# Threshold effects on the mass-scale predictions in SO(10) models and solar-neutrino puzzle

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We compute the threshold uncertainties due to unknown masses of the Higgs bosons on the predictions for the intermediate and unification scales, $M_I$ and $M_U$, respectively, in SO(10) models. We focus on models with separate breaking scales for parity and $SU(2)_R$ symmetries since they provide a natural realization of the seesaw mechanism for neutrino masses. For the two-step symmetry-breaking chains, where left-right-symmetric gauge groups appear at the intermediate scale, we find that parity invariance of the theory at the unification scale drastically reduces the grand-unification-theory (GUT) threshold effects in some cases. Including the effects of the intermediate-scale thresholds, we compute the uncertainty in the above mass scales and study their implications for proton lifetime and neutrino masses. An important outcome of our analysis is that if the Mikheyev-Smirnov-Wolfenstein (MSW) solution to the solar-neutrino puzzle is accepted at the $1\sigma$ level, it rules out $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ as an intermediate symmetry for SO(10) breaking whereas the intermediate symmetry $SU(2)_L \times SU(2)_R \times SU(4)_c$ is quite consistent with it.

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## I. INTRODUCTION

The grand unified theories [1] (GUTs) provide an elegant extension of physics beyond the standard model. The requirement that the gauge couplings constants in these theories become equal at the GUT scale ($M_U$) lends them a predictive power which makes it possible to test them in experiments such as those looking for the decay of the proton. The most predictive such theory is the minimal SU(5) model of Georgi and Glashow [1], where the SU(5) symmetry breaks in one step to the standard model. The only new mass scale in this model is $M_U$ which can be determined by the unification requirement using the low-energy values of any two gauge couplings from the standard model. One then predicts not only $M_U$, but also the remaining low-energy gauge coupling constant (for example, $\sin^2\theta_W$). It is well known that for the minimal SU(5) model they lead to predictions for the proton lifetime as well as $\sin^2\theta_W$, both of which are inconsistent with experiments.

This, however, does not invalidate the idea of grand unification and attention has rightly been focussed on SO(10) [2] GUT models which can accommodate more than one new mass scale. Supersymmetric SU(5) [3] models also belong to this class. In this class of two-mass-scale theories, the values of low-energy gauge coupling constants can determine both the mass scales again making these theories experimentally testable. The determination of the values of the new mass scales become more precise as the low-energy values of the gauge coupling constants become better known. It is therefore not surprising that the recent high precision measurement of $\alpha_{\text{strong}}$ and $\sin^2\theta_W$ at the CERN $e^+e^-$ collider LEP [4] once again revived interest in grand unified theories [5].

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Supersymmetric SU(5) theories have been studied with the goal of predicting the scale of supersymmetry breaking [5]. These models, however, do not have any room for a nonzero neutrino mass nor natural generation of adequate baryon asymmetry, whereas, the SO(10) model is the minimal GUT scheme that provides a framework for a proper understanding of both these problems. In this paper, we concentrate on the SO(10) models with a two-step breaking to the standard model and study the threshold effects on the predictions for the two new mass scales, i.e., $M_U$ and $M_I$. In order to appreciate the significance of our work, it is worth pointing out that in SO(10) models the scale $M_U$ as usual is related to proton decay whereas the intermediate scale is related to neutrino masses if the intermediate symmetry is either of the left-right symmetric [6] groups $SU(2)_L \times SU(2)_R \times G_c$, where $G_c$ is $SU(4)_C$ or $SU(3)_c \times U(1)_{B-L}$. If the neutrino mass is determined independently [for example, from the Mikheyev-Smirnov-Wolfenstein (MSW) solution to the solar neutrino puzzle], then the seesaw mechanism determines the range of the required intermediate mass scale. The viability of a given SO(10) model will then depend on both the value of $M_I$ obtained from renormalization-group analysis as well as the uncertainties in this value arising from threshold corrections.

Let us discuss the kind of SO(10) models we will study here. As is well known, the SO(10) group contains the maximal subgroup $SU(2)_L \times SU(2)_R \times SU(4)_c \times D$, $D$ being a $Z_2$ symmetry which implements the parity transformation (as well as particle-antiparticle transformation). We will refer to this symmetry as $D$ parity. The actual nature of the SO(10) model depends on what symmetry appears at the intermediate scales (i.e., between the GUT scale and $M_W$). The most interesting SO(10) models are the ones where the symmetry breaking to the standard model occurs in two steps, with either of the left-right symmetric groups $SU(2)_L \times SU(2)_R \times SU(4)_c$ or $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ (denoted henceforth by $G_{224}$ and $G_{2213}$ respectively) as the only intermediate symmetry. These are also the theories for which definite predictions can be made. Our work will focus on them. Note the absence of $D$ parity at the intermediate scales. Use of Higgs multiplets belonging to **45** and **210** representations to break SO(10) can lead to such a scenerio, as was pointed out in a series of papers in 1984 by us in collaboration with D. Chang [7]. Let us briefly recapitulate some other motivation for considering such models.

One of the attractive features of the SO(10) models is that they provide a natural understanding of the neutrino masses via the seesaw mechanism [8]. It has, however, been noted that the seesaw mass matrix does not follow naturally in models where $D$-parity and $SU(2)_R$ breaking scales ($M_R$) are the same. On the other hand, if the $D$-parity breaking scale $M_P$ is such that $M_P \gg M_R$ then the seesaw formula emerges naturally [9]. This is perhaps the most compelling motivation for requiring the $D$-parity breaking scale to be significantly larger than $M_R$. There are, however, other motivations from cosmology. If $M_P = M_R$, there arise domain walls bounded by strings at the epoch when $SU(2)_R \times D$ symmetry breaks down. They dominate the mass density of the universe making it hard to understand the successes of the big bang picture. Such problems do not arise in SO(10) models with separate $D$-parity breaking scenarios. Furthermore, exact $D$ parity leads to $n_B = n_{\bar{B}}$. In SO(10) models where the baryon asymmetry of the universe arises from Higgs boson decays, the ratio $n_B / n_\gamma$ receives an additional suppression $(M_P / M_U)^2$ on top of its small value predicted in generic GUT models. This mechanism would prefer scenerios with the $D$-parity breaking above that of $SU(2)_R$ and at the GUT scale. We are of course fully aware that, if baryon asymmetry arises from the decay of heavy Majorana neutrinos, the above constraint does not apply.

