\ &p_1=(E,\mathbf{p}),\quad p_2=(E,-\mathbf{p})\ ,\\ &\mathbf{P}_V\cdot\mathbf{p}=P_Rp\cos\vartheta_{\rm c.m.}\ , \end{aligned} \tag{8}$$

and the helicity eigenstates are given by

$$\begin{aligned} &\epsilon_V(\lambda_V=0)=\frac{1}{M_V}(P,0,0,E_V)\ ,\\ &\epsilon_R(\lambda_R=0)=\frac{1}{M_R}(P,0,0,E_R)\ ,\\ &\epsilon_V(\lambda_V=\pm)=\epsilon_R(\lambda_R=\pm)=\frac{\mp1}{\sqrt{2}}(0,1,\pm i,0)\ . \end{aligned} \tag{9}$$

The invariant mass of the l^+l^- is

$$\begin{aligned} s&=(p_1+p_2)^2=(P_V-P_R)^2\\ &=(E_1+E_2)^2=(E_V-E_R)^2\ , \end{aligned} \tag{10}$$

with

$$\begin{aligned} &s_{\min}=(m_1+m_2)^2\ ,\\ &s_{\max}=(M_V-M_R)^2\ , \end{aligned} \tag{11}$$

and

$$\begin{aligned} &E_V=\frac{M_V^2-M_R^2+s}{2\sqrt{s}}\ ,\\ &E_R=\frac{M_V^2-M_R^2-s}{2\sqrt{s}}\ ,\\ &\mathbf{P}_V^2=\mathbf{P}_R^2=\frac{1}{4s}\Lambda(M_V^2,M_R^2,s)\ . \end{aligned} \tag{12}$$

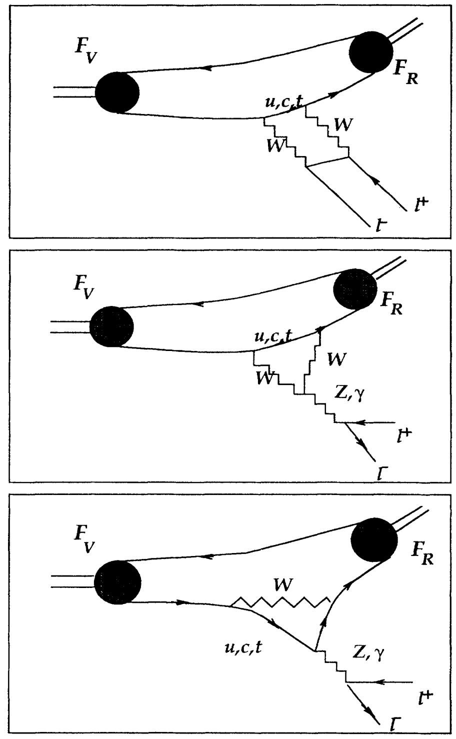

FIG. 2. Decay diagrams for $V\to Rl^+l^-$.

III. $V\to Rl^+l^-$

In the calculations of this section, we model the decay processes on the *b*-quark decay, and we consider only the diagrams shown in Fig. 2. In the *V* decay diagrams, we have introduced the factors $\mathcal{F}$ at each vertex. In general, the $\mathcal{F}$'s are functions of the masses and the decay rates of the mesons as well as the invariant mass of the l^+l^- pair. They can be taken, approximately, to be $\mathcal{F}\simeq2\sqrt{M}\,\psi(0)$, where $\psi(0)$ is the wave function of the resonance at the origin at the relevant vertex.[2] However, since our purpose is to obtain the angular distribution up to an overall factor, we leave them as parts of an overall factor.

We can write the matrix element for the decay in Fig. 2 in the form

$$\mathcal{M}_{V\to Kl^+l^-}=\mathcal{M}_\Box+\mathcal{M}^Z_{\triangle1}+\mathcal{M}^Z_{\triangle2}+\mathcal{M}^\gamma_{\triangle1}+\mathcal{M}^\gamma_{\triangle2}\ , \tag{13}$$

where, for $i=u,c,t$,

$$\mathcal{M}_\Box=\left[\frac{g}{2\sqrt{2}}\right]^4[V_{qi}^\dagger V_{ib}]\left[\frac{\mathcal{F}_V\mathcal{F}_K}{4M_VM_R}\right]\int\frac{d^4k}{(2\pi)^4}\frac{T_\Box^{\mu\nu}t_{\Box\mu\nu}}{(k^2-m_i^2)(k_1^2-m^2)(k_2^2-M_W^2)(k_3^2-M_W^2)}\ ,$$

[2]See [5,6] for the relation between the wave function at the origin and the decay constant.

$$T_{\boxminus}^{\mu\nu} \equiv \mathrm{Tr}[\not{V}_R(\not{P}_R+M_R)\gamma^\mu(1-\gamma_5)(\not{k}+m_i)\gamma^\nu(1-\gamma_5)(\not{P}_V+M_V)\not{\epsilon}_V] ,$$
$$=2k^\alpha \mathrm{Tr}[\not{V}_R(\not{P}_R+M_R)\gamma^\mu(1-\gamma_5)\gamma_\alpha\gamma^\nu(\not{P}_V+M_V)\not{\epsilon}_V] ,$$
$$t_{\boxminus}^{\mu\nu} \equiv [\bar{q}_2\gamma^\nu(1-\gamma_5)(\not{k}_1+m)\gamma^\mu(1-\gamma_5)q_1]=2k_\omega[\bar{q}_2\gamma^\nu\gamma^\omega\gamma^\mu(1-\gamma_5)q_1] , \tag{14}$$
$$\mathcal{M}_{\triangle 1}=\left[\frac{g}{2\sqrt{2}}\right]^2\left[-\frac{g^2}{4}\right][V_{qi}^\dagger V_{ib}]\left[\frac{\mathcal{F}_V\mathcal{F}_K}{4M_VM_R}\right]\int\frac{d^4k}{(2\pi)^4}\frac{\Delta_{\lambda\mu\nu}T_{\triangle 1}^{\nu\lambda}t_{\triangle}^{\mu}}{(k^2-m_i^2)(k_1^2-M_W^2)(k_2^2-M_W^2)(s-M_Z^2)} ,$$
$$\Delta_{\lambda\mu\nu}=[(k_3-k_2)_\lambda g_{\mu\nu}+(k_2-k_1)_\mu g_{\lambda\nu}+(k_1-k_3)_\nu g_{\lambda\mu}] ,$$
$$T_{\triangle 1}^{\mu\nu}=T_{\boxminus}^{\mu\nu} ,\quad t_{\triangle}^{\mu}\equiv[\bar{q}_2\gamma^\mu(a+b\gamma_5)q_1] , \tag{15}$$
$$\mathcal{M}_{\triangle 2}=\left[\frac{g}{2\sqrt{2}}\right]^2\left[\frac{g}{4\cos\theta_W}\right]^2[V_{qi}^\dagger V_{ib}]\left[\frac{\mathcal{F}_V\mathcal{F}_K}{4M_VM_R}\right]\int\frac{d^4k}{(2\pi)^4}\frac{T_{\triangle 2}^{\mu}t_{\triangle\mu}}{(k^2-M_W^2)(k_1^2-m_i^2)(k_2^2-m_i^2)(s-M_Z^2)} ,$$
$$T_{\triangle 2}^{\mu}\equiv \mathrm{Tr}[\not{V}_R(\not{P}_R+M_R)\gamma^\nu(1-\gamma_5)(\not{k}_2+m_i)\gamma^\mu(c+d\gamma_5)(\not{k}_1+m_i)\gamma_\nu(1-\gamma_5)(\not{P}_V+M_V)\not{\epsilon}_V]$$
$$=\mathrm{Tr}[\not{V}_R(\not{P}_R+M_R)(1+\gamma_5)\gamma^\nu\{(c-d)\not{k}_2+(c+d)m_i\}\gamma^\mu(\not{k}_1+m_i)\gamma_\nu(1-\gamma_5)(\not{P}_V+M_V)\not{\epsilon}_V] , \tag{16}$$

where m_i is the mass of the internal quark and m is the mass of the internal lepton. To obtain the Z contribution, we substitute

$$a=-1+4\sin^2\theta_W,\quad b=1 ,$$
$$c=1-\tfrac{8}{3}\sin^2\theta_W,\quad d=1 , \tag{17}$$
$$d-c=\tfrac{8}{3}\sin^2\theta_W ,$$

and to obtain the γ contribution we use

$$a=c=1,\quad b=d=0 ,$$
$$M_Z\to 0,\quad \cos\theta_W\to 1 , \tag{18}$$

together with

$$\frac{g^2}{4}\to e^2 \tag{19}$$

in $\mathcal{M}_{\triangle 1}$ and

$$\left[\frac{g}{4\cos\theta_W}\right]^2\to e^2Q_i \tag{20}$$

in $\mathcal{M}_{\triangle 2}$, where Q_i is the charge of the internal quark line. The dominant term is the γ contribution since $s_{\max}=(M_V-M_R)^2\ll M_W^2,M_Z^2$.

There is also a contribution when the internal quark lines form a resonance. This contribution is small and would produce a kink at $s=M_{\mathcal{R}}^2$. We do the calculations for the resonance contribution in Sec. IV of this paper.

After using the equations in the Appendix, we obtain

$$T_{\boxminus}^{\mu\nu}t_{\boxminus}^{\mu\nu}\to -4k^2\mathcal{R}^\mu[\bar{q}_2\gamma_\mu(1-\gamma_5)q_1] ,$$
$$\Delta_{\lambda\mu\nu}T_{\triangle 1}^{\nu\lambda}\to -6k^2\mathcal{R}_\mu , \tag{21}$$
$$T_{\triangle 2}^{\mu}\to\{(c-d)k^2-2(c+d)m_i^2\}\mathcal{R}^\mu ,$$

where we defined[3]

$$\mathcal{R}^\mu\equiv \mathrm{Tr}[\not{V}_R(\not{P}_R+M_R)\gamma^\mu(1-\gamma_5)(\not{P}_V+M_V)\not{\epsilon}_V] . \tag{22}$$

The matrix element for $V\to Rl^+l^-$ can be written in the form

$$\mathcal{M}=\mathcal{R}^\mu\{\mathcal{V}[\bar{q}_2\gamma_\mu q_1]+\mathcal{A}[\bar{q}_2\gamma_\mu\gamma_5q_1]\} , \tag{23}$$
$$(1\pm\gamma_5)\gamma_5=(\gamma_5\pm 1)=\pm(1\pm\gamma_5) . \tag{24}$$

Therefore, after defining $q_\pm\equiv\frac{1}{2}(1\pm\gamma_5)q$, we have[4]

$$\mathcal{M}=\mathcal{R}^\mu\{\mathcal{V}\pm\mathcal{A}\}[q_{2\pm}^\dagger\gamma_{\mu\mp}q_{1\pm}] , \tag{25}$$

where $\mathcal{V}$ and $\mathcal{A}$ are as given in the Appendix, and[5]

[3]In the following subsections, in order to keep the expressions more general, we introduce the factors ξ,ζ,η. Here we have $\xi=\zeta=\eta=1$.

[4]See [7] for more detailed applications of this method in calculating matrix elements.

[5]Here we used

$$|\uparrow\rangle=\begin{bmatrix}1\\0\end{bmatrix},\quad |\downarrow\rangle=\begin{bmatrix}0\\1\end{bmatrix} \tag{26}$$

and

$$|\uparrow\vartheta\rangle=\begin{bmatrix}\cos\frac{\vartheta}{2}\\ \sin\frac{\vartheta}{2}\end{bmatrix},\quad |\downarrow\vartheta\rangle=\begin{bmatrix}-\sin\frac{\vartheta}{2}\\ \cos\frac{\vartheta}{2}\end{bmatrix} . \tag{27}$$

$$\gamma^{\mu}_{\pm}=[(|\uparrow\rangle\langle\uparrow|+|\downarrow\rangle\langle\downarrow|),\pm(|\downarrow\rangle\langle\uparrow|+|\uparrow\rangle\langle\downarrow|)\ ,\pm i(|\downarrow\rangle\langle\uparrow|-|\uparrow\rangle\langle\downarrow|),\pm(|\uparrow\rangle\langle\uparrow|-|\downarrow\rangle\langle\downarrow|)]\ . \tag{28}$$

The spinors are given by[6]

$$\begin{aligned}
q^{\Uparrow}_{2\pm}&=\frac{1}{\sqrt{2}}(\sqrt{E+m}\pm\sqrt{E-m}\,)|\uparrow\vartheta\rangle,\quad q^{\Uparrow}_{1\pm}=-\frac{1}{\sqrt{2}}(\sqrt{E-m}\mp\sqrt{E+m}\,)|\uparrow\vartheta\rangle\ ,\\
q^{\Downarrow}_{2\pm}&=\frac{1}{\sqrt{2}}(\sqrt{E+m}\mp\sqrt{E-m}\,)|\downarrow\vartheta\rangle,\quad q^{\Downarrow}_{1\pm}=\frac{1}{\sqrt{2}}(\sqrt{E-m}\pm\sqrt{E+m}\,)|\downarrow\vartheta\rangle\ ,
\end{aligned} \tag{29}$$

$$\begin{aligned}
&[q^{\Uparrow\dagger}_{2\pm}\gamma^{\mu}_{\mp}q^{\Uparrow}_{1\pm}]=\pm m\langle\uparrow\vartheta|\gamma^{\mu}_{\mp}|\uparrow\vartheta\rangle=m(\pm1,\sin\vartheta,0,\cos\vartheta)\ ,\\
&[q^{\Downarrow\dagger}_{2\pm}\gamma^{\mu}_{\mp}q^{\Downarrow}_{1\pm}]=\pm m\langle\downarrow\vartheta|\gamma^{\mu}_{\mp}|\downarrow\vartheta\rangle=m(\pm1,-\sin\vartheta,0,-\cos\vartheta)\ ,\\
&[q^{\Uparrow\dagger}_{2\pm}\gamma^{\mu}_{\mp}q^{\Downarrow}_{1\pm}]=2(p\pm E)\langle\uparrow\vartheta|\gamma^{\mu}_{\mp}|\downarrow\vartheta\rangle=(E\pm p)(0,\cos\vartheta,-i,-\sin\vartheta)\ ,\\
&[q^{\Downarrow\dagger}_{2\pm}\gamma^{\mu}_{\mp}q^{\Uparrow}_{1\pm}]=-(p\mp E)\langle\downarrow\vartheta|\gamma^{\mu}_{\mp}|\uparrow\vartheta\rangle=-(E\mp p)(0,\cos\vartheta,i,-\sin\vartheta)\ .
\end{aligned} \tag{30}$$

A. $V\to R_{0^+}l^+l^-$

In this case R is a scalar and $\mathcal{V}_R=1$; thus,

$$\begin{aligned}
\mathcal{R}^{\mu}_{S}&\equiv\mathrm{Tr}[(\not P_R+M_R)\gamma^{\mu}(1-\gamma_5)(\not P_V+M_V)\not\epsilon_V]\ ,\\
\mathcal{R}^{\mu}_{S}&=4\{\xi_S[M_VM_R-(P_R\cdot P_V)]\epsilon^{\mu}_{V}+\zeta_S(P_R\cdot\epsilon_V)P^{\mu}_{V}\}-4i\eta_S\varepsilon^{\alpha\beta\sigma\mu}P_{R\alpha}P_{V\beta}\epsilon_{V\sigma}\ .
\end{aligned} \tag{31}$$

After some algebra we obtain

$$\begin{aligned}
|\mathcal{M}_S|^2=2^8E^2\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]\Bigg\{&\frac{1}{M_V^2}\{\xi_S[M_RM_V-(P_R\cdot P_V)]E_V+\zeta_SP^2(E_R-E_V)\}^2(1-\cos^2\vartheta)\\
&+\{\xi_S^2[M_VM_R-(P_R\cdot P_V)]^2+\eta_S^2P^2(E_V-E_R)^2\}(1+\cos^2\vartheta)\Bigg\}\ .
\end{aligned} \tag{32}$$

The asymmetry factor is given by

$$\alpha_S(s)=\frac{A_S^2-B_S^2}{A_S^2+B_S^2}\ , \tag{33}$$

where

$$\begin{aligned}
A_S^2&=M_V^2\{\xi_S^2[M_VM_R-(P_R\cdot P_V)]^2+\eta_S^2P^2(E_V-E_R)^2\}\ ,\\
B_S^2&=\{\xi_S[M_RM_V-(P_R\cdot P_V)]E_V+\zeta_SP^2(E_R-E_V)\}^2\ .
\end{aligned} \tag{34}$$

The ratio of the contribution from the $\lambda=0$ helicity states to the overall contribution is given by

$$\mathcal{R}^{S}_{\lambda_V=0}=\frac{3}{4}\left[\frac{B_S^2}{2A_S^2+B_S^2}\right](1-\cos^2\vartheta) \tag{35}$$

and the ratio of the contribution from the CP-odd channels is given by

$$\mathcal{R}^{S}_{\mathrm{odd}}=\frac{3}{4}\left[\frac{1}{2A_S^2+B_S^2}\right]\{\xi_S^2[M_VM_R-(P_R\cdot P_V)]^2E_V^2\cos^2\vartheta+\eta_S^2M_V^2P^2(E_V-E_R)^2\}\ . \tag{36}$$

We plot $\mathcal{R}^{S}_{\lambda_V=0},\mathcal{R}^{S}_{\mathrm{odd}},\alpha_S$ in Figs. 3–5 for $B^*(5325)\to K(1430)l^+l^-$ and in Figs. 6–8 for $\Upsilon\to B_{0^+}l^+l^-$ assuming that there is a scalar B_q resonance around 5400 MeV.

