Value of Gottfried sum rule from the current anticommutator on the null plane

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By extending the sum rules from the current anticommutator on the null plane to the $K^{\pm}$ case, we relate the Gottfried sum rule to kaon-nucleon scattering. With the use of this relation, together with Adler-Weisberger relations, we estimate the value of the Gottfried sum rule as 0.26 ± 0.03 which is consistent with the recent experimental value obtained by the New Muon Collaboration.

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I. INTRODUCTION

Recently, a systematic error of the Gottfried sum rule was reduced considerably and its value was reported as [1]

$$\int_0^1 \frac{dx}{x}\{F_2^{ep}(x,Q^2)-F_2^{en}(x,Q^2)\}=0.240\pm0.016(\text{sys}) . \tag{1.1}$$

The correction due to perturbative QCD starts from two loops, and its magnitude is very small [2]. Hence it becomes almost impossible to explain the large deviation from the naively expected value $\frac{1}{3}$ by this correction. Two approaches to explain this defect have appeared: One approach was based on the unmeasured small-x behavior of the structure functions, which attached great importance to physics at high energy [3]; the other approach used pseudo Goldstone bosons such as pions as a substitute for spontaneous chiral-symmetry breaking, where the causes of the defects are considered to lie mainly at low energy [4–6]. In this paper we show that the experimental value of the Gottfried sum rule can be explained by due consideration for the physics in the high-energy region. However, our method differs greatly from the former approach and rather may be related to the latter approach, i.e., spontaneous chiral-symmetry breaking.

Our approach is based on the current anticommutator on the null plane, which was proposed more than 10 years ago [7,8]. Since this method seems to be unfamiliar to many people, we give a review of this subject in Sec. II. Then in Sec. III we derive the modified Gottfried sum rule and estimate its value. In Sec. IV we discuss the physical constraint at high energy which played an important role in deriving this sum rule and the flavor asymmetry of the sea quarks predicted in our approach. In Sec. V we give conclusions.

II. REVIEW OF THE CURRENT ANTICOMMUTATOR ON THE NULL PLANE

Let us first consider the Deser-Gilbert-Sudarshan (DGS) representation of the current commutator [9]. To avoid an inessential complication, we take here the scalar model given by

$$\langle p|[J_a(x),J_b(0)]|p\rangle_c = \int_0^\infty d\lambda^2 \int_{-1}^1 d\beta \exp(i\beta p\cdot x)h_{ab}(\lambda^2,\beta)i\Delta(x,\lambda^2) , \tag{2.1}$$

where c means to take the connected part, $|p\rangle$ is the stable one-particle state, and the scalar current $J_a(x)$ is defined as

$$J_a(x)=\varphi^\dagger(x)\tau_a\varphi(x) . \tag{2.2}$$

The momentum-space representation of (2.1) is

$$\begin{aligned}C_{ab}(p\cdot q,q^2)&=\int d^4x \exp(iq\cdot x)\langle p|[J_a(x),J_b(0)]|p\rangle_c\\&=2\pi\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\delta((q+\beta p)^2-\lambda^2)\\&\quad\times\varepsilon(q^0+\beta p^0)h_{ab}(\lambda^2,\beta) .\end{aligned} \tag{2.3}$$

$C_{ab}(p\cdot q,q^2)$ can be decomposed as

$$\begin{aligned}C_{ab}(p\cdot q,q^2)=&\sum_n(2\pi)^4\delta^4(p+q-n)\\&\times\langle p|J_a(0)|n\rangle\langle n|J_b(0)|p\rangle\\&-\sum_n(2\pi)^4\delta^4(p-q-n)\\&\times\langle p|J_b(0)|n\rangle\langle n|J_a(0)|p\rangle .\end{aligned} \tag{2.4}$$

By taking a frame at rest, $p=(m,0)$, we find that the first and second terms are disconnected under the assumption $m\le(M_s+M_u)/2$, where M_s and M_u are the lowest masses in the s and u channels, respectively. Further, in this frame we see that $(q^0+\beta p^0)$ is equal to $(p\cdot q+\beta m^2)/m$; hence, the sign function $\varepsilon(q^0+\beta p^0)$ in (2.3) can be replaced by $\varepsilon(p\cdot q+\beta m^2)$. Let us now consider an integration path $\sigma=2\beta p\cdot q+q^2$ in the (β,σ) plane with σ defined by $\sigma=\lambda^2-\beta^2m^2$. The support of the weight function $h_{ab}(\lambda^2,\beta)$ is restricted in the region $|\beta|\le1$ and $\sigma\ge0$. Among the regions where the weight function is required to be zero, the region $\omega\le-\beta^2m^2$ corresponding to $\lambda^2\le0$ originates from causality. An important property of the integration path is that the point (β_1,σ_1) where β_1 and σ_1 satisfy $p\cdot q+\beta_1m^2=0$ and $\sigma_1=2\beta_1p\cdot q+q^2$ is always in the causality-forbidden region $\sigma \leq -\beta^2 m^2$. Because of these properties, we can obtain

$$2\pi\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\delta((q+\beta p)^2-\lambda^2)\times h_{ab}(\lambda^2,\beta)\theta(q^0+\beta p^0)=\sum_n(2\pi)^4\delta^4(p+q-n)\times\langle p|J_a(0)|n\rangle\langle n|J_b(0)|p\rangle \quad (2.5)$$

and

$$2\pi\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\delta((q+\beta p)^2-\lambda^2)\times h_{ab}(\lambda^2,\beta)\theta(-(q^0+\beta p^0))=\sum_n(2\pi)^4\delta^4(p-q-n)\times\langle p|J_b(0)|n\rangle\langle n|J_a(0)|p\rangle . \quad (2.6)$$