A complete two-loop analysis of the predictions for $\sin^2\theta_W$ and proton lifetime in this class of SO(10) models was carried out in Ref. [10]. Depending on the nature of the Higgs boson spectrum used to implement the symmetry breaking and the nature of the intermediate symmetry groups, the intermediate scales and the associated physical implications were discussed. The mean values of the mass scales will be taken from this paper.

A basic limitation of all grand unified theories is that all mass-scale predictions are subject to uncertainties arising from Higgs boson thresholds [11]. It has therefore been argued [12] that since the Higgs bosons in question belong to large representations in SO(10) theories, one might worry that the mass-scale predictions derived from two-loop calculations are completely unreliable. In other words, even if $\sin^2\theta_W$ and $\alpha_{\text{strong}}$ are very precisely known, the unification scale $M_U$ and the intermediate scale $M_I$ will have large uncertainties. It was, however, subsequently pointed out that this need not always be true; for instance, if an SO(10) model has an intermediate symmetry group $SU(2)_L \times SU(2)_R \times SU(4)_c \times D$, ($G_{224D}$) the GUT threshold uncertainties in $\sin^2\theta_W$ exactly cancel out [13]. This result of course holds only if the intermediate symmetry is $G_{224D}$ and does not apply to the more interesting models with separate $D$-parity breaking; it also does not say anything about the uncertainties due to intermediate-scale thresholds. It does, however, give rise to the hope that existence of symmetries may reduce the net impact of threshold uncertainties. In any case, if the grand unified theories are to be useful, threshold effects [14] must be calculated. In this paper, we begin this program for the two SO(10) theories and hope to extend it to other models later on. The main results of this paper have already been reported earlier [15].

In Ref. [15] and in the present paper, we adopt the following approach. Using the evolution equation for the coupling constants, we express the $M_U$ and $M_I$ in terms of the known low-energy parameters $\alpha_{\text{strong}}$, $\sin^2\theta_W$, and $\alpha_{\text{em}}$ and the threshold corrections due to Higgs bosons. Since the LEP results have considerably reduced the experimental uncertainty in $\sin^2\theta_W$ as well as $\alpha_s$, the main uncertainty comes from the arbitrariness associated with the Higgs boson masses and the theoretical uncertainties in the scales $M_U$ and $M_I$ can be computed. The final magnitude of the uncertainty depends on how far the scalar masses are split from the symmetry-breaking scale. Using the standard model as a guide, we assume that constraints of one-loop radiative corrections and unitarity bound on tree-level amplitudes would allow the scalar boson masses to be a factor of 10 on either side of the symmetry-breaking scale. We also present results for a wider splitting of $(30)^{\pm 1}$ for illustration even though we believe this to be rather unlikely.

We consider the symmetry-breaking chains:

(A) $SO(10) \to G_{224} \to G_{\text{std}}$ ;

(B) $SO(10) \to G_{2213} \to G_{\text{std}}$ .

We compute the Higgs boson threshold effects on the uncertainty in the intermediate scale $(M_C / M_C^0)$ in case (A) to be $10^{{+2.7} \atop {-1.4}}$ and that in the value of the grand unification scale, i.e., in $(M_U / M_U^0)$ to be $10^{{+0.8} \atop {-1.7}}$. This corresponds to a maximum value for $\tau_p$ in case (A) to be $10^{40}$ years for $\alpha_s = 0.11$ and $10^{38.4}$ years for $\alpha_s = 0.1$. For case (B), we find the uncertainty in $(M_R / M_R^0)$ to be $10^{{+0.6} \atop {-0.3}}$ and that in $(M_U / M_U^0)$ to be at most $10^{\pm 0.2}$. Note that the threshold uncertainties are much less than the estimates of Ref. [12]. For $\alpha_s = 0.11$, we obtain an upper limit on $\tau_p$ in this case to be $5 \times 10^{36}$ years.

Using our results in combination with the seesaw formula for neutrino masses, we find that the presently favored nonadiabatic MSW solution to the solar-neutrino puzzle rules out the SO(10) model (B) which has $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ as an intermediate symmetry. In our opinion this is an important result since this will be the second GUT model that is being definitively ruled out by experiment. The symmetry-breaking chain [model (A)] is, however, quite consistent with data. This paper is organized as follows: in Sec. II, we present a derivation of the formulas for the threshold uncertainties for the model (A); in Sec III, we derive the contribution of the various Higgs multiplets to these uncertainties and give our estimate of these effects. Section IV uses the results of Secs. II and III to derive the same result for the model (B); in Sec. V, we derive the implications of our results for the solar-neutrino puzzle and in Sec. VI, we discuss the implications of our results for proton decay and the effect of adding extra Higgs multiplets on our result. In Sec. VII, we close with some concluding remarks.

## II. THE FORMULA FOR THE THRESHOLD CORRECTIONS: MODEL (A)

Let us now proceed to derive the equations for the threshold uncertainties. We will illustrate the technique for the model (A). We start with the standard renormalization-group equations for the evolution of the gauge coupling constants:

$$\mu\left[\frac{d\alpha_i}{d\mu}\right]=\frac{a_i\alpha_i^2}{2\pi}+\frac{1}{8\pi^2}\sum_j b_{ij}\alpha_i^2\alpha_j \ . \tag{1}$$

In Eq. (1), $\alpha_i=g_i^2/4\pi$ and the one-loop coefficient

$$a_i=-\tfrac{11N}{3}+\tfrac{4}{3}n_g+\tfrac{1}{3}T(s)\ ,$$

where $n_g$ is the number of fermion generations and $T(s)$ is the contribution of the Higgs bosons. The $b_{ij}$ are the two-loop coefficients, which are not needed here. At each symmetry-breaking threshold, we use the following matching conditions [14] (we assume that the group $G_I$ breaks to the group $G_i$ at the scale $M_I$):

$$\frac{1}{\alpha_i(M_I)}=\frac{1}{\alpha_I}-\frac{\lambda_i^I}{12\pi}\ . \tag{2}$$

In Eq. (2), $\lambda_i^I=\mathrm{Tr}(\theta_i^V)^2+\mathrm{Tr}(\theta_i^H)^2\ln(M_H/M_I)$; $\theta_i^V$ are the generators of the lower symmetry $G_i$ for the representations in which the heavy gauge bosons appear; $\theta_i^H$ is the same for the superheavy Higgs bosons.