[6]See, for example, [8].

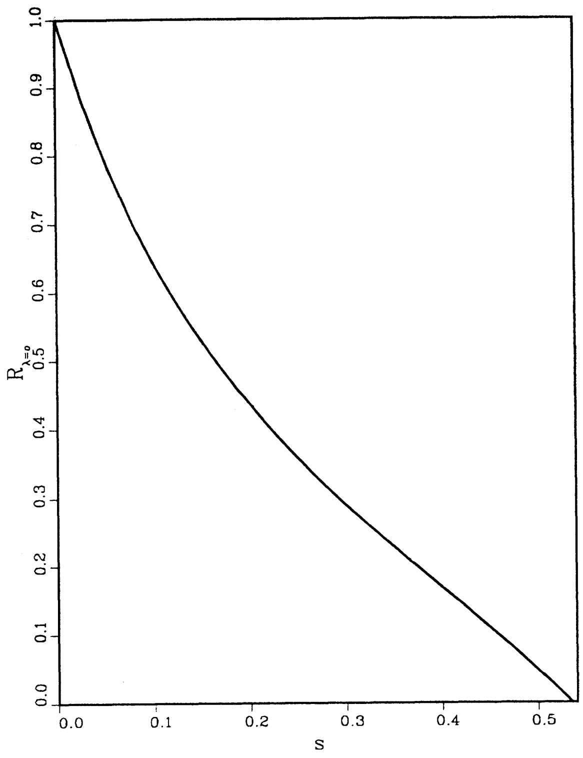

FIG. 3. Plot of $\mathcal{R}^{S}_{\lambda_V=0}$ for $B^*(5325)\to K^*(1430)l^+l^-$.

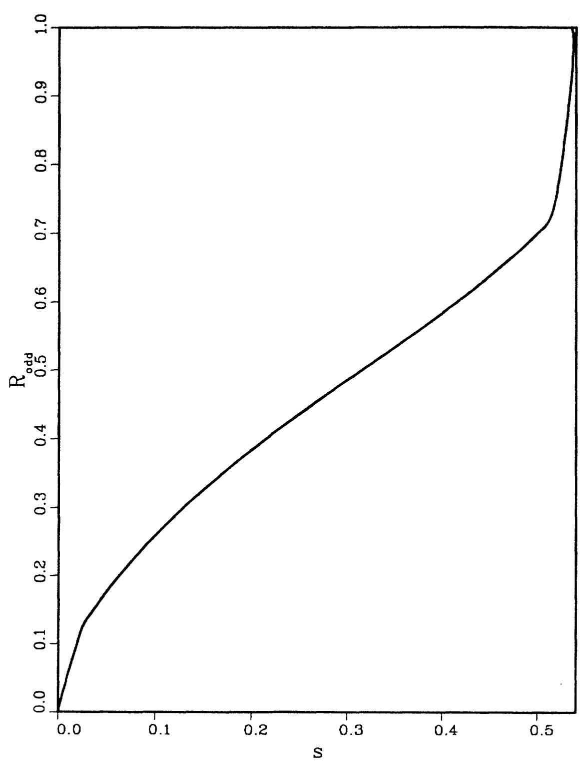

FIG. 4. Plot of $\mathcal{R}^{S}_{\rm odd}$ for $B^*(5325)\to K^*(1430)l^+l^-$.

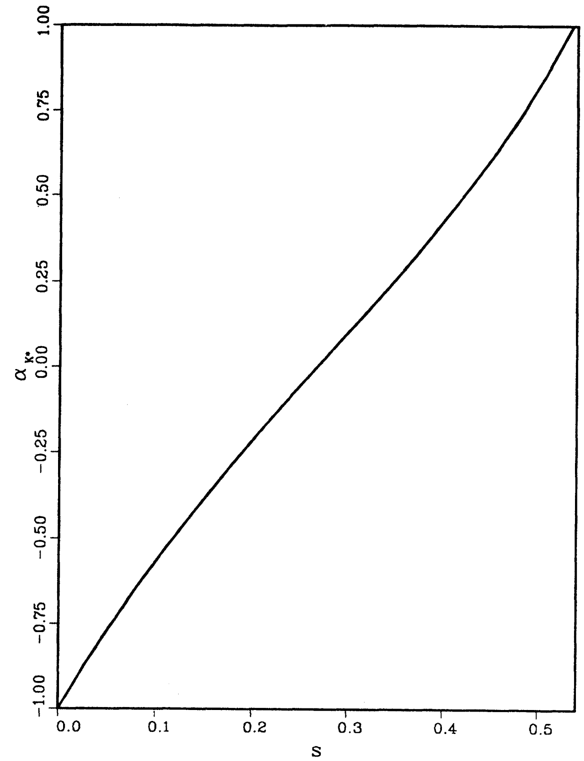

FIG. 5. Plot of α_S for $B^*(5325)\to K^*(1430)l^+l^-$.

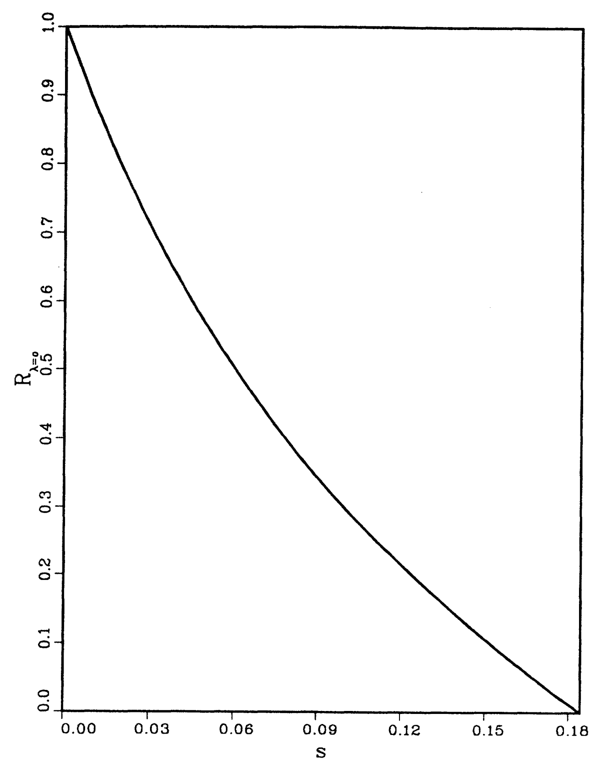

FIG. 6. Plot of $\mathcal{R}^{S}_{\lambda=0}$ for $\Upsilon\to B_{0^+}l^+l^-$.

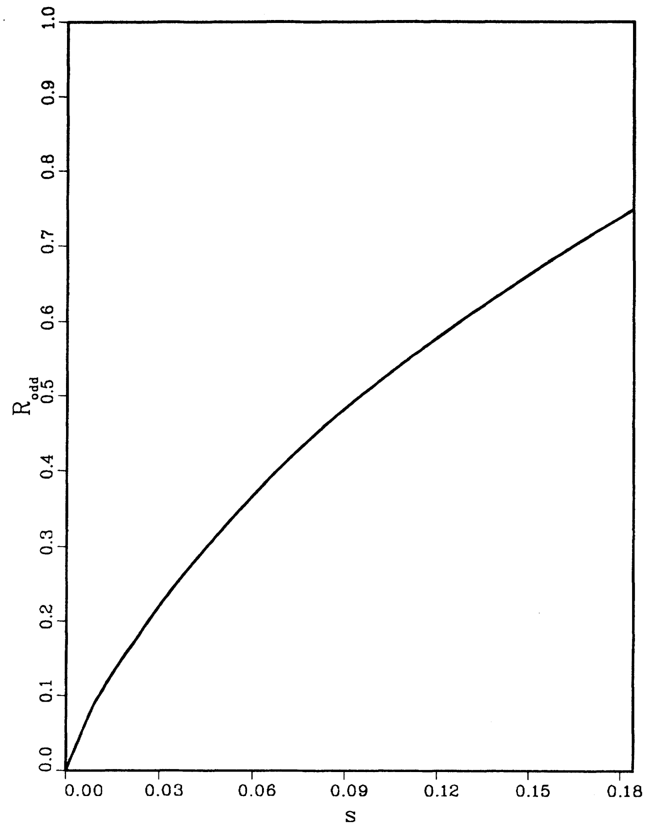

FIG. 7. Plot of $\mathcal{R}^S_{\text{odd}}$ for $\Upsilon \to B_{0^+} l^+ l^-$.

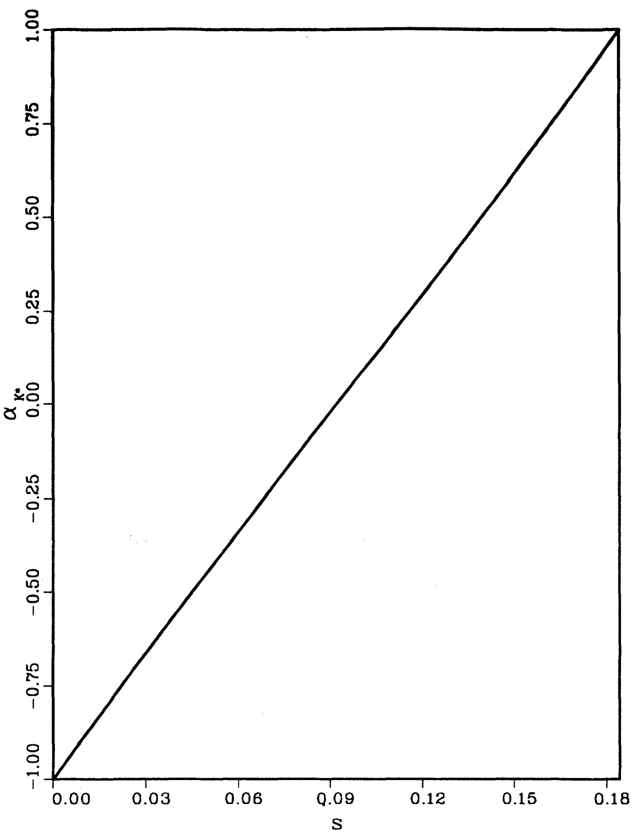

FIG. 8. Plot of α_S for $\Upsilon \to B_{0^+} l^+ l^-$.

B. $V \to R_{0^-} l^+ l^-$

In this case R is a pseudoscalar and $\Gamma_R = \gamma_5$; thus,

$$\mathcal{R}^\mu_P \equiv \text{Tr}[(-\not{P}_R + M_R)\gamma^\mu(1-\gamma_5)(\not{P}_V + M_V)\not{\epsilon}_V] ,$$
$$s\mathcal{R}^\mu_P = 4\{\xi_P[M_V M_R + (P_R \cdot P_V)]\epsilon^\mu_V - \zeta_P(P_R \cdot \epsilon_V)P^\mu_V\} + 4i\eta_P \varepsilon^{\alpha\beta\sigma\mu} P_{R\alpha} P_{V\beta} \epsilon_{v\sigma} . \tag{37}$$

It is sufficient to substitute $M_V M_R \to -M_V M_R$ in the results for the scalar resonance. Hence

$$|\mathcal{M}_P|^2 = 2^8 E^2 \left[|\mathcal{V}|^2 + \frac{p^2}{E^2}|\mathcal{A}|^2 \right] \left\{ \frac{1}{M_V^2}\{\xi_P[M_R M_V + (P_R \cdot P_V)]E_V + \zeta_P P^2(E_R - E_V)\}^2(1-\cos^2\vartheta) \right.$$
$$\left. \times \{\xi_P^2[M_V M_R + (P_R \cdot P_V)]^2 - \eta_P^2 P^2(E_V - E_R)^2\}(1+\cos^2\vartheta) \right\} \tag{38}$$

and the asymmetry factor is

$$\alpha_P(s) = \frac{A_P^2 - B_P^2}{A_P^2 + B_P^2} , \tag{39}$$

where

$$A_P^2 = M_V^2\{\xi_P^2[M_V M_R + (P_R \cdot P_V)]^2 + \eta_P^2 P^2(E_V - E_R)^2\} ,$$
$$B_P^2 = \{\xi_P[M_R M_V + (P_R \cdot P_V)]E_V + \zeta_P P^2(E_V - E_R)\}^2 . \tag{40}$$

The ratio of the contribution from the $\lambda = 0$ helicity states to the overall contribution is given by

$$\mathcal{R}^{S}_{\lambda_V=0}=\frac{3}{4}\left\{\frac{B_P^2}{2A_P^2+B_P^2}\right\}(1-\cos^2\vartheta) \tag{41}$$

and the ratio of the contribution from the *CP*-odd channels is given by

$$\mathcal{R}^{S}_{\text{odd}}=\frac{3}{4}\left\{\frac{1}{2A_S^2+B_S^2}\right\}\{\xi_s^2[M_VM_R+(P_R\cdot P_V)]^2\cos^2\vartheta+\eta_s^2P^2(E_V-E_R)^2\} . \tag{42}$$

As should be, these results are equivalent to the results in [9]. We plot $\mathcal{R}^P_{\lambda_V=0},\mathcal{R}^P_{\text{odd}},\alpha_P$ in Figs. 9–11 for $B^*(5325)\to K(494)l^+l^-$ and in Figs. 12–14 for $\Upsilon\to B_{0^-}l^+l^-$.

