Mass and mixing-angle patterns in the standard model and its minimal supersymmetric extension

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Using renormalization group techniques, we examine several interesting relations among masses and mixing angles of quarks and leptons in the standard model. We extend the analysis to the minimal supersymmetric extension to determine its effect on these mass relations. Remarkably supersymmetry allows for these relations to be satisfied at a single grand unified scale.

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I. INTRODUCTION

Most of the parameters in the standard model are to be found in the Yukawa sector of the theory where they parametrize quark and lepton masses, the interfamily mixings of the quarks, and CP violation. Historically, only one of these 13 parameters was ever predicted [1], the charmed-quark mass, but only after an inspired guess on the value of a strong (i.e., presently incalculable) matrix element.

Theoretical guesses on the nature of physics beyond the standard model abound in the literature. Many use as inspiration the idea of a grand unified theory [2, 3] (GUT) which emerged from the observed pattern of the quantum numbers of the elementary particles. When applied in conjunction with the renormalization group [4], this idea has proven extremely fruitful. Recent work indicates that the experimental values of the gauge couplings are such that all three couplings evolve to the same value [5] at shorter distances only when supersymmetry (SUSY) is included at 1–10 TeV. Without supersymmetry, the gauge couplings meet two at a time, forming a small "GUT triangle" in the plot of their evolution as a function of scale.

This encouraging situation, hinting at a supersymmetric GUT, should be matched by concomitant simplicity in the other parameters of the theory. To that purpose we present a comparative analysis of possible relations among Yukawa couplings at shorter distances both in the standard model itself and in its minimal supersymmetric extension [6].

While we find little evidence to support the view that the standard model is by itself the low-energy manifestation of a pure GUT, we are encouraged by the results of this investigation: Inclusion of supersymmetry allows many possible GUT relations among Yukawa couplings to be satisfied at one appropriate gauge unification scale.

II. MODELS OF THE YUKAWA SECTOR

Many models have been proposed to explain the peculiar structure of the Yukawa couplings. In a certain basis, these Yukawa matrices are well approximated by the matrix [7]

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad (1)$$

which expresses the fact that one family is so much heavier than the other two. The theoretical temptation has been to express the masses of the lighter families and their mixings as radiative effects [8]. Yet this approach has not yielded any satisfactory models. Another has been to use as the starting point the "democratic" matrix where all entries are equal, thereby producing two zero eigenvalues. This type of idea could be implemented by condensates formed by new flavor-blind strong forces [9]. Yet another approach is to think of the "elementary" particles as composites and use the Yukawa matrices as hints to infer their structure [10]. In the following, however, we concentrate on Yukawa matrices that arise naturally in the context of grand unified theories.

In the context of the SU(5) GUT [3], several mass relations arise, based on simple assumptions for the possible Higgs-boson structure. The mass term for the down quarks and leptons comes from the Yukawa interaction of the $\overline{\mathbf{5}}$ and $\mathbf{10}$ of fermions. This leads to relations between the charge $-\frac{1}{3}$ quarks' and the charged leptons' Yukawa couplings. With only a $\overline{\mathbf{5}}$ of Higgs bosons, one obtains equality between the τ-lepton and bottom-quark masses at the GUT scale:

$$m_b = m_\tau \, . \quad (2)$$

To the level of approximation used at the time, this relation was found to be consistent at experimental scales, after taking into account the running of the quark masses [11]. Similar relations apply to the lighter two families, but are clearly incompatible with experiment. To alleviate this, a new scheme was proposed [12] with a slightly more complicated Higgs-boson structure (using a $\mathbf{45}$ representation in conjunction with the $\overline{\mathbf{5}}$). It replaces the above with the more complicated relations for the two lighter families

$$m_d = 3m_e \,,\qquad 3m_s = m_\mu \,. \tag{3}$$

The situation concerning the mixing angles is equally intriguing. There happens to be a near numerical equality between the square of the tangent of the Cabibbo angle and the ratio of the down to the strange quark masses (determined from current algebra). This Gatto-Sartori-Tonin-Oakes (GSTO) relation [13] reads

$$\tan\theta_c \approx \sqrt{\frac{m_d}{m_s}} \,. \tag{4}$$

It has provided the central inspiration in the search for Yukawa matrices. Very general classes of matrices with judiciously chosen textures [14] (i.e., zeros in the right places) could reproduce this relation, at least approximately.

