Spontaneous *CP* violation in supersymmetric theories

Alex Pomarol
Santa Cruz Institute for Particle Physics, University of California,
Santa Cruz, California 95064
(Received 3 August 1992)

We study the minimal version of the supersymmetric standard model with spontaneous *CP* breaking. In this model, the Kobayashi-Maskawa matrix is real and contributions to ε arise from box diagrams involving squarks. We analyze the region of the parameter space which corresponds to values for ε, ε'/ε, and the neutron electric dipole moment (NEDM) in agreement with the experimental data. We show that the *CP*-violating phases must be of order 10^{-2} and the NEDM lies near its present experimental limit.

PACS number(s): 12.15.Cc, 11.30.Er, 11.30.Pb, 11.30.Qc

I. INTRODUCTION

The origin of *CP* nonconservation is not yet fully understood. One major concern in recent times has been to understand why in the standard model (SM) the *CP*-violating phase of QCD, $\bar{\theta}$, is so small (i.e., the strong *CP* problem). Several solutions to this puzzle have been proposed in the literature. The most appealing idea is the Peccei-Quinn (PQ) mechanism [1].

In supersymmetric theories, however, even if one can arrange for $\bar{\theta}\simeq 0$, a new problem arises. It has been known for a long time that the supersymmetric extension of the SM contains a number of new sources of *CP* violation whose contribution to the neutron electric dipole moment (NEDM) is two or three orders of magnitude larger than the experimental limit if the phases that parametrize the *CP* violation, φ, are of order 1 [2–7]. Thus, a fine-tuning of parameters is necessary such that $\varphi \lesssim 10^{-2}-10^{-3}$. Since such *CP*-violating phases arise from different sectors of the supersymmetric model, this multiple fine-tuning appears to be totally unnatural. In fact, it violates 't Hooft's naturalness condition which states that a parameter is only allowed to be very small if setting it to zero increases the symmetry of the theory [8].

One simple and very attractive solution to this problem is to require that *CP* is spontaneously broken. In this case, *CP* invariance is imposed on the initial Lagrangian and it is broken by the ground state along with the gauge symmetry. One example in which such an idea is implemented is the supersymmetric version [9] of the Barr and Nelson models [10]. Models of this type require the existence of exotic superheavy fermions which mix with the standard light fermions. At low energy, such models are indistinguishable from the Kobayashi-Maskawa (KM) model. Another example is given in Ref. [11] where *CP* is spontaneously broken at a high-energy scale inducing complex scalar mass terms at low energy. In such a model, extra color-singlet and -triplet fields are necessary.

In this paper we analyze the minimal version of the supersymmetric standard model with spontaneous *CP* violation (SCPV). In such a model, *CP* violation derives from the phases of the vacuum expectation values (VEV's) of the Higgs bosons. The purpose of this work is to determine whether this model can explain the *CP* nonconservation observed in the K-$\bar{K}$ system while being consistent with the present bounds on the NEDM.

It has been claimed for a long time that supersymmetric models need the KM phase in order to explain the *CP*-violating phenomena [12]. This statement is based on examining spontaneously broken $N=1$ supergravity theories with a flat Kähler metric (i.e., all the scalar kinetic terms are canonical). In these theories,

$$\Delta m_{\tilde{q}}^2 \equiv (m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2) \sim (m_{q_1}^2 - m_{q_2}^2) , \quad (1)$$

where $\tilde{q}_1$ and $\tilde{q}_2$ are the scalar partners (squarks) of the q_1 and q_2 quarks, respectively. Since box diagrams involving superpartners (superbox) are suppressed by a factor $(\Delta m_{\tilde{q}}^2/m_{\tilde{q}}^2)^2$ due to the so-called super-Glashow-Iliopoulos-Maiani (GIM) mechanism, their contributions to ε are negligible for $\varphi \lesssim 10^{-2}$.