C. $V\to R_{1^-}l^+l^-$

In this case R is a vector and $\Gamma_R=\not\epsilon_R$; thus,

$$\begin{aligned}&\mathcal{R}^{\mu}_V\equiv\mathrm{Tr}[\not\epsilon^*_R(\not P_R+M_R)\gamma^\mu(1-\gamma_5)(\not P_V+M_V)\not\epsilon_V] ,\\ &\mathcal{R}^{\mu}_V=4\{\xi_V[(P_R\cdot\epsilon_V)M_V\epsilon^{*\mu}_R+(\epsilon^*_R\cdot P_V)M_R\epsilon^{\mu}_V]-\zeta_V(\epsilon^*_R\cdot\epsilon_V)(M_VP^\mu_R+M_RP^\mu_V)\\ &\qquad -i\eta_V\varepsilon^{\alpha\beta\sigma\mu}\epsilon^*_{R\alpha}(M_VP_{R\beta}+M_RP_{V\beta}\epsilon_{V\sigma})\} .\end{aligned} \tag{43}$$

After some algebra we obtain

$$\begin{aligned}|\mathcal{M}|^2={}&8\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]\Bigg[\Bigg\{2[\zeta_V^2(M_V+M_R)^2P^2+\eta_V^2(M_VE_R+M_RE_V)^2]\\ &+\left[\xi_V(E_V-E_R)\left[\frac{E_V}{M_V}-\frac{E_R}{M_R}\right]+\zeta_V(E_RE_V-P^2)\left[\frac{1}{M_V}+\frac{1}{M_R}\right]\right]^2P^2\Bigg\}(1-\cos^2\vartheta)\\ &+\left\{\xi_V^2(E_V-E_R)^2P^2+\frac{\eta_V^2}{M_V^2}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]^2\right\}(1+\cos^2\vartheta)\\ &+\left\{\xi_V^2(E_V-E_R)^2P^2+\frac{\eta_V^2}{M_R^2}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]^2\right\}(1+\cos^2\vartheta)\Bigg]\\ &+2^9\frac{p}{E}\mathrm{Re}(\mathcal{V}^\dagger\mathcal{A})\xi_V\eta_V(E_V-E_R)\Bigg\{\frac{P}{M_R}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]\\ &-\frac{P}{M_V}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]\Bigg\}\cos\vartheta .\end{aligned} \tag{44}$$

The asymmetry factor is given by

$$\alpha_V(s)=\frac{A_V^2-B_V^2}{A_V^2+B_V^2} , \tag{45}$$

where

$$A_V^2=2\xi_V^2sP^2+\frac{\eta_V^2}{M_V^2}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]^2+\frac{\eta_V^2}{M_R^2}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]^2 ,$$

$$B_V^2=-2[\zeta_V^2(M_V+M_R)^2P^2+\eta_V^2(M_VE_R+M_RE_V)^2]-\left[\xi_V\sqrt{s}\left[\frac{E_V}{M_V}-\frac{E_R}{M_R}\right]+\zeta_V(E_RE_V-P^2)\left[\frac{1}{M_V}+\frac{1}{M_R}\right]\right]^2P^2 .$$

The ratio of the contribution from the $\lambda_V,\lambda_R=0$ channel is given by

$$\mathcal{R}^{V}_{00}=\frac{3}{4}\left\{\frac{1}{2A_V^2+B_V^2}\right\}\left\{\xi_V(E_V-E_R)\left[\frac{E_V}{M_R}-\frac{E_R}{M_V}\right]+\zeta_V(P^2-E_RE_V)\left[\frac{1}{M_V}+\frac{1}{M_R}\right]\right\}^2P^2(1-\cos^2\vartheta) \tag{46}$$

and the ratio of the contribution from the *CP*-odd channel is given by

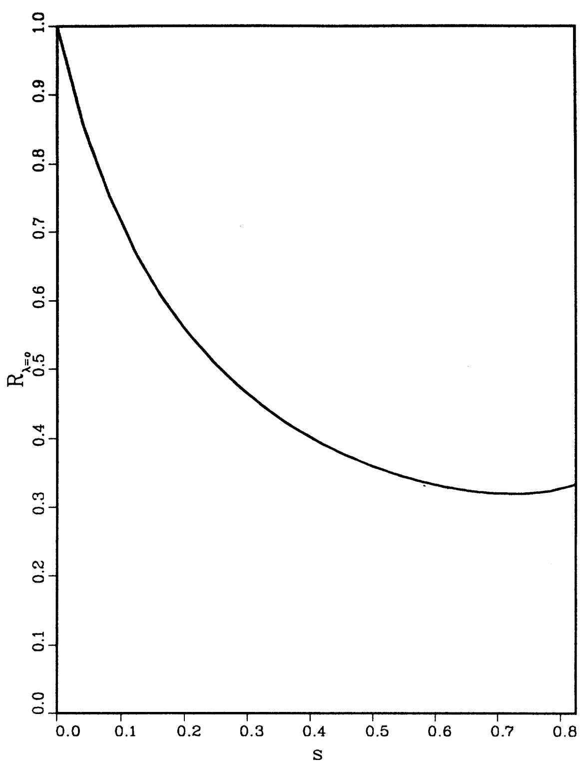

FIG. 9. Plot of $\mathcal{R}^P_{\lambda_V=0}$ for $B^*(5325)\to K(494)l^+l^-$.

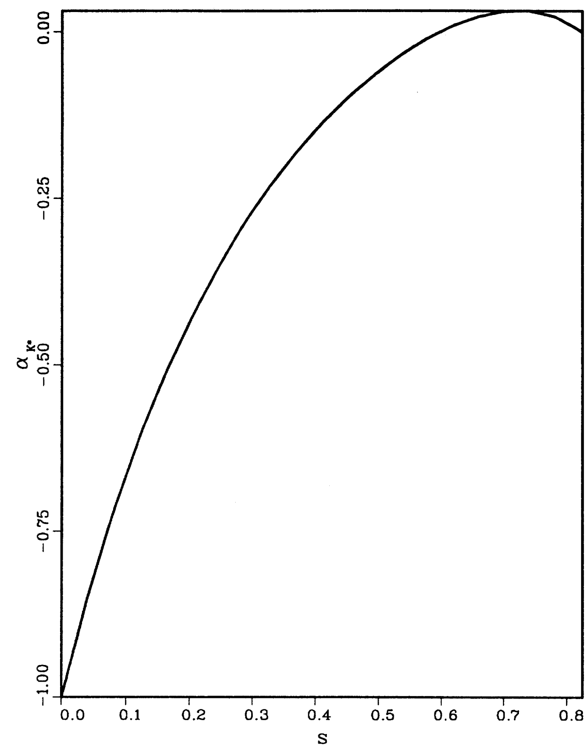

FIG. 11. Plot of α_P for $B^*(5325)\to K(494)l^+l^-$.

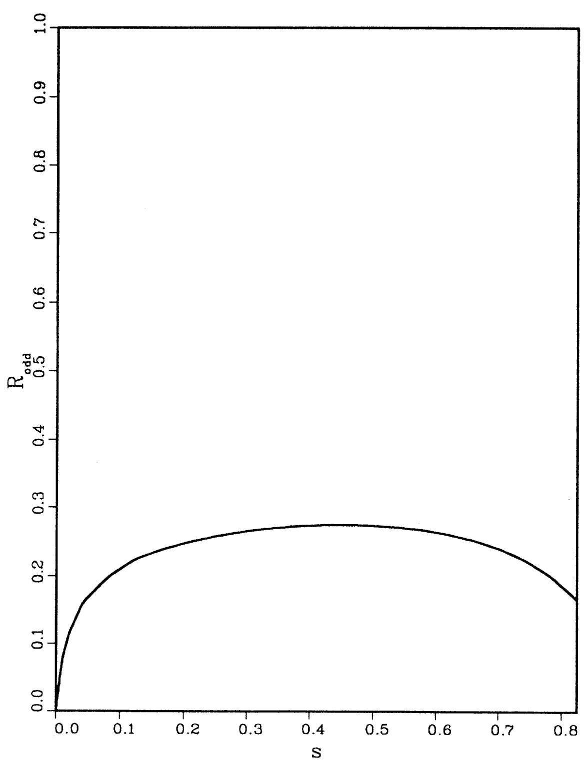

FIG. 10. Plot of $\mathcal{R}^P_{\text{odd}}$ for $B^*(5325)\to K(494)l^+l^-$.

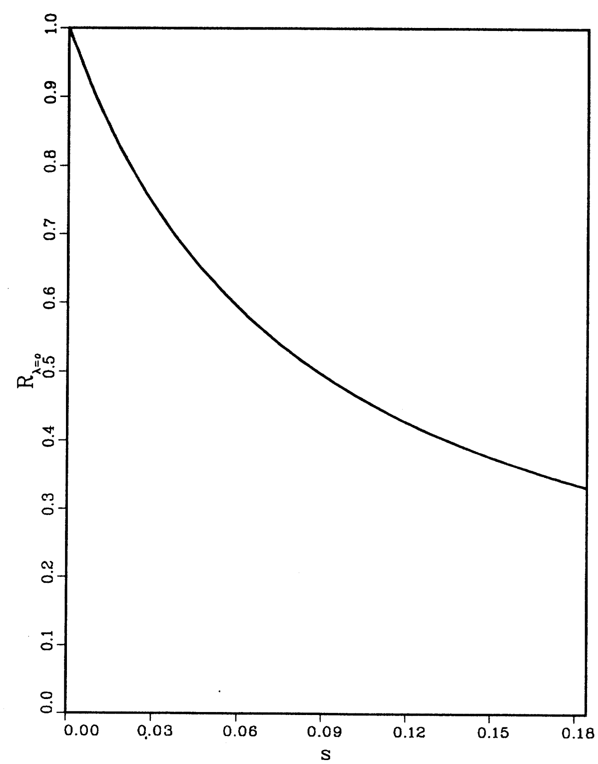

FIG. 12. Plot of $\mathcal{R}^P_{\lambda=0}$ for $\Upsilon\to B_{0^-}l^+l^-$.

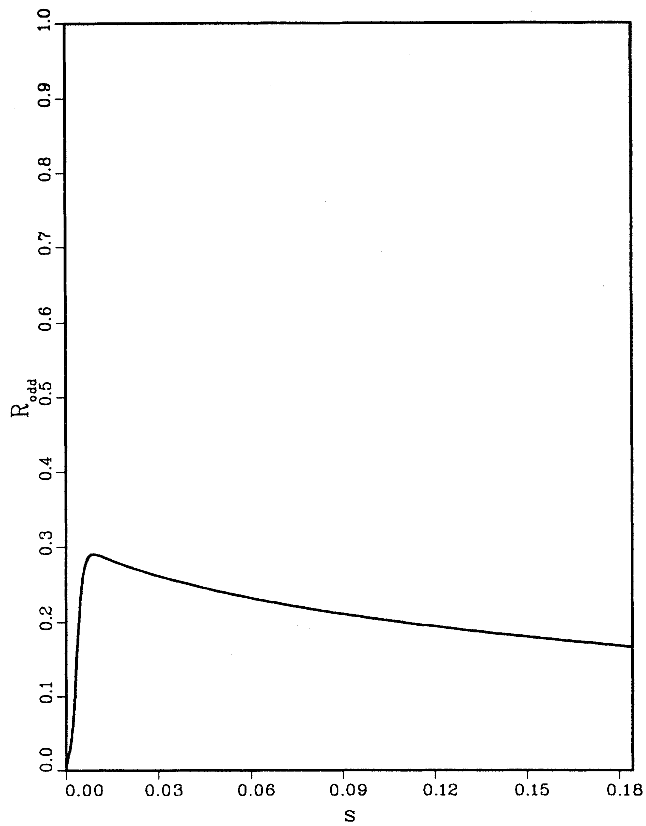

FIG. 13. Plot of $\mathcal{R}^{P}_{\rm odd}$ for $\Upsilon \to B_{0^-} l^+ l^-$.

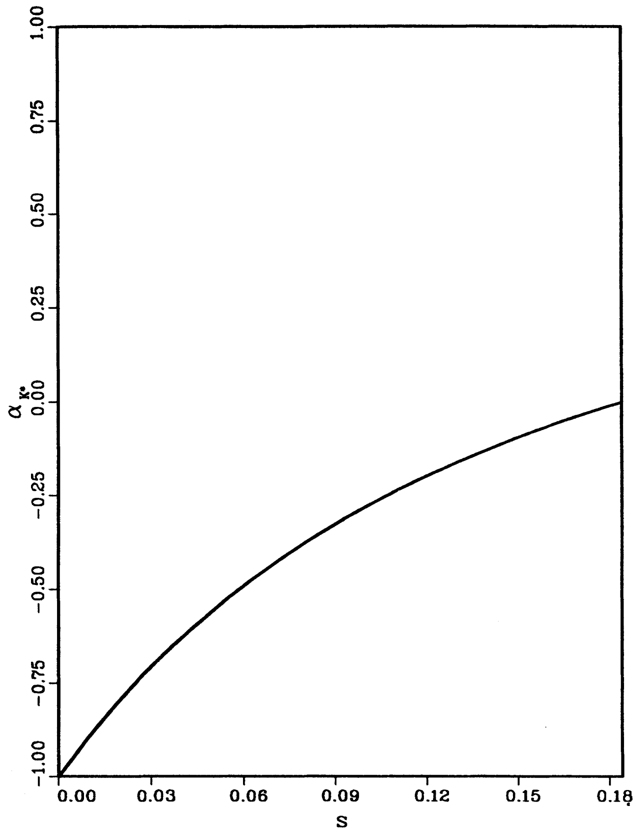

FIG. 14. Plot of α_P for $\Upsilon \to B_{0^-} l^+ l^-$.

$$\mathcal{R}_{\rm odd} = \frac{3}{4}\left\{\frac{1}{2A_V^2+B_V^2}\right\}\left\{2\eta_V^2(M_V E_R + M_R E_V)^2 + \frac{\eta_v^2}{M_V^2}[(M_V+M_R)P^2 - E_V(M_V E_R + M_R E_V)]^2 + \frac{\eta_V^2}{M_R^2}[(M_V+M_R)P^2 - E_R(M_V E_R + M_R E_V)]^2 + [2\xi_V^2(E_V-E_R)^2P^2 - \eta_V^2(M_V E_R + M_R E_V)^2]\cos^2\vartheta\right\} . \tag{47}$$

We plot $\mathcal{R}^V_{\lambda=0}, \mathcal{R}^V_{\rm odd}, \alpha_V$ in Figs. 15–17 for $B^*(5325) \to K^*(892) l^+ l^-$ and in Figs. 18–20 for $\Upsilon \to B_{1^-} l^+ l^-$.

