Weak leptonic decay of light and heavy pseudoscalar mesons in an independent quark model

N. Barik and P. C. Dash
Department of Physics, Utkal University, Bhubaneswar-751004, India
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Weak leptonic decays of light and heavy pseudoscalar mesons are studied in a field-theoretic framework based on the independent quark model with a scalar-vector harmonic potential. Defining the quark-antiquark momentum distribution amplitude obtainable from the bound quark eigenmodes of the model with the assumption of a strong correlation between quark-antiquark momenta inside the decaying meson in its rest frame, we derive the partial decay width with correct kinematical factors from which we extract an expression for the pseudoscalar decay constants f_M. Using the model parameters determined from earlier studies in the light-flavor sector and heavy-quark masses m_c and m_b from the hyperfine splitting of (D^*,D) and (B^*,B), we calculate the pseudoscalar decay constants. We find that while $(f_\pi,f_K)\equiv(138,157\text{ MeV})$; $(f_D,f_{D_s})\equiv(161,205\text{ MeV})$, $(f_B,f_{B_s})\equiv(122,154\text{ MeV})$, and $f_{B_c}=221$ MeV. We also obtain the partial decay widths and branching ratios for some kinematically allowed weak leptonic decay processes.

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I. INTRODUCTION

The discovery of heavy-flavored mesons in the charm and bottom sector has revived overwhelming interest in the study of leptonic weak decays of charged pseudoscalar mesons. These decay rates are usually expressed in terms of a hadronic quantity called the weak decay constant f_M, which is proportional to the matrix element of the quark-antiquark axial-vector current between the vacuum and the pseudoscalar meson (M) state. The decay constants f_M have considerable theoretical and phenomenological importance. Governing the strength of leptonic and nonleptonic decays of pseudoscalar mesons as well as the phenomena such as B^0-$\bar{B}^0$ mixing [1,2], weak decay constants can provide information on the mass of the top quark and on the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix elements. A theoretical estimation of the weak decay constants for the pseudoscalar meson decays of the type $M\rightarrow l\bar{\nu}_l$ requires a rigorous field-theoretic formulation of quark-antiquark annihilation inside the meson bound state to a virtual W boson which subsequently disintegrates to $l\bar{\nu}_l$. But the bound quark-antiquark annihilation within the pseudoscalar meson, like many other low-energy phenomena, cannot be studied in a straightforward manner by the first-principles application of QCD, the underlying theory of strong interaction between quarks and gluons at the structural level of hadrons. Therefore, various phenomenological models [3–7] incorporating the basic features of QCD have been tried to estimate the decay rates and decay constants of these leptonic weak decays. The predictions for the variation of the decay constants with increasing quark masses, as one goes from lighter- to heavier-flavor sectors, are not quite consistent. For example, while many nonrelativistic quark-model calculations [3,4] suggest that $f_K>f_D>f_B$, some of the models based on QCD sum rules [5–7] and lattice calculations [8–10] predict more or less a constant f_M between K and B mesons. Capstick and Godfrey [11] use the so-called "mock-meson" approach to calculate the hadronic matrix elements for obtaining the relativized quark-model expression for f_M. They find $f_{B_c}>f_{D_s}>f_K>f_D>f_{B_s}>f_B>f_\pi$, whereas their value of the ratio $(f_K/f_\pi)\simeq 1.75$, which is much higher than the experimental value of 1.22. The bag model, with its reasonable success in the study of wide-ranging hadronic phenomena, also provides a relativistic framework to estimate the weak decay constants of light as well as heavy pseudoscalar mesons. Donoghue and Johnson [12] had applied the model to light mesons, while Golowich [13] and Claudson [14] extended it to the heavy-meson sector. The predictions of their calculations are more or less reasonable except in the light-meson sector. Thus we find that while there are widely different theoretical predictions for the weak decay constants, experiments in this area have so far made very limited progress. Although in the light-flavor sector the values of f_π and f_K are accurately determined, in the heavy-flavor sector only an upper limit for f_D has been reported [15].

Our motivation in the present work is to investigate the weak leptonic decay of charged pseudoscalar mesons (M) using an alternative scheme of a potential model of relativistic independent quarks. Such a scheme has been very successful in estimating the decay widths and decay constants in the leptonic decay of light neutral vector mesons [16]. It also explains quite satisfactorily the $M1$ transitions among the low-lying vector and pseudoscalar mesons [17]. The potential model adopted here has been used earlier in the study of several low-energy phenomena in the baryonic sector, such as octet baryon masses [18], magnetic moments [19], and weak electric form factors [20], as well as nucleon electromagnetic form factors and charge radii [21]. Application of this model in the ordinary light-meson sector to estimate the $(q\bar{q})$-pion mass consistently with that of PCAC (partial conservation of axial-vector current) pion [22] and $(\rho$-$\pi)$ as well as

(ρ-ω) mass splittings [23] has achieved remarkable success. In view of the success of the model in such wide-ranging phenomena in baryon and meson sectors, we would like to extend here its application to the study of weak leptonic decay of light as well as heavy charged pseudoscalar mesons. Though our primary concern lies with a reliable estimation of the decay constants f_M, we would like to realize first the complete expression for the partial decay width $\Gamma(M \rightarrow l\bar{\nu}_l)$ with correct kinematical factors, from which one can extract the decay constants as well as the branching ratios.