When more general $N=1$ supergravity theories are considered, Eq. (1) is no longer satisfied [13] and superbox diagrams can be phenomenologically important. In fact, in such theories, the squark mass matrix is completely arbitrary, and as a result its diagonalization is independent of the diagonalization of the quark matrix. This implies that the unitary matrices V^a, which characterize the Higgs or gauge fermionic-partner (Higgsino or gaugino, $\tilde{\chi}_a$) interactions with quarks and squarks, i.e.,

$$\mathcal{L}_{\rm int} \propto V_{ij}^a \bar{q}_i \tilde{q}_j (1-\gamma_5)\tilde{\chi}_a + \text{H.c.} , \quad (2)$$

are arbitrary.

In this paper we will work within the context of such general $N=1$ supergravity theories. Our results, however, can be easily generalized to a wide class of effective low-energy supersymmetric models.[1] Following Ref. [15], we will assume approximately diagonal forms for

[1]It has been recently emphasized [14] that the idea of supersymmetry at the weak scale should be tested without regard to the Planck-scale origin of any specific model.

the super-KM matrices, V^a, similar to the standard KM matrix:

$$V^a \simeq \begin{bmatrix} 1 & O(\sin\theta_C) & \sim 10^{-2} \\ O(\sin\theta_C) & 1 & O(\sin\theta_C) \\ \sim 10^{-2} & O(\sin\theta_C) & 1 \end{bmatrix} , \quad (3)$$

where θ_C is the Cabibbo angle. Even with this natural assumption, the contribution of the superbox diagrams to flavor-changing neutral-current (FCNC) processes is too large unless there is some mass degeneracy between squarks. Bounds on $\Delta m_{\tilde q}^2/m_{\tilde q}^2$ from FCNC processes were studied many years ago in Refs. [15,16]. A recent analysis can be found in Ref. [17]. Possible origins for such a degeneracy have been explored in Ref. [18].

II. THE HIGGS SECTOR OF THE MODEL

The minimal supersymmetric extension of the standard model (MSSM) requires two Higgs doublets. The VEV's of the two neutral scalars can be chosen real without loss of generality [19] so that *CP* cannot be spontaneously broken. It has been recently claimed that SCPV can occur in the MSSM when radiative corrections to the Higgs potential are included [20,21]. This, however, requires [21] a Higgs boson lighter than that permitted by the Higgs-boson search at the CERN e^+e^- collider LEP [22]. An extension of the MSSM Higgs sector is thus required if we want to have SCPV in supersymmetric theories.

Let us consider a model with two Higgs doublets $H_1 \equiv (H_1^0, H_1^-)$ and $H_2 \equiv (H_2^+, H_2^0)$ with hypercharges $Y=-1$ and $Y=1$, respectively, and a complex singlet N. Such a Higgs sector has been extensively studied in the literature [23,24] and provides an attractive solution to the μ problem. The most general renormalizable and gauge-invariant superpotential for one quark generation is given by

$$W = \tfrac{1}{3}\lambda_1 N^3 + \lambda_2 H_1 H_2 N + \tfrac{1}{2}\mu_N N^2 + \mu H_1 H_2 + h_d H_1 \tilde{Q}\tilde{D}^c + h_u H_2 \tilde{Q}\tilde{U}^c , \quad (4)$$

where $\tilde{Q}$ is the squark doublet, $\tilde{U}$ and $\tilde{D}$ are the squark singlets, and we have fixed the notation such that $H_1H_2 \equiv H_1^0 H_2^0 - H_1^- H_2^+$. The scalar potential in the supersymmetric limit is given by

$$V = \frac{1}{2}\left[\sum_a \left[\frac{1}{2} g A_i^* \sigma_{ij}^a A_j \right]^2 + \left[\frac{1}{2} g' Y_i A_i^* A_i \right]^2 \right] + \left| \frac{\partial W}{\partial A_i} \right|^2 , \quad (5)$$

where A_i collectively denotes all scalar fields appearing in the theory. After spontaneous supergravity breaking, new terms are induced in the low-energy Higgs potential which softly break global supersymmetry (SUSY). These are given by [13]

$$V_{\text{soft}} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 |N|^2 + m_{12}^2 H_1 H_2 + m_N^2 N^2 + A_N N H_1 H_2 + A_N' N^3 + \text{H.c.} \quad (6)$$