IV. $V \to R_1 R_2$

Now we focus our attention on $V \to R_1 R_2$, where R_1 is a bound state of the $\bar{b}q$ pair and R_2 is a bound state of the $q_1\bar{q}_2$ pair. This process takes place via the decay $b \to q_1 \bar{q}_2 q$ as shown in Fig. 21. The matrix element for the b-quark decay via the exchange of a W boson is given by

$$\mathcal{M}_{b\to q_1\bar{q}_2 q} = \left[\frac{ig}{2\sqrt{2}}\right]^2 \frac{V^\dagger_{qq_2} V_{q_1 b}}{k^2 - M_W^2}[\bar{q}_2\gamma^\mu(1-\gamma_5)b][\bar{q}\gamma_\mu(1-\gamma_5)q_1] . \tag{48}$$

Using the Fierz identity

$$[\bar{q}_2\gamma^\mu(1-\gamma_5)b][\bar{q}\gamma_\mu(1-\gamma_5)q_1] = \tfrac{1}{3}[\bar{q}_2\gamma^\mu(1-\gamma_5)q_1][\bar{q}\gamma_\mu(1-\gamma_5)b] + 2\sum_{a=1}^{8}[\bar{q}_2\gamma^\mu(1-\gamma_5)t^a q_1][\bar{q}\gamma_\mu(1-\gamma_5)t^a b] \tag{49}$$

and dropping the color-octet piece (since we are interested in the color-singlet states only) gives

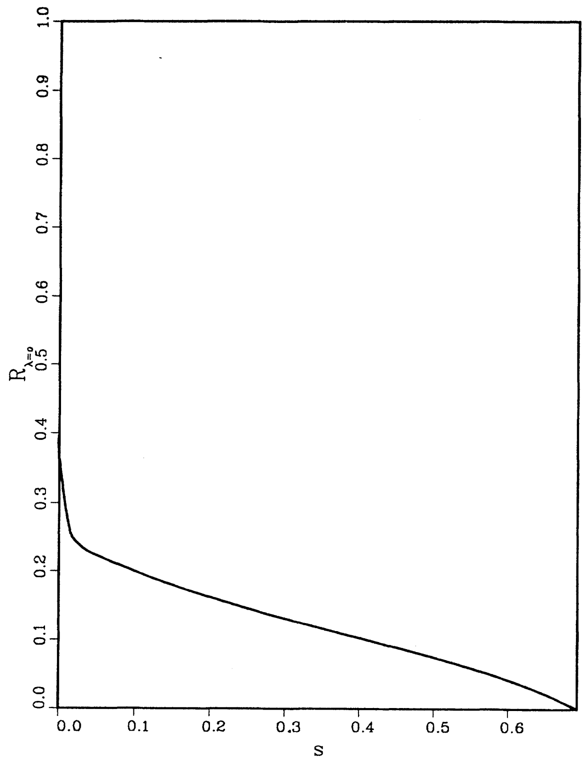

FIG. 15. Plot of $\mathcal{R}^V_{\lambda=0}$ for $B^*(5325)\to K^*(892)l^+l^-$.

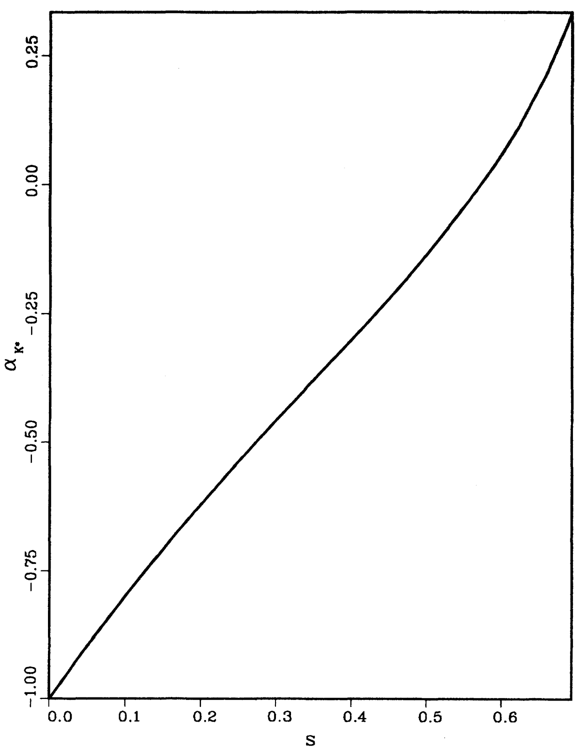

FIG. 17. Plot of α_V for $B^*(5325)\to K^*(892)l^+l^-$.

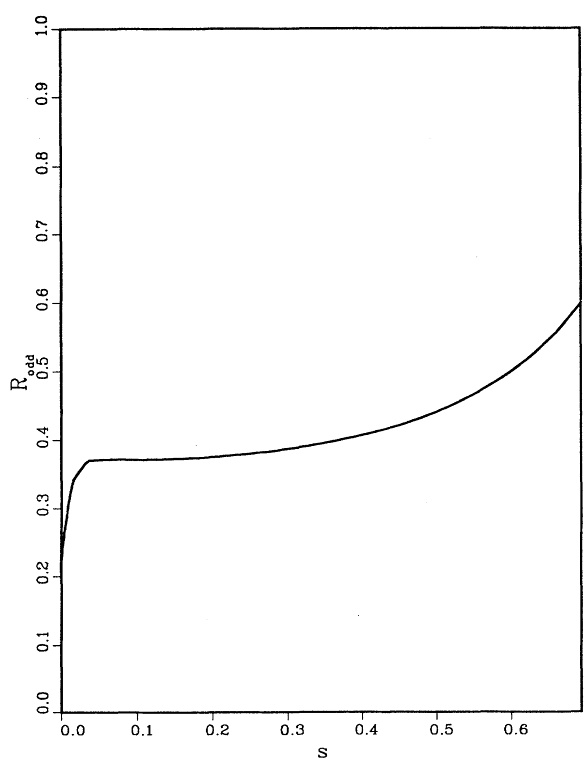

FIG. 16. Plot of $\mathcal{R}^V_{odd}$ for $B^*(5325)\to K^*(892)l^+l^-$.

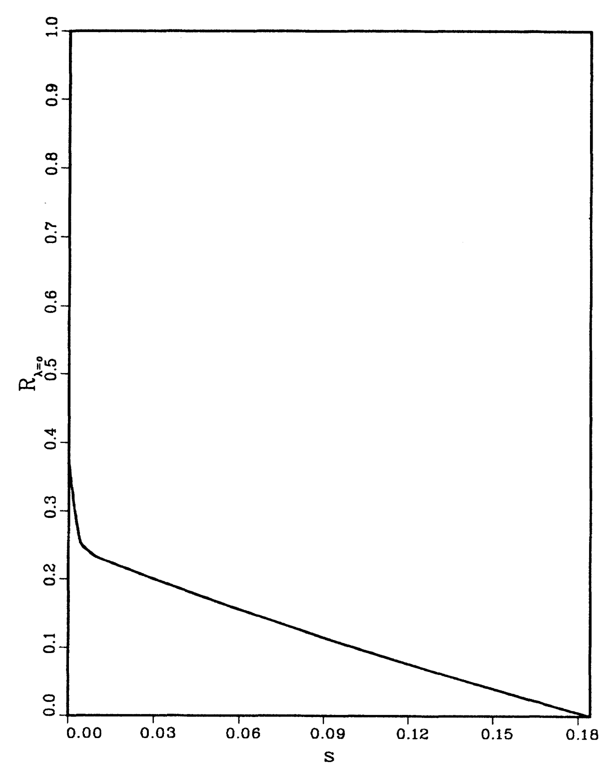

FIG. 18. Plot of $\mathcal{R}^V_{\lambda=0}$ for $\Upsilon\to B_{1^-}l^+l^-$.

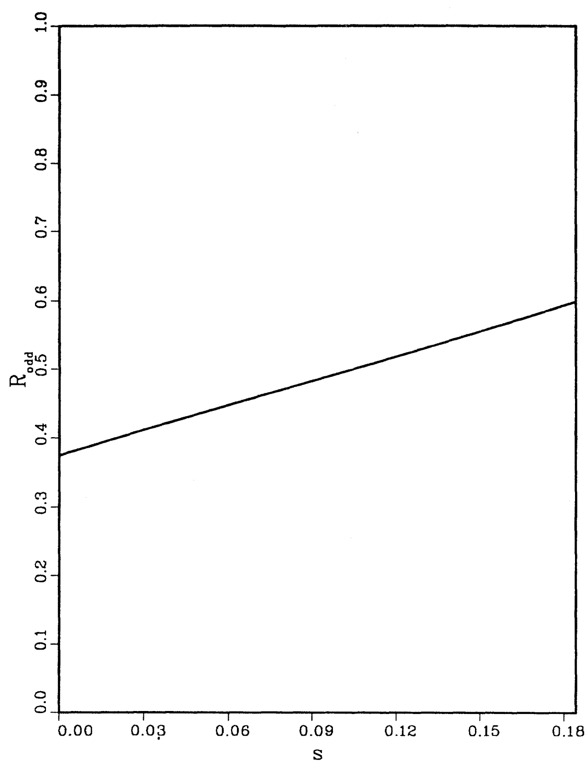

FIG. 19. Plot of $\mathcal{R}^V_{\rm odd}$ for $\Upsilon\to B_{1^-}l^+l^-$.

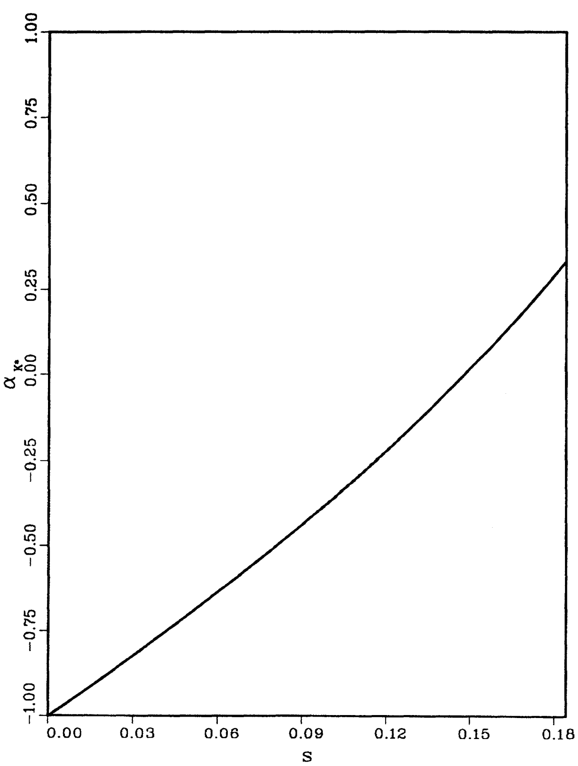

FIG. 20. Plot of α_V for $\Upsilon\to B_{1^-}l^+l^-$.

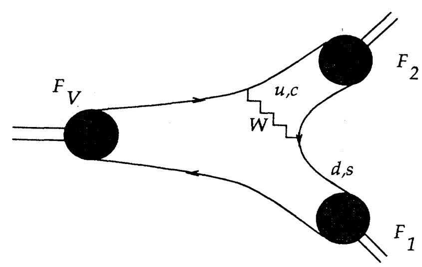

FIG. 21. Decay diagram for $V\to R_1R_2$.

$$\mathcal{M}_{b\to q_1\bar{q}_2 q}=\left[\frac{ig}{2\sqrt{2}}\right]^2\frac{V^\dagger_{qq_2}V_{q_1b}}{k^2-M_W^2}\frac{1}{3}[\bar{q}_2\gamma^\mu(1-\gamma_5)q_1]\times[\bar{q}\gamma_\mu(1-\gamma_5)b]\ . \qquad (50)$$

This is similar to the leptonic decays of mesons. Thus in the following calculations we treat these decays as leptonic decays and assume that the factorization approximation is valid.[7] Now the matrix element for the decay $V\to R_1R_2$ is given by

$$\mathcal{M}_{V\to R_1R_2}=\{\ \}_{1,2}\mathrm{Tr}[\Gamma_2(\not{P}_2+M_2)\gamma^\mu(1-\gamma_5)]\times\mathrm{Tr}[\Gamma_1(\not{P}_1+M_R)\gamma_\mu(1-\gamma_5)(\not{P}_V+M_V)\not{\epsilon}_V]\ , \qquad (51)$$

where

$$\{\ \}_{1,2}\equiv\frac{1}{3}\left[\frac{ig}{2\sqrt{2}}\right]^2\frac{V^\dagger_{qq_2}V_{q_1b}}{k^2-M_W^2}\left[\frac{\mathcal{F}_V\mathcal{F}_1\mathcal{F}_2}{8M_VM_1M_2}\right]\ . \qquad (52)$$

A. $V\to R_1R_{0^+}$ or $V\to R_1R_{0^-}$

We have $\Gamma_2=1,\gamma_5$ and

$$\mathrm{Tr}[(\not{P}_2+M_2)\gamma^\mu(v_2-a_2\gamma_5)]=4v_2P_2^\mu \qquad (53)$$

for the scalar and

$$\mathrm{Tr}[(\not{P}_2+M_2)\gamma^\mu(v_2-a_2\gamma_5)]=4a_2P_2^\mu \qquad (54)$$

for the pseudoscalar. Hence

$$\mathcal{M}_{V\to R_1R_2}=\{\ \}_{1,S}\mathrm{Tr}[\Gamma_1(\not{P}_1+M_1)\not{P}_2(1-\gamma_5)(\not{P}_V+M_V)\not{\epsilon}_V]\ . \qquad (55)$$

[7]For a discussion of this approach, see Ref. [2].

(i) $R_1=R_{0^+}$: $\Gamma_1=1$ and only the $\lambda_V=0$ term contributes:

$$\mathcal{M}_{V\to R_1R_2}=16\{\ \}_{S_1,R_2}\{\xi(M_1M_V-P_1\cdot P_V)(P_2\cdot\epsilon_V)+\zeta[(P_1\cdot P_2)(P_V\cdot\epsilon_V)+(P_2\cdot P_V)(P_1\cdot\epsilon_V)]-i\eta\varepsilon^{\alpha\beta\mu\nu}P_{1\alpha}P_{2\beta}P_{V\mu}\epsilon_{V\nu}\}$$
$$=16\{\ \}_{S_1,R_2}\frac{P}{M_V}\{\xi[M_1M_V-(P_1\cdot P_V)]M_2+\zeta M_2E_V(E_{1-}E_V)\}\ . \tag{56}$$

(ii) $R_1=R_{0^-}$: $\Gamma_1=\gamma_5$ and it is sufficient to substitute $M_1\to -M_1$ in the equation above:

$$\mathcal{M}_{V\to R_1R_2}=-16\{\ \}_{P_1,R_2}\frac{P}{M_V}\{\xi(M_1M_V+P_1\cdot P_V)M_2+\zeta M_2E_V(E_V-E_1)\}\ . \tag{57}$$

(iii) $R_1=R_{1^-}$: $\Gamma_1=\not\epsilon_1$ and

$$\mathcal{M}_{V\to R_1R_2}=16\{\ \}_{V_1,R_2}\{M_V[-\xi(\epsilon_1^*\cdot P_2)(P_1\cdot\epsilon_V)+\zeta(\epsilon_1^*\cdot\epsilon_V)(P_1\cdot P_2)+i\eta\varepsilon^{\alpha\beta\mu\nu}\epsilon_{1\alpha}^*P_{1\beta}P_{2\mu}\epsilon_{V\nu}]$$
$$+M_1[-\xi(\epsilon_1^*\cdot P_V)(P_2\cdot\epsilon_V)+\zeta(\epsilon_1^*\cdot\epsilon_V)(P_2\cdot P_V)-i\eta\varepsilon^{\alpha\beta\mu\nu}\epsilon_{1\alpha}^*P_{2\beta}P_{V\mu}\epsilon_{V\nu}]\}$$
$$=16\{\ \}_{V_1,R_2}M_2\left\{\left[\xi(E_V-E_1)P^2\left[\frac{1}{M_1}-\frac{1}{M_V}\right]+\zeta(P^2-E_VE_1)\left[\frac{E_1}{M_1}+\frac{E_V}{M_V}\right]\right]\delta_{\lambda_V=0}\delta_{\lambda_1=0}\right.$$
$$\left.\pm\eta[(M_VE_1+M_1E_V)+(M_V+M_1)P]\delta_{\lambda_V=\pm}\delta_{\lambda_1=\pm}\right\}, \tag{58}$$

$$\mathcal{R}_{\lambda_1=0}^{V_1,R_2}=M_{(V_1,R_2)}^{-1}\left\{\left[\xi(E_V-E_1)P^2\left[\frac{1}{M_1}-\frac{1}{M_V}\right]+\zeta(P^2-E_VE_1)\left[\frac{E_1}{M_1}+\frac{E_V}{M_V}\right]\right]^2\right\},$$

$$\mathcal{R}_{\rm odd}^{V_1,R_2}=M_{(V_1,R_2)}^{-1}\{\eta^2[(M_VE_1+M_1E_V)+(M_V+M_1)P]^2\}\ , \tag{60}$$

$$M_{(V_1,R_2)}\equiv\left[\xi(E_V-E_1)P^2\left[\frac{1}{M_1}-\frac{1}{M_V}\right]+\zeta(P^2-E_VE_1)\left[\frac{E_1}{M_1}+\frac{E_V}{M_V}\right]\right]^2+\eta^2[(M_VE_1+M_1E_V)+(M_V+M_1)P]^2\ .$$