The assumptions involved in our calculations are mainly as follows. We consider that the quark-antiquark pair inside the pseudoscalar meson annihilates to a single massive virtual W-boson which subsequently gives rise to a lepton pair ($l\bar{\nu}_l$). Although in principle the problem can be formulated for any arbitrary meson momentum $\mathbf{P}$, for the sake of simplicity alone the decay is assumed to take place in the meson rest frame. Thus we are led to believe that a strong correlation exists between the quark-antiquark momenta so as to have their total momenta identically zero in the meson rest frame. With such a consideration, the ground state of the decaying pseudoscalar meson can be suitably represented with an appropriate momentum distribution of the bound quark-antiquark pair in the corresponding SU(6)-spin-flavor configuration. Then the transition probability amplitude for the weak leptonic decay, calculated from an appropriate Feynman diagram, can be expressed effectively as the free quark-antiquark pair annihilation amplitude integrated over the model momentum distribution. There is, of course, an obvious difficulty relating to the energy conservation at the hadron-boson vertex, since the sum total of the kinetic energy of the annihilating quark-antiquark pair in the process is not equal to the rest mass energy of the decaying meson. In the absence of a fully rigorous field-theoretic formulation of the bound quark-antiquark annihilation inside the meson, such difficulty arises as a common feature with all the phenomenological models based on leading-order calculations. We therefore content ourselves with accepting the usual assumption that the differential amount of energy is somehow made available to the boson, when quark-antiquark annihilation occurs with the disappearance of a meson bound state. The outline of the rest of this paper is as follows. In Sec. II, we describe briefly the framework of our model in arriving at an appropriate momentum probability amplitude for the quark-antiquark pair in the ground state of a pseudoscalar meson. We obtain, in Sec. III, the transition matrix element and the partial decay width for the weak leptonic decays of light and heavy charged pseudo-scalar mesons, from which we extract expressions for the corresponding decay constants. In Sec. IV, we draw together our results and conclusions.

II. THE BASIC FRAMEWORK

The study of the weak leptonic decay of pseudoscalar mesons using a field-theoretic calculation requires an appropriate representation of the initial state of the decaying meson in terms of its constituent quark-antiquark pair with their respective spin and momenta. But the bound quark and antiquark inside the meson are in a definite energy state having no definite momenta. Nevertheless, one can consider the momentum probability amplitude for the constituent quark and antiquark inside the meson just before they annihilate to a lepton pair. This can be done by a suitable momentum space projection of the corresponding bound quark orbitals derivable in a model, for which one may have to rely on certain simplifying assumptions. The model adopted for our calculation here merits a brief discussion, as follows.

According to our model, a meson in general is pictured as a color-singlet assembly of a quark and an antiquark independently confined by an average flavor-independent potential of the form [16–22]

$$U(r)=\tfrac{1}{2}(1+\gamma^0)(ar^2+V_0) \ . \tag{1}$$

This form is taken as a phenomenological representation for the confining interaction expected to be generated by the nonperturbative multigluon mechanism. The confining interaction is believed to be the dominant one in the mesonic dimensions involved in the present calculation. The quark-gluon interactions at short distance originating from one-gluon exchange and the quark-pion interaction required in the nonstrange light-flavor sector to preserve chiral symmetry are presumed here to be residual interactions compared to the dominant confining interaction. Although these residual interactions treated perturbatively in the model are crucial in generating meson mass splittings [22,23], their roles in the mesonic decay processes are considered less significant. Therefore, to a first approximation, it is believed that the zeroth-order quark dynamics inside the meson core generated by the confining part of the interaction can provide an adequate description of the meson decay processes. In such a picture, the independent quark Lagrangian density in zeroth order is given as

$$\mathcal{L}_q^0(x)=\bar{\psi}_q(x)\left[\frac{i}{2}\gamma^{\mu}\overleftrightarrow{\partial}_{\mu}-m_q-U(r)\right]\psi_q(x) \ . \tag{2}$$

Then the ensuing Dirac equation with $E_q'=E_q-V_0/2$, $m_q'=m_q+V_0/2$, $\lambda_q=(E_q'+m_q')$, and $r_{0q}=(a\lambda_q)^{-1/4}$ admits static solutions of positive and negative energy in zeroth order. Corresponding to the ground-state mesons, these solutions can be obtained in the form

$$\Phi_{q\lambda}^{(+)}(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} ig_q(r)/r \\ \sigma\cdot\hat{\mathbf{r}}f_q(r)/r\end{bmatrix}\chi_\lambda \ ,$$
$$\Phi_{q\lambda}^{(-)}(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} \sigma\cdot\hat{\mathbf{r}}f_q(r)/r \\ -ig_q(r)/r\end{bmatrix}\tilde{\chi}_\lambda \ . \tag{3}$$

Here the spinors χ_λ and $\tilde{\chi}_\lambda$ stand for

$$\chi_\uparrow=\begin{bmatrix}1\\0\end{bmatrix} , \quad \chi_\downarrow=\begin{bmatrix}0\\1\end{bmatrix} ,$$
$$\text{and } \tilde{\chi}_\uparrow=\begin{bmatrix}0\\-i\end{bmatrix} , \quad \tilde{\chi}_\downarrow=\begin{bmatrix}i\\0\end{bmatrix} ,$$

respectively. The reduced radial parts in the upper and lower component solutions corresponding to the quark flavor q are

$$g_q(r)=\mathcal{N}_q(r/r_{0q})\exp(-r^2/2r_{0q}^2)\ ,$$
$$f_q(r)=-(\mathcal{N}_q/\lambda_q r_{0q})(r/r_{0q})^2\exp(-r^2/2r_{0q}^2)\ , \tag{4}$$

when the normalization factor $\mathcal{N}_q$ is given by

$$\mathcal{N}_q^2=8\lambda_q/[\sqrt{\pi}r_{0q}(3E_q'+m_q')]\ . \tag{5}$$

The quark binding energy of zeroth order in the meson ground state is derivable from the bound-state condition

$$\sqrt{\lambda_q/a}\,(E_q'-m_q')=3\ . \tag{6}$$

It may be possible now to obtain the momentum distribution amplitudes for the bound quark and antiquark in the meson ground state from their corresponding eigenmodes derivable in the model. If $G_q(\mathbf{p},\lambda,\lambda')$ is the amplitude for finding a bound quark of flavor q in its eigenmode $\Phi_{q\lambda}^{(+)}(\mathbf{r})$ in a state of definite momentum $\mathbf{p}$ and spin projection λ', it can be written as

$$\Phi_{q\lambda}^{(+)}(\mathbf{r})=\frac{1}{(2\pi)^3}\sum_{\lambda'}\int\frac{d\mathbf{p}}{\sqrt{2E_p}}G_q(\mathbf{p},\lambda,\lambda')U_q(\mathbf{p},\lambda')\times\exp(i\mathbf{p}\cdot\mathbf{r})\ . \tag{7}$$