CP invariance implies that all couplings and mass parameters are real. In order to have the desired pattern of gauge symmetry breaking, we will assume that only the neutral components of the Higgs bosons develop VEV's:

$$\langle H_1^0 \rangle = v_1, \quad \langle H_2^0 \rangle = v_2 e^{i\rho}, \quad \langle N \rangle = n e^{i\xi} . \quad (7)$$

For $\rho, \xi \neq n\pi (n \in \mathbb{Z})$, *CP* is broken along with the gauge symmetry. The phase-dependent part of the Higgs boson potential can be written as

$$V(\rho,\xi) = A \cos\xi + B \cos 2\xi + C \cos 3\xi + D \cos\rho + E \cos(\rho - 2\xi) + F \cos(\rho + \xi) , \quad (8)$$

where the new A, B, C, D, E, and F quantities can be easily related to the original parameters. It can be shown that there exists a region in the parameter space where the minimum of the potential is at $\rho, \xi \neq n\pi$.[2] From Eq. (8), we see that when ξ is small, ρ must be close to 0 or π. This means that only one fine-tuning, $\xi \ll 1$, will be necessary in order that all *CP*-violating effects are small; we will see that this is required by the NEDM bound. The smallness of ξ does not violate 't Hooft's naturalness condition.

In our model, the *CP*-violating processes will always involve neutral-Higgs-boson couplings. Prior to spontaneous gauge symmetry breaking, the neutral Higgs interactions with fermions are given by (following the notation of Ref. [23])

$$\mathcal{L}_{\text{int}} = -h_u H_2^0 \bar{u}_R u_L - h_d H_1^0 \bar{d}_R d_L - g(H_1^{0*} \bar{\tilde{H}} P_L \tilde{W} + H_2^{0*} \bar{\tilde{W}} P_L \tilde{H}) - (\tfrac{1}{2})^{1/2} (H_1^{0*} \bar{\tilde{H}}_1 - H_2^{0*} \bar{\tilde{H}}_2) P_L (g \tilde{W}_3 - g' \tilde{B}) - \lambda_2 (H_1^0 \bar{\tilde{N}} P_L \tilde{H}_2 + H_2^0 \bar{\tilde{N}} P_L \tilde{H}_1 + N \bar{\tilde{H}}_1 P_L \tilde{H}_2 - N \bar{\tilde{H}} P_L \tilde{H}) - 2\lambda_1 N \bar{\tilde{N}} P_L \tilde{N} + \text{H.c.} , \quad (9)$$

where $P_L = (1-\lambda_5)/2$, and the relevant neutral Higgs interactions with squarks are given by[3]

$$\mathcal{L}_{\text{int}} = h_u (\lambda_2 H_1^{0*} N^* + \mu H_1^{0*} + m_6 A_u H_2^0) \tilde{u}_R^* \tilde{u}_L + h_d (\lambda_2 H_2^{0*} N^* + \mu H_2^{0*} + m_6 A_d H_1^0) \tilde{d}_R^* \tilde{d}_L + \text{H.c.} \quad (10)$$

---

[2]This is only true for the most general superpotential of Eq. (4) or for those theories beyond the minimal $N=1$ supergravity [25].

[3]The coefficients of the soft-breaking terms $H_i \tilde{q}\tilde{q}$ are in principle arbitrary [13]. In agreement with standard theoretical prejudices [14], we will assume that these coefficients are proportional to the Yukawa coupling h_q. Otherwise contributions to the NEDM will be too large.

