

## ***B*- and *L*-violating couplings in the minimal supersymmetric standard model**

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Couplings that may appear in the superpotential of the supersymmetric extension of the standard model but which do not occur in the standard model itself are examined. Experimental constraints on these couplings are examined in the context of natural assumptions on their values. Additional discrete symmetries are considered in cases where the natural values exceed the experimental constraints.

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### **I. COUPLINGS IN THE SUPERSYMMETRIC STANDARD MODEL**

In the supersymmetric extension of the standard model with minimal particle content, the following Yukawa interaction terms are required in the superpotential in order to give the fermions their masses:

$$W \ni h_{ij}^{(1)} L_i e_j H + h_{ij}^{(2)} Q_i d_j H + h_{ij}^{(3)} Q_i u_j \bar{H} . \quad (1)$$

We have adopted the usual notation (see, for example, [1]). Here  $h^{(n)}$  denote the coupling constants. The left-handed SU(2) doublet superfields are  $L$ ,  $Q$ ,  $H$ , and  $\bar{H}$ . They contain the left-handed lepton doublet, the left-handed quark doublet, and the Higgs doublets. The left-handed antiparticles are represented by the chiral superfields  $e$ ,  $d$ , and  $u$ . They contain  $(e_R)^c$ ,  $(d_R)^c$ , and  $(u_R)^c$ . The indices  $i, j$  denote generations. The gauge group indices are suppressed.

The terms in Eq. (1) are necessary to generate masses for the quarks and leptons when the neutral scalar components of  $H$  and  $\bar{H}$  obtain vacuum expectation values (VEV's). These VEV's ( $\langle v \rangle$  and  $\langle \bar{v} \rangle$ ) are constrained by the  $W$  mass to satisfy

$$\langle v \rangle^2 + \langle \bar{v} \rangle^2 = (174 \text{ GeV})^2 . \quad (2)$$

This is determined by minimizing the effective potential of the neutral scalars ( $h$  and  $\bar{h}$ ) in  $H$  and  $\bar{H}$ . This effective potential, including soft supersymmetry-breaking terms (parametrized by masses  $m_1$  and  $m_2$ ), is

$$V = m_1^2 |h|^2 + m_2^2 |\bar{h}|^2 + \frac{g_1^2 + g_2^2}{8} (|h|^2 - |\bar{h}|^2)^2 . \quad (3)$$

This potential contains an additional U(1) (Peccei-Quinn) symmetry [2] under which  $h$  and  $\bar{h}$  undergo independent phase rotations. The breaking of this symmetry due to  $\langle v \rangle \neq 0 \neq \langle \bar{v} \rangle$  produces an unwanted axion. This problem can be eliminated if the superpotential contains a term

$$W \ni \mu H \bar{H} . \quad (4)$$

The effective potential will then most likely contain the supersymmetry-breaking term

$$\mu m_3 h \bar{h} , \quad (5)$$

where  $m_3$  is another supersymmetry-breaking mass. We will require the existence of the term (4) in the following. We will denote all mass parameters that are associated with supersymmetry breaking by  $M_{\text{SUSY}}$ . In order to avoid the experimental constraints on supersymmetric partners,  $M_{\text{SUSY}} \gtrsim 100 \text{ GeV}$ .

While the above couplings are required in the extension of the standard model, they are not the only couplings allowed by the gauge structure. In a natural theory, we must add all possible couplings. Therefore, we consider the following renormalizable terms in the superpotential:

$$W \ni m_i L_i H + \lambda_{ijk}^{(1)} L_i L_j e_k + \lambda_{ijk}^{(2)} Q_i d_j L_k + \lambda_{ijk}^{(3)} u_i d_j d_k . \quad (6)$$