Let us now apply the formulas in Eqs. (1) and (2) to the SO(10) models described earlier as (A). Note that since $D$ parity has been broken in both cases at the GUT scale, the Higgs multiplets needed to implement the symmetry breaking in model (A) are in 210-, 126- and 10-dimensional representations of the SO(10) group. We denote by $M_U$, $M_C$, and $M_Z$ the three symmetry-breaking scales and they arise from the vacuum expectation values (VEV's) of the above three Higgs multiplets, respectively. The $D$-parity breaking manifests itself in the mass of the submultiplet of the **126** representation $\Delta_L$ [transforming as (3,1,10) under $G_{224}$] being different from the submultiplet $\Delta_R$ transforming as $(1,3,\overline{10})$ under the same group. We postpone any further discussion of the Higgs bosons to the next section. Let us now derive the formula for the threshold corrections.

Using Eqs. (1) and (2) and the standard Georgi-Quinn-Weinberg–type analysis, we find the following expressions for $\sin^2\theta_W$ and $\alpha_s$:

$$16\pi\left[\alpha_s^{-1}-\frac{3}{8}\alpha_{\mathrm{em}}^{-1}\right]=A_c\ln\left[\frac{M_c}{M_Z}\right]+A_U\ln\left[\frac{M_U}{M_Z}\right]+F_c+F_\lambda+\Gamma_s\ , \tag{3}$$

where

$$\begin{aligned}A_c&=(8a_3-3a_{2L}-5a_Y-6a_4'+3a_{2R}'+3a_{2L}')\ ,\\ A_U&=(6a_4'-3a_{2R}'-3a_{2L}')\ ,\\ F_c&=3\ln\beta(a_{2L}'-a_{2L}''+a_{2R}'-a_{2R}''-2a_{4c}'+a_{4c}'')\ ,\\ F_\lambda&=-\tfrac{4}{3}(\lambda_{3c}^C-\tfrac{3}{8}\lambda_{2L}^C-\tfrac{5}{8}\lambda_Y^C+\tfrac{3}{4}\lambda_{4c}^U-\tfrac{3}{8}\lambda_{2L}^U-\tfrac{3}{8}\lambda_{2R}^U)\ ,\end{aligned} \tag{4}$$

$$\frac{16\pi}{\alpha_{\mathrm{em}}}\left[\sin^2\theta_W-\frac{3}{8}\right]=B_c\ln\frac{M_c}{M_Z}+B_U\ln\frac{M_U}{M_Z}+G_c+G_\lambda+\Gamma_\theta\ ,$$

where

$$\begin{aligned}B_c&=5(a_{2L}-a_Y)-(5a_{2L}'-2a_{4c}'-3a_{2R}')\ ,\\ B_U&=(5a_{2L}'-2a_{4c}'-3a_{2R}')\ ,\\ G_c&=\ln\beta(5a_{2L}''-3a_{2R}''-2a_{4c}''-5a_{2L}'+3a_{2R}'+2a_{4c}')\ ,\\ G_\lambda&=\tfrac{5}{6}(\lambda_Y^C-\lambda_{2L}^C+\tfrac{3}{5}\lambda_{2R}^U+\tfrac{2}{5}\lambda_{4c}^U-\lambda_{2L}^U)\ .\end{aligned}$$

In the above expressions, $a_i$, $a_i'$ and $a_i''$ denote the evolution coefficient for the gauge couplings between $M_W$ and $M_C$, $M_C$ and $M_P$, and $M_P$ and $M_U$, respectively. $M_P$ is the scale of $D$-parity breaking. The Γ's denote the two-loop contributions, which do not contribute to threshold uncertainties to the leading order and will therefore be omitted henceforth. The values of the $a_i$'s for model (A) are $a_{2L}''=a_{2R}''=a_{2R}'=\frac{11}{3}$, $a_{2L}'=-3$, $a_{4c}'=-\frac{23}{3}$, $a_{4c}''=-\frac{14}{3}$, $a_Y=\frac{41}{10}$, $a_{3c}=-7$.

They will be used in the numerical estimates of the various effects. Using Eqs. (3) and (4), we can express the mass scales $M_C$ and $M_U$ in terms of the low-energy parameters and the threshold contributions. The uncertainties in the low-energy parameters are experimental and can be estimated to be small as we show. The threshold contributions buried in the λ's introduce the theoretical uncertainty having to do with the fact that the heavy Higgs masses are unknown.

Let us first address the uncertainties due to the experimental errors in $\sin^2\theta_W$ and $\alpha_s$ which we take [16] as

$$\begin{aligned}\sin^2\theta_W&=0.2334\pm0.0008,\\ \alpha_s&=0.115\pm0.007\ .\end{aligned} \tag{5}$$

Denoting by $C_0=(16\pi/\alpha_{\mathrm{em}})(\alpha_{\mathrm{em}}/\alpha_s-\frac{3}{8})$ and $C_1=(16\pi/\alpha_{\mathrm{em}})(\sin^2\theta_W-\frac{3}{8})$, we get

$$\left[\Delta\ln\frac{M_C}{M_z}\right]_{\mathrm{expt}}=\Delta\left[\frac{C_0B_U-C_1A_U}{A_cB_U-A_UB_c}\right]\ , \tag{6}$$

$$\left[\Delta\ln\frac{M_U}{M_z}\right]_{\mathrm{expt}}=\Delta\left[\frac{C_0B_c-C_1A_c}{B_UA_c-A_UB_c}\right]\ . \tag{7}$$

Using Eqs. (3) and (4), and the values of various $a_i$'s given above to evaluate the $A$'s and $B$'s, we can now estimate the uncertainty in the quantities $M_C/M_C^0$ and $M_U/M_U^0$ where the quantities with subscripts denote the values corresponding to the mean values of $\sin^2\theta_W$ and $\alpha_s$:

$$\frac{M_C}{M_C^0}=10^{\pm 0.025}\ ,\quad \frac{M_U}{M_U^0}=10^{\pm 0.22}\ . \tag{8}$$

We will find that these uncertainties are small compared to those arising from unknown masses of heavy Higgs bosons, thanks to precision experiments at the LEP $e^+e^-$ collider.

