Gravitational laser backscattering

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A possible way of producing gravitons in the laboratory is investigated. We evaluate the cross section for electron + photon → electron + graviton in the framework of linearized gravitation, and analyze this reaction considering the photon coming either from a laser beam or from a Compton backscattering process.

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The attempts to detect gravitational radiation usually rely on the observation of astrophysical objects. These sources, such as supernovae explosions or coalescing binaries, have in general very uncertain frequency, strength, and spectrum of gravitational waves. A considerable improvement in the search of the gravitational waves would occur if we succeed in producing them in the laboratory. In this case, the energy and propagation direction could, at least in principle, be controlled.

In this paper, a mechanism of graviton production in a linear electron-positron collider is analyzed. Inspired by the Compton laser backscattering process [1], we suggest that the collision of laser photons of few electron volts, at small angle, with an energetic electron beam is able to generate a very collimated graviton beam.

We evaluate, in a first step, the cross section for the process electron + photon → electron + graviton, and estimate the total radiated power assuming that the photons come from a laser beam. The scenario where the photons are harder, as a result of a Compton backscattering process, is examined in a second step.

The coupling of the matter fields with gravity can be obtained using the weak field approximation [2]. The metric is assumed to be close to the Minkowski one ($\eta_{\mu\nu}$):

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \ , \tag{1}$$

where $h_{\mu\nu}$ is the graviton field and $\kappa \equiv \sqrt{32\pi G} = 8.211 \times 10^{-19}$ GeV^{-1} in natural units. The vierbein is expanded as

$$\mathcal{V}^a{}_\mu = \delta^a{}_\mu + \frac{\kappa}{2} h^a{}_\mu - \frac{\kappa^2}{8} h^a{}_\alpha \delta^\alpha{}_b h^b{}_\mu + \cdots \ . \tag{2}$$

The above expressions allow us to write the relevant part of the action as

$$S = \int d^4x \Bigg\{ \frac{\kappa}{2} \Bigg[\Bigg[-\frac{i}{4} \bar{\psi} \left(\gamma^a \partial_\mu + \gamma_\mu \partial^a \right) \psi + \frac{i}{4} \left(\partial_\mu \bar{\psi} \gamma^a + \partial^a \bar{\psi} \gamma_\mu \right) \psi \Bigg] h_a{}^\mu + \left(\frac{i}{2} \bar{\psi} \gamma^\mu \partial_\mu \psi - \frac{i}{2} \partial_\mu \bar{\psi} \gamma^\mu \psi - m \bar{\psi} \psi \right) h \Bigg]$$
$$- \frac{\kappa}{8} \left(\eta^{\mu\alpha} \eta^{\nu\beta} h - 4 h^{\mu\alpha} \eta^{\nu\beta} \right) F_{\mu\nu} F_{\alpha\beta} - \frac{e\kappa}{4} \left(\delta_a{}^\mu h - h_a{}^\mu \right) \bar{\psi} \left(A_\mu \gamma^a + A^a \gamma_\mu \right) \psi \Bigg\} \ , \tag{3}$$

where we omitted the free electron and photon Lagrangians, and the usual QED fermion-photon coupling, and neglected the $O(\kappa^2)$ terms.

In order to evaluate the cross section for the process electron + photon → electron + graviton, we used the Weyl–van der Waerden spinor technique to describe the graviton helicity wave function [3] and the graviton couplings with bosons and fermions [2]. The cross section of the reaction $e^-(p) + \gamma(q) \to e^-(p') + g(k)$ is given by

$$\frac{d\sigma}{dt} = \frac{e^2\kappa^2}{16\pi} \frac{1}{(s-m^2)^2} \frac{(us-m^4)}{t} \left[\left(\frac{m^2}{s-m^2} + \frac{m^2}{u-m^2} \right)^2 + \frac{m^2}{s-m^2} + \frac{m^2}{u-m^2} - \frac{1}{4} \left(\frac{s-m^2}{u-m^2} + \frac{u-m^2}{s-m^2} \right) \right] . \tag{4}$$