Then (2.5) and (2.6) give us the DGS representations of the current anticommutator as

$$W_{ab}(p\cdot q,q^2)=\int d^4x\,\exp(iq\cdot x)\langle p|\{J_a(x),J_b(0)\}|p\rangle_c=2\pi\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\delta((q+\beta p)^2-\lambda^2)h_{ab}(\lambda^2,\beta) . \quad (2.7)$$

This derivation clearly shows that the spectral condition $m \leq (M_s+M_u)/2$ is essential to reach (2.7).

Let us next consider how C_{ab} and W_{ab} can be restricted to the null plane. For this purpose we consider I_{ab} defined as

$$I_{ab}=\lim_{\Lambda\to\infty}\int_{-\infty}^\infty dq^-\exp\left[-\frac{(q^-)^2}{\Lambda^2}\right]H_{ab}(p\cdot q,q^2) , \quad (2.8)$$

where H_{ab} denotes C_{ab} or W_{ab} and the integration is over q^- for a fixed Λ. Since the constraint $(q+\beta p)^2-\lambda^2=0$ is linear with respect to q^- where it changes from $-\infty$ to $+\infty$, we obtain no constraint on β and λ^2 by this integration. Then we consider under what condition on the weight function $h_{ab}(\lambda^2,\beta)$ can we take the limit $\Lambda\to\infty$. Since $p^+>0$ and q^+ are arbitrary parameters, we take $q^+/p^+>1$. Then the integration path $\sigma=2\beta p\cdot q+q^2$ in the (β,σ) plane passes the support of the weight function only for $p\cdot q>0$, and we can safely set $\varepsilon(q^++\beta p^+)=1$ [10]. Hence we find that the condition necessary to take the limit $\Lambda\to\infty$ for the current cummutator is the same as the one for the current anticommutator. The same thing holds for $q^+/p^+<-1$, where in this case $p\cdot q<0$. After all we find the weight function should be $h_{ab}(\lambda^2,\beta)\sim\lambda^{-2-\varepsilon}$ as $\lambda^2\to\infty$, where ε is an arbitrary positive number. Since this constraint has no dependence on p and q, it must hold irrespective of the constraint $|q^+/p^+|>1$. Thus, by taking the limit $\Lambda\to\infty$ under this condition, we obtain

$$\int_{-\infty}^\infty dq^-C_{ab}(p\cdot q,q^2)=2\pi\int d^4x\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\exp[i(\mathbf{q}\cdot\mathbf{x}+\beta\mathbf{p}\cdot\mathbf{x})]h_{ab}(\lambda^2,\beta)\left[-\frac{i}{4}\right]\delta(\mathbf{x}^\perp)\varepsilon(x^-)\delta(x^+) , \quad (2.9)$$

and

$$\int_{-\infty}^\infty dq^-W_{ab}(p\cdot q,q^2)=2\pi\int d^4x\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\exp[i(\mathbf{q}\cdot\mathbf{x}+\beta\mathbf{p}\cdot\mathbf{x})]h_{ab}(\lambda^2,\beta)\left[-\frac{1}{2\pi}\right]\delta(\mathbf{x}^\perp)\ln|x^-|\delta(x^+) . \quad (2.10)$$

where

$$2\ln|x^-|=-\int_{-\infty}^\infty\frac{da}{|a|}\exp(iax^-)$$

and

$$\mathbf{q}\cdot\mathbf{x}=q^+x^--\mathbf{q}^\perp\mathbf{x}^\perp .$$

The weight function in (2.10) appears in the same place as the one in (2.9). In the language of field theory, by using the current commutator abstracted from the canonical quantization on the null plane, we find

$$\int_0^\infty d\lambda^2\int d\beta\exp(i\beta p^+x^-)h_{ab}(\lambda^2,\beta)=\langle p|\{\delta_{ab}S_0(x|0)+i\varepsilon_{abc}A_c(x|0)|p\rangle|_{x^+=\mathbf{x}^\perp=0} , \quad (2.11)$$

where

$$S_0(x|0)=\varphi^\dagger(x)\varphi(0)+\varphi^\dagger(0)\varphi(x) ,$$
$$A_a(x|0)=\varphi^\dagger(x)\tau_a\varphi(0)-\varphi^\dagger(0)\tau_a\varphi(x) , \quad (2.12)$$

and the canonical quantization of the scalar field is assumed as

$$[\varphi^\dagger(x),\varphi(0)]|_{x^+=0}=-\frac{i}{4}\varepsilon(x^-)\delta(\mathbf{x}^\perp) . \quad (2.13)$$