Again using Eqs. (3) and (4) and the expressions for $A$, $B$, $F$, and $G$, we can derive the following expressions for the uncertainties in $M_U$ and $M_C$ arising from threshold effects only:

$$\Delta\ln\left[\frac{M_U}{M_Z}\right]=f_M^P+f_M^U+f_M^C-(A_M/A_\theta)(f_\theta^P+f_\theta^U+f_\theta^C)+(\Delta\ln M_C)_{\rm expt}(\alpha_s,\sin^2\theta_W)\ ,$$

$$\Delta\ln\left[\frac{M_C}{M_Z}\right]=-\frac{1}{A_\theta}(f_\theta^P+f_\theta^U+f_\theta^C)+(\Delta\ln M_U)_{\rm expt}(\alpha_s,\sin^2\theta_W)\ , \tag{9}$$

where

$$f_M^P=\left[1-\frac{a''_{2L}+a''_{2R}-2a''_{4c}}{a'_{2L}+a'_{2R}-2a'_{4c}}\right]\ln\beta\ , \tag{10a}$$

$$f_M^U=\left[\frac{\lambda_{2L}^U-\lambda_{4c}^U}{3(a'_{2L}+a'_{2R}-2a'_{4c})}\right] \tag{10b}$$

and

$$f_M^C=\left[\frac{5\lambda_Y^C+3\lambda_{2L}^C-8\lambda_{3c}^C}{18(a'_{2L}+a'_{2R}-2a'_{4c})}\right]\ , \tag{10c}$$

$$f_\theta^P=-\left[\frac{(a''_{4c}-a''_{2L})(a'_{2R}-a'_{2L})\ln\beta}{(a'_{2L}+a'_{2R}-2a'_{4c})}\right]\ , \tag{10d}$$

$$f_\theta^U=\left[\frac{(\lambda_{4C}^U-\lambda_{2L}^U)(a'_{2R}-a'_{2L})}{6(a'_{2L}+a'_{2R}-2a'_{4c})}\right]\ , \tag{10e}$$

$$f_\theta^C=\frac{1}{6}\left[\frac{\lambda_{2L}^C(a'_{4c}-a'_{2R})+\frac{5}{3}\lambda_Y^C(a'_{2L}-a'_{4c})+\lambda_{3c}^C(a'_{2R}+\frac{2}{3}a_{4c}-\frac{5}{3}a'_{2L})}{a'_{2L}+a'_{2R}-2a'_{4c}}\right]\ , \tag{10f}$$

$$A_M=1-\left[\frac{\frac{5}{3}a_Y+a_{2L}-\frac{8}{3}a_{3c}}{a'_{2L}+a'_{2R}-2a'_{4C}}\right]\ , \tag{10g}$$

$$A_\theta=\frac{5}{8}\left[\frac{1}{a'_{2L}+a'_{2R}-2a'_{4C}}\right]B\ , \tag{10h}$$

where

$$B=(\tfrac{3}{5}a'_{2R}+\tfrac{2}{5}a'_{4C}-a'_{2L})(\tfrac{5}{3}a_Y+a_{2L}-\tfrac{8}{3}a_{3c})-(a'_{2L}+a'_{2R}-2a'_{4C})(a_Y-a_{2L})\ .$$

In Eq. (10), $\beta=M_U/M_P$. If the intermediate symmetry is $G_{224D}$, one can still use the above general expressions, after dropping the $f_\theta^P$ and $f_M^P$ terms in Eqs. (8) and (9). (In this case of course, all $a'=a''$.) Note that in this case $f_\theta^U$ and $f_\theta^P$ vanish. To see that this is what one expects from the results of Parida and Patra (Ref. [13]), we note that in their work (Ref. [13]), the uncertainty in $M_C$ was assumed to be zero. Using this and bringing back the $\sin^2\theta_W$ and $\alpha_s$ terms to the equation (9), $f_\theta^U$ and $f_\theta^P$ terms can be identified as the GUT threshold uncertainty in $\sin^2\theta_W$ and therefore their vanishing in the $G_{224D}$ limit was what was established in Ref. [12].

## III. SURVIVAL HYPOTHESIS AND ESTIMATION OF THE THRESHOLD UNCERTAINTIES

In order to give a numerical estimate of these uncertainties, we need to know the masses of the physical Higgs bosons; more specifically, what submultiplets are at what mass scale. This can be done using the survival hypothesis for the Higgs bosons [17]. The basic assumption of the survival hypothesis is that only a minimal number of fine tunings of the parameters in the Higgs potential are done as required to ensure the hierarchy of the various gauge boson masses. In the case at hand we need to fine tune only two parameters since we have only a two-step breaking. The survival hypothesis then says [17] that a submultiplet of the Higgs multiplet of the GUT group, which acquires a VEV to break a given subgroup $G_i$, is stuck at the symmetry-breaking scale. The other submultiplets which transform as complete irreducible representations under $G_i$ get pushed to the next higher scale. Using this, we find the scales for the Higgs boson masses.

For the case of model (A), the Higgs multiplets needed are 210, 126, and two ten-dimensional multiplets. They are responsible for the three symmetry-breaking scales $M_U$, $M_I = M_C$, and $M_W$, respectively. As is well known, the 210 multiplet also breaks the $D$-parity symmetry. Using the survival hypothesis, the scales of the different Higgs multiplets can be obtained and they are listed in Tables I and II. In Table I, we list the Higgs bosons with masses around $M_U$; in Table II, the Higgs bosons with masses near $M_I$ are listed.

The $U$ submultiplet is the Goldstone mode corresponding to the superheavy gauge bosons and is omitted in computing the threshold uncertainties.