This expression can be written in a more compact form if we define new variables:

$$x = \frac{s-m^2}{m^2} = \frac{2Eq_0}{m^2}(1+\beta\cos\alpha) ,$$
$$y = \frac{-t}{s-m^2} = \frac{k_0}{E}\frac{1+\cos(\theta-\alpha)}{1+\beta\cos\alpha} , \tag{5}$$

where m, E, and $\beta = (1-m^2/E^2)^{1/2}$ are respectively the electron mass, energy, and velocity in the laboratory frame. The laser, of energy q_0, is supposed to make an angle α with the electron beam. The scattered graviton has energy k_0:

$$k_0 = q_0 \frac{1+\beta\cos\alpha}{1-\beta\cos\theta + (q_0/E)\left[1+\cos(\theta-\alpha)\right]}, \tag{6}$$

where θ is the angle between the graviton and the incoming electron. Note that, for $\alpha \simeq 0$, the variable y represents the fraction of the electron energy carried by the graviton in the forward direction ($\theta = 0$).

Equation (4) expressed in terms of x and y becomes

$$\frac{d\sigma}{dy} = \frac{e^2\kappa^2}{64\pi}\left(\frac{1}{y} - \frac{x+1}{x}\right) F(x,y) , \tag{7}$$

with

$$F(x,y) \equiv \left[1 - y + \frac{1}{1-y} - \frac{4y}{x(1-y)} + \frac{4y^2}{x^2(1-y)^2}\right] . \tag{8}$$

Equation (7) shows that the cross section for large values of y is suppressed by an overall factor $(1/y - 1/y_{\max})$ which makes the distribution very sharp for small values of y. This is a major difference from the Compton cross section $e+\gamma_\ell \to e+\gamma_b$, where $\gamma_{\ell(b)}$ is the laser (backscattered) photon. In the latter case, the y distribution assumes its maximum value for $y = y_{\max} = x/(x+1)$, giving rise to a hard photon spectrum.

Let us start by analyzing our result when the initial photon originates from a laser beam. We consider four different designs of e^+e^- linear colliders [4]: SLAC Linear Collider (SLC) ($E = 50$ GeV, $\mathcal{L} = 5\times10^{29}$ cm^{-2} s^{-1}), Palmer-G ($E = 250$ GeV, $\mathcal{L} = 5.85\times10^{33}$ cm^{-2} s^{-1}), Palmer-K ($E = 500$ GeV, $\mathcal{L} = 11.1\times10^{33}$ cm^{-2} s^{-1}), and Serpukhov's VLEPP ($E = 1000$ GeV, $\mathcal{L} = 10^{33}$ cm^{-2} s^{-1}).

Assuming $q_0 = 1$ eV, and $\alpha = 0$, we compare the behavior of the graviton angular distribution $d\sigma/d\cos\theta$ for different electron beam energies. The angular distribution is peaked for very small angles, close to the electron direction [Fig. (1)]. By changing the value of α we are of course able to shift this peak away from the electron beam.

Let us suppose that the laser has flash energy of 2.5 J, the same repetition rate as the electron pulse frequency, and a rms radius of 20μm. In this case, the electron laser luminosity is $\mathcal{L}_{e\gamma_\ell} = \eta\mathcal{L}$, $\mathcal{L}$ being the collider luminosity, and $\eta = A_e N_{\gamma_\ell}/A_{\gamma_\ell} N_e$. Here, $A_{e(\gamma_\ell)}$ is the electron (laser) beam cross section and $N_{e(\gamma_\ell)}$ is the number of electrons (photons) in the bunch (flash).

