Cosmological models in the Schmidt-Greiner-Heinz-Müller theory of gravitation

A. Serna and R. Domínguez-Tenreiro
Departamento de Física Teórica, C-XI, Universidad Autónoma de Madrid, Cantoblanco, 28049-Madrid, Spain
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An exhaustive study of homogeneous and isotropic cosmological models in the Schmidt-Greiner-Heinz-Müller (SGHM) theory of gravitation is performed, where present values of dynamical functions are consistent with astronomical observations and solar-system experiments. We find two types of SGHM models: a first class which is exactly equal to the standard general relativistic (GR) cosmological models, and a second class which never reduces to GR. Some analytical and numerical solutions for the second type of SGHM models are then obtained. The age of the Universe predicted by these models is compatible with the age of old globular clusters except for weak bounds on the Δ_0 parameter defined in this paper.

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I. INTRODUCTION

The Schmidt-Greiner-Heinz-Müller (SGHM) theory of gravitation was proposed in order to prevent the collapse of massive dense objects [1]. In this theory, the gravitational constant depends on a scalar field ϕ which couples to the surrounding masses via the curvature scalar $\mathcal{R}$. This coupling is such that the gravitational interaction decreases with the strength of the scalar field. In principle, if ϕ increases when matter density decreases, the collapse of a massive object could be stopped by the whole system reaching a new stable configuration. However, Schmidt *et al.* [1] have shown that the purpose of their theory was inviable because, even when the coupling constant between scalar and gravitational fields varies within the whole allowed range, the effective gravitational constant only varies within a narrow interval.

Nevertheless, this theory has cosmological interest because, although its predictions at the present time are very close to those of general relativity (GR), the cosmic evolution during earlier epochs can be different from the standard one, giving rise also to different conditions in the early Universe. Banerjee and Santos [2] have obtained some analytical Robertson-Walker models in the framework of the SGHM theory by assuming a particular relationship between the scalar field and the scale factor R. In a class of these models (with negative curvature), the Universe oscillates between finite limits, avoiding a point singularity.

In this work we present a detailed study of SGHM cosmological models, consistent with astronomical observations and solar-system experiments, which does not assume a particular form for any dynamical function of this theory. The aim of this study is to analyze whether these kind of models are able to predict an evolution of the Universe different from that obtained in the standard GR models.

The paper is arranged as follows. We begin outlining the SGHM theory (Sec. II) and then show how to build up homogeneous and isotropic cosmological models in its framework (Sec. III). Some analytical (Sec. IV) and numerical (Sec. V) solutions of SGHM field equations are then obtained. Finally, conclusions and a summary of our results are given in Sec. VI.

II. THE SGHM THEORY

The starting point of the SGHM theory is the conformally invariant equation for a massless scalar field [3], which is generalized by adding a mass term and allowing for an arbitrary coupling constant β between ϕ and the scalar curvature $\mathcal{R}$:

$$(\Box+\tfrac{1}{6}\beta\mathcal{R}+\mu^2)\phi=0\ , \tag{1}$$

where μ has dimensions of $\sec^{-1}$ because it includes a factor c^2/h, h being the Planck constant.

Equation (1) can be obtained from the action integral $I=I_G+I_M$, with I_M as in general relativity, but with

$$I_G=\int(\tfrac{1}{2}\phi_{,\mu}\phi^{,\mu}-\tfrac{1}{2}\mu^2\phi^2-\tfrac{1}{12}\beta\phi^2\mathcal{R}+\gamma\mathcal{R}-2\gamma\Lambda)\sqrt{-g}\,d^4x\ , \tag{2}$$

where

$$\gamma=\frac{c^2}{16\pi G} \tag{3}$$

is half of the inverse gravitational constant, Λ is the cosmological constant, and $g\equiv\det(q_{\mu\nu})$, $g_{\mu\nu}$ being the metric tensor.

From the action integral (2), one deduces that the effective inverse gravitational coupling constant is

$$\gamma_{\rm eff}=\gamma-\tfrac{1}{12}\beta\phi^2\ . \tag{4}$$

That is, the gravitational constant, as measured, e.g., by a Cavendish scale, depends on ϕ and is then a function of space-time coordinates. Note that β has to be negative in order that $\gamma_{\rm eff}$ decreases when ϕ increases.

From (2) we also deduce that the effective mass of the scalar field is now

$$\mu_{\rm eff}=(\mu^2+\tfrac{1}{6}\beta\mathcal{R})^{1/2}\ , \tag{5}$$

and the effective cosmological constant

$$\Lambda_{\rm eff}=\Lambda+\mu^2\phi^2/4\gamma \ . \quad (6)$$

The variation of Eq. (2) with respect to ϕ and $g_{\mu\nu}$ leads to the field equations

$$(\gamma-\tfrac{1}{12}\beta\phi^2)(\mathcal{R}_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}\mathcal{R}) = -\tfrac{1}{2}T_{\mu\nu}-\phi_{,\mu}\phi_{,\nu}-\tfrac{1}{2}g_{\mu\nu}(\phi_{,\alpha}\phi^{,\alpha}-\mu^2\phi^2) + \tfrac{1}{12}\beta[(\phi^2)_{;\mu;\nu}-g_{\mu\nu}(\phi^2)^{;\alpha}_{;\alpha}] \ , \quad (7)$$

$$\Box\phi+\tfrac{1}{6}\beta\mathcal{R}\phi+\mu^2\phi=0 \ , \quad (8)$$

which satisfy the usual conservation law

$$T^{\mu\nu}_{;\nu}=0 \ , \quad (9)$$

where $T^{\mu\nu}$ is the energy-momentum tensor.

