Complete screening of a magnetic flux string by vacuum polarization in 3+1 dimensions

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The feedback effects of the vacuum current induced by a magnetic flux string in 3+1 dimensions are studied. It is found that the induced magnetic field cancels the applied one completely; i.e., a magnetic flux string does not exist in 3+1 dimensions.

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The vacuum polarization around a classical magnetic flux string located at the origin has been studied excluding the feedback effects of induced currents both in 2+1 and 3+1 dimensions [1–4]. It has been shown that the induced current density behaves as $1/r^2$ in 2+1 and $1/r^3$ in 3+1 dimensions respectively near the origin, and decays exponentially at a large radius r with a characteristic length scale $1/m$, m being the fermion mass. The induced charge exhibits a similar distribution in 2+1 dimensions, and its total amount is found to be a topological invariant $Q=-eF/2$ [5], with $e<0$ the fermion charge and F the total magnetic flux, $0 \le F \le 1$; while there is no charge density in 3+1 dimensions. This essential difference is due to the parity violation of the fermion mass term in (2+1)-dimensional Lagrangian.

The feedback interaction of the induced currents in 2+1 dimensions appears as a Chern-Simons term with a dimensional coefficient $e^2/4\pi$ in the Lagrangian [6–8]. This coefficient is equivalent to an effective photon mass, on which the dependence of the self-consistent induced current density $j(r)$ derived with feedback effects included has been investigated in Ref. [9]. It was shown that for the dimensionless coupling constant $g \equiv e^2/m > 1$ the characteristic length scale of $j(r)$ transited from the fermion Compton wavelength $1/m$ to the photon one $1/e^2$. When the photon becomes extremely massive ($g \gg 1$), $j(r)$ is highly concentrated at the origin and its effect cancels the applied magnetic field completely. In 3+1 dimensions a photon does not acquire mass from vacuum polarization. However, the $1/r^3$ divergence of the lowest-order induced current density at the origin gives rise to a similar phenomenon to that associated with $g \gg 1$ in 2+1 dimensions; a magnetic flux string is completely screened by vacuum polarization in an all-order consideration.

It was pointed out that the lowest-order total flux induced by a magnetic flux string was infinite [1,4]. Though this unphysical infinity can be removed by extending the tube containing the applied magnetic field to one with a finite radius $R > 10^{-280}/m$ [4], it indicates that feedback effects are never negligible for R below this scale. Intuitively, it is expected that the inclusion of feedback effects removes the infinity. We shall solve for a self-consistent induced vector potential $A(r)$ satisfying both the Dirac and Maxwell equations, which produce $j(r)$ from $A(r)$, and relate $A(r)$ to $j(r)$ respectively. The finiteness of $A(r)$ will verify our conjecture. Rather than study the system of a flux string directly which involves infinity, we start with an extended magnetic field

$$B(r)=\frac{F\lambda}{e}\frac{e^{-\lambda r}}{r} , \tag{1}$$

which approaches the form of a flux string in the $\lambda \to \infty$ limit. We obtain the self-consistent $A(r)$ under this applied field and study its variation with λ.

The method to calculate $A(r)$ is the same as that developed for the (2+1)-dimensional case [9]. We absorb the combination of the applied and induced vector potentials, both of which are now radius dependent, into the indices of fermion wave functions. Under this approximation the expression for the induced current density, a sum over all wave functions, is simplified to contain only several terms. Then it is possible to solve for $A(r)$ from this expression and the Maxwell equation numerically. We find that the product $-erA(r)$ shows a similar dependence on r to that in 2+1 dimensions, its magnitude increasing from the origin and reaching a plateau of height h. The constant h is always smaller than and approaches F as $\lambda \to \infty$. The slope to attain h becomes infinite in this limit. These imply the existence of a highly concentrated induced current density at the origin. The applied vector potential is then completely canceled by the opposite induced one. Hence a magnetic flux string is in fact "invisible," i.e., does not exist in 3+1 dimensions. Note that our conclusion is applicable to cases with an arbitrary coupling constant e^2. Therefore, assuming a small e^2 suppresses quantum fluctuation, and the assumption of classical background gauge field holds.