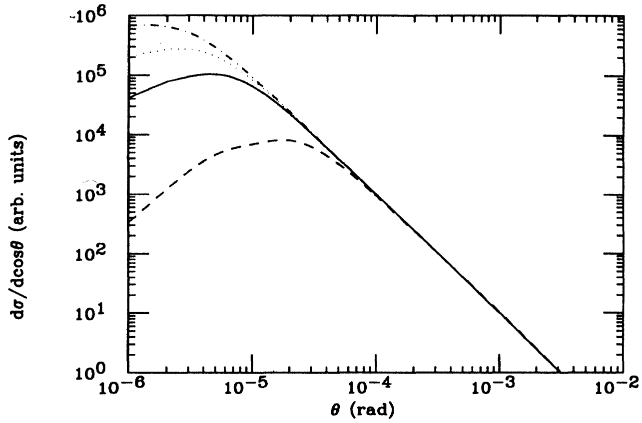

FIG. 1. The angular distribution $d\sigma/\cos\theta$ in arbitrary units versus the angle in radians for the process $e^- + \gamma_\ell \to e^- + g$, where γ_ℓ comes from a laser beam. We assume $q_0 = 1$ eV and plot for different electron beam energies: E=50 GeV (dashed), 250 GeV (solid), 500 GeV (dotted), and 1000 GeV (dot-dashed).

The radiation power emitted in solid angle of semiangle (θ_0) due to the graviton emission is

$$P_\ell(\theta_0) = \mathcal{L}_{e\gamma_\ell}\int_0^{\theta_0} d\theta\ \sin\theta \frac{d\sigma}{d\cos\theta}(\theta)\ k_0(\theta).$$

In Fig. 2 we show the spectral distribution of the emitted radiation

$$\frac{dP_\ell}{dk_0} \propto \frac{k_0}{\beta E - q_0}\left(\frac{\beta E - q_0}{k_0 - q_0} - \frac{x+1}{x}\right) F\left(x, \frac{k_0 - q_0}{\beta E - q_0}\right) .$$

We note that this distribution is maximum for $k_0 = k_0^{\min} = q_0$, and is almost independent of the electron beam energy.

Another way of obtaining the reaction $e+\gamma \to e+g$ is taking advantage of the energetic photons produced by the Compton backscattered process. The energy spectrum of backscattered photons is [1, 5]

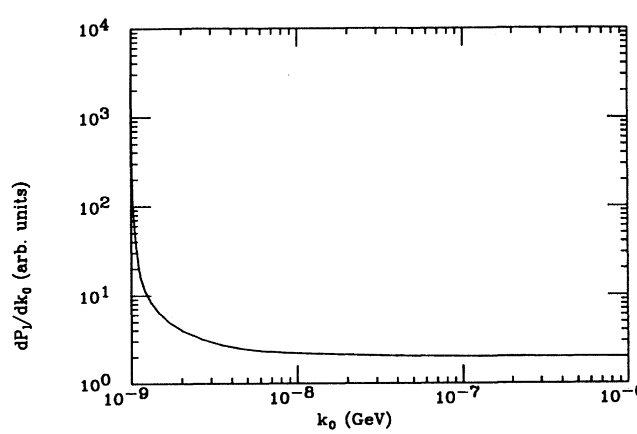

FIG. 2. Spectral distribution of the graviton radiation power dP_ℓ/dk_0 in arbitrary units as a function of k_0, for $E = 50$ GeV.

$$\mathcal{F}_{\gamma/e}(x,z) \equiv \frac{1}{\sigma_c}\frac{d\sigma_c}{dz} = \frac{1}{C(x)}F(x,z), \tag{9}$$

where σ_c is the Compton cross section, and $z = q_\gamma/E$ is the fraction of the initial electron carried by the photon. The function $F(x,z)$ was defined in Eq. (8), with x given by Eq. (5), and

$$C(x) = \left(1 - \frac{4}{x} - \frac{8}{x^2}\right)\ln(1+x) + \frac{1}{2} + \frac{8}{x} - \frac{1}{2(1+x)^2}. \tag{10}$$