III. COSMOLOGICAL MODELS IN SGHM THEORY

A. Cosmological equations

In order to build up cosmological models, we consider a homogeneous and isotropic universe. The line element then has a Robertson-Walker form,

$$ds^2=-dt^2+R^2(t)\left[\frac{dr^2}{1-Kr^2}+r^2d\Omega^2\right] \ , \quad (10)$$

and the energy-momentum tensor corresponds to a perfect fluid:

$$T^{\mu\nu}=(\rho+P/c^2)u_\mu u_\nu+Pg_{\mu\nu} \ ; \quad (11)$$

where $K=0, \pm1$; $R(t)$ is the scale factor; ρ and P are the energy-mass density and pressure, respectively; and u_μ is the four-velocity of the gas.

By defining

$$H\equiv\dot{R}/R \ , \quad (12)$$

$$D\equiv\dot{\phi} \ , \quad (13)$$

where a dot means a time derivative, the homogeneous and isotropic field equations become

$$\dot{H}=\frac{-3}{\beta^2\phi^2+12\gamma_{\rm eff}}\left[P+\tfrac{1}{6}(3-2\beta)D^2+\tfrac{1}{6}(3-2\beta)\mu^2\phi^2+\tfrac{1}{3}\beta\phi DH+(\tfrac{1}{3}\beta^2\phi^2+2\gamma_{\rm eff})\frac{c^2K}{R^2}+(\tfrac{2}{3}\beta^2\phi^2+6\gamma_{\rm eff})H^2\right] \ , \quad (14)$$

$$\dot{D}=\frac{3\beta\phi}{\beta^2\phi^2+12\gamma_{\rm eff}}\left[P+\tfrac{1}{6}(3-2\beta)D^2-2\gamma_{\rm eff}\frac{c^2K}{R^2}-2\gamma_{\rm eff}H^2\right]-\frac{3}{\beta^2\phi^2+12\gamma_{\rm eff}}[(\tfrac{2}{3}\beta^2\phi^2+12\gamma_{\rm eff})DH-(\tfrac{1}{2}\beta\phi^2+4\gamma_{\rm eff})\phi\mu^2] \ , \quad (15)$$

together with the algebraic equation

$$6\gamma_{\rm eff}\frac{c^2K}{R^2}=\rho c^2-6\gamma_{\rm eff}H^2+\tfrac{1}{2}D^2-\tfrac{1}{2}\mu^2\phi^2+\beta\phi DH \ . \quad (16)$$

The dynamical evolution of the temperature T can be obtained from Eq. (9) and the standard state equation [4]:

$$\frac{dT}{dt}=-\frac{3H(\rho_1+P_1/c^2)}{d\rho_1/dt} \ , \quad (17)$$

where $\rho_1=\rho_e+\rho_\gamma$, $P_1=P_e+P_\gamma$, and subscripts e and γ refer to electron-positron and photon, respectively.

Equations (12)–(15) and (17) constitute the basic set of equations to build up cosmological models in SGHM theory. In these equations, we have considered as independent functions the scalar field ϕ, the Hubble parameter $H, D\equiv\dot{\phi}$, and the photon temperature T. The algebraic equation (16) gives, at any time, c^2K/R^2 as a function of ϕ, H, D, and T.

B. Initial conditions

In order to obtain cosmological models compatible with astronomical observations, we take as initial conditions the present values of the dynamical functions (ϕ_0, H_0, D_0, $c^2K/R_0^2, T_0$) and we integrate the field equations backwards in time (for fixed values of the β and μ parameters). By present values we mean their current observational values or their limits from observational data. Some of these initial data can be expressed in terms of the others, or can be directly known from observations. In fact, since the gravitational constant is well known from experiments such as, e.g., the Cavendish scale, the present value of $\gamma_{\rm eff}$ must be equal to the general relativistic γ value. Therefore, if we impose $\gamma_{\rm eff}=\gamma$ at the present time, Eq. (4) implies

$$\phi_0=0 \ . \quad (18)$$

Furthermore, if $\Omega\equiv\rho/\rho_c$, where ρ_c is the critical density needed to close the Universe, Eq. (16) can be written as

$$\rho c^2/\Omega=6\gamma_{\rm eff}H^2-\tfrac{1}{2}D^2+\tfrac{1}{2}\mu^2\phi^2-\beta\phi DH \ , \quad (19)$$

which is a second-order algebraic equation in D with solutions

$$D=-\beta\phi H\pm\left[\beta^2H^2\phi^2+2\left[6\gamma_{\rm eff}H^2+\frac{\mu^2}{2}\phi^2-\frac{\rho c^2}{\Omega}\right]\right]^{1/2} . \quad (20)$$

Notice that there exist two types of SGHM cosmological models.