This example shows that the current commutator on the null plane gives us information of the current anticommutator on it. The fixed-mass sum rules can be obtained from (2.9) and (2.10). As is well known, to reach these sum rules we must assume that we can interchange ν integration and setting $q^+=0$ [11]. Here we need some assumptions. This is known as a class-II graph problem [12]. Since

$$q^2=2(q^+/p^+)(\nu-p^-q^++\mathbf{p}^\perp\mathbf{q}^\perp)-\mathbf{q}^{\perp2} ,$$

q^2 takes a positive value above some ν_0 for $q^+/p^+>0$. However, if we set $q^+=0$ before ν integration, q^2 becomes $-\mathbf{q}^{\perp 2}$, being negative. Hence, to allow for this interchange, we must assume that contributions from the positive-q^2 region are zero. Since $\nu_0\to\infty$ as $q^+\to 0$, these conditions become the ones as $\nu\to\infty$ and are called superconvergence relations. It is at this point where a difference between the current commutator and the current anticommutator appears. We will encounter this as the divergence of the sum rules later in this section.

A generalization of the scalar current to the vector current or the axial-vector current is straightforward, and we obtain

$$\int_{-\infty}^{\infty} dq^- W_{ab}^{++} = 2p^+\int_{-\infty}^{\infty} dx^- \exp(iq^+x^-) P\frac{1}{x^-} G_{ab}(p^+x^-,0) , \tag{2.14}$$

where $W_{ab}^{\mu\nu}$ is given as

$$\begin{aligned} W_{ab}^{\mu\nu} &= \int d^4x \exp(iq\cdot x)\langle p|\{J_a^\mu(x),J_b^\nu(0)\}|p\rangle_c \\ &= \int d^4x \exp(iq\cdot x)\int_0^\infty d\lambda^2\int_{-1}^1 d\beta \exp(i\beta p\cdot x)[(\partial^\mu\partial^\nu-\Box g^{\mu\nu})\{h_1^{ab}+ip\cdot\partial g_1^{ab}\} \\ &\quad +\{-\Box p^\mu p^\nu+p\cdot\partial(p^\mu\partial^\nu+p^\nu\partial^\mu)-g^{\mu\nu}(p\cdot\partial)^2\}h_2^{ab}]\Delta^{(1)}(x,\lambda^2) \end{aligned} \tag{2.15}$$

and $G_{ab}(p\cdot x,0)$ is given by

$$G_{ab}(p\cdot x,0) = -i\int_0^\infty d\lambda^2\int_{-1}^1 d\beta \exp(i\beta p\cdot x)\beta\{h_1^{ab}+m^2(h_2^{ab}-\beta g_1^{ab})\} . \tag{2.16}$$

Here we assumed

$$\delta(x^+)[J_a^+(x),J_b^+(0)] = if_{abc}J_c^+(0)\delta^4(x) . \tag{2.17}$$

A similar consideration holds for the axial-vector current, and we obtain

$$\int_{-\infty}^{\infty} dq^-\{W_{ab}^{++}-W_{ab}^{5++}\}=0 , \tag{2.18}$$

where W_{ab}^{5++} is defined similarly to W_{ab}^{++}, and we assume

$$\delta(x^+)[J_a^{5+}(x),J_b^{5+}(0)] = if_{abc}J_c^+(0)\delta^4(x) . \tag{2.19}$$

In the case of the current commutator, $P(1/x^-)$ in (2.14) is replaced by $\delta(x^-)$; hence, $G_{ab}(p^+,x^-,0)$ is restricted at $x^-=0$. The condition (2.17) or (2.19) plays a role to select out the nonzero part in the DGS representation. Thus we can identify the other parts in this representation as zero. Then a similar discussion as the one developed in the paragraph before (2.9) shows that the corresponding parts in the DGS representation of the current anticommutator to these parts are also zero. Hence we can select out the nonzero part in this representation. Since the singularity structure of this part is not $\delta(x^-)$ but $P(1/x^-)$, we encounter the nonlocal quantity $G_{ab}(p^+x^-,0)$, as in (2.14). It is possible to have a field-theoretical representation of this quantity as in the scalar current model. However, we do not need such a representation. What is really needed is a group-theoretical structure of $G_{ab}(p;x,0)$. Let us take the SU(3)-octet currents and the state $|p\rangle$ belonging in the SU(3) octet. Then the terms which remain after taking the matrix element are two octet pieces and one singlet piece in the current anticommutator. Hence we can decompose $G_{ab}(p;x,0)$ as

$$G_{ab}(p\cdot x,0) = d_{abc}A_c(p\cdot x,0)+f_{abc}S_c(p\cdot x,0) . \tag{2.20}$$

This nonlocal quantity in general contains not only singular pieces as $p\cdot x\to 0$ [13], but also contributions from higher-twist terms.