The first term here can be eliminated by a redefinition of  $L$  and  $H$ . It is assumed that this redefinition has been performed, and this term will henceforth be omitted. Notice that because of the antisymmetric contraction of the SU(2) indices in the second term,  $i$  and  $j$  cannot be equal, and in the fourth term  $j$  and  $k$  cannot be equal; otherwise those terms are zero. Furthermore, we can add higher-dimension (nonrenormalizable) operators, that arise from new physics at some scale  $\Lambda$ . The dimension-5 operators that violate baryon or lepton number and that are allowed by the gauge symmetry to appear in the superpotential are

$$\begin{aligned} W \ni & \frac{\kappa_{ijkl}^{(1)}}{\Lambda} Q_i Q_j Q_k L_l + \frac{\kappa_{ijkl}^{(2)}}{\Lambda} u_i u_j d_k e_l \\ & + \frac{\kappa_{ijk}^{(3)}}{\Lambda} Q_i Q_j Q_k H + \frac{\kappa_{ijk}^{(4)}}{\Lambda} Q_i u_j e_k H \\ & + \frac{\kappa_{ij}^{(5)}}{\Lambda} L_i L_j \bar{H} \bar{H} + \frac{\kappa_i^{(6)}}{\Lambda} L_i H \bar{H} \bar{H} . \end{aligned} \quad (7)$$

Again the gauge indices are suppressed. Here and in the following the value of  $\Lambda$  will be taken to be of order of a grand-unified theory (GUT) scale ( $\sim 10^{15} \text{ GeV}$ ). In the next section we will review the experimental limits that can be obtained on these lepton and baryon-number-violating terms. We shall then survey these limits in light of the values that can be expected for the couplings based on natural assumptions about the Higgs sector. We shall

see that only in those cases where the couplings can lead to proton decay are the expected values in conflict with the experimental limits. Processes that violate only lepton or baryon number (but not both) will have expected rates below the limits. We then examine what discrete symmetries are required to eliminate the unacceptable terms.

## II. EXPERIMENTAL LIMITS

If both  $\lambda^{(2)}$  and  $\lambda^{(3)}$  are nonzero then lepton and baryon numbers are not conserved and the proton can decay. A possible decay diagram is shown in Fig. 1. Using  $M_{\text{GUT}} \approx 10^{15}$  GeV and the experimental limit on the proton lifetime [3], we obtain a limit on the couplings:

$$\sqrt{\lambda^{(2)}\lambda^{(3)}} \lesssim \left[ \frac{M_{\text{SUSY}}}{\text{TeV}} \right] \times 10^{-12}. \quad (8)$$

For  $M_{\text{SUSY}} \sim 1$  TeV, we find

$$\sqrt{\lambda^{(2)}\lambda^{(3)}} \lesssim 10^{-12}. \quad (9)$$

In addition, the first two terms in Eq. (7) can contribute to proton decay. In this case a loop involving superpartners such as a  $W$ -ino is required. An example of a contributing Feynman diagram is given in Fig. 2. From its contribution to proton decay, we obtain a limit on  $\kappa^{(1)}$ :

$$\frac{\kappa_{1121}^{(1)}}{\Lambda} \sim \frac{\kappa_{1122}^{(1)}}{\Lambda} \lesssim (16\pi^2) \frac{M_{\text{SUSY}}}{M_{\text{GUT}}^2} \sim \frac{10^{-11}}{10^{15} \text{ GeV}}. \quad (10)$$

Proton decay via a diagram involving  $\kappa^{(2)}$  that only involves right-handed superfields cannot proceed via  $W$ -ino exchange unless helicity is flipped in the graph (e.g., Fig. 3). Each helicity flip adds a factor of  $M_{\text{quark}}/M_{\text{SUSY}}$ , and hence no meaningful constraint on  $k^{(2)}$  is obtained. By this we mean that if  $k^{(2)}$  were to be as large as possible, consistent with perturbative estimates of the decay process, then the estimate by this mechanism of the proton lifetime would be much longer than the experimental limit. Alternatively, a Higgsino can be used to complete the graph. Then the suppression contains two factors of  $m_{\text{quark}}/v$ , where  $v$  is either  $\langle v \rangle$  or  $\langle \bar{v} \rangle$ . It is conceivable that the constraint (2) on the VEV's may allow such a factor to be large, but for the minimal Higgs content, the ratio of  $\langle \bar{v} \rangle$  and  $\langle v \rangle$  is constrained to be between 1 and  $m_t/m_b$  [6], and hence no meaningful constraint on  $\kappa^{(2)}$  can be obtained.