(i) If we impose the condition that SGHM theory reduces to GR in the limit $\beta=\mu=0$, then Eq. (20) implies

$$\Omega = \frac{\rho c^2}{6\gamma H^2} = \Omega^{\mathrm{FRW}} \tag{21}$$

(FRW refers to the Friedmann-Robertson-Walker model) and, from Eqs. (18) and (15), we obtain

$$D_0 = 0 \ , \quad \dot{D}_0 = 0 \ . \tag{22}$$

As a consequence, $\phi=0$ for every time, and Eqs. (14)–(16) become the usual GR field equations. That is, these types of models are exactly the standard ones.

(ii) If we consider Ω_0 as a free parameter ($\Omega_0 \neq \Omega_0^{\mathrm{FRW}} \equiv \rho_0 c^2/6\gamma H_0^2$), the SGHM theory never reduces to GR, and we find

$$D_0 = -\sqrt{12\gamma H_0^2(1-\Omega_0^{\mathrm{FRW}}/\Omega_0)} \ , \tag{23}$$

where we have taken the negative solution for D because ϕ must grow with ρ [1]. In order to have a real D_0, Ω_0 must be greater than Ω_0^{FRW}:

$$\Omega_0 > \Omega_0^{\mathrm{FRW}} \ . \tag{24}$$

Since these models only reduce to GR when $\Omega_0 \to \Omega_0^{\mathrm{FRW}}$, we can always find, for any β or μ, a value of Ω_0 close enough to Ω_0^{FRW} to imply cosmological models compatible with observations. Consequently, no bounds on β and μ can be obtained from observations.

We must also note from Eq. (23) that flat or closed universes are possible even for $\Omega_0^{\mathrm{FRW}} \ll 1$, that is, even for small ρ_0 values.

The difference between type (i) and type (ii) models can be parametrized by $\Delta_0 \equiv 1-\Omega_0^{\mathrm{FRW}}/\Omega_0$. Only models with $\Delta_0 \neq 0$ will be considered in this paper.

C. Observational limits on the initial data

Since the post-Newtonian parameters of the SGHM theory are [1]

$$\gamma_{\mathrm{PPN}}=1, \quad \beta_{\mathrm{PPN}} = 1+\frac{\beta\phi_0^2}{24\gamma_{\mathrm{eff}\,0}}, \quad \alpha_i=\zeta_i=\xi=0 \ , \tag{25}$$

the condition (18) implies that these parameters reduce to the standard ones and, consequently, this theory is compatible with solar-system experiments for any β. Also, Eqs. (4) and (18) imply a vanishing present value of $\dot{\gamma}_{\mathrm{eff}}$ (even for $D_0 \neq 0$) and, hence, we cannot use the experimental bounds on $\dot{G}_0$ to obtain a constraint on β or ϕ. Moreover, we cannot set bounds on μ from the present value of Λ_{eff} because, from Eqs. (6) and (18), it reduces to the standard cosmological constant.

In conclusion, there does not exist any observational limit on the SGHM parameters β and μ. This is consistent with the fact that SGHM theory reduces to GR for $\Omega_0 = \Omega_0^{\mathrm{FRW}}$ even if β and μ are nonvanishing.

The observational bounds on the remaining initial data are as follows.

The Hubble constant is usually taken as [5]

$$H_0 = 100h \ \mathrm{km\,sec^{-1}\,Mpc^{-1}} \quad (\tfrac{1}{2} \le h \le 1) \ . \tag{26}$$

The spectrum of cosmic microwave background can be fitted by a blackbody at [6]

$$T_0 = 2.735 \pm 0.017 \ \mathrm{K} \ . \tag{27}$$

The values obtained for Ω_0 from several dynamical methods increase with scale, from $(0.0016\pm0.0008)h^{-1}$ in the solar-system neighborhood to as large as unity on scales of $\geq 100h^{-1}$ Mpc [7].

In order to obtain an observational bound on the SGHM parameter Δ_0 we can use the deceleration parameter $q \equiv -R\ddot{R}/\dot{R}^2$ to eliminate $\dot{H}$ in Eq. (14). By combining this result with Eq. (16) and evaluating at t_0, we obtain

$$q_0 = \frac{\Omega_0^{\mathrm{FRW}}}{2} + \frac{4\pi G\rho_{R_0}}{3H_0^2} + (2-\beta)\Delta_0 \ , \tag{28}$$

where ρ_{R_0} is the present value of the relativistic energy density.