The considered system is described by the Dirac Hamiltonian

$$H=\boldsymbol{\alpha}\cdot(\mathbf{p}-e\,\mathbf{A}_{\rm app}-e\,\mathbf{A})+\beta m , \tag{2}$$

where the matrices β and α_i, $i=1-3$, are in the standard representation. The applied vector potential $\mathbf{A}_{\rm app}$ corresponding to the extended magnetic field is given in cylindrical coordinates by

$$er\,\mathbf{A}_{\rm app}=F(1-e^{-\lambda r})\hat{\boldsymbol{\phi}} , \tag{3}$$

with $F \equiv (e/2\pi)\oint \mathbf{A}_{\rm app}\cdot d\mathbf{l}$ and $\hat{\boldsymbol{\phi}}$ the unit vector in the azimuthal direction. The induced vector potential $\mathbf{A}$ is derived from the induced current density $j(r)$ through the Maxwell equation

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rA)\right]=-4\pi j(r)\ . \tag{4}$$

An induced scalar potential does not appear in Eq. (2) due to the absence of the induced charge density.

The induced current density is expressed as a sum over all products of fermion partial waves [5]:

$$e\langle j^{\mu}\rangle=\frac{-e}{2}\sum_{p_z,k,\nu}\mathrm{sgn}(E)\bar{\psi}_{p_zk\nu}\gamma^{\mu}\psi_{p_zk\nu}\ , \tag{5}$$

with the appropriate regularization [10,11]. The function $\psi_{p_zk\nu}$ is the normalized solution to the Dirac Hamiltonian in Eq. (2) with the index p_z for fermion z-component momentum, $k=(E^2-p_z^2-m^2)^{1/2}$ for the radial one and $\nu=n-F$ for angular momentum, $n\in\mathbb{Z}$. Only the induced current density in the azimuthal direction is nonvanishing from rotational symmetry, and it has been derived for the case of a magnetic flux string excluding $A(r)$ [2]:

$$j(r)=\frac{-e(mr)^4\sin(F\pi)}{4\pi^3r^3}[\tfrac{1}{3}K_{2+F}(mr)K_{2-F}(mr)-\tfrac{4}{3}K_{1+F}(mr)K_{1-F}(mr)+K_F^2(mr)]\ , \tag{6}$$

where K_F is the Fth modified Bessel function.

For the present system with feedback effects taken into account, we rewrite Eq. (6) by substituting the combination $\bar{\nu}(r)\equiv er[A_{\mathrm{app}}(r)+A(r)]$ for the index F as in Ref. [9]:

$$j(r)=\frac{-e(mr)^4\sin[\bar{\nu}(r)\pi]}{4\pi^3r^3}[\tfrac{1}{3}K_{2+\bar{\nu}(r)}(mr)K_{2-\bar{\nu}(r)}(mr)-\tfrac{4}{3}K_{1+\bar{\nu}(r)}(mr)K_{1-\bar{\nu}(r)}(mr)+K_{\bar{\nu}(r)}^2(mr)]\ . \tag{7}$$

Equation (7) is an approximate expression for the induced current density, in which the contribution from the derivative of the total vector potential $d\bar{\nu}/dr$ has been neglected. We shall find that as $\lambda\to\infty$ Eq. (7) describes the system of a magnetic flux string appropriately.

We analyze the case of $F=1$ with $-e$ and m set to be 1 as an example. We solve Eqs. (4) and (7) for $rA(r)$ using an iterative method. Results for different λ are presented in Fig. 1. It is observed that $rA(r)$ grows from zero and reaches a constant at large r, which approaches F with λ. As $\lambda=10^7$ $rA(r)$ rises so rapidly that it is essentially equal to F everywhere except the tiny region characterized by $r\sim 1/\lambda$. On this λ scale both the applied and induced magnetic fields are in the form of a flux string, and Eq. (7) gives an exact expression for the induced current density. It is evident that the applied and induced vector potentials will cancel each other completely if λ is large enough. For smaller e^2 or larger m the same conclusion is achieved but the complete cancellation happens at higher λ.