Here $E_p=[(\mathbf{p}^2+m_q^2)]^{1/2}$ and $U_q(\mathbf{p},\lambda')$ is the usual free Dirac spinor which is normalized according to the relations

$$U_q^\dagger(\mathbf{p},\lambda_1)U_q(\mathbf{p},\lambda_2)=2E_p\delta_{\lambda_1\lambda_2}=V_q^\dagger(\mathbf{p},\lambda_1)V_q(\mathbf{p},\lambda_2)\ ,$$
$$\bar{U}_q(\mathbf{p},\lambda_1)U_q(\mathbf{p},\lambda_2)=2m_q\delta_{\lambda_1\lambda_2}=\bar{V}_q(\mathbf{p},\lambda_1)V_q(\mathbf{p},\lambda_2)\ . \tag{8}$$

The corresponding projection operators are

$$\sum_\lambda U_q(\mathbf{p},\lambda)\bar{U}_q(\mathbf{p},\lambda)=(\not{p}+m_q)\ ,$$
$$\sum_\lambda V_q(\mathbf{p},\lambda)\bar{V}_q(\mathbf{p},\lambda)=(\not{p}-m_q)\ . \tag{9}$$

Then Eq. (7) can be inverted to give

$$G_q(\mathbf{p},\lambda,\lambda')=\frac{U_q^\dagger(\mathbf{p},\lambda')}{\sqrt{2E_p}}\int d\mathbf{r}\,\Phi_{q\lambda}^{(+)}(\mathbf{r})\exp(-i\mathbf{p}\cdot\mathbf{r})\ . \tag{10}$$

Now, using $\Phi_{q\lambda}^{(+)}(\mathbf{r})$ as provided in Eqs. (3)–(6), one can find

$$G_q(\mathbf{p},\lambda,\lambda')=G_q(\mathbf{p})\delta_{\lambda\lambda'}\ , \tag{11}$$

where, with $\alpha_q=1/2r_{0q}^2$,

$$G_q(\mathbf{r})=\frac{i\pi\mathcal{N}_q}{2\alpha_q\lambda_q}\sqrt{(E_p+m_q)/E_p}(E_p+E_q)\exp(-p^2/4\alpha_q)\ . \tag{12}$$

Thus $G_q(\mathbf{p})$ essentially provides the momentum probability amplitude for a quark q in its eigenmode $\Phi_{q\lambda}^{(+)}(\mathbf{r})$ to have a definite momentum $\mathbf{p}$ inside the meson. In a similar manner, one can obtain the momentum probability amplitude $\tilde{G}_q(\mathbf{p})$ for an antiquark in its eigenmode $\Phi_{q\lambda}^{(-)}(\mathbf{r})$ to realize that, for like flavors, $\tilde{G}_q(\mathbf{p})=G_q^*(\mathbf{p})$.

Since any scattering or decay process occurs physically in definite momentum eigenstates of the participating particles, the crux of the problem studied here lies in expressing the initial state of the decaying pseudoscalar meson in a suitable manner so as to reflect an appropriate momentum distribution of the constituent quark-antiquark pair in its corresponding spin-flavor configuration. In view of this, we represent the initial state of the decaying pseudoscalar meson $M\equiv(q_1\bar{q}_2)$ with an arbitrary momentum $\mathbf{P}$ as

$$|M(\mathbf{P})\rangle=\frac{\sqrt{3}}{\sqrt{N(\mathbf{P})}}\sum\zeta_{q_1q_2}^M(\lambda_1,\lambda_2)\times\int d\mathbf{p}_1d\mathbf{p}_2\delta^{(3)}(\mathbf{p}_1+\mathbf{p}_2-\mathbf{P})\times G_M(\mathbf{p}_1,\mathbf{p}_2)b_{q_1}^\dagger(\mathbf{p}_1,\lambda_1)\times\tilde{b}_{q_2}^\dagger(\mathbf{p}_2,\lambda_2)|0\rangle\ , \tag{13}$$

where $b_{q_1}^\dagger(\mathbf{p}_1,\lambda_1)$ and $\tilde{b}_{q_2}^\dagger(\mathbf{p}_2,\lambda_2)$ are, respectively, the quark and antiquark creation operators. The factor $\sqrt{3}$ is due to the effective color-singlet configuration of the meson. $\sum\zeta_{q_1q_2}^M(\lambda_1,\lambda_2)$ stands for the appropriate SU(6)-spin-flavor coefficients for the pseudoscalar meson M. $N(\mathbf{P})$ represents the overall normalization, which after imposing the meson state normalization

$$\langle M(\mathbf{P})|M(\mathbf{P}')\rangle=(2\pi)^3\delta^{(3)}(\mathbf{P}-\mathbf{P}') \tag{14}$$

can be obtained in an integral form as

$$N(\mathbf{P})=\frac{1}{(2\pi)^3}\int d\mathbf{p}_1|G_M(\mathbf{p}_1,\mathbf{P}-\mathbf{p}_1)|^2\ . \tag{15}$$