In the context of SO(10) [15], these three different relations could all be obtained in one model [16], with the required texture enforced naturally by discrete symmetries at the GUT scale. In this model the mixing of the third family with the two lighter ones is dictated exclusively by the the charge $\frac{2}{3}$ quarks' Yukawa matrix. There ensues an GSTO-like relation for the mixing of the second and third families [16]

$$V_{cb} = \sqrt{\frac{m_c}{m_t}} \,, \tag{5}$$

which provides a relation between the top-quark mass and the lifetime of the B meson.

These four relations can all be obtained if one takes the Yukawa mixing matrices to be of the form [12, 16] (shown here in a specific basis)

$$\mathbf{Y}_u = \begin{pmatrix} 0 & P & 0 \\ P & 0 & Q \\ 0 & Q & V \end{pmatrix} , \tag{6}$$

$$\mathbf{Y}_d = \begin{pmatrix} 0 & R & 0 \\ R & S & 0 \\ 0 & 0 & T \end{pmatrix} ; \quad \mathbf{Y}_e = \begin{pmatrix} 0 & R & 0 \\ R & -3S & 0 \\ 0 & 0 & T \end{pmatrix} . \tag{7}$$

This form has been recently rediscovered by several groups [17, 18], and some of our analysis overlaps with their work.

Although derived with specific and sometimes complicated Higgs-boson structures in mind [as in the SO(10) model], these relations may well prove sturdier than the theories which generated them. In the following, we first examine the relations in the context of the standard model at varying scales all the way to the Planck scale. We then extend the analysis to the minimal supersymmetric extension of the standard model, and compare the effect of this extension on their compatibility at some unified scale. A more thorough treatment of the SUSY extension is in preparation [19].

III. THE RENORMALIZATION GROUP

We outline and will use the numerical techniques and routines developed in our previous work [20, 21]. We first use experiments to fix the parameters of the standard model at lower energies. We then use these values as initial conditions in the renormalization group running to shorter length scales, using the $\overline{\text{MS}}$ scheme. In the standard model, we use two-loop renormalization group equations in evolving the couplings. In the supersymmetric extension, we work to one loop. In each case, we include a proper treatment of thresholds and make no approximations in the Yukawa sector [23]. Our incomplete knowledge of the standard model parameters forces us to repeat the analysis for a range of allowed values of the top quark and Higgs-boson masses. Although the top-quark mass is not exactly known, it is believed to lie somewhere between 91 GeV (Ref. [24]) and 200 GeV, the lower limit being set by direct experimental searches, the upper one by the radiative effect of the top-quark mass on the ratio of neutral- to charged-current processes (ρ parameter). In these runs, we take $g_3(M_Z) = 1.191$ and the physical bottom-quark mass $M_b = 4.89$ GeV.

Let us summarize the salient features of the renormalization group running in the standard model. At the one-loop level, the gauge couplings are unaffected by the other couplings in the theory. On the other hand, the Yukawa couplings are affected at one loop by both the gauge and Yukawa couplings. Since the top Yukawa coupling is at least as big as the gauge couplings at low energy, that means the running of the Yukawa couplings is sensitive to mostly the top Yukawa coupling and the QCD gauge couplings. Thus we can expect the mass and mixing relations we have just described to be sensitive to the value of the top-quark mass. The Higgs quartic self-coupling enters in the running of the other couplings only at the two-loop level, so that its effect on the quark and lepton parameters is small. However, its own running is very sensitive to the top-quark mass; it can become negative as easily as it can blow up, corresponding to vacuum instability or to strong self-interaction of the Higgs boson (triviality bound), respectively [25]. The discovery of the Higgs boson with mass outside these bounds would be a signal for physics beyond the standard model. The graphs in Fig. 1 summarize these bounds for representative values of the top-quark mass. For example, if $M_t = 150$ GeV, we see from the corresponding plot that a Higgs-boson mass between 95 and 150 GeV need not imply any new physics up to the Planck scale. However, if the Higgs boson were observed outside of this range, then some new physics must appear at the scale indicated by the curve, either because of vacuum instability if $M_H < 95$ GeV or because the Higgs-boson interaction becomes too strong if $M_H > 150$ GeV. It is amusing to note that it is for comparable values of the top and Higgs-boson masses that these bounds are least restrictive, but it is important to emphasize that a high value of the top mass with a relatively low value of the Higgs-boson mass necessarily indicates the presence of new physics within reach of the Superconducting Super Collider (SSC). Subsequently, when examining the mass and mixing-angle