The determination of q_0 through a Hubble diagram is not accurate because of the galaxy photometric evolution [8]. It is usual to take

$$0 \le q_0 \le 2 \ , \tag{29}$$

which implies

$$\Delta_0 \le \frac{1}{2-\beta}\left[2-\frac{\Omega_0^{\mathrm{FRW}}}{2}-\frac{4\pi G\rho_{R_0}}{3H_0^2}\right] . \tag{30}$$

IV. ANALYTICAL SOLUTIONS

The only analytical solutions to the SGHM cosmological equations which can be found in the literature are those obtained by Banerjee and Santos [2] assuming $\phi=\phi_0(R_0/R)^n$ and then allowing for a difference between the present value of γ_{eff} and the general relativistic γ value.

Here we present a different analytical solution which does not assume a particular form for any dynamical function and which is compatible with the boundary conditions quoted in Sec. III B. This solution is valid when $\beta=\mu=0$ and is exact for pure matter or pure radiation universes with flat geometries. For a mixed gas and nonflat spaces, we can also obtain exact expressions for $D(T)$, $H(T)$, and $R(T)$, but only semianalytical expressions for $\phi(T)$ and $t(T)$.

By using T as the independent variable in Eqs. (12)–(17), and taking $\beta=\mu=0$, we obtain

$$H' = \frac{1}{4}\left[\frac{1}{3}\frac{\rho_{R_0}c^2}{\gamma}e^{4X}+\frac{1}{2\gamma}D^2+2\frac{c^2K}{R^2}\frac{1}{H}+6H\right] , \tag{31}$$

$$D' = 3D \ , \tag{32}$$

$$\phi' = -D/H \ , \tag{33}$$

$$t' = 1/H \ , \tag{34}$$

$$R' = -R \ , \tag{35}$$

where primes denote differentiation with respect to $X \equiv \ln(T/T_0)$.

The solutions of Eqs. (32) and (35) are straightforward:

$$D/D_0=(T/T_0)^3\ , \tag{36}$$

$$R/R_0=T_0/T\ . \tag{37}$$

Substituted into Eq. (31), these lead to

$$H^2=\left[\frac{T}{T_0}\right]^2\left[\frac{D_0^2}{12\gamma}\left[\frac{T}{T_0}\right]^4+\frac{\rho_{R_0}c^2}{6\gamma}\left[\frac{T}{T_0}\right]^2+\frac{\rho_{b_0}c^2}{6\gamma}\frac{T}{T_0}-\frac{c^2K}{R_0^2}\right]\ , \tag{38}$$

and Eqs. (33) and (34) become

$$\phi=-\sqrt{6\gamma}D_0\int\frac{X\,dX}{\sqrt{AX^4+BX^2+CX-D}}\ , \tag{39}$$

$$t=\sqrt{6\gamma}\int\frac{Y^2dY}{\sqrt{-DY^4+CY^2+BY+A}}\ , \tag{40}$$

where $X\equiv T/T_0\equiv 1/Y$, $A\equiv D_0^2/2$, $B\equiv\rho_{R_0}c^2$, $C=\rho_{b_0}c^2$, and $D\equiv 6\gamma c^2K/R_0^2$.

The integrals in Eqs. (39) and (40) are elliptic. In particular, the integral in Eq. (39) has the same mathematical form as that appearing in the function $t(R)$ of standard FRW models with a nonvanishing cosmological constant. Consequently, the solution of Eq. (39) is similar, but with different definitions of constants and variables than those obtained by Edwards [9] for $t^{\rm FRW}(R)$ in terms of Jacobian elliptic functions.

The integration of Eq. (40) between 0 and 1 gives us the age of the Universe. A semianalytical expression for t_0 can be obtained by developing the integrand of Eq. (40) in a series of A (a similar procedure to evaluate t_0 was used by Agnese *et al.* [10] in FRW models with $\Lambda_0\neq 0$). We obtain

$$\begin{aligned}t_0&=\sum_{n=0}^{\infty}\frac{(2n-1)!!}{2^n n!}(-A)^nB^{-(n+1/2)}\int_0^1 Y^{1-2n}(1-uY)^{-(n+1/2)}(1-vY)^{-(n+1/2)}dY\\&=\sum_{n=0}^{\infty}\frac{(2n-1)!}{(2-2n)n!}(-A)^nB^{-(n+1/2)}F_D(2-2n,n+\tfrac{1}{2},n+\tfrac{1}{2},3-2n,u,v)\ ,\end{aligned} \tag{41}$$

where F_D is the hypergeometric function of two variables, and

$$\begin{aligned}u&=-\frac{1}{2B}(C+\sqrt{C^2+4BD}),\\v&=-\frac{1}{2B}(C-\sqrt{C^2+4BD})\ .\end{aligned} \tag{42}$$

Since these solutions are too complex to be used, we will consider two limit cases where simple analytical expressions for $\phi(T)$ and $t(T)$ can be found. The other cases will be solved by numerical integration of Eqs. (12)–(17).