When the neutral Higgs bosons develop VEV's, the interactions of Eq. (9) and Eq. (10) induce complex mass terms for the gauginos, Higgsinos, quarks, and squarks.[4] Since the phases ρ and ξ cannot be rotated away, *CP* is violated by the fermion and scalar propagators. The gauginos and Higgsinos mix with each other, and the resulting mass eigenstates are called charginos ($\tilde{\chi}^+$) and neutralinos ($\tilde{\chi}^0$). Notice that our model is similar to the supersymmetric model with explicit *CP* violation. However, there are some important differences. The KM matrix is now real [26] as is the gluino mass (m_g). Moreover, all *CP*-violating phases φ can be written as a function of only the two phases ρ and ξ, i.e., $\varphi=\varphi(\rho,\xi)$.

III. CONTRIBUTIONS TO ε, ε′ AND THE NEDM

In this section we calculate the predictions of the model described in the previous section for the ε and ε′ parameters and the NEDM. Following the notation of Ref. [27], we have

$$\varepsilon=\frac{1}{\sqrt{2}}e^{i\pi/4}\left[\frac{1}{2}t_m+t_0\right] , \qquad (11)$$

$$\varepsilon'=-\frac{1}{\sqrt{2}}ie^{i(\delta_2-\delta_0)}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}t_0 , \qquad (12)$$

where A_i are the weak-decay amplitudes of the neutral kaon to two pions of isospin i, δ_i are the corresponding phases from strong interactions and

$$t_i=\frac{\mathrm{Im}A_i}{\mathrm{Re}A_i},\quad t_m=\frac{\mathrm{Im}M_{12}}{\mathrm{Re}M_{12}} , \qquad (13)$$

where M_{ij} is the neutral kaon mass matrix in the $K^0-\bar{K}^0$ basis. We have used the phase convention such that $t_2=0$. From the experimental values [22,28]

$$\begin{aligned}&|\varepsilon|\simeq 2.26\times10^{-3} ,\\ &\varepsilon'/\varepsilon\lesssim 1.45\times10^{-3} ,\\ &|A_2/A_0|\simeq\tfrac{1}{22} ,\\ &\delta_2-\delta_0\simeq -53^\circ ,\end{aligned} \qquad (14)$$

we have

$$\begin{aligned}&t_m\simeq 2\sqrt{2}|\varepsilon|\simeq 6\times10^{-3} ,\\ &t_0\simeq\sqrt{2}|A_0/A_2||\varepsilon'|\lesssim 10^{-4} .\end{aligned} \qquad (15)$$

To begin with, let us consider the contribution to t_m. The only diagrams that considerably contribute to $\mathrm{Im}M_{12}$ are those involving phases in the propagators of the superpartners. In order to have a complex $\tilde{\chi}^0$ or $\tilde{\chi}^+$ propagator, it is easy to see from Eq. (9) that mixing between gauginos and Higgsinos is required. As a result, the superbox diagrams must involve a quark-squark-

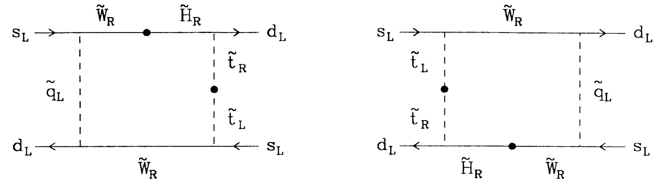

FIG. 1. Dominant one-loop contribution to $\mathrm{Im}M_{12}$. We denote by ● a $\tilde{H}$-$\tilde{W}$ or $\tilde{t}_L$-$\tilde{t}_R$ mixing.