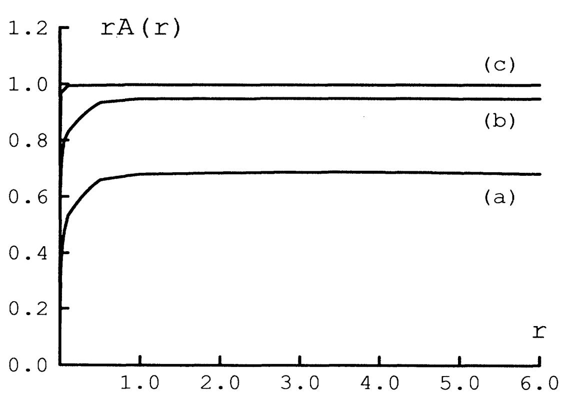

FIG. 1. The r dependence of the self-consistent induced vector potential $rA(r)$ for (a) $\lambda=10^3$, (b) $\lambda=10^5$, and (c) $\lambda=10^7$ with $-e=m=1$ and $F=1$.

To realize the cancellation analytically, we propose a model form for $-erA(r)$ with two parameters h and s which characterize its height and the slope to reach this height. They are determined by the limiting behaviors of $rA(r)$ and $j(r)$ at the origin and infinity. Assuming $rA(r)\sim r$ at small r and $\sim h$ at large r we have, from Eq. (7), $j(r)\sim 1/r^2$ and $\sim\exp(-2mr)/r^2$ as $r\to 0$ and $\to\infty$, respectively. The above limits still hold when they are substituted into Eq. (4), so self-consistent solutions should satisfy these conditions. A possible choice for the model of $rA(r)$ is given by

$$-erA(r)=h\left[1-\frac{\exp(-2mr)}{1+sr}\right]\ . \tag{8}$$

Inserting Eqs. (7) and (8) into the right-hand and left-hand sides of (4) respectively, we obtain a set of coupled equations for h and s corresponding to the $r\to 0$ and $\to\infty$ limits:

$$\begin{aligned}h(2m+s)&=\frac{4e^2}{3\pi}[F\lambda-h(2m+s)]\ ,\\ \frac{h}{s}&=\frac{e^2\sin[(F-h)\pi]}{4\pi m}\ .\end{aligned} \tag{9}$$

Solving Eq. (9) we find, as $\lambda\to\infty$,

$$\begin{aligned}h&=F\left[1-\frac{m(3\pi+4e^2)}{e^4}\frac{1}{\lambda}\right]\ ,\\ s&=\frac{4e^2}{3\pi+4e^2}\lambda\ .\end{aligned} \tag{10}$$

It is obvious that h and s exhibit the desired dependence on λ; they approach F and infinity, respectively, as $\lambda\to\infty$. Their values evaluated at $\lambda=10^7$ are consistent with those shown in Fig. 1. The induced vector potential tends to the form of a flux string, which is in the opposite direction to the applied one. The applied magnetic flux string is thus completely screened. Note that the cancellation becomes exact only when $\lambda \gg m$, or the applied magnetic field is confined in a region with radius much smaller than the fermion Compton wavelength. For a real magnetic flux tube corresponding to $\lambda \ll m$ feedback effects are negligible [4]. The expressions for h and s also possess the right dependence on e^2 and m; $rA(r)$ approaches F at higher λ for smaller e^2 or larger m.