Now the fixed-mass sum rules can be obtained from (2.14) and (2.18) by using the method of Dicus, Jackiw, and Teplitz [11]. Corresponding to the Adler-Weisberger sum rule, we obtain

$$g_A^2(0)+\frac{2f_\pi^2}{\pi}\int_{\nu_0^\pi}^\infty \frac{d\nu}{\nu}\{\sigma^{\pi^+p}(\nu)+\sigma^{\pi^-p}(\nu)\} = \frac{1}{2\pi}P\int_{-\infty}^\infty \frac{d\alpha}{\alpha}\left\{\frac{2\sqrt{6}}{3}A_0(\alpha,0)+\frac{2\sqrt{3}}{3}A_8(\alpha,0)\right\} , \tag{2.21}$$

where Cabibbo angle is set to zero, $\sigma^{\pi p}$ means the total cross section of the πp scattering at $q^2=0$, $\nu=p\cdot q$, and $\nu_0^\pi=m_Nm_\pi+m_\pi^2/2$. Corresponding to the Adler sum rule, we obtain

$$\int_0^1 \frac{dx}{2x}\{F_2^{\bar\nu p}(x,Q^2)+F_2^{\nu p}(x,Q^2)\} = \frac{1}{2\pi}P\int_{-\infty}^\infty \frac{d\alpha}{\alpha}\left\{\frac{2\sqrt{6}}{3}A_0(\alpha,0)+\frac{2\sqrt{3}}{3}A_8(\alpha,0)\right\} , \tag{2.22}$$

where $q^2=-Q^2$ and $x=Q^2/(2\nu)$. From (2.21) and (2.22), we obtain the relation

$$g_A^2(0)+\frac{2f_\pi^2}{\pi}\int_{\nu_0^\pi}^\infty \frac{d\nu}{\nu}\{\sigma^{\pi^+p}(\nu)+\sigma^{\pi^-p}(\nu)\} = \int_0^1 \frac{dx}{2x}\{F_2^{\bar\nu p}(x,Q^2)+F_2^{\nu p}(x,Q^2)\} . \tag{2.23}$$

All the sum rules (2.21)–(2.23) diverge logarithmically if the leading high-energy behavior is given by the Pomeron with its intercept $\alpha_P(0)=1$. This fact is very important as we explain in the following. The sum rule (2.21) is derived by the anticommutator of the lightlike axial charge where we take $m_\pi \neq 0$. Since this charge is not conserved as a result of spontaneous chiral-symmetry breaking [14], states with different four-momenta are connected, and high-mass states can contribute to the intermediate states in the sum rule. Thus all the intermediate states transmit information of the spontaneous chiral-symmetry breaking of the vacuum, and this makes the sum rule divergent. In this sense we take the view that the Pomeron is closely related to spontaneous chiral-symmetry breaking of the vacuum. This thought was already discussed many years ago by Weinberg in the context of chiral dynamics [15] and also by Pagels [16]. It seems there were many people who considered it. Recently, Bjorken took again the Manohar-Georgi viewpoint [17] to give a physical background of the soft Pomeron and indicated it might give us a good reason for the badly broken Gottfried sum rule [18]. In fact, the sum rule (2.23) relates the behavior due to spontaneous chiral-symmetry breaking to the small-x behavior of the structure function, and it gives us good evidence that the Gottfried sum rule may be related to this symmetry breaking.

Now to get a meaningful result we should start from a finite quantity. At first, $\alpha_P(0)=1-\varepsilon$, where ε is an arbitrary positive number, was assumed [7]. Then the discussion was generalized to the nonforward matrix element to treat the case $\alpha_P(0)\geq 1$ [8]. By an analytical continuation from the nonforward matrix element to the forward one, it was found that the sum rule had a finite ambiguous term due to the regularization. This part plays an important role in reaching the modified Gottfried sum rule. However, to arrange all the kinematical preliminaries for this argument is very complex and long. Because of this fact, we reproduce it here by assuming $\alpha_P(0)=1-\varepsilon$ for the forward matrix element. The essential steps for reaching the modified Gottfried sum rule cannot be lost in this simplified version. We assumed the leading high-energy behavior of $\sigma^{\pi p}$ and $F_2=\nu W_2$ as

$$\{\sigma^{\pi^+p}(\nu)+\sigma^{\pi^-p}(\nu)\}\sim\beta_{\pi N}\left[\frac{\nu}{a}\right]^{\alpha_P(0)-1}, \tag{2.24}$$

$$\nu\{W_2^{\bar\nu p}(\nu,Q^2)+W_2^{\nu p}(\nu,Q^2)\}\sim\left[\frac{Q_0^2}{Q^2}\right]^{\alpha_P(0)-1}\beta_{\nu N}(Q^2,1-\alpha_P(0))\left[\frac{\nu}{a}\right]^{\alpha_P(0)-1}, \tag{2.25}$$

where $Q_0^2=1\ \mathrm{GeV}^2$. By expanding $\beta_{\nu N}(Q^2,\varepsilon)$ as

$$\beta_{\nu N}(Q^2,\varepsilon)=\beta_{\nu N}^0(Q^2)+\varepsilon\beta_{\nu N}^1(Q^2)+O(\varepsilon^2), \tag{2.26}$$

we obtain

$$\frac{2f_\pi^2\beta_{\pi N}}{\pi}\int_{\nu_0\pi}^\infty\frac{d\nu}{\nu}\left[\frac{\nu}{a}\right]^{-\varepsilon}=\frac{2f_\pi^2\beta_{\pi N}}{\pi}\left[\frac{1}{\varepsilon}+\ln\left[\frac{a}{\nu_0^\pi}\right]\right]+O(\varepsilon), \tag{2.27}$$

$$\frac{1}{2}\beta_{\nu N}(Q^2,\varepsilon)\left[\frac{1}{Q^2}\right]^{-\varepsilon}\int_{Q^2/2}^\infty\frac{d\nu}{\nu}\left[\frac{\nu}{a}\right]^{-\varepsilon}=\frac{1}{2\varepsilon}\{\beta_{\nu N}^0+\varepsilon[\beta_{\nu N}^1+\ln(2a)]\}+O(\varepsilon). \tag{2.28}$$