Higgsino coupling so that their contribution to $\mathrm{Im}M_{12}$ is suppressed by a factor m_q/m_W. If the phases arise from a squark propagator, then $\tilde{q}_L-\tilde{q}_R$ mixing is necessary [see Eq. (10)]. In this case, superbox diagrams receive a suppression factor $m_q/m_{\tilde{q}}$. Thus, only superbox diagrams involving $\tilde{t}$ are non-negligible. The largest of these contributions arise from the diagrams shown in Fig. 1. In addition, there are two more diagrams like those of Fig. 1 but with $\tilde{H}$ and $\tilde{W}$ interchanged in the $\tilde{H}$-$\tilde{W}$ fermion line, and contributing an opposite phase. Since these diagrams involve different super-KM matrix elements, there will be only a partial cancellation, which we denote by S. In order that the contribution of the diagrams of Fig. 1 be large enough, we need a small mass for the lightest chargino and squark. In such a case, the contribution to $\mathrm{Re}M_{12}$ given by the diagram shown in Fig. 2 is also large and a degeneracy between $\tilde{u}_L$ and $\tilde{d}_L$ is required in order to be consistent with the experimental value.[5] From Ref. [15], we have, for $m_{\tilde{\chi}^+}\sim m_{\tilde{q}}\sim 100$ GeV,

$$\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2}\lesssim\frac{1}{30} . \qquad (16)$$

When the bound (16) is saturated, the value of t_m is given by the ratio between the diagrams of Figs. 1 and 2. A rough calculation gives

$$t_m\simeq\frac{(m_t/\sqrt{2}m_W\sin\beta)V_{13}S\sin\varphi}{\Delta m_{\tilde{q}}^2/m_{\tilde{q}}^2} , \qquad (17)$$

where $\tan\beta=v_2/v_1$ and V_{13} is, according to Eq. (3), $\sim10^{-2}$. We have assumed the maximal $\tilde{H}$-$\tilde{W}$ and $\tilde{t}_L$-$\tilde{t}_R$ mixing. This is a natural assumption for $m_{\tilde{\chi}^+}\sim m_t\sim 100$ GeV. The fact that the diagrams of Fig. 1 involve the $\bar{d}P_R\tilde{H}^c\tilde{t}_R$ coupling which is proportional to $m_t/\sqrt{2}m_W\sin\beta$ is crucial: the super-GIM mechanism does not apply and such diagrams receive only one power of the suppression factor $\Delta m_{\tilde{q}}^2/m_{\tilde{q}}^2$. For $\tan\beta\simeq1$, $m_t\simeq 2m_W$, and $S\simeq1/2$, we have

$$t_m\simeq 3\times10^{-1}\sin\varphi . \qquad (18)$$

Since this is the maximal contribution to t_m, we have

[4]Neutral Higgs complex mass terms are also induced. For small φ, however, they do not give rise to any significant phenomenological implication.

[5]The contribution of $\mathrm{Re}M_{12}$ from superbox diagrams involving neutralinos and gluinos can be neglected if the masses of these particles are larger than 200 GeV [17].

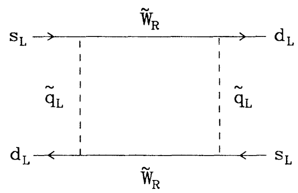

FIG. 2. Dominant one-loop contribution to ReM_{12}.

from Eq. (15) the lower bound

$$\varphi \gtrsim 2\times 10^{-2} . \tag{19}$$

To estimate t_0, we assume that A_0 is dominated by the penguin diagrams [29]. The largest contribution to ImA_0 arises from penguin diagrams involving charginos and top squarks (Fig. 3). The chargino penguin diagrams also contribute to ReA_0. The dominant contribution is shown in Fig. 4 and leads to the effective Lagrangian (for $m_{\tilde{\chi}^+}\sim m_{\tilde{q}}$) [30]

$$\mathcal{L}_{\rm sp}\simeq \frac{\alpha_s\alpha_W}{24m_{\tilde{q}}^2}\sin\theta_C\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2}\mathcal{O}_{LR}+\text{H.c.} , \tag{20}$$

where

$$\mathcal{O}_{LR}=(\bar{s}_L\gamma_\mu T^a d_L)(\bar{q}_R\gamma^\mu T^a q_R) ,$$

and T^a is the Hermitian SU(3)$_c$ generator. Considering only the chargino contribution, the ratio t_0 is found to be of the same order of the ratio t_m given in Eq. (17). However, the dominant contribution to ReA_0 arises from the standard penguin diagram:

$$\mathcal{L}_p\simeq \frac{\alpha_s\alpha_W}{3m_W^2}\sin\theta_c\ln\frac{m_c^2}{m_K^2}\mathcal{O}_{LR}+\text{H.c.} \tag{21}$$