If lepton number is conserved and baryon number is not, then the relevant experimental constraint arises from the absence of neutron-antineutron oscillations. The leading contribution to such oscillations is from a dia-

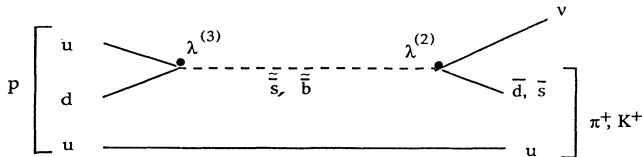


FIG. 1. Possible contribution to proton decay, involving  $\lambda^{(2)}$  and  $\lambda^{(3)}$ .

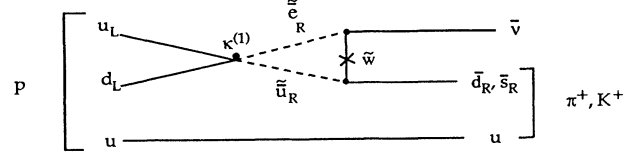


FIG. 2. Possible contribution to proton decay, involving  $\kappa^{(1)}$ .

gram involving four  $\lambda^{(3)}$  vertices (Fig. 4). A limit can be obtained from the oscillation time of the neutron into an antineutron as obtained by the Fréjus Collaboration results [4]. They find the oscillation time in the environment of a nucleus to be

$$\tau_{\text{nucl}} \geq 6.5 \times 10^{31} \text{ yr}. \quad (11)$$

Using nuclear corrections [5], the nuclear stability measurements give a free oscillation time of

$$\tau_{n \leftrightarrow \bar{n}} \geq 1.2 \times 10^8 \text{ s}. \quad (12)$$

This results in a limit on the coupling

$$\lambda_{112}^{(3)} \lesssim \left[ \frac{M_{\text{SUSY}}}{\text{TeV}} \right]^{5/2} \times 10^{-5}. \quad (13)$$

No meaningful constraint is obtained for neutron oscillation by  $\kappa^{(3)}$  due to the additional powers of  $m_p/\Lambda$  that are present in this case.

If lepton number is violated while baryon number is not, constraints arise from the absence of such processes as  $\mu \rightarrow 3e$ . This process is allowed if  $\lambda^{(1)}$  is present. A tree diagram involving  $(\tau)$  sneutrino exchange (Fig. 5), together with the experimental limit on the branching ratio  $B(\mu \rightarrow 3e) \lesssim 1.0 \times 10^{-12}$  from the SINDRUM Collaboration [7], gives this limit on the coupling constants:

$$\lambda^{(1)} \lesssim \left[ \frac{M_{\text{SUSY}}}{\text{TeV}} \right] \times 10^{-3}. \quad (14)$$

A similar limit can be obtained using  $\mu \rightarrow e\gamma$  and the limit from the Crystal Box Collaboration [8].

Unfortunately, the decay  $\mu \rightarrow e\gamma$  via two vertices involving  $\kappa^{(6)}$  and VEV insertions (Fig. 6, for example) gives no meaningful constraint on  $\kappa^{(6)}$ .

The term involving  $\kappa^{(4)}$  violates lepton number while respecting baryon number. It can mediate exotic decays of the  $\tau$  such as  $\tau \rightarrow \pi^0 \mu \nu \chi_0$ , where  $\chi_0$  is the lightest supersymmetric partner (see, for example, Fig. 7). Using an upper bound on exotic decays of the  $\tau$  of about 1%, we

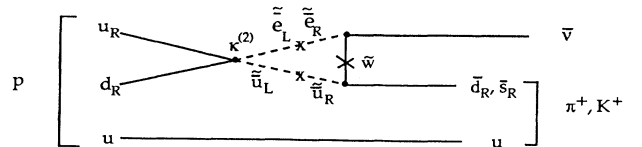


FIG. 3. Possible contribution to proton decay, involving  $\kappa^{(2)}$ .

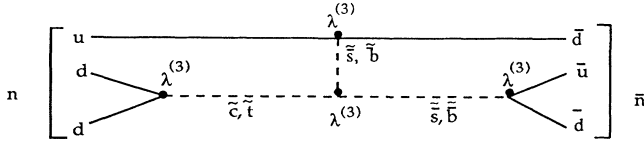


FIG. 4. Possible contribution to neutron oscillation, involving  $\lambda^{(3)}$ .

obtain no meaningful constraint on  $\kappa^{(4)}$ .