B. $V\to R_1R_{1^-}$

We have $\Gamma_2=\not\epsilon_2$ and

$$\mathrm{Tr}[\not\epsilon_2(\not P_2+M_2)\gamma^\mu(1-\gamma_5)]=4M_2\epsilon_2^\mu\ . \tag{60}$$

(i) $R_1=R_{0^+}$: $\Gamma_1=1$ and

$$\mathcal{M}_{V\to R_1R_2}=16\{\ \}_{S_1,V_2}\{\xi(M_1M_V-P_1\cdot P_V)(\epsilon_2^*\cdot\epsilon_V)+\zeta[(P_1\cdot\epsilon_2^*)(P_V\cdot\epsilon_V)+(\epsilon_2^*\cdot P_V)(P_1\cdot\epsilon_V)]-i\eta\varepsilon^{\alpha\beta\mu\nu}P_{1\alpha}\epsilon_{2*\beta}P_{V\mu}\epsilon_{V\nu}\}$$
$$=16\{\ \}_{S_1,V_2}\left\{[-\xi(M_1M_V-P_1\cdot P_V)\mp\eta(E_V-E_1)P]\delta_{\lambda_V=\pm}\delta_{\lambda_2=\pm}\right.$$
$$\left.+\frac{1}{M_V}[\xi(M_1M_V-P_1\cdot P_V)E_V+\zeta(E_V-E_1)P^2]\delta_{\lambda_V=0}\delta_{\lambda_2=0}\right\}, \tag{61}$$

$$\mathcal{R}_{\lambda_2=0}^{S_1,V_2}=M_{(S_1,V_2)}^{-1}\left\{\frac{1}{M_V^2}[\xi(M_1M_V-P_1\cdot P_V)E_V+\zeta(E_V-E_1)P^2]^2\right\},$$

$$\mathcal{R}_{\rm odd}^{S_1,V_2}=M_{(S_1,V_2)}^{-1}\{\eta^2(E_V-E_1)^2P^2]\}\ , \tag{62}$$

$$M_{(S_1,V_2)}\equiv\left\{[\xi^2(M_1M_V-P_1\cdot P_V)^2+\eta^2(E_V-E_1)^2P^2]+\frac{1}{M_V^2}[\xi(M_1M_V-P_1\cdot P_V)E_V+\zeta(E_V-E_1)P^2]^2\right\}.$$

(ii) $R_1=R_{0^-}$: $\Gamma_1=\gamma_5$ and it is sufficient to substitute $M_1\to -M_1$ in the equation above:

$$\mathcal{M}_{V\to R_1R_2}=16\{\ \}_{P_1,V_2}M_2\left\{[\xi(M_1M_V+P_1\cdot P_V)\mp\eta(E_V-E_1)P^2]\delta_{\lambda_V=\pm}\delta_{\lambda_2=\pm}\right.$$
$$\left.+\frac{P}{M_V}[-\xi(M_1M_V+P_1\cdot P_V)E_V+\zeta(E_V-E_1)P^2]\delta_{\lambda_V=0}\delta_{\lambda_2=0}\right\}, \tag{63}$$

$$\mathcal{R}^{P_1,V_2}_{\lambda_2=0}=M^{-1}_{(P_1,V_2)}\left\{\frac{1}{M_V^2}[\xi(M_1M_V+P_1\cdot P_V)E_V+\zeta(E_V-E_1)P^2]^2\right\},$$
$$\mathcal{R}^{P_1,V_2}_{\rm odd}=M^{-1}_{(P_1,V_2)}\{\eta^2(E_V-E_1)^2P^2]\},\qquad(64)$$
$$M_{(P_1,V_2)}\equiv[\xi^2(M_1M_V+P_1\cdot P_V)^2+\eta^2(E_V-E_1)^2P^2]+\frac{1}{M_V^2}[\xi(M_1M_V+P_1\cdot P_V)E_V+\zeta(E_V-E_1)P^2]^2 .$$

(iii) $R_1=R_{1^-}$: $\Gamma_1=\not\epsilon_1$ and

$$\mathcal{M}_{V\to R_1R_2}=16\{\ \}_{V_1,V_2}M_2\{M_V[-\xi(\epsilon_1^*\cdot\epsilon_2^*)(P_1\cdot\epsilon_V)+\zeta(\epsilon_1^*\cdot\epsilon_V)(P_1\cdot\epsilon_2^*)+i\eta\varepsilon^{\alpha\beta\mu\nu}\epsilon_{1*\alpha}P_{1\beta}\epsilon^*_{2\mu}\epsilon_{V\nu}]$$
$$+M_1[-\xi(\epsilon_1^*\cdot P_V)(\epsilon_2^*\cdot\epsilon_V)+\zeta(\epsilon_1^*\cdot\epsilon_V)(\epsilon_2^*\cdot P_V)-i\eta\varepsilon^{\alpha\beta\mu\nu}\epsilon_{1\alpha}\epsilon^*_{2\beta}P_{V\mu}\epsilon_{V\nu}]\}$$
$$=16\{\ \}_{V_1,V_2}M_2\left[M_V\left\{P\left[(\zeta-\xi)(E_VE_1-P^2)\left[\frac{1}{M_V}+\frac{1}{M_1}\right]+\xi(M_V+M_1)\right]\delta_{\lambda_V=0}\delta_{\lambda_1=0}\delta_{\lambda_2=0}\right.\right.$$
$$+[\xi P(E_V-E_1)\pm\eta(E_VE_1-P^2+M_VM_1)]\delta_{\lambda_V=0}\delta_{\lambda_1=\pm}\delta_{\lambda_2=\mp}$$
$$+[\xi P(E_V-E_1)\mp\eta(E_VE_1-P^2+M_VM_1)]\delta_{\lambda_V=\pm}\delta_{\lambda_1=0}\delta_{\lambda_2=\pm}$$
$$\left.\left.+[\xi P(M_V+M_1)\pm\eta(E_VM_1+M_VE_1)]\delta_{\lambda_V=\pm}\delta_{\lambda_1=\pm}\delta_{\lambda_2=0}\right\}\right],\qquad(65)$$

$$\mathcal{R}^{V_1,V_2}_{\lambda_V,\lambda_1,\lambda_2=0}=M^{-1}_{(V_1,V_2)}\left\{P^2[(\zeta-\xi)(E_VE_1-P^2)\left[\frac{1}{M_V}+\frac{1}{M_1}\right]+\xi(M_V+M_1)]^2\right\},$$
$$\mathcal{R}^{V_1,V_2}_{\rm odd}=M^{-1}_{(V_1,V_2)}\{\eta^2[2(E_VE_1-P^2+M_VM_1)^2+(E_VM_1+M_VE_1)^2]\},$$
$$M_{(V_1,V_2)}\equiv P^2\left[(\zeta-\xi)(E_VE_1-P^2)\left[\frac{1}{M_V}+\frac{1}{M_1}\right]+\xi(M_V+M_1)\right]^2+\xi^2[2P^2(E_V-E_1)^2+P^2(M_V+M_1)^2]$$
$$+\eta^2[2(E_VE_1-P^2+M_VM_1)^2+(E_VM_1+M_VE_1)^2] .\qquad(66)$$

In Tables I–IV we apply our results to rare decays of B^* and Υ.

V. CONCLUSION

In this paper we calculated the matrix elements and the differential decay rates for rare decays of vector mesons, $V\to Rl^+l^-$ and $V\to R_1R_2$, when R's are scalars, pseudoscalars, and vectors. We extracted the ratios of the contributions from the zero helicity states and the ratios the contributions from the CP-odd and CP-even channels. We also obtained the asymmetry factor $\alpha_R(s)$ for each case. Our calculations show that for $V\to R_{0^+}l^+l^-$, the decay is dominated by the $\lambda=0$ channel and the even CP channel for the small s values and by the odd CP channel for large s values. The asymmetry factor is negative for $s<(M_V-M_R)^2/2$ and positive for $s>(M_V-M_R)^2/2$. For $V\to R_{0^-}l^+l^-$, the decay is dominated by the even CP channel accounting for close to 80% of the decay rate. For $V\to R_{1^-}l^+l^-$, both the even and odd CP channels have substantial contributions. It is important to know the behavior of the asymmetry factor as a function of the invariant mass of the l^+l^- pair for experimental reasons. For example, for $B\to Xl^+l^-$, in the case of the Collider Detector at Fermilab Collaboration (CDF) the efficiency for $\alpha=+1$ is about 75% of the efficiency for $\alpha=-1$.[8] We do not expect the QCD corrections to affect $\mathcal{R}_{\rm odd}$ since QCD conserves CP, but we do expect them to effect $\mathcal{R}_{\lambda=0}$ since only the total angular momentum is conserved in QCD in general.

TABLE I. Our results applied to the rare decays of B^*.

X	$B^*\to K^*(1430)X$ R_0	$R_{\rm odd}$	$B^*\to K(494)X$ R_0	$R_{\rm odd}$
D^{0*}_{2010}	0.003	0.997	0.048	0.952
J/ψ	0.018	0.982	0.037	0.9623

TABLE II. Same as Table I for $B^*\to K^*(892)X$.

X	$B^*\to K^*(892)X$ R_0	$R_{\rm odd}$
$\eta_c(0^-)$	0.228	0.772
$D^0_u(0^-)$	0.208	0.792
$\xi_{c0}(0^+)$	0.243	0.757
$D^*_{2010}(1^-)$	0.683	0.033
$J/\psi(1^-)$	0.411	0.079

[8]Private conversation with J. Mueller.

TABLE III. Our results applied to the rare decay of Υ.

	$\Upsilon\to B_{s0^+}(\approx 5400)X$		$\Upsilon\to B_{s0^-}(\approx 5400)X$		$\Upsilon\to B_{s1^-}(\approx 5400)X$	
X	R_0	$R_{\rm odd}$	R_0	$R_{\rm odd}$	R_0	$R_{\rm odd}$
J/ψ	0.017	0.983	0.049	0.951	0.2739	0.166

APPENDIX

When calculating the matrix elements, we used the following relations for the Dirac γ matrices:

$$\begin{aligned}&\{\gamma_\nu\gamma_\beta\gamma_\mu(1-\gamma_5)\}\{\gamma^\mu\gamma^\beta\gamma^\nu(v-a\gamma_5)\}\\&\quad=\{\gamma_\mu(1-\gamma_5)\}\{10[\gamma^\mu(v-a\gamma_5)]-6[\gamma^\mu(a-v\gamma_5)]\}\ ,\\&\gamma_\nu\gamma_\beta\gamma_\mu=g_{\nu\beta}\gamma_\mu+g_{\mu\beta}\gamma_\nu-g_{\nu\mu}\gamma_\beta-i\epsilon_{\nu\beta\mu\alpha}\gamma_5\gamma^\alpha\ ,\\&-6i\gamma_5\gamma^\alpha=\epsilon^{\nu\beta\mu\alpha}\gamma_\nu\gamma_\beta\gamma_\mu\ .\end{aligned}\tag{A1}$$

1. Loop integrals

$$\begin{aligned}\mathcal{J}&=\int\frac{d^4k}{(2\pi)^4}\frac{1}{\prod_i[k_i^2-a_i^2]}\ ,\\ \mathcal{J}^\mu&=\int\frac{d^4k}{(2\pi)^4}\frac{k_i^\mu}{\prod_i[k_i^2-a_i^2]}\ ,\\ \mathcal{J}^{\mu\nu}&=\int\frac{d^4k}{(2\pi)^4}\frac{k_i^\mu k_j^\nu}{\prod_i[k_i^2-a_i^2]}\ .\end{aligned}\tag{A2}$$

In the limit where the external momenta are negligible with respect to the internal momenta, $k_i\to k$ up to an overall sign, and these integrals reduce to, up to an overall sign,

$$\begin{aligned}\mathcal{J}&=\int\frac{d^4k}{(2\pi)^4}\frac{1}{\prod_i[k^2-a_i^2]}\ ,\\ \mathcal{J}^\mu&=\int\frac{d^4k}{(2\pi)^4}\frac{k^\mu}{\prod_i[k^2-a_i^2]}=0\ ,\\ \mathcal{J}^{\mu\nu}&=\int\frac{d^4k}{(2\pi)^4}\frac{k^\mu k^\nu}{\prod_i[k^2-a_i^2]}\\&=\frac{1}{4}g^{\mu\nu}\int\frac{d^4k}{(2\pi)^4}\frac{k^2}{\prod_i[k^2-a_i^2]}\ .\end{aligned}\tag{A3}$$

In the text we also used the definitions for the loop integrals

$$\begin{aligned}\mathcal{J}_\square&\equiv\int\frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2-M_W^2)^2(k^2-m_I^2)(k^2-m^2)}\ ,\\ \mathcal{J}_{\triangle 1}&\equiv\int\frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2-M_W^2)^2(k^2-m_I^2)}\ ,\\ I_{\triangle 2}&\equiv\int\frac{d^4k}{(2\pi)^4}\frac{(c-d)k^2-2(c+d)m_I^2}{(k^2-M_W^2)(k^2-m_I^2)^2}\ .\end{aligned}\tag{A4}$$

We also used the fact that, after using the limit where the external momenta are negligible compared to the internal momenta, we have

$$\int\frac{d^4k}{(2\pi)^4}\Delta_{\lambda\mu\nu}\gamma^\nu k\!\!\!/\gamma_\lambda\to-3k^2\gamma^\mu\ .\tag{A5}$$

2. Intermediate steps for the matrix elements in $V\to Rl^+l^-$

$$\mathcal{M}=\frac{g^2}{4}\left[\frac{g}{2\sqrt{2}}\right]^2[V_{qi}^\dagger V_{ib}]\left[\frac{\mathcal{F}_V\mathcal{F}_K}{4M_VM_R}\right]\left\{\left[6\mathcal{J}_{\triangle I}+\frac{\mathcal{J}_{\triangle 2}}{4\cos^2\theta_W}\right]\frac{\bar q_2\gamma^\mu(a+b\gamma_5)q_1}{s-M_Z^2}-2\mathcal{J}_\square[\bar q_2\gamma_\mu(1-\gamma_5)q_1]\right\}\mathcal{R}^\mu\ .\tag{A6}$$

Thus

$$\begin{aligned}\mathcal{V}_R&=\frac{g^2}{4}\left[\frac{g}{2\sqrt{2}}\right]^2[V_{qi}^\dagger V_{ib}]\left[\frac{\mathcal{F}_V\mathcal{F}_K}{4M_VM_4}\right]\left\{\frac{a}{s-M_Z^2}\left[6\mathcal{J}_{\triangle 1}+\frac{\mathcal{J}_\triangle 2}{4\cos^2\theta_W}\right]-2\mathcal{J}_\square\right\}\ ,\\ \mathcal{A}_R&=\frac{g^2}{4}\left[\frac{g}{2\sqrt{2}}\right]^2[V_{qi}^\dagger V_{ib}]\left[\frac{\mathcal{F}_V\mathcal{F}_K}{4M_VM_R}\right]\left\{\frac{b}{s-M_Z^2}\left[6\mathcal{J}_{\triangle 1}+\frac{\mathcal{J}_{\triangle 2}}{4\cos^2\theta_W}\right]+2\mathcal{J}_\square\right\}\ .\end{aligned}\tag{A7}$$

TABLE IV. Same as Table III for $\Upsilon\to B_{0^-}X$.