Therefore,

$$t_0\simeq \frac{m_W^2}{16m_{\tilde{q}}^2}\frac{(m_t/\sqrt{2}m_W\sin\beta)V_{13}S\sin\varphi}{\ln(m_c^2/m_K^2)} . \tag{22}$$

For the same values of the parameters considered in obtaining Eq. (18), we have $t_0\sim 6\times 10^{-6}$ in agreement with the experimental limit given in Eq. (15). We must remark that the predictions for t_0 have large uncertainties [31] and cannot be considered a precision test of the model.

Constraints from the NEDM, d_n, are more severe. The predictions of our model for d_n can be estimated using previous calculations of the NEDM in supersymmetric models with explicit *CP* violation. Such calculations can

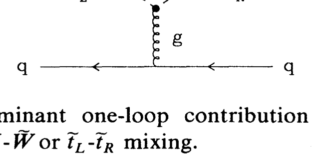

FIG. 3. Dominant one-loop contribution to ImA_0. We denote by ● a $\tilde{H}$-$\tilde{W}$ or $\tilde{t}_L$-$\tilde{t}_R$ mixing.

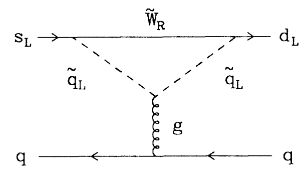

FIG. 4. Chargino one-loop contribution to ReA_0.

be found in Refs. [2–4]; more recent analyses are given in Refs. [5–7]. The dominant contribution to d_n arises from diagrams involving gluinos. Although the *CP*-violating phase that appears in such diagrams is different from the phase that appears in Fig. 1, both are of the same order (assuming no accidental cancellation). Different contributions to the NEDM arising from the induced quark electric dipole moment, quark chromoelectric dipole moment, Weinberg's three-gluon operator and one-photon-three-gluon operator have been considered in Ref. [6]. Using the experimental value [32] $|d_n|<1.2\times 10^{-25}$ e cm, the tightest bound found in Ref. [6] for $m_g\sim m_{\tilde{q}}\sim m_Z$ is $\varphi\lesssim 7.5\times 10^{-3}$.

The next most important contribution to the NEDM comes from diagrams involving charginos (e.g., Fig. 5). For $m_{\tilde{d}}\sim m_{\tilde{\chi}^+}$, this contribution is given by [3]

$$d_n\simeq \frac{eg^2}{36\sqrt{2}\pi^2 m_W\cos\beta}\frac{m_d}{m_{\tilde{d}}}\sin\varphi , \tag{23}$$

which also gives rise to an upper bound for φ of $\sim 10^{-2}$. The fact that these bounds are so close to that of Eq. (19) suggests that a more rigorous calculation should be carried out in order to determine whether this model is ruled out. Notice, however, that we still have enough freedom in the parameter space to decrease the contribution to the NEDM without decreasing the contribution to t_m coming from the diagrams of Fig. 1. For example, constraints on φ from gluino contributions to the NEDM can be made less severe by taking a larger gluino mass (if $m_g\gtrsim 300$ GeV, the bound relaxes to $\varphi\lesssim 10^{-1}$). In the chargino case, contributions to the NEDM are smaller in the region of small tanβ or small soft-supersymmetry-breaking gaugino mass term, $M<m_W$.[6] In addition, con-

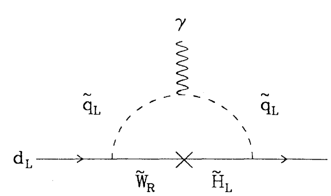

FIG. 5. One-loop chargino diagram contributing to the NEDM. We denote by a cross a mass insertion in the fermion line.