Finally, $G_M(\mathbf{p}_1,\mathbf{p}_2)$ in Eq. (13) provides the effective momentum distribution amplitude for the quark and antiquark inside the meson. In an independent-particle picture of the present model, $G_M(\mathbf{p}_1,\mathbf{p}_2)$ can be expressed in terms of individual momentum distribution amplitudes $G_{q_1}(\mathbf{p}_1)$ and $\tilde{G}_{q_2}(\mathbf{p}_2)$ of the quark q_1 and antiquark $\bar{q}_2$, respectively. In doing so, we resort to an ansatz that the effective momentum distribution amplitude $G_M(\mathbf{p}_1,\mathbf{p}_2)$ is simply the geometric mean of the individual quark and antiquark momentum distribution amplitudes $G_{q_1}(\mathbf{p}_1)$ and $\tilde{G}_{q_2}(\mathbf{p}_2)$ and is given by

$$G_M(\mathbf{p}_1,\mathbf{p}_2)=[G_{q_1}(\mathbf{p}_1)\tilde{G}_{q_2}(\mathbf{p}_2)]^{1/2}\ . \tag{16}$$

This is a straightforward extension of the ansatz we have followed for the leptonic decay of neutral vector mesons [16] in accordance with the idea of Margolis and Mendel [24]. Such a choice would in fact imply the existence of a strong correlation between the quark-antiquark momenta inside the meson so as to have their total momenta identically zero in the meson rest frame. Now the description of the initial meson state in its rest frame would follow directly from Eqs. (15) and (16), so that

$$|M(0)\rangle = \frac{\sqrt{3}}{\sqrt{N(0)}} \sum \zeta^M_{q_1 q_2}(\lambda_1,\lambda_2) \times \int d\mathbf{p}_1 [G_{q_1}(\mathbf{p}_1)\tilde{G}_{q_2}(-\mathbf{p}_1)]^{1/2} \times b^\dagger_{q_1}(\mathbf{p}_1,\lambda_1) \times \tilde{b}^\dagger_{q_2}(-\mathbf{p}_1,\lambda_2)|0\rangle \ . \quad (17)$$

Thus the decaying pseudoscalar meson state represented in the model by the expression in Eq. (17) with the quark momentum distribution amplitude given in Eq. (12) can enable one to determine the transition probability amplitude for weak leptonic decay.

III. DECAY WIDTHS AND DECAY CONSTANTS

We consider here the weak leptonic decay of charged pseudoscalar mesons such as $\pi^\pm$, $K^\pm$, $D^\pm$, $D_s^\pm$, $B^\pm$, and $B_c^\pm$. Assuming that the main contribution to the weak leptonic decay processes comes from the single virtual boson annihilation of the bound quark-antiquark $(q_1\bar{q}_2)$ pair inside the pseudoscalar meson M, we can illustrate it by the corresponding Feynman diagram in Fig. 1 from which the S-matrix element in the configuration space is written effectively as

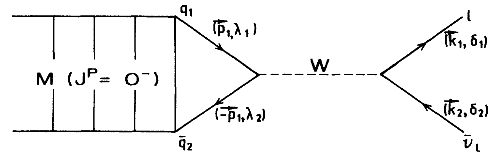

FIG. 1. One-boson contribution to the weak leptonic decay of charged pseudoscalar mesons.

$$S_{f_i} = \langle l(\mathbf{k}_1,\delta_1)\bar{\nu}_l(\mathbf{k}_2,\delta_2)|(-iG_F/\sqrt{2}) \times \int d^4x\, \bar{\psi}^{(-)}_{\nu_l}(x)\gamma^\mu(1-\gamma^5)\psi^{(-)}_l(x) \times \sum_{q_m q_n} \mathcal{V}_{q_m q_n} \bar{\psi}^{(+)}_{q_m}(x)\gamma_\mu(1-\gamma^5) \times \psi^{(+)}_{q_n}(x)|M(0)\rangle \ . \quad (18)$$

Here G_F is the Fermi coupling constant and $\mathcal{V}_{q_m q_n}$ are the CKM matrix elements. The quark and lepton field expansions are taken as

$$\psi_q(x) = \sum_{\lambda'} \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{p}}{\sqrt{2E_{p'}}} [b_q(\mathbf{p}',\lambda')U_q(\mathbf{p}',\lambda')\exp(-ip'x) + \tilde{b}^\dagger_q(\mathbf{p}',\lambda')V_q(\mathbf{p}',\lambda')\exp(ip'x)] \ , \quad (19)$$

$$\psi_l(x) = \sum_{\delta'} \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{k}'}{\sqrt{2E_{k'}}} [d_l(\mathbf{k}',\delta')U_l(\mathbf{k}',\delta')\exp(-ik'x) + \tilde{d}^\dagger_l(\mathbf{k}',\delta')V_l(\mathbf{k}',\delta')\exp(ik'x)] \ . \quad (20)$$

Now, simplifying the leptonic and hadronic parts separately by a vacuum insertion technique and using the initial meson state as per Eq. (17), one can obtain, with $\hat{O} \equiv (1,0,0,0)$ and $(E_{p_1}+E_{p_2}) \simeq M_P$,

$$S_{fi} = -i(2\pi)^4\delta^{(4)}(k_1+k_2-\hat{O}M_P)\mathcal{M}(k_1,k_2,\delta_1,\delta_2) \ , \quad (21)$$

where $\mathcal{M}(k_1,k_2,\delta_1,\delta_2)$ is the transition matrix element for the decay process and M_P is the pseudoscalar meson mass. We must mention the difficulty as well as the assumptions involved here in extracting the correct δ function relating to the energy-momentum conservation. In a zeroth-order description, such as the present one, the total energy available to the lepton pair produced comes out to be the sum-total kinetic energies $(E_{p_1}+E_{p_2})$ of the annihilating quark-antiquark pair, which is not equal to the rest energy of the decaying pseudoscalar meson. This difficulty is commonly encountered in all such leading-order calculations. Here one usually assumes that the differential amount of energy is somehow made available to the intermediate boson, when quark-antiquark annihilation occurs with the vanishing of the meson bound state. With this consideration, $(E_{p_1}+E_{p_2})$ in the argument of the δ function in Eq. (21) has been replaced by the pseudoscalar meson mass M_P. If we write