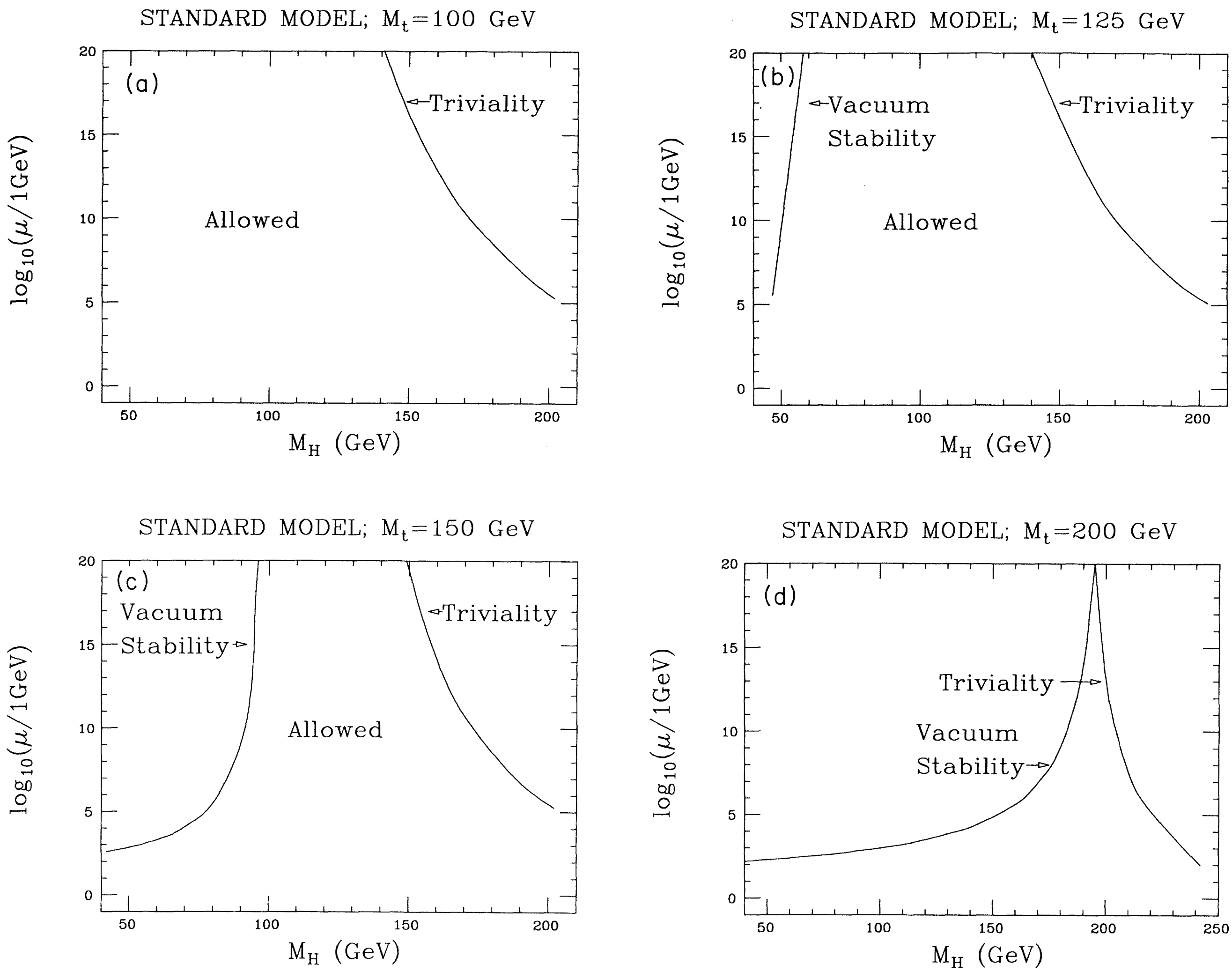

FIG. 1. Vacuum stability and triviality bounds on the Higgs-boson mass for (a) M_t = 100 GeV, (b) M_t = 125 GeV, (c) M_t = 150 GeV, and (d) M_t = 200 GeV, giving scales of expected new physics beyond the standard model.

relations in the context of the standard model, we will make the choices for M_t and M_H in our renormalization group runs consistent with these bounds. For a chosen value of M_t, varying M_H within the vacuum stability and triviality bounds does not affect any of our results, and we will therefore choose a corresponding, representative value of M_H.

As is well known, the standard model shows no apparent inconsistencies until perhaps the Planck scale, where quantum gravity enters the picture. The nature of the physics to be found between our scale and the Planck scale is a matter of theoretical taste. At one extreme, because of the values of the gauge couplings, new phenomena may be inferred every two orders of magnitude. At the other, there is the possible desert suggested by GUT's; however, the absence of new phenomena over many orders of magnitude cannot be understood (perturbatively) unless one generalizes the standard model in some way to solve the hierarchy problem. Supersymmetrizing the standard model at an experimentally accessible scale can accomplish this. This particular scenario is bolstered by the fact that with such "low-energy" supersymmetry, the three gauge couplings of the standard model meet at one scale ($\sim 10^{16}$ GeV) at the perturbative value of $\sim \frac{1}{26}$ [5]. The collapse of the GUT triangle in the supersymmetric extension fixes two scales, the one at which the gauge couplings unify, the other at the threshold of supersymmetry. Minimal