[6]Note that in the limit $M\to 0$ the phases in diagram shown in Fig. 5 can be rotated away giving a zero contribution to the NEDM.

tributions to t_m depend strongly on the super-KM matrices [see Eq. (17)] which are in principle arbitrary.

Let us finally notice that our model does not provide a solution to the strong *CP* problem. After the Higgs bosons develop VEV's, $\bar{\theta}$ is induced at the tree level. We find a value for $\bar{\theta}$ of $O(\rho) \sim 10^{-2}$.

IV. CONCLUSIONS

Even if $\bar{\theta}$ is small due to a PQ symmetry, a massless quark or some other possible mechanism, supersymmetric theories must still face the problem of having additional *CP*-violating phases that induce a too large d_n.

In this paper we have proposed a supersymmetric model where *CP* is broken spontaneously. For this purpose, an additional scalar singlet is required. Phases in the VEV's of the neutral Higgs bosons are then responsible for all *CP*-violating phenomena. They induce at low energy complex mass matrices for squarks, charginos, and neutralinos. The main contribution to ε arises from superbox diagrams (Fig. 1). We showed that if $\Delta m_{\tilde{q}}^2 / m_{\tilde{q}}^2$ saturates the bound derived from experimental limits from FCNC processes, such diagrams can explain the experimental value of ε. Contributions to ε'/ε and the NEDM are in agreement with the experimental bounds if the gluino mass is larger than about 200 GeV and the *CP*-violating phases are $\sim 10^{-2}$. The smallness of these phases is natural in the sense of 't Hooft.

Deviations from the SM predictions are expected to be important in *CP*-violating *B* decays. Such processes will be crucial for revealing the detailed structure of this model. An analysis of the impact of different classes of supersymmetric models on *B* decays can be found in Ref. [33].

We must admit that our model suffers from the usual domain wall problems just like most models with SCPV. A possible solution which avoids such problems has been recently suggested in Ref. [34].

Finally, it is interesting to note that the above analysis shows that *CP* violation can arise generically as a supersymmetric effect at low energy. In other words, the KM phase is not strictly necessary to explain the experimental observed *CP*-violating phenomena. However, such a picture is consistent only for a small region of the parameter space of the supersymmetric model.

ACKNOWLEDGMENTS

I would like to thank F. del Aguila, M. Dine, H. Haber, and Y. Nir for helpful discussion and comments. I am also grateful to M. Dine and H. Haber for a critical reading of the manuscript. This work was supported by the MEC (Spain).

[1] R. P. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. D **16**, 1791 (1977).

[2] J. Ellis, S. Ferrara, and D. V. Nanopoulos, Phys. Lett. **114B**, 231 (1982); W. Buchmüller and D. Wyler, *ibid.* **121B**, 321 (1983).

[3] J. Polchinski and M. B. Wise, Phys. Lett. **125B**, 393 (1983).

[4] F. del Aguila, M. B. Gavela, J. A. Grifols, and A. Méndez, Phys. Lett. **126B**, 71 (1983); D. V. Nanopoulos and M. Srednicki, *ibid.* **128B**, 61 (1983); A. I. Sanda, Phys. Rev. D **32**, 2992 (1985).

[5] E. Braaten, C. S. Li, and T. C. Yuan, Phys. Rev. Lett. **64**, 1709 (1990); J. Dai *et al.*, Phys. Lett. B **237**, 216 (1990); M. Dine and W. Fischler, *ibid.* **242**, 239 (1990); R. Arnowitt, J. L. Lopez, and D. V. Nanopoulos, Phys. Rev. D **42**, 2423 (1990); R. Arnowitt, M. J. Duff, and K. S. Stelle, *ibid.* **43**, 3085 (1991).