	$\Upsilon\to B_{0^-}X$	
X	R_0	$R_{\rm odd}$
D^{0*}	0.037	0.963
	$\Upsilon\to B_{1^-}^*X$	
X	R_0	$R_{\rm odd}$
D^0	0.672	0.041
D^{0*}	0.629	0.044

a. Scalar

$$\mathcal{R}_S^{\mu}=4\{\xi_s[M_V M_R-(P_R\cdot P_V)]\epsilon_V^{\mu}+\zeta_s(P_R\cdot\epsilon_V)P_V^{\mu}\}-4i\eta_S\varepsilon^{\alpha\beta\sigma\mu}P_{R\alpha}P_{V\beta}\epsilon_{V\sigma}\ , \tag{A8}$$

$$\mathcal{R}_S\cdot[q_{2+}^{\Downarrow\dagger}\gamma_- q_{1+}^{\Uparrow}]=4m\left\{\{\xi_S[M_R M_V-(P_R\cdot P_V)](P-E_V\cos\vartheta)+\zeta_S(E_R-E_V)P(E_v-P\cos\vartheta)\}\frac{\delta_{\lambda=0}}{M_V}\right.$$
$$\left.+\{\pm\xi_S[M_V M_R-(P_R\cdot P_V)]+\eta_S P(E_R-E_V)\}\sin\vartheta\frac{\delta_{\lambda=\pm}}{\sqrt{2}}\right\},$$

$$\mathcal{R}_S\cdot[q_{2-}^{\Downarrow\dagger}\gamma_+ q_{1-}^{\Uparrow}]=4m\left\{\{\xi_S[M_R M_V-(P_R\cdot P_V)](-P-E_V\cos\vartheta)+\zeta_S(E_V-E_R)P(E_V+P\cos\vartheta)\}\frac{\delta_{\lambda=0}}{M_V}\right.$$
$$\left.+\{\pm\xi_S[M_V M_R-(P_R\cdot P_V)]+\eta_S P(E_R-E_{V)}\}\sin\vartheta\frac{\delta_{\lambda=\pm}}{\sqrt{2}}\right\},$$

$$\mathcal{R}_S\cdot[q_{2+}^{\Downarrow\dagger}\gamma_- q_{1+}^{\Downarrow}]=4m\left\{\{\xi_S[M_R M_V-(P_R\cdot P_V)](P=E_V\cos\vartheta)+\zeta_S(E_V-E_R)P(E_V-P\cos\vartheta)\}\frac{\delta_{\lambda=0}}{M_V}\right.$$
$$\left.+\{\xi_S[M_V M_R-(P_R\cdot P_V)]\mp\sin\vartheta)-\eta_S P(E_R-E_V\sin\vartheta\}\frac{\delta_{\lambda=\pm}}{\sqrt{2}}\right\},$$

$$\mathcal{R}_S\cdot[q_{2-}^{\Downarrow\dagger}\gamma_+ q_{1-}^{\Downarrow}]=4m\left\{\{\xi_S[M_R M_V-(P_R\cdot P_V)](-P+E_V\cos\vartheta)+\zeta_S(E_V-E_R)P(E_V-P\cos\vartheta)\}\frac{\delta_{\lambda=0}}{M_V}\right.$$
$$\left.+\{\xi_S[M_V M_R-(P_R\cdot P_V)](\mp\sin\vartheta)-\eta_S P(E_R-E_V\sin\vartheta\}\frac{\delta_{\lambda=\pm}}{\sqrt{2}}\right\},$$

$$\mathcal{R}_S\cdot[q_{2+}^{\Uparrow\dagger}\gamma_- q_{1+}^{\Downarrow}]=4(E+p)\left\{\{\xi_S[M_R M_V-(P_R\cdot P_V)]E_V+\zeta_S(E_R-E_V)\}\sin\vartheta\frac{\delta_{\lambda=0}}{M_V}\right.$$
$$\left.+\{\xi_S[M_V M_R-(P_R\cdot P_V)](1\pm\cos\vartheta)+\eta_S P(E_V-E_R)(\cos\vartheta\mp1)\}\frac{\delta_{\lambda=\pm}}{\sqrt{2}}\right\},$$

$$\mathcal{R}_S\cdot[q_{2-}^{\Uparrow\dagger}\gamma_+ q_{1-}^{\Downarrow}]=4(E-p)\left\{\{\xi_S[M_R M_V-(P_R\cdot P_V)]E_V+\zeta_S P^2(E_R-E_V)\}\sin\vartheta\frac{\delta_{\lambda=0}}{M_V}\right.$$
$$\left.+\{\xi_S[M_V M_R-(P_R\cdot P_V)](1\pm\cos\vartheta)+\eta_S P(E_V-E_R)(\cos\vartheta\mp1)\}\frac{\delta_{\lambda=\pm}}{\sqrt{2}}\right\},$$

$$\mathcal{R}_S\cdot[q_{2+}^{\Downarrow\dagger}\gamma_- q_{1+}^{\Uparrow}]=-4(E-p)\left\{\{\xi_S[M_R M_V-(P_R\cdot P_V)]E_V+\zeta_S P^2(E_R-E_V)\}\sin\vartheta\frac{\delta_{\lambda=0}}{M_V}\right.$$
$$\left.+\{\xi_S[M_V M_R-(P_R\cdot P_V)](-1\pm\cos\vartheta)+\eta_S P(E_V-E_R)(\cos\vartheta\pm1)\}\frac{\delta_{\lambda=\pm}}{\sqrt{2}}\right\},$$

$$\mathcal{R}_S\cdot[q_{2-}^{\Downarrow\dagger}\gamma_+ q_{1-}^{\Uparrow}]=-4(E+p)\left\{\{\xi_S[M_R M_V-(P_R\cdot P_V)]E_V+\zeta_S P^2(E_R-E_V)\}\sin\vartheta\frac{\delta_{\lambda=0}}{M_V}\right.$$
$$\left.+\{\xi_S[M_V M_R-(P_R\cdot P_V)](-1\pm\cos\vartheta)+\eta_S P(E_V-E_R)(\cos\vartheta\pm1)\}\frac{\delta_{\lambda=\pm}}{\sqrt{2}}\right\}, \tag{A9}$$

$$H_0^{\Downarrow\Uparrow}=8\frac{m}{M_V}\{\xi_S[M_R M_V-(P_R\cdot P_V)](\mathcal{A}P-\mathcal{V}E_V\cos\vartheta)+\zeta_S(E_R-E_V)P(\mathcal{A}E_V-\mathcal{V}P\cos\vartheta)\}\ ,$$

$$H_+^{\Downarrow\Uparrow}=\frac{8}{\sqrt{2}}m\mathcal{V}\{\xi_S[M_V M_R-(P_R\cdot P_V)]+\eta_S P(E_R-E_V)\}\sin\vartheta\ ,$$

$$H_-^{\Downarrow\Uparrow}=\frac{8}{\sqrt{2}}m\mathcal{V}\{-\xi_S[M_V M_R-(P_R\cdot P_V)]+\eta_S P(E_R-E_V)\}\sin\vartheta\ ,$$

$$
\begin{aligned}
H_0^{\Downarrow\Downarrow} &= 8\frac{m}{M_V}\{\xi_S[M_R M_V-(P_R\cdot P_V)](\mathcal{A}P+\mathcal{V}E_V\cos\vartheta)+\zeta_S(E_R-E_V)P(\mathcal{A}E_V+\mathcal{V}P\cos\vartheta)\}\ ,\\
H_+^{\Downarrow\Downarrow} &= -\frac{8}{\sqrt{2}}m\mathcal{V}\{\xi_S[M_V M_R-(P_R\cdot P_V)]+\eta_S P(E_R-E_V)\}\sin\vartheta\ ,\\
H_-^{\Downarrow\Downarrow} &= -\frac{8}{\sqrt{2}}m\mathcal{V}\{-\xi_S[M_V M_R-(P_R\cdot P_V)]+\eta_S P(E_R-E_V)\}\sin\vartheta\ ,\\
H_0^{\Uparrow\Downarrow} &= \frac{8E}{M_V}\left[\mathcal{V}+\frac{p}{E}\mathcal{A}\right]\{\xi_S[M_R M_V-(P_R\cdot P_V)]E_V+\zeta_S P^2(E_R-E_V)\}\sin\vartheta\ ,\\
H_+^{\Uparrow\Downarrow} &= \frac{8E}{\sqrt{2}}\left[\mathcal{V}+\frac{p}{E}\mathcal{A}\right]\{\xi_S[M_V M_R-(P_R\cdot P_V)](1+\cos\vartheta)-\eta_S P(E_V-E_R)(1-\cos\vartheta)\}\ ,\\
H_-^{\Uparrow\Downarrow} &= \frac{8E}{\sqrt{2}}\left[\mathcal{V}+\frac{p}{E}\mathcal{A}\right]\{\xi_S[M_V M_R-(P_R\cdot P_V)](1-\cos\vartheta)+\eta_S P(E_V-E_R)(1+\cos\vartheta)\}\ ,\\
H_0^{\Downarrow\Uparrow} &= \frac{-8E}{M_V}\left[\mathcal{V}-\frac{p}{E}\mathcal{A}\right]\{\xi_S[M_R M_V-(P_R\cdot P_V)]E_V+\zeta_S P^2(E_R-E_V)\}\sin\vartheta\ ,\\
H_+^{\Downarrow\Uparrow} &= \frac{8E}{\sqrt{2}}\left[\mathcal{V}-\frac{p}{E}\mathcal{A}\right]\{\xi_S[M_V M_R-(P_R\cdot P_V)](1-\cos\vartheta)-\eta_S P(E_V-E_R)(1+\cos\vartheta)\}\ ,\\
H_-^{\Downarrow\Uparrow} &= \frac{8E}{\sqrt{2}}\left[\mathcal{V}-\frac{p}{E}\mathcal{A}\right]\{\xi_S[M_V M_R-(P_R\cdot P_V)](1+\cos\vartheta)+\eta_S P(E_V-E_R)(1-\cos\vartheta)\}\ ,
\end{aligned}
\tag{A10}
$$

$$
\begin{aligned}
|H_0|^2 = {} & \frac{2^7E^2}{M_V^2}\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]\{\xi_S[M_R M_V-(P_R\cdot P_V)]E_V+\zeta_S P^2(E_R-E_V)\}^2(1-\cos^2\vartheta)\\
& +2^7\frac{m^2}{M_V^2}\{\xi_S^2[M_R M_V-(P_R\cdot P_V)]^2(|\mathcal{A}|^2P^2+|\mathcal{V}|^2E_V^2\cos^2\vartheta)+\zeta_S^2(E_R-E_V)^2P^2(|\mathcal{A}|^2E_V^2+|\mathcal{V}|^2P^2\cos^2\vartheta)\\
& \qquad +2\xi_S\zeta_S[M_R M_V-(P_R\cdot P_V)](E_R-E_V)P^2E_V(|\mathcal{A}|^2+|\mathcal{V}|^2\cos^2\vartheta)\}\ ,\\
|H_+|^2 = {} & 2^7E^2\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right](\{\xi_S^2[M_V M_R-(P_R\cdot P_V)]^2+\eta_S^2P^2(E_V-E_R)^2\}(1+\cos^2\vartheta)\\
& \qquad -2\xi_S[M_V M_R-(P_R\cdot P_V)]\eta_S P(E_V-E_R)(1-\cos^2\vartheta))\\
& +2^7E^2\frac{p}{E}\mathrm{Re}(\mathcal{V}^\dagger\mathcal{A})\{\xi_S^2[M_V M_R-(P_R\cdot P_V)]^2\}\cos\vartheta\\
& +2^6|\mathcal{V}^2|\frac{m^2}{M_V^2}\{\xi_S[M_V M_R-(P_R\cdot P_V)]+\eta_S P(E_R-E_V)\}^2\sin^2\vartheta\ ,\\
|H_-|^2 = {} & 2^7E^2\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right](\{\xi_S^2[M_V M_R-(P_R\cdot P_V)]^2+\eta_S^2P^2(E_V-E_R)^2\}(1+\cos^2\vartheta)\\
& \qquad +2\xi_S[M_V M_R-(P_R\cdot P_V)]\eta_S P(E_V-E_R)(1-\cos^2\vartheta))\\
& -2^7E^2\frac{p}{E}\mathrm{Re}(\mathcal{V}^\dagger\mathcal{A})\{\xi_S^2[M_V M_R-(P_R\cdot P_V)]^2\}\cos\vartheta\\
& +2^6|\mathcal{V}|^2\frac{m^2}{M_V^2}\{\xi_s[M_V M_R-(P_R\cdot P_V)]-\eta_S P(E_R-E_V)\}^2\sin^2\vartheta\ ,
\end{aligned}
\tag{A11}
$$

$$
\begin{aligned}
|\mathcal{M}_S|^2 = {} & 2^7E^2\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]\left\{\frac{1}{M_V^2}\{\xi_S[M_R M_V-(P_R\cdot P_V)]E_V+\zeta_S P^2(E_R-E_V)\}^2(1-\cos^2\vartheta)\right.\\
& \qquad \left.+\{\xi_S^2[M_V M_R-(P_R\cdot P_V)]^2+\eta_S^2P^2(E_V-E_R)^2\}(1+\cos^2\vartheta)\right\}\\
& +2^7\left\{\frac{m^2}{M_V^2}\{\xi_S^2[M_R M_V=(P_R\cdot P_V)]^2(|\mathcal{A}|^2P^2+|\mathcal{V}|^2E_V^2\cos^2\vartheta)+\zeta_S^2(E_R-E_V)^2P^2(|\mathcal{A}|^2E_V^2+|\mathcal{V}|^2P^2\cos^2\vartheta)\right.
\end{aligned}
$$

$$+2\xi_S\zeta_S[M_RM_V-(P_R\cdot P_V)](E_R-E_V)P^2E_V(|\mathcal{A}|^2+|\mathcal{V}|^2\cos^2\vartheta)\Bigg\}$$

$$+|\mathcal{V}|^2\frac{m^2}{M_V^2}\{\xi_S^2[M_VM_R-(P_R\cdot P_V)]^2+\eta_S^2P^2(E_R-E_V)^2\}\sin^2\vartheta\} \ . \qquad \text{(A12)}$$

In the limit $m\to 0$,

$$\alpha_S(s)=\frac{\{\xi_S^2[M_VM_R-(P_R\cdot P_V)]^2+\eta_S^2P^2(E_V-E_R)\}^2-(1/M_V^2)\{\xi_S[M_RM_V-(P_R\cdot P_V)]E_V+\zeta_SP^2(E_R-E_V)\}^2}{\{\xi_S^2[M_VM_R-(P_R\cdot P_V)]^2+\eta_S^2P^2(E_V-E_R)\}^2+(1/M_V^2)\{\xi_S[M_RM_V-(P_R\cdot P_V)]E_V+\zeta_SP^2(E_R-E_V)\}^2} \ ,$$

$$\mathcal{R}_{\rm odd}^S=\frac{\frac{3}{4}\{\xi_S^2[M_VM_R-(P_R\cdot P_V)]^2\cos^2\vartheta+\eta_S^2P^2(E_V-E_R)^2\}}{(1/M_V^2)\{\xi_S[M_RM_V-(P_R\cdot P_V)]E_V+\zeta_SP^2(E_R-E_V)\}^2+2\{\xi_S^2[M_VM_R-(P_R\cdot P_V)]^2+\eta_S^2P^2(E_V-E_R)^2\}} \ .$$

b. Pseudoscalar

$$\mathcal{R}_P^\mu=4\{\xi_P[M_VM_R+(P_R\cdot P_V)]\epsilon_V^\mu-\zeta_P(P_R\cdot\epsilon_V)P_V^\mu\}+4i\eta_P\varepsilon^{\alpha\beta\sigma\mu}P_{R\alpha}P_{V\beta}\epsilon_{V\sigma} \ . \qquad \text{(A13)}$$