supersymmetry implies two Higgs-boson doublets and eliminates the feisty quartic self-coupling of the standard model. But there appears an extra parameter, the ratio of the vacuum values of these two doublets, parametrized by an angle β, $\tan\beta = v_u/v_d$, where v_u (v_d) is the vacuum expectation value of the Higgs-boson field that gives mass to the charge $\frac{2}{3}$ ($-\frac{1}{3}$, -1) fermions. In the following [26], we examine certain relations among masses and mixing angles in the context of the standard model itself and in its minimal supersymmetric extension. In the latter, we only treat the case of one light Higgs doublet.

IV. RUNNING THE RELATIONS IN THE STANDARD MODEL

We now proceed to run these relations using only standard model physics. The three gauge couplings semiconverge in forming the GUT triangle around 10^{16} GeV.

A. Relation (I): $m_b = m_\tau$

This relation is the most natural one in the SU(5) theory, and it could be expected to be valid at scales where the standard model gauge couplings are the closest to one another. We examine its validity for three different physical values of the top and Higgs-boson masses in the standard model. The results are summarized in Fig. 2. The noteworthy feature of the figure is that this simplest of the SU(5) relations is valid at an energy scale many orders of magnitude removed from that at which the gauge couplings tend to converge [21]. Our result is vastly different from that of the original investigations in Ref. [11]. We have improved on their work by including two-loop effects in the running of the quark Yukawas couplings, by taking into account the full Yukawa sector, and most importantly by incorporating QCD corrections in the extraction of the bottom-quark mass [20].