$$l^\mu(k_1,k_2,\delta_1,\delta_2) = \bar{U}_{\nu_l}(\mathbf{k}_1,\delta_1)\gamma^\mu(1-\gamma^5)V_l(\mathbf{k}_2,\delta_2)/(4E_{k_1}E_{k_2})^{1/2} \quad (22)$$

and

$$h_\mu = \sum \zeta^M_{q_1 q_2}(\lambda_1,\lambda_2) \times \int d\mathbf{p}_1[G_{q_1}(\mathbf{p}_1)\tilde{G}_{q_2}(-\mathbf{p}_1)]^{1/2} \times V_{q_2}(-\mathbf{p}_1,\lambda_2)\gamma_\mu(1-\gamma^5)U_{q_1}(\mathbf{p}_1,\lambda_1) \ , \quad (23)$$

then the transition matrix element in Eq. (21) can be written as

$$\mathcal{M}(k_1,k_2,\delta_1,\delta_2) = \frac{\sqrt{3}}{(2\pi)^3} G_F \mathcal{V}_{q_1 q_2} l^\mu(k_1,k_2,\delta_1,\delta_2) h_\mu / \sqrt{2N(0)} \ . \quad (24)$$

In fact, the spacelike component of h_μ in Eq. (23) vanishes due to angular part integration for the spin-singlet configuration of the pseudoscalar mesons, since

$$\int d\Omega[\bar{V}_{q_2}(-\mathbf{p},\downarrow)\gamma_i(1-\gamma^5)U_{q_1}(\mathbf{p},\uparrow) - \bar{V}_{q_2}(-\mathbf{p},\uparrow)\gamma_i(1-\gamma^5)U_{q_1}(\mathbf{p},\downarrow)] = 0 \ . \quad (25)$$

On the other hand, the nonvanishing timelike component h_0 derives its contribution from the current combination

$$[\bar{V}_{q_2}(-\mathbf{p},\downarrow)\gamma_0(1-\gamma^5)U_{q_1}(\mathbf{p},\uparrow)-\bar{V}_{q_2}(-\mathbf{p},\uparrow)\gamma_0(1-\gamma^5)U_{q_1}(\mathbf{p},\downarrow)] = -2i[(E_1+m_{q_1})(E_2+m_{q_2})-p^2]/[(E_1+m_{q_1})(E_2+m_{q_2})]^{1/2}\,, \quad (26)$$

where $E_i=(p^2+m_{q_i}^2)^{1/2}$. Hence the transition matrix element in Eq. (24) can effectively become

$$\mathcal{M}(k_1,k_2,\delta_1,\delta_2) = \frac{\sqrt{3}}{(2\pi)^3}G_F\mathcal{V}_{q_1q_2}l^0(k_1,k_2,\delta_1,\delta_2)h_0/\sqrt{2N(0)}\,. \quad (27)$$

Then the decay width can be calculated from the expression

$$\Gamma(M\to l\bar{\nu}_l)=\frac{1}{(2\pi)^2}\int d\mathbf{k}_1 d\mathbf{k}_2\delta^{(4)}(k_1+k_2-\hat{O}M_P) \times\sum_{\delta_1,\delta_2}|\mathcal{M}(k_1,k_2,\delta_1,\delta_2)|^2\,, \quad (28)$$

where $\sum_{\delta_1,\delta_2}$ stands for the sum over the final-state lepton spins (δ_1,δ_2). Equation (28) can be simplified to be written as

$$\Gamma(M\to l\bar{\nu}_l)=\frac{3G_F^2|\mathcal{V}_{q_1q_2}|^2}{2(2\pi)^8N(0)}L^{00}H_{00}\,, \quad (29)$$

where the leptonic part

$$L^{00}=\int\frac{d\mathbf{k}_1 d\mathbf{k}_2}{4E_{k_1}E_{k_2}}\delta^{(4)}(k_1-k_2-\hat{O}M_P) \times\mathrm{Tr}[(\not{k}_1+m_l)\gamma^0(1-\gamma^5) \times(\not{k}_2-m_{\bar{\nu}_l})\gamma^0(1-\gamma^5)] \quad (30)$$

and the hadronic part

$$H_{00}=\tfrac{1}{8}\int\frac{d\mathbf{p}\,d\mathbf{p}'}{(E_1E_2E_1'E_2')^{1/2}}[G_{q_1}(\mathbf{p})\tilde{G}_{q_2}(-\mathbf{p})G_{q_1}(\mathbf{p}')\tilde{G}_{q_2}(-\mathbf{p}')]^{1/2} \times[\bar{V}_{q_2}(-\mathbf{p},\downarrow)\gamma_0(1-\gamma^5)U_{q_1}(\mathbf{p},\uparrow)-\bar{V}_{q_2}(-\mathbf{p},\uparrow)\gamma_0(1-\gamma^5)U_{q_1}(\mathbf{p},\downarrow)] \times[\bar{U}_{q_1}(\mathbf{p}',\uparrow)\gamma_0(1-\gamma^5)V_{q_2}(-\mathbf{p}',\downarrow)-\bar{U}_{q_1}(\mathbf{p}',\downarrow)\gamma^0(1-\gamma^5)V_{q_2}(-\mathbf{p}',\uparrow)]\,. \quad (31)$$