Defining the ratio of electron-photon invariant mass squared ($\hat{s} = 4Eq_\gamma$) to invariant mass squared of the collider,

$$\tau \equiv \frac{\hat{s}}{s} = \frac{q_\gamma}{E},$$

and assuming a conversion coefficient (the average number of converted photons per electron) $\xi = 0.65$ [6], the $e\gamma_b$ luminosity can be written in terms of the machine luminosity as $\mathcal{L}_{e\gamma_b} = \xi\mathcal{L}$. The luminosity distribution of the backscattered photons is

$$\frac{d\mathcal{L}}{d\tau} = \xi\,\mathcal{L}\,\mathcal{F}_{e/\gamma}(x,\tau). \tag{11}$$

In this case, the radiation power, emitted in solid angle of semiangle (θ_0), is

$$P_b(\theta_0) = \int_0^{\tau_{\max}} d\tau \frac{d\mathcal{L}}{d\tau}\int_0^{\theta_0} d\theta\,\sin\theta \frac{d\hat{\sigma}}{d\cos\theta}(\theta,\tau)\,k_0(\theta,\tau),$$

where $\tau_{\max} = x/(x+1)$, and $\hat{\sigma}$ is the elementary cross section for the process $e + \gamma \to e + g$ evaluated at $s = \hat{s}$.

In Table I we present the total radiation power for different values of θ_0, and for a laser energy (q_0) of 1 eV. Since x increases when we increase either the laser energy or the electron beam energy, the different choices of colliders considered here give a reasonable idea of the laser backscattering capabilities to produce gravitons in the laboratory.

As pointed out before, gravitons can be produced via $e + \gamma_\ell \to e + g$, where γ_ℓ is a laser photon, and via $e + e(+\gamma_\ell) \to e + \gamma_b \to e + g$, where γ_b is the laser backscattered photon. In the former case, the radiation power P_ℓ is almost independent of the particular choice of θ_0, since the angular distribution of the graviton peaks at very small angles relative to the electron beam direction. On the other hand, in the latter case, the backscattered photon is very hard, giving rise to a flatter angular distribution and making the dependence of P_b on θ_0 more evident. The total forward emitted power becomes, therefore, small for hard photons.

TABLE I. Total graviton radiation power emitted in a solid angle of semiangle θ_0. P_ℓ refers to the direct laser photon process and P_b to the backscattered photon one. We consider in both cases a laser photon with energy of 1 eV.

Collider	P_ℓ (eV/s)	θ_0	P_b (eV/s)
		10°	1.69×10^{-28}
SLC	1.17×10^{-20}	20°	4.48×10^{-28}
		30°	7.50×10^{-28}
		10°	5.02×10^{-24}
Palmer-G	2.32×10^{-19}	20°	1.47×10^{-23}
		30°	2.66×10^{-23}
		10°	1.51×10^{-23}
Palmer-K	6.70×10^{-19}	20°	4.57×10^{-23}
		30°	8.47×10^{-23}
		10°	2.26×10^{-24}
VLEPP	1.53×10^{-18}	20°	6.94×10^{-24}
		30°	1.32×10^{-23}

In a recent paper, Chen [7] has examined the production of gravitons due to the interaction between the beams near the interaction point in an e^+e^- collider (gravitational beamstrahlung). His results for the total radiation power, at the SLC, is roughly one order of magnitude below the one for the gravitational laser backscattering process. For the next generation of linear e^+e^- colliders, the gravitational beamstrahlung yields a larger radiation power. Nevertheless, the coherent contribution is again smaller than the one coming from the laser backscattering.

We should stress that, in spite of the small emitted radiation power, the gravitational laser backscattering process yields a very collimated beam of gravitational waves with a defined energy spectrum (see Fig. 2). These points could be of great help for the complex issue of gravitational wave detection [8].

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