Using (2.27) and (2.28), we obtain from (2.23),

$$4f_\pi^2\beta_{\pi N}=\pi\beta_{\nu N}^0 \tag{2.29}$$

and

$$g_A^2(0)+\frac{2f_\pi^2}{\pi}\int_{\nu_0^\pi}^\infty\frac{d\nu}{\nu}\{\sigma^{\pi^+p}(\nu)+\sigma^{\pi^-p}(\nu)-\beta_{\pi N}\}+\frac{2f_\pi^2\beta_{\pi N}}{\pi}\ln\left[\frac{1}{2\nu_0^\pi}\right]=\int_0^1\frac{dx}{2x}\{F_2^{\bar\nu p}(x,Q^2)+F_2^{\nu p}(x,Q^2)-\beta_{\nu N}^0\}+\frac{1}{2}\beta_{\nu N}^1. \tag{2.30}$$

We assume the smooth extrapolation of the off-shell pion-nucleon total cross section to the on-shell one. Then (2.20) together with the experimental values $f_\pi\sim 0.094$ GeV and $\beta_{\pi N}\sim 109$ determines $\beta_{\nu N}^0$ as 1.22 [19], and this value corresponds to the small-x behavior of the sea-quark distributions. We will discuss this point in Sec. IV. The sum rule (2.30) has an ambiguous part $\beta_{\nu N}^1$. This part was missed in Ref. [7] and found in Ref. [8] by an analytical continuation from the nonforward matrix element. In the case of the method in Ref. [8], the part corresponding to $\beta_{\nu N}^1$ was not necessarily related to the intercept of the Pomeron, and it entered naturally through the t dependence of the residue of the Pomeron. In the following section we explain how this $\beta_{\nu N}^1$ can be determined, and in so doing, we show that the value of the Gottfried sum rule can be obtained.

III. RELATING THE GOTTFRIED SUM RULE TO KAON-NUCLEON SCATTERING

From (2.14) we obtain the sum rule for the sp scattering as

$$\int_{Q^2/2}^\infty d\nu\, W_2^{ep}(\nu,Q^2)=\frac{1}{6\pi}P\int_{-\infty}^\infty\frac{d\alpha}{\alpha}\left\{\frac{2\sqrt{6}}{3}A_0(\alpha,0)+A_3(\alpha,0)+\frac{\sqrt{3}}{3}A_8(\alpha,0)\right\}. \tag{3.1}$$

As in Sec. II, we take the leading high-energy behavior of νW_2^{ep} as

$$\nu W_2^{ep}(\nu,Q^2)\sim\left[\frac{Q_0^2}{Q^2}\right]^{\alpha_P(0)-1}\beta_{ep}(Q^2,1-\alpha_P(0))\left[\frac{\nu}{a}\right]^{\alpha_P(0)-1}. \tag{3.2}$$

Since the right-hand side of the sum rule (3.1) is Q^2 independent, we obtain

$$\int_0^1 \frac{dx}{x}\{F_2^{ep}(x,Q^2)-\beta_{ep}^0(Q^2)\}+\beta_{ep}^1(Q^2)$$
$$=\int_0^1 \frac{dx}{x}\{F_2^{ep}(x,Q_0^2)-\beta_{ep}^0(Q_0^2)\}+\beta_{ep}^1(Q_0^2)$$
$$\equiv C_p \tag{3.3}$$

and

$$\beta_{ep}^0(Q^2)=\beta_{ep}^0(Q_0^2)\ , \tag{3.4}$$

where

$$\beta_{ep}(Q^2,\varepsilon)=\beta_{ep}^0(Q^2)+\varepsilon\beta_{ep}^1(Q^2)+O(\varepsilon^2)\ .$$

Now we assume that the residue of the Pomeron β_{ep} and $\beta_{\nu N}$ comes only from the singlet piece. By comparing the coefficient of $A_0(\alpha,0)$ in the sum rules (2.22) and (3.1), we find the relation $\beta_{\nu N}=6\beta_{ep}$ and obtain

$$\beta_{\nu_N}^0=6\beta_{ep}^0,\quad \beta_{\nu N}^1=6\beta_{ep}^1\ . \tag{3.5}$$