It is sufficient to substitute $M_VM_R\to -M_VM_R$ in the results for the scalar resonance. Hence

$$|\mathcal{M}_P|^2=2^8E^2\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]\Bigg\{\frac{1}{M_V^2}\{\xi_S[M_RM_V+(P_R\cdot P_V)]E_V+\zeta_SP^2(E_R-E_V)\}^2(1-\cos^2\vartheta)$$

$$+\{\xi_S^2[M_VM_R+(P_R\cdot P_V)]^2-\eta_S^2P^2(E_V-E_R)^2\}(1+\cos^2\vartheta)\Bigg\}$$

$$+2^7\Bigg\{\frac{m^2}{M_V^2}\{\xi_S^2[M_RM_V+(P_R\cdot P_V)]^2(|\mathcal{A}|^2P^2+|\mathcal{V}|^2E_V^2\cos^2\vartheta)+\zeta_S^2(E_R-E_V)^2P^2(|\mathcal{A}|^2E_V^2+|\mathcal{V}|^2P^2\cos^2\vartheta)$$

$$-2\xi_S\zeta_S[M_RM_V+(P_R\cdot P_V)](E_R-E_V)P^2E_V(|\mathcal{A}|^2+|\mathcal{V}|^2\cos^2\vartheta)\Bigg\}$$

$$+|\mathcal{V}|^2\frac{m^2}{M_V^2}\{\xi_S^2[M_VM_R+(P_R\cdot P_V)]^2+\eta_S^2P^2(E_R-E_V)^2\}\sin^2\vartheta\} \ , \qquad \text{(A14)}$$

$$\alpha_P(s)=\frac{\{\xi_S^2[M_VM_R+(P_R\cdot P_V)]^2+\eta_S^2P^2(E_V-E_R)\}^2-(1/M_V^2)\{\xi_S[M_RM_V+(P_R\cdot P_V)]E_V-\zeta_SP^2(E_R-E_V)\}^2}{\{\xi_S^2[M_VM_R+(P_R\cdot P_V)]^2+\eta_S^2P^2(E_V-E_R)\}^2+(1/M_V^2)\{\xi_S[M_RM_V+(P_R\cdot P_V)]E_V-\zeta_SP^2(E_R-E_V)\}^2} \ ,$$

$$\mathcal{R}_{\rm odd}^P=\frac{\frac{3}{4}\{\xi_S^2[M_VM_R+(P_R\cdot P_V)]^2\cos^2\vartheta+\eta_S^2P^2(E_V-E_R)^2\}}{(1/M_V^2)\{\xi_S[M_RM_V+(P_R\cdot P_V)]E_V-\zeta_SP^2(E_R-E_V)\}^2+2\{\xi_S^2[M_VM_R+(P_R\cdot P_V)]^2+\eta_S^2P^2(E_V-E_R)^2\}} \ .$$

c. Vector

In this section, because of the length of the expressions, we do the calculations in the limit $m_l\to 0$:

$$\mathcal{R}_V^\mu=4\{\xi_V[(P_R\cdot\epsilon_V)M_V\epsilon_R^{*\mu}+(\epsilon_R^*\cdot P_V)M_R\epsilon_V^\mu]-\zeta_V(\epsilon_R^*\cdot\epsilon_V)(M_VP_R^\mu+M_RP_V^\mu)-i\eta_V\varepsilon^{\alpha\beta\sigma\mu}\epsilon_{R\alpha}^*(M_VP_{R\beta}+M_RP_{V\beta}\epsilon_{V\sigma})\} \ ,$$

$$\text{(A15)}$$

$$\mathcal{R}_V\cdot[q_{2+}^\dagger\gamma_-q_{1+}]=4(E+p)\Bigg\{\left[\xi_V(E_V-E_R)\left[\frac{E_V}{M_V}-\frac{E_R}{M_R}\right]+\zeta_V(E_RE_V-P^2)\left[\frac{1}{M_V}+\frac{1}{M_R}\right]\right]P\sin\vartheta\,\delta_{\lambda_V=0}\delta_{\lambda_R=0}$$

$$+\xi_V(E_V-E_R)\frac{P}{\sqrt{2}}[(1\pm\cos\vartheta)\delta_{\lambda_V=\pm}\delta_{\lambda_R=0}+(1\mp\cos\vartheta)\delta_{\lambda_V=0}\delta_{\lambda_R=\pm})]$$

$$+\eta_V\left[(M_V+M_R)\frac{P^2}{\sqrt{2}M_V}-\frac{E_V}{\sqrt{2}M_V}(M_VE_R+M_RE_V)\right](-\cos\vartheta\pm 1)\delta_{\lambda_V=0}\delta_{\lambda_R=\pm}$$

$$+\eta_V\left[(M_V+M_R)\frac{P^2}{\sqrt{2}M_R}-\frac{E_R}{\sqrt{2}M_R}(M_VE_R+M_RE_V)\right](\cos\vartheta\pm 1)\delta_{\lambda_V=\pm}\delta_{\lambda_R=0}\Bigg\}$$

$$+[\zeta_V(M_V+M_R)P\pm\eta_V(M_VE_R+M_RE_V)]\sin\vartheta\,\delta_{\lambda_V=\pm}\delta_{\lambda_R=\pm} \ ,$$

$$\mathcal{R}_V\cdot[q_{2-}^{\Uparrow\dagger}\gamma_+q_{1-}^{\Downarrow}=4(E-p)\left\{\left[\xi_V(E_V-E_R)\left[\frac{E_V}{M_V}-\frac{E_R}{M_R}\right]+\zeta_V(E_RE_V-P^2)\left[\frac{1}{M_V}+\frac{1}{M_R}\right]\right]P\sin\vartheta\,\delta_{\lambda_V=0}\delta_{\lambda_R=0}\right.$$

$$+\xi_V(E_V-E_R)\frac{P}{\sqrt{2}}[(1\pm\cos\vartheta)\delta_{\lambda_V=\pm}\delta_{\lambda_R=0}+(1\mp\cos\vartheta)\delta_{\lambda_V=0}\delta_{\lambda_R=\pm})]$$

$$+\eta_V\left[(M_V+M_R)\frac{P^2}{\sqrt{2}M_V}-\frac{E_V}{\sqrt{2}M_V}(M_VE_R+M_RE_V)\right](-\cos\vartheta\pm1)\delta_{\lambda_V=0}\delta_{\lambda_R=\pm}$$

$$\left.+\eta_V\left[(M_V+M_R)\frac{P^2}{\sqrt{2}M_R}-\frac{E_R}{\sqrt{2}M_R}(M_VE_R+M_RE_V)\right](\cos\vartheta\pm1)\delta_{\lambda_V=\pm}\delta_{\lambda_R=0}\right\}$$

$$+[\zeta_V(M_V+M_R)P\pm\eta_V(M_VE_R+M_RE_V)]\sin\vartheta\,\delta_{\lambda_V=\pm}\delta_{\lambda_R=\pm}\ ,$$

$$\mathcal{R}_V\cdot[q_{2+}^{\Downarrow\dagger}\gamma_-q_{1+}^{\Uparrow}]=-4(E-p)\left\{\left[\xi_V(E_V-E_R)\left[\frac{E_V}{M_V}-\frac{E_R}{M_R}\right]+\zeta_V(E_RE_V-P^2)\left[\frac{1}{M_V}+\frac{1}{M_R}\right]\right]P\sin\vartheta\,\delta_{\lambda_V=0}\delta_{\lambda_R=0}\right.$$

$$+\xi_V(E_V-E_R)\frac{P}{\sqrt{2}}[(-1\pm\cos\vartheta)\delta_{\lambda_V=\pm}\delta_{\lambda_R=0}+(-1\mp\cos\vartheta)\delta_{\lambda_V=0}\delta_{\lambda_R=\pm})]$$

$$+\eta_V\left[(M_V+M_R)\frac{P^2}{\sqrt{2}M_V}-\frac{E_V}{\sqrt{2}M_V}(M_VE_R+M_RE_V)\right](-\cos\vartheta\mp1)\delta_{\lambda_V=0}\delta_{\lambda_R=\pm}$$

$$\left.+\eta_V\left[(M_V+M_R)\frac{P^2}{\sqrt{2}M_R}-\frac{E_R}{\sqrt{2}M_R}(M_VE_R+M_RE_V)\right](\cos\vartheta\mp1)\delta_{\lambda_V=\pm}\delta_{\lambda_R=0}\right\}$$

$$+[\zeta_V(M_V+M_R)P\pm\eta_v(M_VE_R+M_RE_V)]\sin\vartheta\,\delta_{\lambda_V=\pm}\delta_{\lambda_R=\pm}\ ,$$

$$\mathcal{R}_V\cdot[q_{2-}^{\Downarrow\dagger}\gamma_+q_{1-}^{\Uparrow}]=-4(E+p)\left\{\left[\xi_V(E_V-E_R)\left[\frac{E_V}{M_V}-\frac{E_R}{M_R}\right]+\zeta_V(E_RE_V-P^2)\left[\frac{1}{M_V}+\frac{1}{M_R}\right]\right]P\sin\vartheta\,\delta_{\lambda_V=0}\delta_{\lambda_R=0}\right.$$

$$+\xi_V(E_V-E_R)\frac{P}{\sqrt{2}}[(-1\pm\cos\vartheta)\delta_{\lambda_V=\pm}\delta_{\lambda_R=0}+(-1\mp\cos\vartheta)\delta_{\lambda_V=0}\delta_{\lambda_R=\pm})]$$

$$+\eta_V\left[(M_V+M_R)\frac{P^2}{\sqrt{2}M_V}-\frac{E_V}{\sqrt{2}M_V}(M_VE_R+M_RE_V)\right](-\cos\vartheta\mp1)\delta_{\lambda_V=0}\delta_{\lambda_R=\pm}$$

$$\left.+\eta_V\left[(M_V+M_R)\frac{P^2}{\sqrt{2}M_R}-\frac{E_R}{\sqrt{2}M_R}(M_VE_R+M_RE_V)\right](\cos\vartheta\mp1)\delta_{\lambda_V=\pm}\delta_{\lambda_R=0}\right\}$$

$$+[\zeta_V(M_V+M_R)P\pm\eta_V(M_VE_R+M_RE_V)]\sin\vartheta\,\delta_{\lambda_V=\pm}\delta_{\lambda_R=\pm}\ , \tag{A16}$$

$$H_{00}^{\Uparrow\Downarrow}=8E\left[\mathcal{V}+\frac{p}{E}\mathcal{A}\right]\left\{\left[\xi_V(E_V-E_R)\left[\frac{E_V}{M_V}-\frac{E_R}{M_R}\right]+\zeta_V(E_RE_V-P^2)\left[\frac{1}{M_V}+\frac{1}{M_R}\right]\right]P\sin\vartheta\right\}\ ,$$

$$H_{0+}^{\Uparrow\Downarrow}=8E\left[\mathcal{V}+\frac{p}{E}\mathcal{A}\right]\left\{\xi_V(E_V-E_R)\frac{P}{\sqrt{2}}+\frac{\eta_V}{\sqrt{2}M_V}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]\right\}(1-\cos\vartheta)\ ,$$

$$H_{0-}^{\Uparrow\Downarrow}=8E\left[\mathcal{V}+\frac{p}{E}\mathcal{A}\right]\left\{\xi_V(E_V-E_R)\frac{P}{\sqrt{2}}-\frac{\eta_V}{\sqrt{2}M_V}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]\right\}(1+\cos\vartheta)\ ,$$

$$H_{+0}^{\Uparrow\Downarrow}=8E\left[\mathcal{V}+\frac{p}{E}\mathcal{A}\right]\left\{\xi_V(E_V-E_R)\frac{P}{\sqrt{2}}+\frac{\eta_V}{\sqrt{2}M_R}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]\right\}(1+\cos\vartheta)\ ,$$

$$H_{-0}^{\Uparrow\Downarrow}=8E\left[\mathcal{V}+\frac{p}{E}\mathcal{A}\right]\left\{\xi_V(E_V-E_R)\frac{P}{\sqrt{2}}+\frac{\eta_V}{\sqrt{2}M_R}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]\right\}(1-\cos\vartheta)\ ,$$

$$H_{++}^{\Uparrow\Downarrow}=8E\left[\mathcal{V}+\frac{p}{E}\mathcal{A}\right]+[\zeta_V(M_V+M_R)P+\eta_V(M_VE_R+M_RE_V)]\sin\vartheta\ ,$$

$$H^{\Uparrow\Downarrow}_{--}=8E\left[\mathcal{V}+\frac{p}{E}\mathcal{A}\right]+[\zeta_V(M_V+M_R)P-\eta_V(M_VE_R+M_RE_V)]\sin\vartheta\ , \tag{A17}$$

$$H^{\Downarrow\Uparrow}_{00}=-8\left[\mathcal{V}-\frac{p}{E}\mathcal{A}\right)\left\{\left[\xi_V(E_V-E_R)\left[\frac{E_V}{M_V}-\frac{E_R}{M_R}\right]+\zeta_V(E_RE_V-P^2)\left[\frac{1}{M_V}+\frac{1}{M_R}\right]\right]P\sin\vartheta\right\}\ ,$$

$$H^{\Downarrow\Uparrow}_{0+}=-8\left[\mathcal{V}-\frac{p}{E}\mathcal{A}\right]\left\{\xi_V(E_V-E_R)\frac{P}{\sqrt{2}}+\frac{\eta_V}{\sqrt{2}M_V}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]\right\}(1+\cos\vartheta)\ ,$$

$$H^{\Downarrow\Uparrow}_{0-}=8E\left[\mathcal{V}-\frac{p}{E}\mathcal{A}\right]\left\{\xi_V(E_V-E_R)\frac{P}{\sqrt{2}}-\frac{\eta_V}{\sqrt{2}M_V}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]\right\}(1-\cos\vartheta)\ ,$$

$$H^{\Downarrow\Uparrow}_{+0}=8E\left[\mathcal{V}-\frac{p}{E}\mathcal{A}\right]\left\{\xi_V(E_V-E_R)\frac{P}{\sqrt{2}}+\frac{\eta_V}{\sqrt{2}M_R}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]\right\}(1-\cos\vartheta)\ ,$$

$$H^{\Downarrow\Uparrow}_{-0}=8E\left[\mathcal{V}-\frac{p}{E}\mathcal{A}\right]\left\{\xi_V(E_V-E_R)\frac{P}{\sqrt{2}}-\frac{\eta_V}{\sqrt{2}M_R}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]\right\}(1+\cos\vartheta)\ ,$$

$$H^{\Uparrow\Downarrow}_{++}=-8\left[\mathcal{V}-\frac{p}{E}\mathcal{A}\right]+[\zeta_V(M_V+M_R)P+\eta_V(M_VE_R+M_RE_V)]\sin\vartheta\ ,$$

$$H^{\Uparrow\Downarrow}_{--}=-8\left[\mathcal{V}-\frac{p}{E}\mathcal{A}\right]+[\zeta_V(M_V+M_R)P-\eta_V(M_VE_R+M_RE_V)]\sin\vartheta\ , \tag{A18}$$