The term  $\kappa^{(5)}$  can induce  $\mu \rightarrow e \gamma$  as well as neutrino oscillations. Here experiment [9] constrains the mass difference  $\Delta m_\nu$  and the mixing angle between different neutrino species. If we assume maximal mixing then

$$\Delta m_\nu \sim \left[ \frac{\kappa_{12}^{(5)}}{\Lambda} \right]^2 (250 \text{ GeV})^3, \quad (15)$$

which yields no useful constraint on  $\kappa^{(5)}$ .

### III. APPROXIMATE FLAVOR SYMMETRIES AND NATURAL VALUES FOR COUPLINGS

Fermion masses are (mostly) much smaller than the scale at which the electroweak symmetry breaks and generates their mass. This is indicative of approximate symmetries which involve the separate rotations of the phases of the matter fields [10]. Since the Yukawa coupling constants  $h^{(n)}$  in Eq. (1) are proportional to the quark or lepton masses (divided by the VEV's), we expect that the constants  $\lambda_{ijk}^{(n)}$  in Eq. (6) be

$$\lambda_{ijk}^{(n)} \sim \sqrt{m_i m_j m_k / v^3}, \quad (16)$$

where  $v$  is taken to be 123 GeV, corresponding to  $\langle v \rangle \simeq \langle \bar{v} \rangle$  in Eq. (2). This we call the natural value of the coupling constant  $\lambda_{ijk}^{(n)}$ . Similarly, the natural value of  $\kappa_{i_1, \dots, i_m}^{(n)}$  is

$$\kappa_{i_1, \dots, i_m}^{(n)} \sim \prod_{p=1}^m \sqrt{m_{i_p} / v}. \quad (17)$$

We may now consider the natural values of the couplings which are constrained by experiment in Sec. II. Clearly, our prescription for assigning natural values to the coupling constants gives the largest couplings for the vertices involving third-generation (s)particles. For example, proton decay is most likely to occur to the final state  $K\mu$  or  $K\nu_\mu$ . The natural values for the leading contributions to proton decay are then

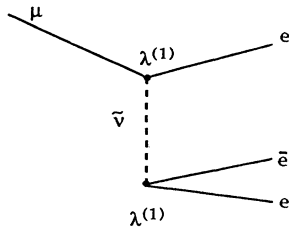


FIG. 5. Possible contribution to muon decay, involving  $\lambda^{(1)}$ .

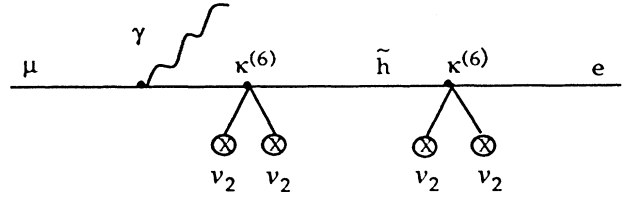


FIG. 6. Possible contribution to muon decay, involving  $\kappa^{(6)}$ .

$$\lambda_{121}^{(2)} \sim \lambda_{112}^{(3)} \sim 10^{-5}, \quad (18)$$

$$\kappa_{1122}^{(1)} \sim 10^{-7}. \quad (19)$$

For neutron oscillation also we must consider  $\lambda_{112}^{(3)}$ , whose natural value is given above. And for  $\mu$  decay to  $3e$  we have the natural value

$$\lambda_{123}^{(1)} \sim 10^{-5}. \quad (20)$$

As we can see, the natural values for the couplings are well below the current experimental limits, with the exception of those involved in proton decay. The combination of  $\lambda^{(2)}\lambda^{(3)}$  in proton decay has a natural value much larger than the experimental limit. If the scale of new physics  $\Lambda$  that appears in the dimension-five couplings is near the GUT scale, then  $\kappa^{(1)}$  may also have a natural value that is not allowed by experiment. Hence if either but not both lepton and baryon number is violated by terms having natural strength, the resulting rates will be too small to have been seen in current experiments.

If both  $\lambda^{(2)}$  and  $\lambda^{(3)}$ , or  $\kappa^{(1)}$ , are present with their natural values then fast proton decay would occur. We now turn to the possible discrete symmetries that would forbid these terms [11].