Then the regularization-dependent term $\beta_{\nu N}^1$ in the sum rule (2.30) can be related to the β_{ep}^1 in the sum rule (3.3), and we obtain

$$I_\pi-3C_p=\int_0^1 \frac{dx}{2x}\{F_2^{\bar{\nu}p}(x,Q^2)+F_2^{\nu p}(x,Q^2)-6F_2^{ep}(x,Q^2)\}\ , \tag{3.6}$$

$$I_\pi=g_A^2(0)+\frac{2f_\pi^2}{\pi}\int_{\nu_0^\pi}^\infty \frac{d\nu}{\nu^2}\{(\nu^2-m_\pi^2m_N^2)^{1/2}\times[\sigma^{\pi^+p}(\nu)+\sigma^{\pi^-p}(\nu)]-\beta_{\pi N}\}+\frac{2f_\pi^2\beta_{\pi N}}{\pi}\ln\left[\frac{1}{2\nu_0^\pi}\right]\ , \tag{3.7}$$

where we assumed here the smooth extrapolation to the on-shell pion-nucleon scattering amplitude. For the neutron target, since we take the Cabibbo angle as zero, we obtain

$$I_\pi-3C_n=\int_0^1 \frac{dx}{2x}\{F_2^{\bar{\nu}p}(x,Q^2)+F_2^{\nu p}(x,Q^2)-6F_2^{en}(x,Q^2)\}\ , \tag{3.8}$$

where C_n is defined similarly as C_p. The sum rules (3.7) and (3.8) give us

$$\int_0^1 \frac{dx}{x}\{F_2^{ep}(x,Q^2)-F_2^{en}(x,Q^2)\}=C_p-C_n\ . \tag{3.9}$$

The sum rule (3.9) takes the form of the Gottfried sum rule, but in our case it does not necessarily take the value $\frac{1}{3}$. From the derivation we see the value of the Gottfried sum rule (C_p-C_n) depends heavily on the small-x behavior of the structure function.

Let us now extend the argument to the kaon to get the explicit value of C_p and C_n. In this case we obtain two other new sum rules as

$$I_K^p-3C_p=\int_0^1 \frac{dx}{x}[3\{F_2^{ep}(x,Q^2)+F_2^{en}(x,Q^2)\}-\{F_2^{\bar{\nu}p}(x,Q^2)+F_2^{\nu p}(x,Q^2)\}]\ , \tag{3.10}$$

$$I_K^n-3C_n=\int_0^1 \frac{dx}{x}[3\{F_2^{ep}(x,Q^2)+F_2^{en}(x,Q^2)\}-\{F_2^{\bar{\nu}p}(x,Q^2)+F_2^{\nu p}(x,Q^2)\}]\ , \tag{3.11}$$

where

$$I_K^p=[g_A^{p\Sigma^0}(0)]^2+[g_A^{p\Lambda}(0)]^2+\frac{2f_K^2}{\pi}\int_{\nu_0^K}^\infty \frac{d\nu}{\nu^2}\{(\nu^2-m_N^2m_K^2)^{1/2}[\sigma^{K^+p}(\nu)+\sigma^{K^-p}(\nu)]-\beta_{KN}\}+\frac{2f_K^2\beta_{KN}}{\pi}\ln\left[\frac{1}{2\nu_0^K}\right]+U_p\ , \tag{3.12}$$

$$I_K^n=[g_A^{n\Sigma^-}(0)]^2+\frac{2f_K^2}{\pi}\int_{\nu_0^K}^\infty \frac{d\nu}{\nu^2}\{(\nu^2-m_N^2m_K^2)^{1/2}[\sigma^{K^+n}(\nu)+\sigma^{K^-n}(\nu)]-\beta_{KN}\}+\frac{2f_K^2\beta_{KN}}{\pi}\ln\left[\frac{1}{2\nu_0^K}\right]+U_n\ , \tag{3.13}$$

and the symmetry relation as

$$f_\pi^2\beta_{\pi N}=f_K^2\beta_{KN}\ . \tag{3.14}$$

Here U_p and U_n are contributions below the $\bar{K}N$ threshold. The sum rules (3.8), (3.10), and (3.11) give us the relations

$$C_p=\frac{1}{9}(2I_K^p-I_K^n+2I_\pi)\ , \tag{3.15}$$

$$C_n=\frac{1}{9}(-I_K^p+2I_K^n+2I_\pi)\ , \tag{3.16}$$

$$C_p-C_n=\tfrac{1}{3}(I_K^p-I_K^n)\ . \tag{3.17}$$

The last relation (3.17) relates the Gottfried sum rule to the kaon-nucleon scatterings. Now in the I_K^p and I_K^n Born terms, U_p and U_n originate in the $\bar{K}N$ reactions; hence, we can express them as [20]

$$[g_A^{p\Sigma^0}(0)]^2+[g_A^{p\Lambda}(0)]^2+U_p=2-\frac{2f_K^2}{\pi}\int_{\nu_0^K}^{\infty}\frac{d\nu}{\nu^2}(\nu^2-m_N^2m_K^2)^{1/2}\{\sigma^{K^-p}(\nu)-\sigma^{K^+p}(\nu)\}\ , \tag{3.18}$$

$$[g_A^{n\Sigma^-}(0)]^2+U_n=1-\frac{2f_K^2}{\pi}\int_{\nu_0^K}^{\infty}\frac{d\nu}{\nu^2}(\nu^2-m_N^2m_K^2)^{1/2}\{\sigma^{K^-n}(\nu)-\sigma^{K^+n}(\nu)\}\ . \tag{3.19}$$

From (3.12), (3.13), and (3.17)–(3.19), we obtain

$$C_p-C_n=\frac{1}{3}\left[1-\frac{4f_K^2}{\pi}\int_{\nu_0^K}^{\infty}\frac{d\nu}{\nu^2}(\nu^2-m_N^2m_K^2)^{1/2}\{\sigma^{K^+n}(\nu)-\sigma^{K^+p}(\nu)\}\right]\ . \tag{3.20}$$