After some standard algebra in evaluating the trace followed by the integration in Eq. (30), one can easily find, after taking $m_{\bar{\nu}_l}=0$,

$$L^{00}=2\pi m_l^2(1-m_l^2/M_P^2)^2\,. \quad (32)$$

Using Eq. (12) for the quark and/or antiquark momentum distribution amplitudes and Eq. (26), it is also straightforward to express Eq. (31) as

$$H_{00}=\left[\frac{2\mathcal{N}_{q_1}\mathcal{N}_{q_2}\pi^4}{\alpha_{q_1}\alpha_{q_2}\lambda_{q_1}\lambda_{q_2}}\right]I_M^2 \quad (33)$$

when

$$I_M=\int_0^\infty dp\,p^2\mathcal{A}(p)\exp[-p^2(1/\alpha_{q_1}+1/\alpha_{q_2})/8] \quad (34)$$

and

$$\mathcal{A}(p)=\frac{[(E_1+E_{q_1})(E_2+E_{q_2})]^{1/2}}{[E_1^3E_2^3(E_1+m_{q_1})(E_2+m_{q_2})]^{1/4}} \times[(E_1+m_{q_1})(E_2+m_{q_2})-p^2]\,. \quad (35)$$

The overall normalization factor $N(0)$ for the initial meson state considered in the center-of-mass frame can be obtained from Eq. (15) in the form

$$N(0)=\left[\frac{\mathcal{N}_{q_1}\mathcal{N}_{q_2}}{8\alpha_{q_1}\alpha_{q_2}\lambda_{q_1}\lambda_{q_2}}\right]J_M \quad (36)$$

when

$$J_M=\int_0^\infty dp\,p^2\mathcal{B}(p)\exp[-p^2(1/\alpha_{q_1}+1/\alpha_{q_2})/4] \quad (37)$$

and

$$\mathcal{B}(p)=[(E_1+m_{q_1})(E_2+m_{q_2})/(E_1E_2)]^{1/2} \times(E_1+E_{q_1})(E_2+E_{q_2})\,. \quad (38)$$

Now, substituting Eqs. (32), (33), and (36) into Eq. (29), one can arrive at the final expression of the weak leptonic decay width of a pseudoscalar meson $M\equiv(q_1\bar{q}_2)$ in its usual form as

$$\Gamma(M\to l\bar{\nu}_l)=\frac{G_F^2}{8\pi}|\mathcal{V}_{q_1q_2}|^2M_Pm_l^2(1-m_l^2/M_P^2)^2f_M^2\,, \quad (39)$$

where the decay constant f_M finds expression in the form

$$f_M^2=3I_M^2/(2\pi^2M_PJ_M)\,. \quad (40)$$

Here the integrals I_M and J_M as defined through Eq. (34)–(38) can be calculated numerically by the standard Gaussian quadrature technique. Although we extract here the expression for the weak decay constant f_M

through the calculation of the corresponding decay width, it is always possible to obtain it directly from the hadronic part of the transition matrix element. Therefore, we can use Eq. (40) as a general expression for the weak constant even for a neutral heavy pseudoscalar meson such as B_s^0.

IV. RESULTS AND DISCUSSION

In this section we evaluate the decay constants f_M, and the partial decay widths $\Gamma(M\to l\bar{\nu}_l)$ for the leptonic weak decays of light- as well as heavy-flavored pseudoscalar mesons such as π, K, D, D_s, B, and B_c, using expressions in Eqs. (39) and (40) derived in Sec. III. The calculation primarily involves the potential parameters of the model (a,V_0) and the quark masses ($m_u=m_d$, m_s, m_c, and m_b). The potential parameters (a,V_0) are taken from the earlier applications of the model to baryon and meson sectors [16–22]. They are

$$(a,V_0)\equiv(0.017\,166\ \text{GeV}^3\,,-0.1375\ \text{GeV})\ . \tag{41}$$

The light-quark masses $m_u=m_d$ including the strange quark mass m_s along with the corresponding quark binding energies $E_u=E_d$ and E_s also follow from our earlier work in baryon sector [18]. Such parameters have also been tested in the $(\rho-\pi)$ mass splitting [22] yielding $M_\rho=785.3$ MeV and $M_\pi=140$ MeV and in the leptonic decay [16] of neutral vector mesons (ρ,ω,ϕ) providing their electromagnetic decay constants f_V as 0.22, 0.07, and 0.09, respectively, in very good agreement with the experiment. In order to get a reasonable set of mass parameters in the heavier-flavor sector, we generate the ground-state hyperfine splitting of (D^*,D) and (B^*,B) by appropriately taking into account the center-of-mass correction and subsequently the one gluon exchange correction as per Ref. [22]. The ground-state mass values obtained in this manner for (D^*,D), (D_s^*,D_s), (B^*,B), (B_s^*,B_s), and (B_c^*,B_c) are provided in Table I in comparision with the available experimental data. The quark masses m_q and their corresponding binding energies E_q, which in fact play the role of the effective constituent quark masses in a relativistic model, are provided in Table II. Thus the quark-mass parameters used here can be recorded as,

$$\begin{aligned}(m_u=m_d,m_s)&\equiv(78.75\ \text{MeV},\ 315.75\ \text{MeV})\ ,\\(m_c,m_b)&\equiv(1.493\ \text{GeV},\ 4.777\ \text{GeV})\ .\end{aligned}\tag{42}$$

Although the pseudoscalar meson masses obtained here show a reasonable degree of agreement with the experimental values, the theoretical uncertainty due to the perturbative calculation used here cannot be overlooked. Therefore, in our present calculation of the decay constants and the partial decay widths, we would prefer to use the experimental meson masses. However, in the case of B_c, where the experimental data are not available, we would use the model mass as per Table I. The integrals I_M and J_M in Eqs. (34)–(38) evaluated numerically with the help of the standard Gaussian quadrature technique are provided in Table II. Then it becomes straightforward to calculate the weak decay constants f_M from the expression in Eq. (40). Table III provides the results of our calculations in comparison with the available experimental data [15] as well as the predictions of other models. Capstick and Godfrey [11] have estimated the decay constants using a relativized quark model and have compared their results with those of several others referred to therein. A comparison of results of almost all model calculations available in the literature except a few such as [6,25,26] indicate that f_B is smaller than both f_π and f_D. The present calculation not only confirms this, but at the same time shows a remarkable agreement of f_π and f_K with the corresponding experimental values. This agreement in the light-flavor sector lends credence to our model predictions in the heavier-flavor sector, where experimental data are still not available. It may be noted that our prediction of f_D is well within the experimental upper limit so far available [15].