B. Relations (II): $m_d = 3m_e$, $3m_s = m_\mu$

We now turn to the more complicated relations among masses of the two lighter families. There are large theoretical uncertainties in the extraction of the masses of the three lightest quarks from experiment, although the mass ratios are known more accurately. Following Refs. [27, 20], we take their values to be $m_d/m_u = 1.8$ and $m_s/m_d = 21$, so that specifying m_s fixes m_d and m_u. We note that m_s/m_d and m_μ/m_e effectively do not run. Therefore, given this value for m_s/m_d, we do not expect relations (II) to be both satisfied exactly, since $m_\mu/9m_e \approx 23$. The uncertainties in the light-quark masses are accounted for by examining the ratios $m_d/3m_e$ and $3m_s/m_\mu$ for a range of m_s(1 GeV) values from 140 to 250 MeV. We have run these same ratios for representative values of the top and Higgs-boson masses but find the results to be fairly insensitive to the value of the top. Therefore, in Fig. 3, we only present results for top and Higgs-boson masses of 190 GeV and 180 GeV, respectively. Unlike relation (I) which holds only at $\sim 10^7$ GeV, we see that relations (II) can hold within $\sim 5\%$ at 10^{16} GeV for acceptable values of the light-quark masses.

C. Relation (III): $\tan\theta_c = (m_d/m_s)^{1/2}$

We find the GSTO relation to be quite independent of scale. The reason is that the Cabibbo angle effectively

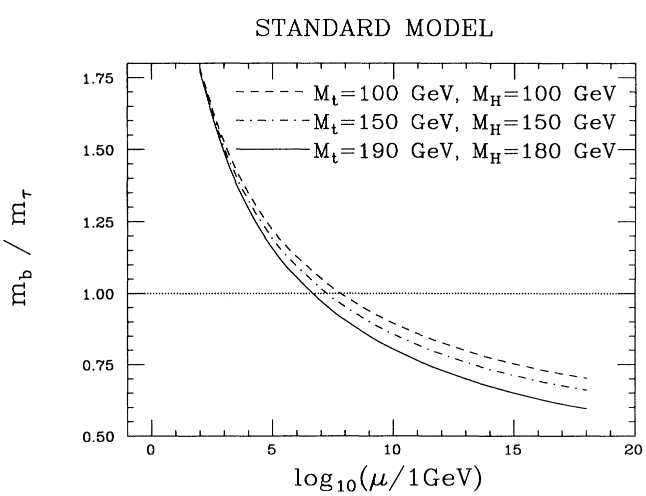

FIG. 2. Plot of m_b/m_τ as a function of scale in the standard model for various top and Higgs-boson masses.