Let us now give their contributions to the various $\lambda^U$'s and to final uncertainties. Defining $\eta_i = \ln M_i / M_U$, we find

$$\lambda_{2L}^U = \lambda_{2R}^U = 6 + 30\eta_{\zeta_0} + 30\eta_\Sigma + 20\eta_\zeta \, . \tag{11}$$

Here, we have assumed that $M_{\Sigma_L} = M_{\Sigma_R}$ by left-right symmetry. Similarly, $\zeta_1$ and $\zeta_2$ effects are combined and denoted by $\zeta$. The coefficients in front of the different $\eta$'s are simply the Dynkin indices of the different multiplets under the different gauge groups. For instance, for $\eta_{2L}$, the Dynkin index is that of the gauge group SU(2), etc:

$$\lambda_{4C}^U = 4 + 2\eta_H + 2\eta_S + 32\eta_{\zeta_0} + 24\eta_\Sigma + 24\eta_\zeta + 4\eta_{\zeta_3} \, . \tag{12}$$

Repeating the similar procedure for the intermediate scale, we find the contribution to $\lambda^C$'s to be as follows:

$$\lambda_Y^C = \tfrac{1}{5}(3\eta_\phi + 2\eta_{R_1} + 4\eta_{R_2} + 16\eta_{R_3} + 32\eta_{R_4} + 64\eta_{R_5} + 24\eta_{R_6} + 14) \, , \tag{13}$$

$$\lambda_{2L}^C = \eta_\phi \, , \tag{14}$$

$$\lambda_{3c}^C = 1 + \eta_{R_1} + 5\eta_{R_2} + 5\eta_{R_3} + \eta_{R_4} + 5\eta_{R_5} \, . \tag{15}$$

The threshold contributions to $\Delta \ln(M_C/M_Z)$ and $\Delta \ln(M_U/M_Z)$ can now be written as

$$\Delta \ln(M_C/M_Z) = 1.29 \ln\beta + 0.0259(-2 + 2\eta_{10} + 4\eta_{126} + 2\eta_{210}) + 0.0017(65 - 15\eta_\phi + 769\eta_R) \, , \tag{16}$$

$$\Delta \ln(M_U/M_Z) = -0.92 \ln\beta - 0.0621\eta_{10} - 0.124\eta_{126} - 0.062\eta_{210} + 0.031\eta_\phi - 0.496\eta_R \, . \tag{17}$$

TABLE I. The Higgs bosons at mass scale $M_U$.

| SO(10) representation | $G_{224}$ submultiplet |
|---|---|
| 10 | $H(1,1,6)$ |
| 126 | $\zeta_0(2,2,15)$, $S(1,1,6)$, $\Delta_L(3,1,10)$ |
| 210 | $\Sigma_L(3,1,15)$, $\Sigma_R(1,3,15)$, $\zeta_1(2,2,10)$, $\zeta_2(2,2,\overline{10})$ $\zeta_3(1,1,15)$, $S'(1,1,1,)$, $U(2,2,6)$ |

TABLE II. The Higgs bosons with masses at $M_C$. The numbers within the parentheses refer to the representation content under $SU(2)_L \times U(1)_Y \times SU(3)_C$. The multiplet $\phi$ arises from the $\phi(2,2,0)$ and the $R$ multiplets arise from the multiplet $\Delta_R(1,3,\overline{10})$.

| SO(10) representation | $G_{213}$ submultiplet at $M_C$ |
|---|---|
| 10 | $\phi(2,-\sqrt{\tfrac{3}{5}}\tfrac{1}{2},1)$ |
| 126 | $R_1(1,\sqrt{\tfrac{3}{5}}\tfrac{1}{3},\bar{3})$ |
| | $R_2(1,\sqrt{\tfrac{3}{5}}\tfrac{1}{3},\bar{6})$ |
| | $R_3(1,-\sqrt{\tfrac{3}{5}}\tfrac{2}{3},\bar{6})$ |
| | $R_4(1,-\sqrt{\tfrac{3}{5}}\tfrac{4}{3},\bar{3})$ |
| | $R_5(1,\sqrt{\tfrac{3}{5}}\tfrac{4}{3},\bar{6})$ |
| | $R_6(1,2\sqrt{\tfrac{3}{5}},1)$ |

In deriving Eqs. (16) and (17), we have assumed that the Higgs multiplets belonging to a single SO(10) supermultiplet have the same mass at a given scale [18]. This implies that $\eta_{\zeta_0} = \eta_S = \eta_{126}$, $\eta_\Sigma = \eta_\zeta$, and all $\eta_{R_i}$ are equal. While there can be deviations from this degeneracy assumption, their contributions will not change our results noticeably. Secondly, we allow the $e^\eta$'s to be between $\frac{1}{10}$ and 10 as well as $\frac{1}{30}$ and 30. The first case corresponds to allowing the Higgs self-scalar couplings to range from $10^{-2}$ to $10^2$ times the gauge coupling and is better motivated by the analogy to the standard model than the second choice, although we present the results for both cases. As mentioned above, the value of $M_{\Delta_L}$ is always kept lower than $M_U$, which corresponds to taking $\beta$ less

TABLE III. In this table, we present our results for the threshold uncertainties in the intermediate scale and the unification scale for different values of $e^\eta$. The first four lines correspond to the case where the uncertainty in $M_I$ is maximized whereas the last four lines correspond to the case where the uncertainty in $M_U$ is maximized.

| Symmetry-breaking chain | $M_H/M_U$ or $M_H/M_I$ | $M_I/M_I^0$ | $M_U/M_U^0$ |
|---|---|---|---|
| $G_{224}$ | $\frac{1}{30}$ to 30 | $\overset{+4}{10^{-2.1}}$ | $\overset{+1.2}{10^{-2.5}}$ |
| $G_{2213}$ | | $\overset{+0.9}{10^{-0.4}}$ | $\overset{+0.1}{10^{-0.2}}$ |
| $G_{224}$ | $\frac{1}{10}$ to 10 | $\overset{+2.7}{10^{-1.4}}$ | $\overset{+0.8}{10^{-1.7}}$ |
| $G_{2213}$ | | $\overset{+0.6}{10^{-0.3}}$ | $10^{\pm 0.1}$ |
| $G_{224}$ | $\frac{1}{30}$ to 30 | $\overset{+4.2}{10^{-2.2}}$ | $\overset{+1.2}{10^{-2.5}}$ |
| $G_{2213}$ | | $\overset{+0.5}{10^{-0.2}}$ | $10^{\pm 0.2}$ |
| $G_{224}$ | $\frac{1}{10}$ to 10 | $\overset{+2.8}{10^{-1.5}}$ | $\overset{+0.8}{10^{-1.7}}$ |
| $G_{2213}$ | | $\overset{+0.3}{10^{-0}}$ | $10^{\pm 0.2}$ |

than one. Using Eqs. (16) and (17), we compute the uncertainties in the intermediate mass scales as well as the $M_U$. We present these results in Table III. There are two possibilities; one when the uncertainty in $M_C(=M_I)$ is maximized and another when the uncertainty in $M_U$ is maximized. We find the maximal uncertainty in $M_U/M_U^0$ to be $10^{-1.7}{}^{+0.8}$ whereas that in $M_C/M_C^0$ to be $10^{-1.5}{}^{+2.8}$ from Higgs boson threshold effects alone. We will study the impact of our results on the predictions of the SO(10) model in Sec. V.