A. Pure matter universe with flat geometry

In a pure matter universe ($\rho_{R_0}=0$) with flat geometry ($K=0$), Eq. (38) becomes

$$H^2=H_0^2X^3[\Delta_0(X^3-1)+1]\ , \tag{43}$$

and Eqs. (39) and (40) lead to

$$\phi=4\sqrt{\tfrac{1}{3}\gamma}\ \ln\left[\frac{\sqrt{\Delta_0(X^3-1)+1}+\sqrt{\Delta_0X^3}}{1+\sqrt{\Delta_0}}\right]\ , \tag{44}$$

$$t=\frac{2}{3H_0}\left[\frac{\sqrt{(1-\Delta_0)Y^3+\Delta_0}-\sqrt{\Delta_0}}{1-\Delta_0}\right]\ . \tag{45}$$

For $T=T_0$ (that is, $X=Y=1$), the last equation gives the age of the Universe:

$$t_0=\frac{2}{3H_0}\frac{1}{1+\sqrt{\Delta_0}}\ , \tag{46}$$

which implies that SGHM models are younger than the standard ones.

We must note that the limits of $H(T)$, $t(T)$, and $R(T)$ for $T\rightarrow\infty$ or $T\rightarrow 0$ are the same as in Friedman-Robertson-Walker (FRW) models. However, the limit of $\phi(T)$ when $T\rightarrow 0$ is a negative constant.

Obviously, for $\Delta_0=0$ (that is, $D_0=0$), all the dynamical functions reduce to their usual FRW expressions, in accordance with what has been explained in Sec. III B.

B. Pure radiation universe with flat geometry

In a pure radiation universe with flat geometry, Eq. (38) becomes

$$H^2=H_0^2X^4[\Delta_0(X^2-1)+1]\ , \tag{47}$$

and Eqs. (39) and (40) give

$$\phi=\sqrt{12\gamma}\ \ln\left[\frac{\sqrt{\Delta_0(X^2-1)+1}+\sqrt{\Delta_0X^2}}{1+\sqrt{\Delta_0}}\right]\ , \tag{48}$$

$$t=Y\frac{\sqrt{(1-\Delta_0)Y^2+\Delta_0}}{2(1-\Delta_0)H_0}+\frac{\Delta_0}{2H_0(1-\Delta_0)^{3/2}}\times\ln\left[\frac{\sqrt{\Delta_0}}{\sqrt{(1-\Delta_0)Y^2+\Delta_0}+\sqrt{(1-\Delta_0)Y^2}}\right]\ . \tag{49}$$

The age of a radiation universe is then

$$t_0=\frac{1}{2(1-\Delta_0)H_0}+\frac{\Delta_0}{2H_0(1-\Delta_0)^{3/2}}\ln\left[\frac{\sqrt{\Delta_0}}{1+\sqrt{1-\Delta_0}}\right]. \tag{50}$$

In the limit $\Delta_0\to 0$, Eqs. (47)–(50) reduce again to the FRW expressions.

Since a pure radiation universe is only a good approximation for high temperatures, Eq. (50) is not a realistic evaluation of t_0. However, we also obtain younger universes than in radiation FRW models.

V. NUMERICAL SOLUTIONS

In order to obtain SGHM cosmological models where the present values of the dynamical functions are compatible with observations, we have taken these values as the initial data for Eqs. (12)–(17), and we have performed numerical integration backwards in time.

The qualitative behavior of SGHM models mainly depends on Ω_0, μ, and β, so that here we only show our results for $h=\frac{1}{2}$, $T_0=2.735$ K, and $\eta_{10}=3$ in order to avoid an excessive number of free parameters.

Figure 1 displays the ratio $\xi\equiv H/H^{\rm FRW}$ at $T=10^{10}$ K as a function of Ω_0 (parametrized by Δ_0) for several values of β and μ. Since the ratio ξ is a measure of the difference between the universe dynamics at a given temperature as described by SGHM or FRW models, its value is very close to unity for $\Delta_0\to 0$ (and any β or μ values), but the expansion rate is always greater than in the FRW case for a nonvanishing Δ_0. In particular, ξ_{10} increases when μ or Δ_0 increase. However, the dependence of ξ_{10} on β is qualitatively different. In fact, for high Δ_0 values (that is, $\Delta_0\to 1$), an increase of $|\beta|$ implies an increase of ξ_{10} and, in consequence, greater $|\beta|$ values need smaller Δ_0 values in order that SGHM theory reduces to GR.

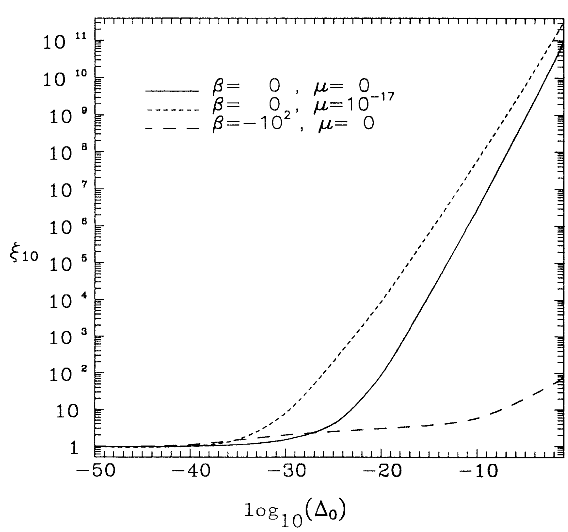

FIG. 1. Expansion rate $\xi=H/H^{\rm FRW}$ at $T=10^{10}$ K as a function of $\log_{10}\Delta_0$ for several values of β and μ.