### IV. DISCRETE SYMMETRIES AND PROTON DECAY

Motivated by the need to suppress proton decay, we consider discrete  $Z_N$  symmetries, possibly resulting from the breaking of some continuous  $U(1)$  symmetry. An analysis of discrete symmetries was given in [11]. The possible symmetries will be characterized by the charges of the chiral superfields under those symmetries, and by the terms in Eqs. (6) and (7) that are allowed.

We write the charges on the superfields as a vector:

$$\alpha = (\alpha_Q, \alpha_u, \alpha_d, \alpha_L, \alpha_e, \alpha_H, \alpha_{\bar{H}}), \quad (21)$$

where the transformation of a superfield  $\Phi$  with  $Z_N$  charge  $\alpha_\Phi$  under the discrete symmetry is

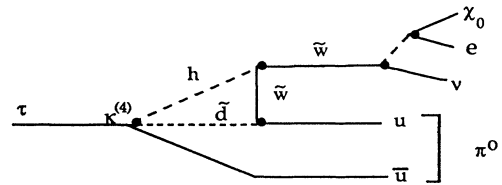


FIG. 7. Possible contribution to tau decay, involving  $\kappa^{(4)}$ .

$$\Phi \rightarrow e^{(2\pi i/N)\alpha_\Phi} \Phi. \quad (22)$$

Due to weak hypercharge invariance of all of the terms in the superpotential, we can assign the charge of one field (choose  $Q$ ) to be zero. Because we require the presence of the Yukawa couplings (1), we have the following conditions on the  $Z_N$  charges of the Higgs:

$$\begin{aligned} \alpha_H &= -\alpha_Q - \alpha_d \pmod{N}, \\ \alpha_H &= -\alpha_L - \alpha_e \pmod{N}, \\ \alpha_{\bar{H}} &= -\alpha_Q - \alpha_u \pmod{N}. \end{aligned} \quad (23)$$

The first two of Eqs. (23) lead to a condition on the charges of the matter superfields:

$$\alpha_Q + \alpha_d = \alpha_L + \alpha_e \pmod{N}. \quad (24)$$

These conditions reduce the number of independent charges to three. Thus we can choose a convenient basis in which the charge of any field is given in terms of three integers ( $m, n$ , and  $p$ ):

$$\begin{aligned} \alpha_R &= (0, -1, 1, 0, 1, -1, 1), \\ \alpha_A &= (0, 0, -1, -1, 0, 1, 0), \end{aligned} \quad (25)$$

and

$$\alpha_L = (0, 0, 0, -1, 1, 0, 0).$$

Now the total charges may be written as

$$\alpha = m\alpha_R + n\alpha_A + p\alpha_L. \quad (26)$$

Notice that the last of the vectors in (25) is equivalent to antilepton number.

We now examine the symmetries that will allow which of the dimension-four and -five couplings (6) and (7). That the  $L$ -violating couplings  $LLe$  and  $QdL$  ( $\lambda^{(1)}$  and  $\lambda^{(2)}$ ) be allowed is the condition

$$m - 2n - p = 0 \pmod{N}. \quad (27)$$

That the  $B$ -violating coupling  $udd$  ( $\lambda^{(3)}$ ) be allowed is the condition

$$m - 2n = 0 \pmod{N}. \quad (28)$$

That the term  $QQQL$  ( $\kappa^{(1)}$ ) be allowed is the condition

$$n - p = 0 \pmod{N}. \quad (29)$$

Requiring that the Higgs mass term (4) be allowed implies that

$$\alpha_H + \alpha_{\bar{H}} = 0 \pmod{N},$$

or  $(30)$

$$n = 0 \pmod{N}.$$

Since we expect that the discrete symmetries will be remnants of a gauged  $U(1)$  symmetry, we may also impose anomaly-cancellation conditions on the charges of the superfields [12]. Transformation (22) tells us that the  $U(1)$  charges are

$$q_i = \alpha_i + \beta_i N, \quad (31)$$

where the  $\beta_i$  are integers. The anomalies of the  $U(1)$  symmetry will cancel if constraints on the charges of the various fermions in the theory are satisfied. For example, the  $Z_N^3$  anomaly-cancellation condition is