## IV. THRESHOLD CORRECTIONS FOR MODEL (B)

Let us now turn our attention to discussing model (B), where the SO(10) symmetry first breaks down at the scale $M_U$ to $SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_c$ which subsequently breaks down at scale $M_R$ to the standard model. Again as before the $D$-parity symmetry is broken at the GUT scale. The Higgs multiplets necessary to implement this chain are 45-, 54-, 126- and 10-dimensional ones. The $D$-odd component of the 45-dimensional Higgs multiplet breaks the GUT symmetry. It has been pointed out that [19], without the presence of the **54**, the **45** will break SO(10) down to $SU(5)\times U(1)$ rather than $G_{2231}$. However, as far as the threshold corrections are concerned, in the limit of exact degeneracy the contribution of the submultiplets of the **54** exactly cancel. Let us now present the equations for the threshold corrections to $M_U$ and $M_R$ in this case. Using the same notation as in Eqs. (8) and (9) [except that we replace $M_C$ by $M_R$ in Eq. (9)], we give the expressions for the various $f$'s and $A$'s below. Defining $A'_U=8a'_{3c}-3a'_{2L}-3a'_{2R}-2a'_{BL}$ and $B'_U=5a'_{2L}-3a'_{2R}-2a'_{BL}$, we get

$$A_\theta=\frac{1}{A'_U}\left[8a_{3c}-3a_{2L}-5a_Y-\frac{5}{B'_U}(a_{2L}-a_Y)\right], \quad (18a)$$

$$A_M=\left[1-\frac{1}{A'_U}(8a_{3c}-3a_{2L}-5a_Y)\right], \quad (18b)$$

$$f_\theta^P=\ln\beta\left[\frac{1}{A'_U}(8a''_{3c}-6a''_{2L}-2a''_{BL})-\frac{2}{B'_U}(a''_{2L}-a''_{BL})\right], \quad (18c)$$

$$f_\theta^U=\frac{-1}{6A'_U}(8\lambda_{3c}^U-6\lambda_{2L}^U-2\lambda_{BL}^U)+\frac{2}{6B'_U}(\lambda_{2L}^U-\lambda_{BL}^U), \quad (18d)$$

$$f_\theta^C=\frac{-1}{6A'_U}(8\lambda_{3c}^R-3\lambda_{2l}^R-5\lambda_Y^R)+\frac{5}{6B'_U}(\lambda_{2L}^R-\lambda_Y^R), \quad (18e)$$

$$f_M^P=\ln\beta\left[1-\frac{1}{A'_U}(8a''_{3c}-6a''_{2L}-2a''_{BL})\right], \quad (18f)$$

$$f_M^U=\frac{1}{6A'_U}(8\lambda_{3c}^U-6\lambda_{2L}^U-2\lambda_{BL}^U), \quad (18g)$$

$$f_{M_C}=\frac{1}{6A'_U}(8\lambda_{3c}^R-3\lambda_{2L}^R-5\lambda_Y^R). \quad (18h)$$

In order to evaluate the threshold corrections, we need the values of gauge coupling evolution coefficients $a_i$'s as well the mass scales of the physical Higgs bosons. We have $a''_{2L}=a''_{2R}=a'_{2R}=-\frac{7}{3}$, $A''_{BL}=7$, $a'_{3c}=a''_{3c}=-7$, $a'_{2L}=-3$, $a'_{BL}=\frac{11}{2}$, $a_{3c}=-7$, $a_{2L}=-\frac{19}{6}$, $a_Y=\frac{41}{10}$.

Next we will use the survival hypothesis to determine the mass scales of the various Higgs bosons. In Table IV, we give the Higgs bosons with masses of order $M_U$. We do not include the components of the 54-dimensional multiplet since in the degenerate multiplet approximation their effects cancel out exactly. The Higgs fields with masses of order $M_R$ are only two in number and are therefore listed in the text.

There are only two Higgs multiplets at scale $M_R$; they are $\Delta_R^{++}(1,2\sqrt{\frac{3}{5}},1)$ and $\phi(2,-\frac{1}{2}\sqrt{\frac{3}{5}},1)$ where the numbers within the parentheses refer to their transformation property under the standard model. Their contribution to the various $\lambda$'s are given below:

$$\lambda_{2L}^U(126)=24\eta_{H_{1L}}+12\eta_{H_{2L}}+6\eta_{H_3}+6\eta_{H_4}+16\eta_{H_5}+2\eta_{H_6}, \quad (19a)$$

$$\lambda_{2R}^U(126)=\lambda_{2L}^U(126),$$

$$\lambda_{3c}^U(126)=30\eta_{H_1}+6\eta_{H_2}+4\eta_{H_3}+4\eta_{H_4}+24\eta_{H_5}+\eta_{H_7}+\eta_{H_8}, \quad (19b)$$

$$\lambda_{BL}^U(126)=12\eta_{H_1}+6\eta_{H_2}+16\eta_{H_3}+16\eta_{H_4}+\eta_{H_7}+\eta_{H_8}, \quad (19c)$$

$$\lambda_{2L}^U(10)=\lambda_{2R}^U(10)=0, \quad (19d)$$

$$\lambda_{BL}^U(10)=\lambda_{3c}^U(10)=\tfrac{1}{2}(\eta_{T_1}+\eta_{T_2}), \quad (19e)$$

$$\lambda_{2L}^U(45)=2\eta_{S_2}, \quad (19f)$$

$$\lambda_{2R}^U(45)=2\eta_{S_3}, \quad (19g)$$

TABLE IV. Higgs bosons with masses of order $M_U$.