$$|H_{00}|^2=2^7\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]\left[\xi_V(E_V-E_R)\left[\frac{E_V}{M_V}-\frac{E_R}{M_R}\right]+\zeta_V(E_RE_V-P^2)\left[\frac{1}{M_V}+\frac{1}{M_R}\right]\right]^2P^2(1-\cos^2\vartheta)\ ,$$

$$|H_{0+}|^2=2^6\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]\left\{\xi_V^2(E_V-E_R)^2P^2+\frac{\eta_V^2}{M_V^2}[(M_V+M_R)P-E_V(M_VE_R+M_RE_V)]^2\right\}(1+\cos^2\vartheta)$$
$$-2^7\frac{p}{E}\mathrm{Re}(\mathcal{V}^\dagger\mathcal{A})\xi_V\eta_V\left\{(E_V-E_R)\frac{P}{M_V}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]\cos\vartheta\right\}\ ,$$

$$|H_{0-}|^2=2^6\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]\left\{\xi_V^2(E_V-E_R)^2P^2+\frac{\eta_V^2}{M_V^2}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]^2\right\}(1+\cos^2\vartheta)$$
$$-2^7\frac{p}{E}\mathrm{Re}(\mathcal{V}^\dagger\mathcal{A})\xi_V\eta_V\left\{(E_V-E_R)\frac{P}{M_V}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]\cos\vartheta\right\}\ ,$$

$$|H_{+0}|^2=2^6\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]\left\{\xi_V^2(E_V-E_R)^2P^2+\frac{\eta_V^2}{M_R^2}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]^2\right\}(1+\cos^2\vartheta)$$
$$+2^7\frac{p}{E}\mathrm{Re}(\mathcal{V}^\dagger\mathcal{A})\xi_V\eta_V\left\{(E_V-E_R)\frac{P}{M_R}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]\cos\vartheta\right\}\ ,$$

$$|H_{-0}|^2=2^6\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]\left\{\xi_V^2(E_V-E_R)^2P^2+\frac{\eta_V^2}{M_R^2}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)^2]\right\}(1+\cos^2\vartheta)$$
$$+2^7\frac{p}{E}\mathrm{Re}(\mathcal{V}^\dagger\mathcal{A})\xi_V\eta_V\left\{(E_V-E_R)\frac{p}{M_R}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]\cos\vartheta\right\}\ ,$$

$$|H_{++}|^2=2^7\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]+[\zeta_V(M_V+M_R)P+\eta_V(M_VE_R+M_RE_V)]^2(1-\cos^2\vartheta)\ ,$$

$$|H_{--}|^2=2^7\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]+[\zeta_V(M_V+M_R)P-\eta_V(M_VE_R+M_RE_V)]^2(1-\cos^2\vartheta)\ , \tag{A19}$$

$$|\mathcal{M}|^2=2^7\left[|\mathcal{V}|^2+\frac{p^2}{E^2}|\mathcal{A}|^2\right]\left[\left\{2[\zeta_V^2(M_V+M_R)^2P^2+\eta_V^2(M_VE_R+M_RE_V)^2]\right.\right.$$
$$\left.+\left[\xi_V(E_V-E_R)\left(\frac{E_V}{M_V}-\frac{E_R}{M_R}\right)+\zeta_V(E_RE_V-P^2)\left(\frac{1}{M_V}+\frac{1}{M_R}\right)\right]^2P^2\right\}(1-\cos^2\vartheta)$$
$$+\left\{\xi_V^2(E_V-E_R)^2P^2+\frac{\eta_V^2}{M_V^2}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]^2\right\}(1+\cos^2\vartheta)$$
$$\left.+\left\{\xi_V^2(E_V-E_R)^2P^2+\frac{\eta_V^2}{M_R^2}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]^2\right\}(1+\cos^2\vartheta)\right]$$
$$+2^8\frac{p}{E}\mathrm{Re}(\mathcal{V}^\dagger\mathcal{A})\xi_V\eta_V(E_V-E_R)\left\{\frac{P}{M_R}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]\right.$$
$$\left.-\frac{P}{M_V}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]\right\}\cos\vartheta\ , \tag{A20}$$

$$\alpha_V(s)=\frac{2^7(|\mathcal{V}|^2+[p^2/E^2]|\mathcal{A}|^2)}{[|\mathcal{M}|^2]_{\cos\vartheta=0}}\left[\left\{\xi_V^2(E_V-E_R)^2P^2+\frac{\eta_V^2}{M_V^2}[(M_V+M_R)P^2-E_V(M_VE_R+M_RE_V)]^2\right\}\right.$$
$$+\left\{\xi_V^2(E_V-E_R)^2P^2+\frac{\eta_V^2}{M_R^2}[(M_V+M_R)P^2-E_R(M_VE_R+M_RE_V)]^2\right\}$$
$$-\left\{2[\zeta_V^2(M_V+M_R)^2P^2+\eta_V^2(M_VE_R+M_RE_V)^2]\right.$$
$$\left.\left.+\left[\xi_V(E_V-E_R)\left(\frac{E_V}{M_V}-\frac{E_R}{M_R}\right)+\zeta_V(E_RE_V-P^2)\left(\frac{1}{M_V}+\frac{1}{M_R}\right)\right]^2P^2\right\}\right]\ . \tag{A21}$$

3. Intermediate steps for the matrix elements in $V\to R_1R_2$

$$\mathcal{M}_{V\to R_1R_2}=\{\ \}_{12}\mathrm{Tr}[\Gamma_2(\not P_2+M_2)\gamma^\mu(1-\gamma_5)]\mathrm{Tr}(\Gamma_1(\not P_1+M_R)\gamma_\mu(1-\gamma_5)(\not P_V+M_V)\not\epsilon_V]\ , \tag{A22}$$

where

$$\{\ \}_{12}\equiv\frac{1}{3}\left[\frac{ig}{2\sqrt{2}}\right]^2\frac{V_{qq_2}^\dagger V_{q_1b}}{k^2-M_W^2}\left[\frac{\mathcal{F}_V\mathcal{F}_1\mathcal{F}_2}{8M_VM_1M_2}\right]\ . \tag{A23}$$

a. $V\to R_IR_{0^+}$ *or* $V\to R_IR_{0^+}$

We have $\Gamma_2=1,\gamma_5$ and

$$\mathrm{Tr}[(\not P_2+M_2)\gamma^\mu(v_2-a_2\gamma_5)]=4v_2P_2^\mu \tag{A24}$$

for the scalar and

$$\mathrm{Tr}[(\not P_2+M_2)\gamma^\mu(v_2-a_2\gamma_5)]=4a_2P_2^\mu \tag{A25}$$

for the pseudoscalar. Hence

$$\mathcal{M}_{V\to R_1R_2}=\{\ \}_{1,S}\mathrm{Tr}[\Gamma_1(\not P_1+M_1)\not P_2(1-\gamma_5)(\not P_V+M_V)\not\epsilon_V]\ . \tag{A26}$$

(i) $R_1=R_{0^+}$: $\Gamma_1=1$ and only the $\lambda_V=0$ term contributes:

$$\mathrm{Tr}[(\not P_1+M_1)\not P_2(1-\gamma_5)(\not P_V+M_V)\not\epsilon_V]=\mathrm{Tr}[\not P_1\not P_2\not P_V\not\epsilon_V(1-\gamma_5)]+M_1M_V\mathrm{Tr}[\not P_2\not\epsilon_V]$$
$$=4\{\xi(M_1M_V-P_1\cdot P_V)(P_2\cdot\epsilon_V)+\zeta[(P_1\cdot P_2)(P_V\cdot\epsilon_V)+(P_2\cdot P_V)(P_1\cdot\epsilon_V)]$$
$$-i\eta\varepsilon^{\alpha\beta\mu\nu}P_{1\alpha}P_{2\beta}P_{V\mu}\epsilon_{V\nu}\}$$
$$=\frac{4P}{M_V}\{\xi[M_1M_V-(P_1\cdot P_V)]M_2+\zeta M_2E_V(E_1-E_V)\}\ . \tag{A27}$$

(ii) $R_1 = R_{0^-}$: $\Gamma_1 = \gamma_5$ and it is sufficient to substitute $M_1 \to -M_1$ in the equation above:

$$\mathrm{Tr}[(\not P_1 - M_1)\not P_2(1-\gamma_5)(\not P_V + M_V)\not\epsilon_V] = \frac{-4P}{M_V}\{\xi(M_1 M_V + P_1\cdot P_V)M_2 + \zeta M_2 E_V(E_V - E_1)\} \ . \tag{A28}$$

(iii) $R_1 = R_{1^-}$: $\Gamma_1 = \not\epsilon_1$ and

$$\begin{aligned}\mathrm{Tr}[\not\epsilon_1^*(\not P_1 + M_1)\not P_2(1-\gamma_5)(\not P_V + M_V)\not\epsilon_V] &= M_V \mathrm{Tr}[\not\epsilon_1^* \not P_1 \not P_2 \not\epsilon_V(1+\gamma_5)] + M_1 \mathrm{Tr}[\not\epsilon_1^* \not P_2 \not P_V \not\epsilon_V(1-\gamma_5)] \\ &= 4\{M_V[-\xi(\epsilon_1^*\cdot P_2)(P_1\cdot\epsilon_V) + \zeta(\epsilon_1^*\cdot\epsilon_V)(P_1\cdot P_2) + i\eta\varepsilon^{\alpha\beta\mu\nu}\epsilon_{1\alpha}^* P_{1\beta}P_{2\mu}\epsilon_{V\nu}] \\ &\quad + M_1[-\xi(\epsilon_1^*\cdot P_V)(P_2\cdot\epsilon_V) + \zeta(\epsilon_1^*\cdot\epsilon_V)(P_2\cdot P_V) - i\eta\varepsilon^{\alpha\beta\mu\nu}\epsilon_{1\alpha}^* P_{2\beta}P_{V\mu}\epsilon_{V\nu}]\} \\ &= 4M_2\left\{\left[\xi(E_V - E_1)P^2\left[\frac{1}{M_1} - \frac{1}{M_V}\right] \right.\right. \\ &\qquad \left. + \zeta(P^2 - E_V E_1)\left[\frac{E_1}{M_1} + \frac{E_V}{M_V}\right]\right]\delta_{\lambda_V=0}\delta_{\lambda_1=0} \\ &\qquad \left. - \eta[(M_V E_1 + M_1 E_V) + (M_V + M_1)P]\delta_{\lambda_V=\pm}\delta_{\lambda_1=\pm}\right\} \ .\end{aligned} \tag{A29}$$

b. $V \to R_1 R_{1^-}$

We have $\Gamma_2 = \not\epsilon_2$ and

$$\mathrm{Tr}[\not\epsilon_2(\not P_2 + M_2)\gamma^\mu(1-\gamma_5)] = 4M_2\epsilon_2^\mu \ . \tag{A30}$$

(i) $R_1 = R_{0^+}$: $\Gamma_1 = 1$ and

$$\begin{aligned}\mathrm{Tr}[(\not P_1 + M_1)\not\epsilon_2^*(1-\gamma_5)(\not P_V + M_V)\not\epsilon_V] &= \mathrm{Tr}[\not P_1 \not\epsilon_2^* \not P_V \not\epsilon_V(1-\gamma_5)] + M_1 M_V \mathrm{Tr}[\not\epsilon_2^* \not\epsilon_V] \\ &= 4\{\xi(M_1 M_V - P_1\cdot P_V)(\epsilon_2^*\cdot\epsilon_V) + \zeta[(P_1\cdot\epsilon_2^*)(P_V\cdot\epsilon_V) + (\epsilon_2^*\cdot P_V)(P_1\cdot\epsilon_V)] \\ &\quad - i\eta\varepsilon^{\alpha\beta\mu\nu}P_{1\alpha}\epsilon_{2*\beta}P_{V\mu}\epsilon_{V\nu}\} \\ &= 4\{-\xi(M_1 M_V - P_1\cdot P_V) \mp \eta(E_V - E_1)P\}\delta_{\lambda_V=\pm}\delta_{\lambda_2=\pm} \\ &\quad + \frac{4}{M_V}\{\xi(M_1 M_V - P_1\cdot P_V)E_V + \zeta(E_V - E_1)P^2\}\delta_{\lambda_V=0}\delta_{\lambda_2=0} \ .\end{aligned} \tag{A31}$$

(ii) $R_1 = R_{0^-}$: $\Gamma_1 = \gamma_5$ and it is sufficient to substitute $M_1 \to -M_1$ in the equation above:

$$\begin{aligned}\mathrm{Tr}[(\not P_1 - M_1)\not\epsilon_2^*(1-\gamma_5)(\not P_V + M_V)\not\epsilon_V] &= 4\{\xi(M_1 M_V + P_1\cdot P_V) \mp \eta(E_V - E_1)P^2\}\delta_{\lambda_V=\pm}\delta_{\lambda_2=\pm} \\ &\quad + \frac{4P}{M_V}\{-\xi(M_1 M_V + P_1\cdot P_V)E_V + \zeta(E_V - E_1)P^2\}\delta_{\lambda_V=0}\delta_{\lambda_2=0} \ .\end{aligned} \tag{A32}$$

(iii) $R_1 = R_{1^-}$: $\Gamma_1 = \not\epsilon_1$ and

$$\begin{aligned}\mathrm{Tr}[\not\epsilon_1^*(\not P_1 + M_1)\not\epsilon_2^*(1-\gamma_5)(\not P_V + M_V)\not\epsilon_V] &= M_V \mathrm{Tr}[\not\epsilon_1^* \not P_1 \not\epsilon_2^* \not\epsilon_V(1+\gamma_5)] + M_1 \mathrm{Tr}[\not\epsilon_1^* \not\epsilon_2^* \not P_V \not\epsilon_V(1-\gamma_5)] \\ &= 4\{M_V[-\xi(\epsilon_1^*\cdot\epsilon_2^*)(P_1\cdot\epsilon_V) + \zeta(\epsilon_1^*\cdot\epsilon_V)(P_1\cdot\epsilon_2^*) + i\eta\varepsilon^{\alpha\beta\mu\nu}\epsilon_{1*\alpha}P_{1\beta}\epsilon_{2\mu}^*\epsilon_{V\nu}] \\ &\quad + M_1[-\xi(\epsilon_1^*\cdot P_V)(\epsilon_2^*\cdot\epsilon_V) + \zeta(\epsilon_1^*\cdot\epsilon_V)(\epsilon_2^*\cdot P_V) - i\eta\varepsilon^{\alpha\beta\mu\nu}\epsilon_{1\alpha}\epsilon_{2\beta}^* P_{V\mu}\epsilon_{V\nu}]\} \\ &= 4\left\{P\left[(\zeta - \xi)(E_V E_1 - P^2)\left[\frac{1}{M_V} + \frac{1}{M_1}\right] + \xi(M_V + M_1)\right]\delta_{\lambda_V=0}\delta_{\lambda_1=0}\delta_{\lambda_2=0} \right. \\ &\quad + [\xi P(E_V - E_1) \pm \eta(E_V E_1 - P^2 + M_V M_1)]\delta_{\lambda_V=0}\delta_{\lambda_1=\pm}\delta_{\lambda_2=\mp} \\ &\quad + [\xi P(E_V - E_1) \mp \eta(E_V E_1 - P^2 + M_V M_1)]\delta_{\lambda_V=\pm}\delta_{\lambda_1=0}\delta_{\lambda_2=\pm} \\ &\quad \left. + [\xi P(M_V + E_1) \pm \eta(E_V M_1 + M_V M_1)]\delta_{\lambda_V=\pm}\delta_{\lambda_1=\pm}\delta_{\lambda_2=0}\right\} \ .\end{aligned} \tag{A33}$$

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