The deviation from $\frac{1}{3}$ is clearly expressed in this expression, and the experimental fact convinces us $(C_p-C_n)<\frac{1}{3}$. Further, it should be noted that this expression does not depend on a particular parameter to reach it. Let us try to estimate this magnitude of the deviation. We use the recent parameters given in Ref. [21] in the region $4<p_K<300$ GeV/c, where p_K is the momentum of the kaon in the laboratory frame. In the region $0.6<p_K<4$ GeV/c, we estimate its contribution directly from cross-section data [21,22] with neutron data extrapolation down to $p_K=0.6$ GeV/c, and in the region below 0.6 GeV/c we estimate it by setting $(\sigma^{K^+n}-\sigma^{K^+p})\sim1.8$ mb, where this value is the extrapolated one at $p_K=0.6$ GeV/c. Then, by using the experimental value of $f_K\sim0.11356$ GeV, we obtain $(C_P-C_n)=0.26$, where from the region $4<p_K<300$ GeV/c we get the contribution -0.028, from $0.6<p_K<4$ GeV/c, -0.039, and from $0<p_K<0.6$ GeV/c, -0.006. In this estimate we neglected the contribution above $p_K>300$ GeV/c, which might be a small negative quantity. Let us now consider the systematic error of this estimate. The error due to the extrapolation to the on-shell quantity for the Adler-Weisberger sum rule for the pion is about 5–10 %. For the kaon this kind of error may be large, in general. However, in the case of the Adler-Weisberger-type sum rule, a particular cancellation of the extrapolation factors exist [23]; hence, we can expect that the error will be reduced. Thus we take here this kind of the error as 20%. As to the error from the fit to data, we consider it as follows. We estimated contributions above $p_K=4$ GeV/c by using the parameter in Ref. [19] and found that its contribution to (C_P-C_n) was -0.016. Compared with the fit by the parameter in Ref. [21], this pushed up the value of the Gottfried sum rule by 0.012. Now the parameters in Ref. [19] are old and the fact $\sigma^{K^+n}>\sigma^{K^+p}$ at high energy is not reflected well in them. Hence this value will be reduced. Thus we take that the error above $p_K>4$ GeV/c is ±0.008. Concerning the error from the extrapolated region, we consider it to be small, because the KN reactions behave smoothly in this energy region as opposed to $\bar{K}N$ reactions. However, considering the crude estimate in this region we take the error as ±0.003. Then, since the error from the region $0.6<p_K<4$ GeV/c can be expected to be about 10%, i.e., ±0.004, we can estimate that the net error from the fit to the data is about 20%. Then the total systematical error can be less than 40% at most. Therefore we obtain $(C_p-C_n)=0.26\pm0.03$.

IV. SYMMETRY RELATIONS AT HIGH ENERGY AND FLAVOR ASYMMETRY OF THE SEA QUARKS

Here we first discuss the symmetry relations at high energy. From (2.29) and (3.4) together with the fact that the Pomeron is flavor singlet, we obtained [7]

$$\lim_{x\to0}x\lambda_d(x,Q^2)=\lim_{x\to0}x\lambda_u(x,Q^2)=\lim_{x\to0}x\lambda_s(x,Q^2)=a\sim0.15\ , \tag{4.1}$$

where λ_i denotes the sea-quark distribution in the proton. It is important to note this constraint is Q^2 independent. The value 0.15 is consistent with the old estimate based on low-Q^2 data. However, in almost all the recent phenomenological sea-quark distributions, this constraint is not maintained at high Q^2. This may be no problem if the small-x region which we discuss here is far from the experimentally accessible region. However, considering the role played by this constraint in the derivation of the Gottfried sum rule, it may have phenomenological significance at ongoing experiments at the DESY ep collider HERA and future CERN and Superconducting Super Collider (SSC) experiments. Now we can represent the constraint (4.1) as the one on the Pomeron-meson couplings. By denoting them as γ_i^0 where the superscript 0 meant the off-shell ($q^2=0$) quantity, we obtained [7]

$$f_\pi^2\gamma_\pi^0=f_K^2\gamma_K^0=f_K^2\gamma_K^0=f_D^2\gamma_D^0=f_F^2\gamma_F^0\ , \tag{4.2}$$

where in this case the discussion was generated to SU(4). This means that the scale of these couplings is determined by the decay constants of pseudoscalar mesons, which is a signal of spontaneous chiral-symmetry breaking of the vacuum at high energy. It is interesting to note that Pagels already obtained a similar relation to (4.2) by a completely different method many years ago [16]. His result was $f_\pi\gamma_\pi=$const, where γ_π is the on-shell Pomeron-pion coupling. We can have another representation of the constraint (4.1) or (4.2). We denote the structure function of the sea quark in the Pomeron as $P_i(x)$. Then, according to the method in Ref. [24], we obtain, from (4.1),

$$\lim_{x\to0}xP_u=\lim_{x\to0}xP_d=\lim_{x\to0}xP_s\sim0.0087\ . \tag{4.3}$$

Let us turn now to the flavor asymmetry of the sea quark. By the same kind of reasoning as in Sec. III, we find $C_p\sim1.32$ and $C_n\sim1.06$ from the relations (3.15) and (3.16). Then the sum rule (3.6) becomes

$$\tfrac{2}{3}\int_0^1 dx\,(\lambda_d-\lambda_u)+\tfrac{2}{3}\int_0^1 dx\,(\lambda_d-\lambda_s)\sim 0.51\ . \qquad (4.4)$$