| SO(10) representation | $G_{2213}$ content of the heavy boson |
|---|---|
| 10 | $T_1(1,1,\frac{1}{3}\sqrt{\frac{3}{2}},3)$ |
| | $T_2(1,1,-\frac{1}{3}\sqrt{\frac{3}{2}},\bar{3})$ |
| 126 | $H_{1L}(3,1,-\frac{1}{3}\sqrt{\frac{3}{2}},6)$ |
| | $H_{1R}(1,3,+\frac{1}{3}\sqrt{\frac{3}{2}},\bar{6})$ |
| | $H_{2L}(3,1,-\frac{1}{3}\sqrt{\frac{3}{2}},3)$ |
| | $H_{2R}(1,3,+\frac{1}{3}\sqrt{\frac{3}{2}},\bar{3})$ |
| | $H_3(2,2,-\frac{2}{3}\sqrt{\frac{3}{2}},3)$ |
| | $H_4(2,2,+\frac{2}{3}\sqrt{\frac{3}{2}},\bar{3})$ |
| | $H_5(2,2,0,8)$, $H_6(2,2,0,1)$ |
| | $H_7(1,1,\frac{1}{3}\sqrt{\frac{3}{2}},3)$ |
| | $H_8(1,1,-\frac{1}{3}\sqrt{\frac{3}{2}},\bar{3})$ |
| 45 | $S_1(1,1,0,8)$, $S_2(3,1,0,1)$, $S_3(1,3,0,1)$ |

$$\lambda_{3c}^{U}(45)=3\eta_{S_1} , \quad (19h)$$

$$\lambda_{BL}^{U}(45)=0 , \quad (19i)$$

$$\lambda_{Y}^{R}=\tfrac{6}{5}+\tfrac{24}{5}\eta_{R_1}+\tfrac{3}{5}\eta_{\phi} , \quad (19j)$$

$$\lambda_{2L}^{R}=\eta_{\phi} , \quad (19k)$$

$$\lambda_{3c}^{R}=0 . \quad (19l)$$

Using Eqs. (18) and (19) and the values of $a_i$'s we get for the threshold contribution to the uncertainties in $M_U$ and $M_R$ the following expressions:

$$\Delta \ln(M_U/M_Z)=0.0685\ln\beta-0.171\eta_{126}-0.049\eta_{45}-0.039\eta_{10}-0.177\eta_{\phi}-0.146\eta_{R_1} , \quad (20)$$

$$\Delta \ln(M_R/M_Z)=0.095\ln\beta-0.083\eta_{126}+0.033\eta_{45}+0.062\eta_{10}-0.06\eta_{\phi}+0.22\eta_{R_1} . \quad (21)$$

In order to evaluate the possible uncertainties, as before we keep $\beta$ less than one and allow $e^{\eta}$ to vary between $\frac{1}{10}$ and 10 in one case and $\frac{1}{30}$ and 30 in the second case. The results are given in Table III. We see that in this case the threshold uncertainties are much less than in model (A). The maximal uncertainity in $M_R/M_R^0$ is $10^{{}^{+0.6}_{-0.3}}$ whereas that in $M_U/M_U^0$ is $10^{\pm 0.2}$.

We also wish to note at this point the uncertainties in $M_U$ and $M_R$ arising from the errors in $\alpha_s$ and $\sin^2\theta_W$: Using the same formula as in Eqs. (6) and (7), we find for model (B)

$$M_U/M_U^0=10^{\pm 0.25} , \quad M_R/M_R^0=10^{\pm 0.18} . \quad (22)$$

## V. SOLAR NEUTRINO PUZZLE AND SO(10)

In this section, we study the implications of the results derived in this paper for the solar neutrino puzzle. As is well known, one of the most interesting resolutions of the solar neutrino deficit is the so-called MSW matter oscillation mechanism [20]. In this mechanism, resonant enhancement of the oscillation $\nu_e$ to either $\nu_\mu$ or $\nu_\tau$ takes place in the solar core for a range of values of the $\Delta m^2$ and the mixing angle $\theta$. In the so-called high-mass (adiabatic) solution, the value of $\Delta m^2$ is of order $10^{-4}$ eV$^2$ with $\sin^2\theta \simeq 0.02-0.6$ whereas in the nonadiabatic solution, we instead have $\Delta m^2 \sin^2 2\theta \simeq 4\times 10^{-8}$eV$^2$ with $\Delta m^2 \simeq 10^{-6}$ eV$^2-8\times 10^{-8}$ eV$^2$. The combination of chlorine [21], kamiokande [22], and initial gallium data [23] seems to point towards the nonadiabatic solution [20]. Furthermore, in SO(10) models, it is not easy without unnatural choice of parameters to get $\nu_e-\nu_\mu$ mixing angles larger than the Cabibbo angle. In this case, the nonadiabatic branch is automatically picked, which we assume below. This case seems to fit quite well with the seesaw picture for the neutrino masses in the SO(10) model [24] provided one assumes $D$-parity breaking [9]. In the presence of $D$-parity breaking, the hierarchical quadratic mass formula for neutrino masses follows naturally (a fact, which appears not to have been well appreciated by many theorists). The point, briefly, is that due to the presence of couplings of Higgs bosons of type $\mathbf{126}\times\mathbf{126}\times\mathbf{10}\times\mathbf{10}$, a VEV of $\Delta_R$ induces a VEV of the **126** submultiplet $\Delta_L$ of order $m_W^2/v_R$. This leads to a direct mass for all left-handed neutrinos of the same order invalidating the conventional seesaw mechanism formulas. If, however, $D$ parity is broken at the GUT scale, the $\Delta_L$ VEV becomes only of order [9] $m_W^2 v_R/M_U^2$ which is smaller than the seesaw contribution to the neutrino masses.