$$\sum_i q_i^3 = rN + s \frac{N^3}{8}, \quad (32)$$

where the sum on the left-hand side of this equation runs over all of the quark, lepton, and Higgsino fields of charge  $q_i$ . The right-hand side of this equation arises from fermions that have mass of order of the scale at which the  $U(1) \rightarrow Z_N$  breaking takes place. Such fermions can either have Dirac or Majorana masses. In order to have a Majorana mass a particle must have a  $Z_N$  charge of 0 or  $N/2$ . The integer  $s$  is the number of these Majorana fermions with charge  $N/2$ . Two chiral fermions of charge  $a_j$  and  $a'_j$  can couple to generate one Dirac particle provided that  $a_j + a'_j = z_j N$ , where  $z_j$  is an integer. In this case

$$r = \sum_{\text{Dirac pairs}} z_j (-N^2 z_j^2 + 3a_j N z_j - 3a_j^2). \quad (33)$$

The triangle anomaly-cancellation conditions of involving both  $Z_N$  and  $U(1)_Y$  are [ $Z_N$ - $U(1)$ - $U(1)$  and  $Z_N^2$ - $U(1)$ ]

$$\begin{aligned} \sum_i y_i^2 q_i &= - \sum_{\text{heavy pairs}} (a_j + a_{j'}) y_j^2, \\ \sum_i y_i q_i^2 &= - \sum_{\text{heavy pairs}} (a_j^2 - a_{j'}^2) y_j, \end{aligned} \quad (34)$$

where  $y_i$  are the  $U(1)_Y$  charges. Note that the Majorana fermions (if any) must have  $y_j = 0$ , and the pairs of Dirac fermions have  $y_j = -y_{j'}$ . The  $Z_N$ - $SU(M)$ - $SU(M)$  anomaly-cancellation condition is

$$\sum_i T_i q_i = - \sum_{\text{heavies}} T_j q_j, \quad (35)$$

where  $T_i$  are the Casimir operators of  $SU(M)$  (normalized such that the value on the fundamental representation is  $1/2$ ), and the right-hand side is for heavy fermions. Note that only Majorana particles in real representations of  $SU(M)$  can contribute. The Dirac fermions cancel in pairs. The  $Z_N$ -gravity-gravity anomaly-cancellation condition is

$$\sum_i q_i = r'N + s' \frac{N}{2}, \quad (36)$$

where  $r'$  and  $s'$  have the same origins as  $r$  and  $s$  in Eq. (32). In terms of the exponents  $m, n$ , and  $p$  in Eq. (26) we write these conditions for the  $Z_N$ -gravity-gravity,  $Z_N$ - $SU(2)$ - $SU(2)$ , and  $Z_N$ - $SU(3)$ - $SU(3)$  anomalies as

$$\begin{aligned} (m - 5n - p)N_G + 2nN_H &= tN + s \frac{N}{2}, \\ -(p + n)N_G + nN_H &= t'N, \\ -nN_G &= t''N. \end{aligned} \quad (37)$$

Here  $N_G$  denotes the number of generations of quarks

and leptons, and  $N_H$  the number of sets of  $H$  and  $\bar{H}$ . The integers  $t, t'$ , and  $t''$  depend on the  $\beta_i$  in Eq. (31) and on  $\sum_{\text{heavies}}$  and  $r'$  in Eqs. (35) and (36):

$$\begin{aligned} t &= \sum_i \beta_i + r' , \\ t' &= \sum_i \beta_i - \frac{1}{N} \sum_{\text{heavies}} T_j^{\text{SU}(2)} q_j , \\ t'' &= \sum_i \beta_i - \frac{1}{N} \sum_{\text{heavies}} T_j^{\text{SU}(3)} q_j . \end{aligned} \quad (38)$$

Note that if  $N$  is odd, then we must take  $s$  to be even, and the last term in the first of Eq. (37) may be absorbed into  $tN$ . Note also that as a result of Eq. (30), the Higgs superfields do not contribute to any of the anomalies provided that  $\beta_H$  and  $\beta_{\bar{H}}$  are zero, which we now assume.