The dependence of ξ on temperature is shown in Fig. 2 for different values of Δ_0, β, and μ. As we can see from this figure, SGHM and FRW predictions are quite similar over a certain temperature interval, but diverge at high temperatures. The coincidence interval is larger for lower Δ_0, β, and μ values. Again, the behavior of $\xi(T)$ with respect to β is qualitatively different: for high β and T values, $\xi(T)$ becomes a constant greater than unity [see Fig. 2(c)]. The curve for $\Delta_0=10^{-1}$ has been included in Fig. 2(c) in order to illustrate the dependence of $\xi(T)$ on Δ_0. However, we must note that these values of Δ_0 and β imply, according to Eq. (28), a deceleration parameter incompatible with observations.

Figure 3 shows the scalar field as a function of temperature for the same Δ_0, β, and μ values as in previous figures. As can be seen from Figs. 3(a) and 3(b), ϕ grows very quickly with T until a temperature T_* is reached, and then the growth of ϕ becomes considerably slower. The value of T_* and the slope of $\log_{10}\phi$ vs $\log_{10}T$ for high temperatures decrease when Δ_0 increases. However, the scalar field is greater for higher Δ_0 values. Also, for high $|\beta|$ values, the curves $\log_{10}\phi$ vs $\log_{10}T$ reach the same slope at high temperatures for any Δ_0, but the growth in the scalar field is smaller than in the $\beta=0$ case (except for very low Δ_0 values) [see Fig. 3(c)].

The above-mentioned dependence of ξ and ϕ on Δ_0 and T can be understood from Eqs. (43) and (44) or (47) and (48) in the $K=0$ case. According to these equations, both ξ and ϕ are monotone increasing functions of T and Δ_0. In particular, for a pure matter universe, Eq. (43) implies that ξ^2 is dominated by the unity term for small Δ_0 and T values. As the temperature grows, the $\Delta_0(T/T_0)^3$ term becomes more and more important and ξ^2 separates from unity. Obviously, higher Δ_0 values imply that $\Delta_0(T/T_0)^3$ dominates at lower temperatures. Moreover, Eq. (44) implies a slow variation of ϕ with T at high temperatures as corresponds to a logarithmic behavior. The ϕ value at these temperatures is dominated by the big multiplicative factor $4\sqrt{\gamma/3}$. However, as T approaches T_0, the scalar field quickly drops to zero. This behavior is smoother as $\Delta_0\to 0$. Similar features are found from Eqs. (47) and (48).

Another important output of our numerical integration is the age of the Universe, taken as the time elapsed since the e^+e^- annihilation at $T\simeq 5\times 10^9$ K up to now. Figure 4 shows the dependence of H_0t_0 on Δ_0 for different values of β and μ. Note that SGHM cosmological models are always younger than standard models, in accordance with Eqs. (46) and (50) for $K=0$. The age of the Universe decreases when Δ_0, μ, or $|\beta|$ increase, and tends to $t_0^{\rm FRW}$ when $\Delta_0\to 0$ (for any β and μ). Since t_0 must be greater than the age of globular clusters ($1.4-1.9\times 10^{10}$ yr [11]), we can obtain upper limits on Δ_0 for given values of β and μ. Obviously, the least restrictive bound on Δ_0 is found when $\beta=\mu=0$ (and $h=\frac{1}{2}$). For these values of β, μ, and h, Fig. 4 implies that Δ_0 must be smaller than 10^{-2}. This is not a very strong bound on SGHM cosmological models because, from Figs. 1–3, the predicted