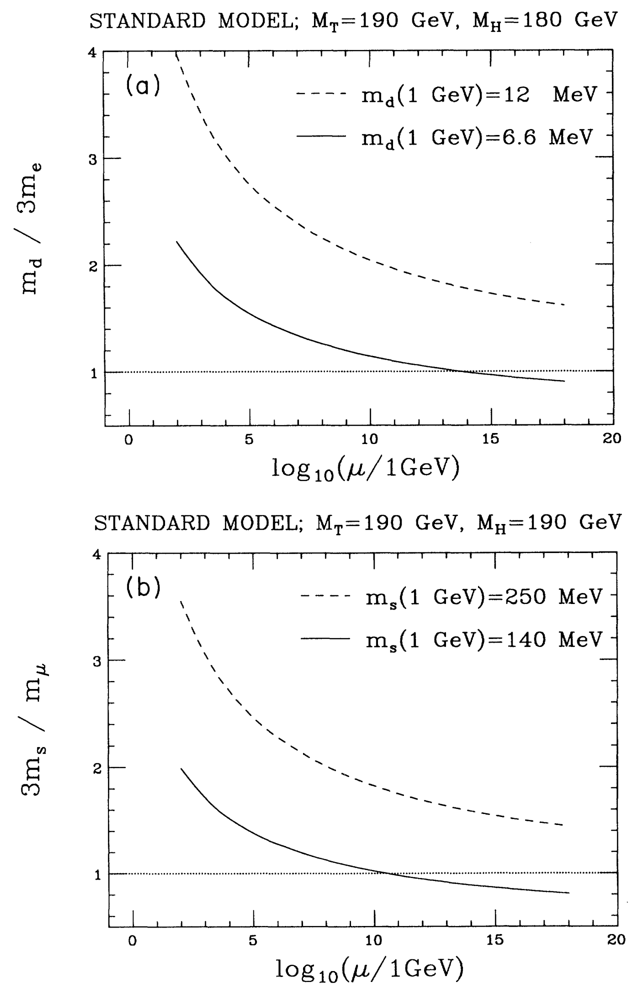

FIG. 3. Plots of (a) $m_d/3m_e$ and (b) $3m_s/m_\mu$ as a function of scale in the standard model for $M_t = 190$ GeV and $M_H = 180$ GeV.

does not run [21], and the ratio of light quarks is essentially unaffected by QCD, since both are far away from the Pendleton-Ross [28] infrared fixed point. Further, one finds that their numerical values are fairly independent of the value of the top-quark mass and of the Higgs-boson mass. The agreement is spectacular, hovering around the 4% level. For example, for $M_t = 100$ GeV and $M_H = 100$ GeV, we find that $\tan\theta_c/\sqrt{m_d/m_s} = 1.038$ from M_Z to the Planck scale.

D. Relation (IV): $V_{cb} = (m_c/m_t)^{1/2}$

This expression involves the top-quark mass directly, which may thus be predicted from this relation. On the other side of the equation, the experimental value of the "23" element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, V_{cb}, is known only to within $\sim 10\%$ [29], $V_{cb} = 0.043 \pm 0.006$. The value of V_{cb} at all scales is obtained by running the CKM angles. These numerical results do not depend on the value of the CP-violating phase.

We note that because of the Pendleton-Ross fixed point, the ratio of the two quark masses runs appreciably in the infrared region. We find that for a top quark in its lower allowed range, 91 − 150 GeV, this relation fails over all scales. Accordingly, we present our results for values of V_{cb} and M_t for which the relation can be satisfied below the Planck scale. We use for V_{cb} values ranging from its central value of 0.043 to 0.050. Here and in the following, we take the value of m_c at 1 GeV to be 1.41 GeV.

The results of our runs can be summarized in Fig. 4 in which we plot both V_{cb} and $\sqrt{m_c/m_t}$ as a function of scale. From the first plot, we see that the top quark has to be at least 178 GeV for $\sqrt{m_c/m_t}$ to meet V_{cb} at the Planck scale. The second plot shows that $M_t = 180$ GeV allows for this relation to be easily satisfied at 10^{16} GeV, if $V_{cb}(M_Z) = 0.05$. This means that a few GeV difference in M_t changes the meeting of the curves (both of which are affected by M_t) by three orders of magnitude. Finally, in the third plot, we see that for this relation to be valid at the unification scale, using the central value of V_{cb}, a 197 GeV top quark is needed. We conclude that, given the uncertainties in the value of V_{cb}, this relation may well be valid as long as $M_t > 175$ GeV.