If we wish to have a discrete symmetry which is anomaly-free for each generation, we can set  $N_G$  to 1 in Eq. (37). Only one  $Z_2$  symmetry,  $R_2(m=1, n=p=0)$ , survives. The anomaly cancellation for this symmetry requires the addition of one heavy Majorana particle with  $Z_N$  charge 1 per generation. Its restriction on the superpotential is the same as that of conventional  $R$  parity, i.e., it forbids all of the  $B$ - and  $L$ -violating terms in (6). However, it still allows the dangerous term in (7) involving  $\kappa^{(1)}$ . If we extend this case to arbitrary  $N_G$ , then we find an anomaly-free  $R_2$  symmetry, with the addition of  $N_G$  heavy Majorana fermions.

In the case of  $Z_3$ , the anomaly conditions of  $Z_N \times \text{SU}(2) \times \text{SU}(2)$  and  $Z_N \times \text{SU}(3) \times \text{SU}(3)$  are trivially satisfied for the phenomenologically relevant case  $N_G=3$  and  $N_H=1$  for  $n=0$  and  $m$  and  $p$  unconstrained by appropriate choices of the  $\beta_i$ . So we can consider the inequivalent set  $R_3, L_3, R_3 L_3$ , and  $R_3 L_3^2$ . When we require that the other anomaly conditions be satisfied only  $R_3 L_3$  survives. Additional heavy fields are required to achieve anomaly cancellation. Several choices are possible. For example, choosing the  $\beta_i$  so that the charges of  $(Q, u, d, L, e, H, \bar{H})$  are  $(0, -1, 1, -1, 2, -1, 1)$ , we need the following set of heavy fields: an  $\text{SU}(3)$ -singlet  $\text{SU}(2)$ -doublet Dirac pair with  $(Z_N, \text{U}(1)_Y)$  charges  $(1, 1)$  and  $(2, -1)$ , an  $\text{SU}(3) \times \text{SU}(2)$ -singlet Dirac pair with  $(Z_N, \text{U}(1)_Y)$  charges  $(-1, 2)$  and  $(-2, 2)$ , and an  $\text{SU}(3) \times \text{SU}(2)$ -singlet Dirac pair with  $(Z_N, \text{U}(1)_Y)$  charges  $(0, 0)$  and  $(-3, 0)$  [13].

This symmetry satisfies our requirements regarding proton decay. That is, it allows the coupling  $QdL$ , but not  $udd$ . Furthermore, the term in  $\kappa^{(1)}$  is also disallowed. That  $L$ -violating terms may still be allowed is not troublesome in light of the naturalness assumptions on their coupling constants, as seen in Sec. III.

Note that for  $N$  higher than 3, the conditions (37) require additional generations of particles. We will therefore not consider them.

## V. GENERALIZED $R$ PARITIES

Discrete symmetries in which the fermionic measure also carries a  $Z_N$  charge can also be imposed on supersymmetric theories:

$$d\theta^2 \rightarrow e^{-(2\pi i/N)\alpha_W} d\theta^2 . \quad (39)$$

Under these *generalized  $R$  parities* the superpotential must transform with the opposite charge:

$$W_F \rightarrow e^{(2\pi i/N)\alpha_W} W_F . \quad (40)$$

Such discrete symmetries cannot arise as subgroups of gauge symmetries and hence we can have no requirement of anomaly cancellation. We would expect therefore that there would be many more possible symmetries [11]. Here we will discuss the case of  $Z_2$ .

In order that the mass terms for the quarks and leptons [Eq. (1)] be allowed in the superpotential, the following must be satisfied:

$$\begin{aligned} \alpha_Q + \alpha_d + \alpha_H &= 1 \pmod{2} , \\ \alpha_Q + \alpha_u + \alpha_{\bar{H}} &= 1 \pmod{2} , \\ \alpha_L + \alpha_e + \alpha_H &= 1 \pmod{2} . \end{aligned} \quad (41)$$