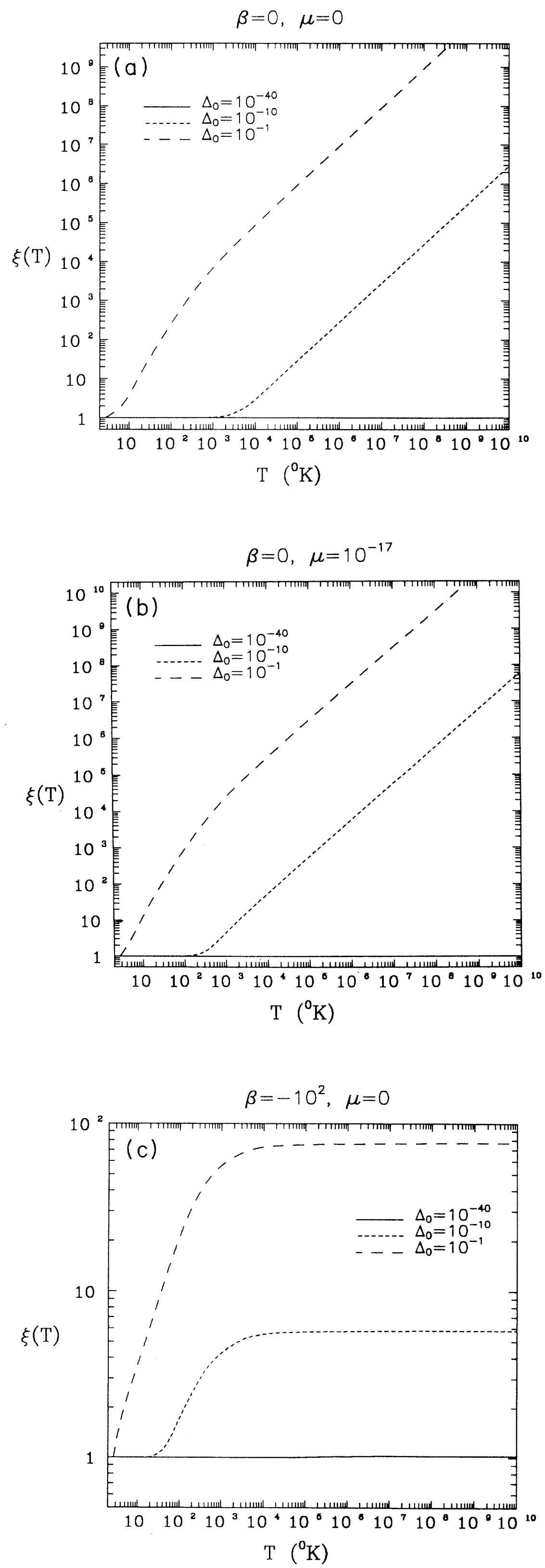

FIG. 2. Dependence of ξ on temperature for $\Delta_0=10^{-1}$, 10^{-10}, 10^{-40} and (a) $\beta=0$, $\mu=0$, (b) $\beta=0$, $\mu=10^{-17}$, (c) $\beta=-10^2$, $\mu=0$.

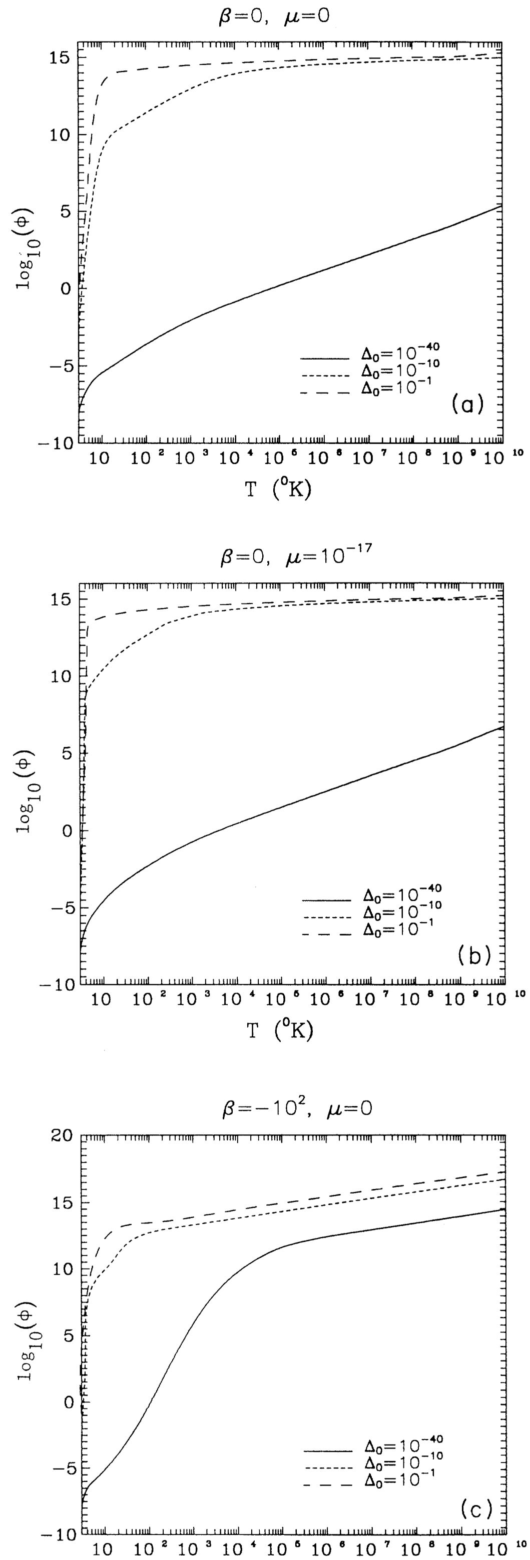

FIG. 3. The scalar field as a function of temperature for $\Delta_0=10^{-1}$, 10^{-10}, 10^{-40} and (a) $\beta=0$, $\mu=0$, (b) $\beta=0$, $\mu=10^{-17}$, (c) $\beta=-10^2$, $\mu=0$.