We shall make more comments on the particular case of $F=1$. Since the lowest-order induced current for a magnetic flux string with integer F vanishes, it seems that this special flux can never be canceled by vacuum polarization due to the absence of feedback effects. However, starting with an extended system we observe different distribution for the lowest-order induced current. For arbitrary F the current density behaves as $F\lambda/r^2$ in the region of $r \ll 1/\lambda$ from Eq. (7) with $\bar{v}(r)$ replaced by the applied vector potential in Eq. (3), and decreases outside the above region, following the lowest-order expression Eq. (6). As $\lambda \to \infty$ the current density is recovered to that described by Eq. (6) for $F<1$. For $F=1$ the current density vanishes for all $r>0$, consistent with Eq. (6), but tends to be divergent at the origin in the $\lambda \to \infty$ limit. It is this singularity, not appearing in a direct analysis for the system of a magnetic flux string, that gives the required feedback effects. Hence the two limits $F \to 1$ and $\lambda \to \infty$ do not commute. If we allow $\lambda \to \infty$ at first, the current density, always divergent at the origin for $0<F<1$, drops to zero at $F=1$. This drop is not observed if the limit $\lambda \to \infty$ is applied later, because the current density remains singular there. A similar difference happens in the calculation for the total induced charge Q in $2+1$ dimensions. Setting $F=1$ at the beginning leads to vanishing induced charge density and Q, while evaluating Q at first and then letting $F=1$ gives $Q=-e/2$. Therefore we have explored different structure through an extended configuration. For $2+1$ dimensions a flux string and an extended field lead to the same limits in the sense that feedback effects due to the lowest-order induced current vanish in both systems as $F \to 1$.

Cases with other values of F and several extended forms of magnetic field have been studied. Similar features to those exhibited in Fig. 1 are obtained. It is found that for $F=1$ the self-consistent induced current density is more singular near the origin and decays faster at large r than for $F=0.5$. This result is consistent with the above statement; for the former case no self-consistent induced current exists at large r where the lowest-order one vanishes, and feedback effects arise completely from the region of small r so that a more divergent current density is required there.

To generalize the conclusion to F beyond 1, say, $F=1+\epsilon$ with $0<\epsilon<1$, we design a special procedure of preparing the magnetic field, in which the applied flux is increased from 0 to 1 in the form of a string, and then from 1 to F in the extended form. At the former stage the effect of the self-consistent induced current cancels the applied field completely, and the space is left empty. Hence, at the latter stage the system can be treated as an extended magnetic field with total flux ϵ only. As $\lambda \to \infty$ a complete screening of the magnetic flux string with $F>1$ is observed. In fact, this is the only possible solution, because any nonzero net line field leads to an infinite total flux which is not self-consistent. Therefore an accumulation of singular induced current proportional to F, similar to Q in $2+1$ dimensions, is associated with the magnetic flux string. Based on the above analysis, it is concluded that the periodicity of the lowest-order induced current distribution in F is lost when feedback effects are taken into account.

Finally we shall examine the influences of contributions from higher derivatives of $\bar{v}(r)$. Following the same procedures as in Ref. [9] it is straightforward to derive the first-derivative correction to $j(r)$:

$$\Delta j(r)=\frac{m^2e^2}{2\pi^3}\frac{d\bar{v}(r)}{dr}\frac{d}{d\nu}\{\sin(\nu\pi)[K_\nu^2(mr)-K_{1+\nu}(mr)K_{1-\nu}(mr)]\}_{\nu=\bar{v}(r)}\sim\frac{-e}{2\pi^2}\frac{F\lambda-h(2m+s)}{r^2}\ ,\quad r\to 0 \tag{11}$$

where Eqs. (3) and (8) have been inserted to obtain the second expression. At large r the correction $\Delta j(r)$ vanishes much faster than $j(r)$ due to the factor $d\bar{v}/dr$, while at small r the former behaves like the latter as shown in Eq. (11). If $\Delta j(r)$ is included into Eq. (9) for analysis, we shall find that the cancellation is improved, because the induced vector potential approaches the applied one at smaller λ. The corrections from the second derivative or the square of the first derivative of $\bar{v}(r)$ are less important due to their $1/r$ behavior in the $r \to 0$ limit. The higher-derivative corrections are also negligible by similar reasoning. Therefore these corrections do not modify the conclusion that a magnetic flux string is completely screened by vacuum polarization.

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