In order to retain the Higgs mass term (4), we require

$$\alpha_H + \alpha_{\bar{H}} = 1 \pmod{2} . \quad (42)$$

As before, we can select a basis

$$\begin{aligned} \alpha_R &= (0, 1, 1, 0, 1, 0, 0) , \\ \alpha_A &= (0, 0, 1, 1, 0, 0, 1) , \end{aligned} \quad (43)$$

and

$$\alpha_L = (0, 0, 0, 1, 1, 1, 1) ,$$

so that the total charges can be written as

$$\alpha = m\alpha_R + n\alpha_A + p\alpha_L . \quad (44)$$

Using Eq. (42), we see that the  $H\bar{H}$  term requires  $n=1$ . The  $L$ -violating conditions that allow  $LLe$  and  $QdL$  are

$$m - 2n - p = 1 \pmod{2} . \quad (45)$$

And the  $B$ -violating condition that allows  $udd$  becomes

$$m - 2n = 1 \pmod{2} . \quad (46)$$

For the  $QQQL$  term to be allowed we need

$$n + p = 1 \pmod{2} . \quad (47)$$

Since these equations can be solved by  $p=0, n=1, m=1$ , we are able to find a  $Z_2$  symmetry that is phenomenologically unacceptable, in contrast to the case of Sec. IV. Conventional  $R$  parity is equivalent to the case  $n=1, m=p=0$ , which forbids all of the dimension-four  $B$ - and  $L$ -violating terms.

## VI. CONCLUSIONS

We have examined possible terms that are allowed by gauge invariance in the supersymmetric standard model and that violate lepton and baryon number. The experimental constraints on these terms have been discussed. Using an assumption on the size of these couplings that is inspired by the size of the couplings that give quark and

lepton masses, we have established natural values for the strength of these couplings. Only those terms that can lead to proton decay have natural values that are larger than the experimental constraints.

There is only one possible anomaly-free  $Z_2$  discrete symmetry that can exist without the addition of new particles having masses of order the electroweak scale. This symmetry requires the existence of heavy Majorana fermions and forbids all of the renormalizable  $B$ - and  $L$ -violating terms. However it allows the term  $QQQL$  which produces proton decay at an unacceptable rate and does not constrain the number of generations. It is equivalent to conventional  $R$  parity.

There is only one possible anomaly-free  $Z_3$  discrete symmetry that can exist without the addition of new particles having masses of order the electroweak scale. This

symmetry requires that there be three generations of quarks and leptons and that there exist heavy fermions. It forbids all of the renormalizable  $B$ -violating terms, as well as the term  $QQQL$ . Lepton number is violated in this case. Decays such as  $\mu \rightarrow e\gamma$  are expected to occur at rates below the current limits if the couplings have the values that we expect.

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- [1] I. Hinchliffe, *Annu. Rev. Nucl. Part. Sci.* **36**, 505 (1986).
  - [2] R. Peccei and H. R. Quinn, *Phys. Rev. D* **16**, 1791 (1977).
  - [3] IMB Collaboration, R. Becker-Szendy *et al.*, *Phys. Rev. D* **42**, 2974 (1990).
  - [4] Fréjus Collaboration, Ch. Berger *et al.*, *Phys. Lett. B* **240**, 237 (1990).
  - [5] C. B. Dover, A. Gal, and J. M. Richard, *Phys. Rev. D* **27**, 1090 (1983); *Phys. Rev. C* **31**, 1423 (1985).
  - [6] G. Altarelli, Report No. CERN-TH.6305/91 (unpublished).
  - [7] SINDRUM Collaboration, U. Bellgardt *et al.*, *Nucl. Phys. B* **299**, 1 (1988).
  - [8] Crystal Box Collaboration, R. D. Bolton *et al.*, *Phys. Rev. D* **38**, 2077 (1988).
  - [9] B. Borodovsky *et al.*, *Phys. Rev. Lett.* **68**, 274 (1992).
  - [10] A. Antaramian, L. J. Hall, and A. Rašin, *Phys. Rev. Lett.* **69**, 1871 (1992).
  - [11] L. E. Ibáñez and G. G. Ross, *Phys. Lett. B* **260**, 291 (1991); *Nucl. Phys. B* **368**, 3 (1992).
  - [12] L. Krauss and F. Wilczek, *Phys. Rev. Lett.* **62**, 1221 (1989).
  - [13] This symmetry is equivalent to the “baryon parity” of [11]. Note however that we disagree with the claim of [11] that the symmetry is anomaly-free with the minimal particle content of the standard model.