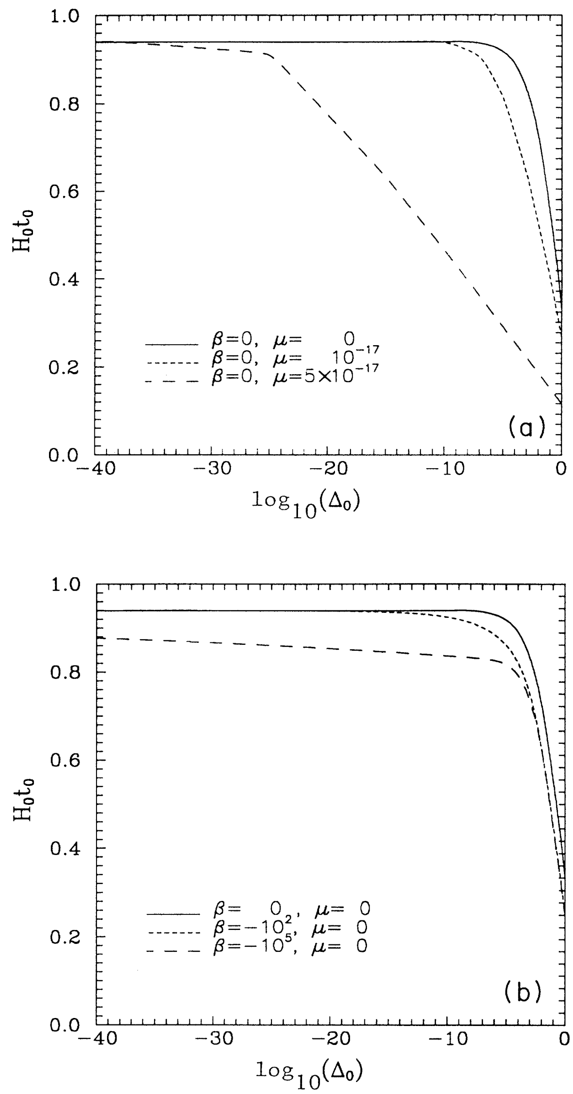

FIG. 4. The age of the Universe as a function of Δ_0 for (a) $\beta=0$ and different values of μ, and (b) $\mu=0$ and different values of β.

evolution of a universe with $\Delta_0=10^{-2}$ is very different from a standard behavior (with $\Delta_0=0$). For example, the expansion rate of the Universe when $\Delta_0=10^{-2}$ is much faster than in a FRW model. In particular, when $T=10^{10}$ K, Fig. 1 implies that $H/H^{\rm FRW}\simeq 10^{10}$. Consequently, there exist SGHM cosmological models which are able to predict a matter-dominated evolution different from the standard behavior and which are compatible with the age of globular clusters and the present values of dynamical functions.

VI. SUMMARY AND CONCLUSIONS

An exhaustive study of cosmological models in SGHM theory, consistent with astronomical observations and solar-system experiments, has been performed.

We have found two types of SGHM models: those which reduce to GR in the limit $\beta=\mu=0$, and those which do not tend to GR in this limit.

In the first type of model, we find that $\Omega_0=\Omega_0^{\rm FRW}$, and solutions are exactly the standard FRW ones at any temperature. Therefore, this kind of model cannot be considered as an alternative to GR.

In the second type of SGHM model, the density parameter Ω_0 depends on the present value of ϕ and is therefore an additional free parameter due to the presence of the scalar field. This kind of model only reduces to GR when $\Omega_0\rightarrow\Omega_0^{\rm FRW}$ (for any β and μ). Consequently, it is not possible to obtain observational bounds on β and μ, because we can always find a Ω_0 close enough to $\Omega_0^{\rm FRW}$ to imply cosmological models compatible with observations. The difference between SGHM and FRW models can be then parametrized by $\Delta_0\equiv 1-\Omega_0^{\rm FRW}/\Omega_0$, and only the observational bounds on Δ_0 can be used to restrict possible deviations from GR.

The extra degree of freedom introduced by the scalar field allows for flat or closed universes even with small ρ_0 (or $\Omega_0^{\rm FRW}$) values. That is, SGHM models with $K=0$ can be compatible even with the measurements of the mass-luminosity relation at small scales.

We have obtained analytical solutions valid for flat universes with $\beta=\mu=0$ and a pure matter or radiation cosmic gas. These solutions allow us to understand the dynamics of SGHM models. More general cases require numerical integration. This has been carried out for different values of Δ_0, β, and μ. The following features have been found. SGHM and FRW models are quite similar over a certain temperature range whose length depends on Δ_0, β, and μ. However, at high temperatures, the expansion rate of the Universe is always faster than in the FRW case and, consequently, SGHM models are younger than the standard ones. The age of globular clusters does not imply a very strong bound on Δ_0 ($\Delta_0\lesssim 10^{-2}$ in the most favorable case when $\beta=\mu=0$ and $h=\frac{1}{2}$). Therefore, it is possible to find SGHM models that yield present values of their dynamical functions and a predicted age of the Universe which are compatible with observations, but which predict a cosmological evolution considerably different from that obtained in FRW models.

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