It is probably more reliable to evaluate the ratios of the decay constants without allowing any scope for possible model constraints to creep in. The recent evaluation of the decay constants by Rosner [27], in which the quark mass limit is combined with a factorization hypothesis, yields sufficiently high values for the decay constants. Nevertheless, the ratios of the decay constants evaluated in this calculation agree well with that of lattice calculations [8,28]. More recently, O'Donnell [29] in a framework of a linear potential model has shown the ratios of the decay constants in agreement with those of lattice calculations. In Table IV, we provide a comparative picture of the ratios of the decay constants evaluated in the present model with several others. It can be observed that our predictions are quite consistent with those of lattice as well as other model calculations. This model, like

TABLE I. Hyperfine splitting of the ground-state heavy-flavored mesons with $\alpha_s=0.37$.

Meson	Spin-averaged mass $\bar{M}$ (GeV)		Meson mass M_P (GeV)	
M	Theory	Expt.	Theory	Expt.
$D^{\pm *}$	1.9751	1.9749	2.0159	2.0101
$D^{\pm}$			1.8526	1.8694
$B^{\pm *}$	5.3140	5.3131	5.3306	5.3246
$B^{\pm}$			5.2641	5.2786
$D_s^{\pm *}$	2.0658	2.0752	2.1074	2.1103
$D_s^{\pm}$			1.9503	1.9690
$B_s^{0 *}$	5.3868	5.4138	5.4032	5.4256
B_s^{0}			5.3378	5.3786
$B_c^{\pm *}$	6.2969		6.3078	
$B_c^{\pm}$			6.2642	

TABLE II. The quark mass m_q and the corresponding binding energy E_q of pseudoscalar mesons together with I_M and J_M.

Meson M	m_{q_1} (GeV)	m_{q_2} (GeV)	E_{q_1} (GeV)	E_{q_2} (GeV)	I_M (GeV)^4	J_M (GeV)^5
$\pi^{\pm}$	0.078 75	0.078 75	0.471 25	0.471 25	0.014 31	0.011 76
$K^{\pm}$	0.078 75	0.315 75	0.471 25	0.591	0.041 42	0.021 38
$D^{\pm}$	0.078 75	1.492 76	0.471 25	1.579 51	0.175 65	0.096 81
$D_s^{\pm}$	1.492 76	0.315 75	1.579 51	0.591	0.315 79	0.184 02
$B^{\pm}$	0.078 75	4.776 59	0.471 25	4.766 33	0.438 86	0.375 62
B_s^0	0.315 75	4.776 59	0.591	4.766 33	0.788 95	0.737 05
$B_c^{\pm}$	1.492 76	4.776 59	1.579 51	4.766 33	2.913 08	4.225 76

TABLE III. Decay constants of pseudoscalar mesons in MeV in comparison with the predictions of other models together with the experiment.

Model	f_π	f_K	f_D	f_{D_s}	f_B	f_{B_s}	f_{B_c}
Expt. [15]	131.73±0.15	160.6±1.3	<310				
Present work	138	157	161	205	122	154	221
Potential [11], Set 2^b	100	153	149	160	96	111	141
Relativized quark [11], Set 3	79	138	131	175	83	119	204
Potential [30]	139	176	150	210	125	175	425
Bag [13]	178	182	148	166	98		
Bag [14]			172	196	149	170	255
Lattice [28]			174±53	234±72	105±34	155±75	
Lattice [31]	141±21	155±21	282±28		183±28		

TABLE IV. Ratios of the decay constants of pseudoscalar mesons in comparison with those of other model calculations.

Model	f_K/f_π	f_π/f_B	f_B/f_D	f_{B_s}/f_{D_s}	f_{B_s}/f_B	f_{D_s}/f_D
Present work	1.14	1.13	0.75	0.76	1.27	1.27
Potential [11], set 2^b	1.53	1.04	0.64	0.69	1.16	1.07
Relativized quark [11], set 3	1.75	0.95	0.63	0.68	1.43	1.34
Potential [29]			0.63	0.67	1.26	1.19
Potential [30]	1.27	1.11	0.83	0.83	1.40	1.40
Factorization [27]			0.68	0.68	1.25	1.25
Bag [13]	1.02	1.82	0.66			1.12
Bag [14]			0.87	0.87	1.13	1.14
Sum rule [6]		1.59	1.10	0.92	1.07	1.25
Lattice [8]			0.62	0.70	1.25	1.11
Lattice [28]			0.60	0.66	1.48	1.34

TABLE V. Partial decay width $\Gamma(M\to l\bar{\nu}_l)$ and the branching ratio $B(M\to l\bar{\nu}_l)$ of pseudoscalar mesons in comparison with the experiment.