[6] A. De Rújula, M. B. Gavela, O. Pène, and F. J. Vegas, Phys. Lett. B **245**, 640 (1990).

[7] Y. Kizukuri and N. Oshimo, Phys. Rev. D **45**, 1806 (1992).

[8] G. 't Hooft, in *Recent Advances in Gauge Theories,* Proceedings of the Cargese Summer Institute, Cargese, France, 1979, edited by G. 't Hooft *et al.*, NATO Advanced Study Institute Series B: Physics Vol. 59 (Plenum, New York, 1980).

[9] S. M. Barr and A. Masiero, Phys. Rev. D **38**, 366 (1988).

[10] A. Nelson, Phys. Lett. **136B**, 387 (1984); **143B**, 65 (1984); S. M. Barr, Phys. Rev. Lett. **53**, 329 (1984); Phys. Rev. D **30**, 1805 (1984).

[11] A. Dannenberg, L. Hall, and L. Randall, Nucl. Phys. **B271**, 574 (1986).

[12] F. del Aguila *et al.*, Phys. Lett. **129B**, 77 (1983); J. M. Frère and M. B. Gavela, *ibid.* **132B**, 107 (1983); J. M. Gérard, W. Grimus, A. Raychaudhuri, and G. Zoupanos, *ibid.* **140B**, 349 (1984); P. Langacker and B. Sathiapalan, *ibid.* **144B**, 401 (1984).

[13] S. K. Soni and H. A. Weldon, Phys. Lett. B **216**, 215 (1983).

[14] L. J. Hall and L. Randall, Phys. Rev. Lett. **65**, 2939 (1990).

[15] J. Ellis and D. V. Nanopoulos, Phys. Lett. **110B**, 44 (1982).

[16] M. J. Duncan and J. Trampetic, Phys. Lett. **134B**, 439 (1984).

[17] F. Gabbiani and A. Masiero, Nucl. Phys. **B322**, 235 (1989).

[18] M. Dine, A. Kagan, and S. Samuel, Phys. Lett. B **243**, 250 (1990).

[19] See, for example, J. F. Gunion, G. L. Kane, H. E. Haber, and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Reading, MA, 1990), for a comprehensive review and a guide to the literature.

[20] N. Maekawa, Phys. Lett. B **282**, 387 (1992).

[21] A. Pomarol, Phys. Lett. B **287**, 331 (1992).

[22] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).

[23] J. F. Gunion and H. E. Haber, Nucl. Phys. **B272**, 1 (1986).

[24] J. Ellis *et al.*, Phys. Rev. D **39**, 844 (1989); M. Drees, Int. J. Mod. Phys. A **4**, 3635 (1989); P. Binétruy and C. A. Savoy, Phys. Lett. B **277**, 543 (1992).

[25] J. C. Romão, Phys. Lett. B **173**, 309 (1986).

[26] G. C. Branco, Phys. Rev. Lett. **44**, 504 (1980).

[27] L. L. Chau, Phys. Rep. **95**, 1 (1983).

[28] J. M. Gérard, in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics,* Geneva, Switzerland, 1991, edited by S. Hegarty, K. Potter, and E. Quercigh (World Scientific,

Singapore, 1992).
[29] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B120**, 316 (1997); F. J. Gilman and M. B. Wise, Phys. Lett. **83B**, 83 (1979).
[30] P. Langacker and B. Sathiapalan, Phys. Lett. **144B**, 395 (1984).
[31] G. Buchalla, A. J. Buras, and M. K. Harlander, Nucl. Phys. **B337**, 313 (1990).
[32] I. S. Altarev *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 360 (1986) [JETP Lett. **44**, 460 (1986)]; K. F. Smith *et al.*, Phys. Lett. B **234**, 191 (1990).
[33] I. I. Bigi and F. Gabbiani, Nucl. Phys. **B352**, 309 (1991).
[34] L. M. Krauss and S. J. Rey, Phys. Rev. Lett. **69**, 1308 (1992).