After the radiative corrections are taken into account [25], the formulas for neutrino masses are (assuming generation mixings to be small)

$$m_{\nu_e}=0.5\frac{m_u^2}{M_N} , \quad (23a)$$

$$m_{\nu_\mu}=0.7\frac{m_c^2}{M_N} , \quad (23b)$$

$$m_{\nu_\tau}=0.18\frac{m_t^2}{M_N} . \quad (23c)$$

In order to find the neutrino masses, we need to know $M_N$ which is given by $M_N=(f/g)M_R$. It can be argued on the basis of vacuum stability [26] that $f\leq g$. The mean value of $M_R$ has been obtained from two-loop analysis of the two SO(10) models in Refs. [10,27]. For case (A) we have $M_C^0\simeq 10^{11.5}$ GeV, whereas for model (B), we have $M_R^0\simeq 10^9$ GeV. The uncertainty in the exponent due to the error in $\alpha_s$ and $\sin^2\theta_W$ is about $\pm 0.025$ in model (A) and $\pm 0.18$ in model (B). Including this and the threshold uncertainties, we find the minimum value of neutrino masses (corresponding to $f=g$) to range between the following values:

Model (A): $m_{\nu_e}=6\times 10^{-12}$ eV$-2\times 10^{-7}$ eV ;

$m_{\nu_\mu}=4\times 10^{-7}$ eV$-10^{-2}$ eV ;

$m_{\nu_\tau}=0.006$ eV$-180$ eV .

Model (B): $m_{\nu_e}=10^{-7}$ eV$-10^{-6}$ eV ;

$m_{\nu_\mu}=10^{-2}$ eV$-10^{-1}$ eV ,

$m_{\nu_\tau}=100$ eV$-1$ keV .

We see that if the adiabatic solution is ruled out as is currently believed, then model (B) will be ruled out by the solar-neutrino experiments and only model (A) will be acceptable. This is an important result in our opinion since there are no other uncertainties one can hide behind to save this model.

Two remarks are now in order:

In the framework of general SO(10) models, the relations in Eq. (23) can be turned into lower bounds when the same maximum value is chosen for the heavy neutrino masses and our conclusions remain unchanged. To see this note that the general seesaw formula leads to $m_{\nu\nu}=m_u^T(M_N)^{-1}m_u$. When all eigenvalues of $M_N$ acquire their maximum values, i.e., $M_N=M_I$, we have the inequality $m_{\nu\nu}\geq(m_u^T m_u)/M_I$. On diagonalizing both sides, we get $m_{\nu_\mu} \geq m_c^2/M_I$ at the GUT scale. This only strengthens our conclusion.

In deriving our conclusion in case (B), we assumed all components of the 54-dimensional Higgs boson to have the same mass. Let us see what uncertainties are introduced if this assumption is relaxed. Under $G_{2213}$, the 54-dimensional representation breaks up as $S_1'(2,2,-\sqrt{\frac{3}{2}}\frac{1}{3},\bar{3})$; $\bar{S}_1'$; $S_3'(1,1,\frac{2}{3}\sqrt{\frac{3}{2}},6)$; $\bar{S}_3'$; $S_5'(1,1,0,8)$; $S_6'(3,3,0,1)$. Their contribution to the uncertainity in $M_R$ is given by

$$\Delta \ln \left[\frac{M_U}{M_Z}\right] = -0.73\eta_{S_6'} + 0.25\eta_{S_5'} + 0.73\eta\eta_{S_3'} - 0.25\eta_{S_1'} \ . \tag{24}$$

In order to see how big this effect is, we have analyzed the most general SO(10) invariant Higgs potential involving the 45- and 54-dimensional Higgs multiplets. For reasonable values of the self-scalar couplings (i.e., $\lambda \leq 1$ or so), the various masses are within a factor of 2–4 of each other. Allowing therefore a liberal spread of value for $e^{\eta}$ between $\frac{1}{10}$ and 1, we find the maximum uncertainity in $M_R$ induced by a nondegenerate 54 to be at most a factor of 10. This would put a lower bound on $m_{\nu_\mu}$ of $5\times10^{-3}$ eV which is bigger than the required upper limit of $3\times10^{-3}$ eV. The splitting of the 45 multiplets does not affect the results as can be seen from Eq. (21) due to their small coefficients. We note that in deriving our conclusion, we have taken the data at the $1\sigma$ level.

## VI. HIGGS BOSON RELATED UNCERTAINTY IN PROTON DECAY AND STABILITY OF THE THRESHOLD CALCULATIONS

In this section, we discuss two questions: (i) the uncertainty in the predictions for proton decay and (ii) the effect of adding extra Higgs bosons to a GUT theory on the above calculations. First, we discuss the predictions for proton in the two SO(10) models under discussion. Again using the results of Refs. [10,27] we find that for $\alpha_s = 0.11$, the value of $M_U = 10^{15.8}$ GeV. The uncertainty in $\alpha_s$ leads to an uncertainty of order $10^{\pm 0.22}$ [see Eqs. (8) and (22)] multiplying the above value. We predict the proton lifetime for the model (A) to be $\tau_p = 1.6\times10^{35\pm0.7\pm0.9^{+3.2}_{-6.8}}$ years. For the model (B), we find, $\tau_p = 1.6\times10^{35\pm0.7\pm1.0\pm0.8}$ years.

Let us now turn to the question of the stability of our results. It is sometimes stated that any additional Higgs multiplet added to a GUT model will add to the already existing uncertainty. However, in a recent paper [28], it has been shown by one of the authors that if the additional Higgs multiplet does have a VEV or has VEV in a gauge direction which has been broken by a Higgs field with the same representation content, then threshold effects from such multiplets always cancel in $\sin^2\theta_W$ or the intermediate scales. This lends a degree of stability to the above calculations.

## VII. CONCLUSIONS

In conclusion, we have presented a detailed analysis of the threshold effects due to the unknown masses of the Higgs bosons and shown their effect on the numerical predictions for the values of the unification and the intermediate scales in two SO(10) models with a two-step breaking. To the best of our knowledge, this is the first time such an analysis has been carried out for the SO(10) models. An interesting outcome of this analysis is that the nonadiabatic MSW solution to the solar-neutrino puzzle is inconsistent at the $1\sigma$ level with the predictions of model (B) which has an intermediate symmetry $SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_c$. We wish to emphasize that a key assumption in our analysis is the survival hypothesis used to estimate the rough order of magnitude of the superheavy Higgs boson masses. This hypothesis is valid as long as one sticks to the minimal fine-tuning hypothesis according to which we fine-tune parameters only as many times as needed to obtain the gauge hierarchies (in our case two). Our results will therefore be modified if extra fine-tunings are made. However, our formulas [Eqs. (8) and (9)] can be used for estimating those changes. The extension of our discussions to the supersymmetric SO(10) models, which are less open to the charge of unnaturalness, are presently under consideration.

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