Physical process	$\Gamma(M\to l\bar{\nu}_l)$ MeV	$B(M\to l\bar{\nu}_l)$	Expt. $B(M\to l\bar{\nu}_l)$ [15]
$\pi^{\pm}\to\mu^{\pm}\bar{\nu}_\mu$	0.277×10^{-13}	1.094±0.001	0.999 8782±0.000 0014
$K^{\pm}\to\mu^{\pm}\bar{\nu}_\mu$	0.326 ± 10^{-13}	0.613±0.002	0.635±0.002
$D^{\pm}\to\mu^{\pm}\bar{\nu}_\mu$	1.210×10^{-13}	$(0.196\pm0.004)\times10^{-3}$	$<0.72\times10^{-3}$
$D_s^{\pm}\to\mu^{\pm}\bar{\nu}_\mu$	0.495×10^{-11}	$(0.338\pm0.022)\times10^{-2}$	$<3\times10^{-2}$
$B^{\pm}\to\mu^{\pm}\bar{\nu}_\mu$	0.957×10^{-16}	$(0.188\pm0.007)\times10^{-6}$	
$B_c^{\pm}\to\mu^{\pm}\bar{\nu}_\mu$	0.341×10^{-13}		
$D^{\pm}\to\tau^{\pm}\bar{\nu}_\tau$	2.762×10^{-13}	$(0.447\pm0.0096)\times10^{-3}$	
$D_s^{\pm}\to\tau^{\pm}\bar{\nu}_\tau$	4.547×10^{-11}	$(3.109\pm0.193)\times10^{-2}$	
$B^{\pm}\to\tau^{\pm}\bar{\nu}_\tau$	0.214×10^{-13}	$(4.199\pm0.163)\times10^{-5}$	
$B_c^{\pm}\to\tau^{\pm}\bar{\nu}_\tau$	0.821×10^{-11}		

many others, concludes that $f_{B_c} > f_{B_s} > f_B$, $f_{D_s} > f_D$, $f_D > f_B$, and $f_\pi > f_B$.

We can also calculate the partial decay widths of leptonic weak decays using the expression in Eq. (39) and hence the branching ratios. The branching ratio for the weak leptonic decay process of a heavy pseudoscalar meson of $(Q\bar{q})$ type can be expressed as

$$B(M \rightarrow l\bar{\nu}_l) = \frac{G_F^2}{8\pi}\tau_M |\mathcal{V}_{Qq}|^2 M_P m_l^2 (1 - m_l^2/M_P^2)^2 f_M^2 \,, \tag{43}$$

where τ_M is the mean lifetime of the meson M of mass M_P, and $\mathcal{V}_{Qq}$ is the CKM matrix element. Using $|\mathcal{V}_{Qq}|$ and τ_M as per Ref. [15] and the model estimated f_M values, the branching ratios can be calculated from Eq. (43). Table V provides our results for the partial decay widths and the branching ratios of several kinematically allowed weak decay processes of the type $M \rightarrow \mu\bar{\nu}_\mu$ and $M \rightarrow \tau\bar{\nu}_\tau$. We again find that the model predictions are very close to the available experimental values in the light-flavor sector.

Thus we conclude that in view of the overwhelming agreement with other reliable calculations available in the literature, the present model proves to be a very good alternative scheme to study the light as well as heavy pseudoscalar mesons. This model can therefore be further extended to other interesting areas involving B and D mesons, such as their semileptonic and radiative decays. The work on this line is being pursued towards this end.

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[1] ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **192**, 245 (1987).
[2] CLEO Collaboration, M. Artuso *et al.*, Phys. Rev. Lett. **62**, 2233 (1989).
[3] S. N. Sinha, Phys. Lett. B **178**, 110 (1986).
[4] S. Godfrey, Phys. Rev. D **33**, 1391 (1986).
[5] L. J. Reinders, Phys. Rev. D **38**, 947 (1988).
[6] S. Narison, Phys. Lett. B **198**, 104 (1987); **285**, 141 (1992).
[7] C. A. Dominguez and N. Paver, Phys. Lett. B **197**, 423 (1987).
[8] M. B. Gavela *et al.*, Phys. Lett. B **206**, 113 (1988).
[9] T. A. DeGrand and R. D. Loft, Phys. Rev. D **38**, 954 (1988).
[10] R. M. Woloshyn *et al.*, Phys. Rev. D **39**, 978 (1989).
[11] S. Capstick and S. Godfrey, Phys. Rev. D **41**, 2856 (1990).
[12] J. F. Donoghue and K. Johnson, "The pion and an improved static bag," MIT report, 1979 (unpublished).
[13] E. Golowich, Phys. Lett. **91B**, 271 (1980).
[14] M. Claudson, Harvard Report No. 91-546, 1981 (unpublished).
[15] Review of Particle Properties, Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S11 (1992).
[16] N. Barik, P. C. Dash, and A. R. Panda, Phys. Rev. D **47**, 1001 (1993).
[17] N. Barik, P. C. Dash, and A. R. Panda, Phys. Rev. D **46**, 3856 (1992).
[18] N. Barik and B. K. Dash, Phys. Rev. D **33**, 1925 (1986).
[19] N. Barik and B. K. Dash, Phys. Rev. D **34**, 2803 (1986).
[20] N. Barik, B. K. Dash, and M. Das, Phys. Rev. D **23**, 1725 (1985).
[21] N. Barik and B. K. Dash, Phys. Rev. D **34**, 2092 (1986).
[22] N. Barik, B. K. Dash, and P. C. Dash, J. Phys. **29**, 543 (1987).
[23] B. E. Palldino and P. Leal Ferreira, IFT Sao Paulo Report No. IFT/p-35/88 (unpublished).
[24] B. Margolis and R. R. Mendel, Phys. Rev. D **28**, 468 (1983).
[25] P. Cea, P. Colangelo, L. Cosmai, and G. Nardulli, Phys. Lett. B **206**, 691 (1988).
[26] V. S. Mathur and T. Yamawaki, Phys. Lett. **107B**, 127 (1981); Phys. Rev. D **29**, 2057 (1984).
[27] J. Rosner, Phys. Rev. D **42**, 3732 (1990).
[28] C. Bernard, T. Draper, G. Hockney, and A. Soni, Phys. Rev. D **38**, 3540 (1988).
[29] P. J. O'Donnell, Phys. Lett. B **261**, 136 (1991).
[30] H. Krasemann, Phys. Lett. **96B**, 397 (1980).
[31] H. W. Hamber, Phys. Rev. D **39**, 896 (1989).