From the Gottfried sum rule, we obtain

$$A\equiv\int_0^1 dx\,(\lambda_d-\lambda_u)\sim 0.11\ . \qquad (4.5)$$

Then the sum rules (4.4) and (4.5) give us

$$B\equiv\int_0^1 dx\,(\lambda_d-\lambda_s)\sim 0.66\ . \qquad (4.6)$$

Thus the ratio of the SU(2) symmetry breaking to the SU(3) one, A/B, is about 0.17. Since the distribution of the sea quarks are flavor symmetric near $x=0$, this ratio means the strange sea quark is suppressed very steeply as we go to large x compared with the up and down sea quarks. Although it may be a mere coincidence, this ratio is about the same order as the Cabibbo angle $\sin\theta_C$.

V. CONCLUSIONS

The experimental value of the Gottfried sum rule was explained by the sum rules based on the current anticommutator on the null plane. It was stressed that the symmetry constraints at high energy played an important role in obtaining this value and that these relations reflected the spontaneous chiral-symmetry breaking of the vacuum. Thus this approach may have some relevance to the current popular approach at relatively low energy, which is also based on this symmetry breaking [4–6]. Using another sum rule derived by the current anticommutator on the null plane, we showed that flavor asymmetry of the sea quarks could be explained naturally. We gave this as the ratio of the SU(2) symmetry breaking to the SU(3) one and showed that its value was about 0.17.

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[1] P. Amaudruz *et al.,* Phys. Rev. Lett. **66**, 2712 (1991).
[2] D. A. Ross and C. T. Sachrajda, Nucl. Phys. **B149**, 497 (1978).
[3] A. D. Martin, W. J. Stirling, and R. G. Roberts, Phys. Lett. B **252**, 653 (1990); P. N. Harriman, A. D. Martin, W. J. Stirling, and R. G. Roberts, Phys. Rev. D **42**, 798 (1990).
[4] E. M. Henley and G. A. Miller, Phys. Lett. B **251**, 453 (1990); S. Kumano and J. T. Londergan, Phys. Rev. D **44**, 717 (1991); A. Signal, A. W. Schreiber, and A. W. Thomas, Mod. Phys. Lett. A **6**, 271 (1991).
[5] J. Stern and G. Clement, Phys. Lett. B **264**, 426 (1991); M. Wakamatsu, Phys. Rev. D **44**, R2631 (1991); Phys. Lett. B **269**, 394 (1991).
[6] E. J. Eichten, I. Hinchliffe, and C. Quigg, Phys. Rev. D **45**, 2269 (1992).
[7] S. Koretune, Phys. Lett. **83B**, 218 (1979); Phys. Rev. D **21**, 820 (1980); Phys. Lett. **113B**, 335 (1982).
[8] S. Koretune, Prog. Theor. Phys. **72**, 821 (1984).
[9] S. Deser, W. Gilbert, and E. C. G. Sudarshan, Phys. Rev. **115**, 731 (1959).
[10] We can replace the sign function $\varepsilon(a^0)$ by $\varepsilon(a^+)$ for any timelike vector a.
[11] D. A. Dicus, R. Jackiw, and V. L. Teplitz, Phys. Rev. D **4**, 1733 (1971).
[12] S. L. Adler and R. F. Dashen, *Current Algebra and Application to Particle Physics* (Benjamin, New York, 1968), p. 345.
[13] The limit may be regular. A mathematical example of this statement is $f(x)=x\ln|x|$. This function contains the singular piece $\ln|x|$ as $x\to 0$, but it has a regular limit in this limit.
[14] M. Ida, Prog. Theor. Phys. **51**, 1521 (1974); R. Carlitz, D. Heckathorn, J. Kaur, and W. K. Tung, Phys. Rev. D **11**, 1234 (1975), and references cited therein.
[15] S. Weinberg, in *Lectures on Elementary Particles and Quantum Field Theory,* edited by S. Deser, M. Grisaru, and H. Pendelton (MIT Press, Cambridge, MA, 1970), p. 283.
[16] H. Pagels, Nuovo Cimento A **17**, 1 (1973).
[17] A. Manohar and H. Georgi, Nucl. Phys. **B243**, 189 (1984).
[18] J. D. Bjorken, "Soft and hard pomerons: Is there a distinction?," Report No SLAC-PUB-5608 (unpublished).
[19] V. D. Barger and D. B. Cline, *Phenomenological Theories of High Energy Scattering* (Benjamin, New York, 1969), p. 91.
[20] W. I. Weisberger, Phys. Rev. **143**, 1302 (1966).
[21] Particle Data Group, J. J. Hernández *et al.,* Phys. Lett. B **239**, 1 (1990), pp. III.77–III.81.
[22] Y. Sumi, Prog. Theor. Phys. Suppl. **41&42**, 15 (1967).
[23] See Ref. [12], p. 50.
[24] E. l. Berger, J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. **B286**, 704 (1987).