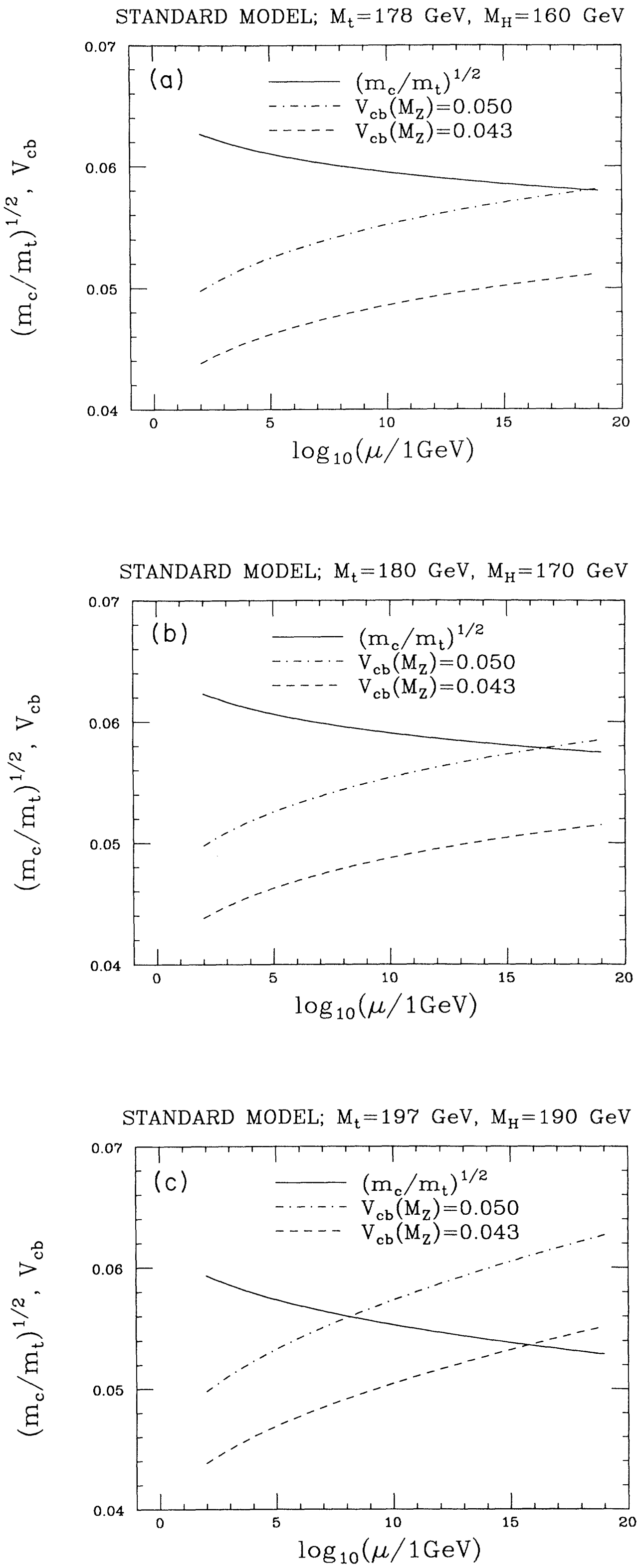

FIG. 4. Plot of V_{cb} and $\sqrt{m_c/m_t}$ as a function of scale in the standard model for (a) $M_t = 178$ GeV, (b) $M_t = 180$ GeV, and (c) $M_t = 197$ GeV, for both the central value (0.043) and the maximal value (0.050) of $V_{cb}(M_Z)$.

E. Other possible mass relations

Another interesting mass relation involves the ratio of the determinants of the charge $-\frac{1}{3}$ to charge −1 mass matrices and should equal one if relations (I) and (II) are valid. We note that, independent of the top mass (91–200 GeV), this weaker (less predictive) relation ($m_d m_s m_b = m_e m_\mu m_\tau$) can be satisfied at 10^{16} GeV for quark masses within the range stated above.

The relations considered so far have been motivated by specific theoretical models. There are other relations which are not similarly motivated but which may nevertheless hint at an underlying unified structure. In our search for simple relations among the quark masses, we considered an appealing "geometric mean" relation in the up sector, namely

$$m_u m_t = m_c^2 \ . \tag{8}$$

This relation favors higher top-quark masses and can be satisfied well at 10^{16} GeV in the $M_t = 190$ GeV, $M_H = 180$ GeV scenario with an up-quark mass compatible with that needed to satisfy relations (II) at 10^{16} GeV.

A similar relation involving the down-type quarks was tested

$$m_d m_b = m_s^2 \ . \tag{9}$$

This relation favors higher top-quark masses as well. In fact, in order to satisfy Eqs. (8) and (9) at 10^{16} GeV, as well as relations (II), given a fixed value for m_s(1 GeV) within the range cited, the top-quark mass would have to be larger than 200 GeV. Fortuitously, such a value would also favor relation (IV). These geometric mean relations have been discussed in the literature recently [30].

To conclude our analysis of the standard model case, we see that it is hard to arrive at a unified picture. The scale at which relation (I) tends to be satisfied does not coincide with that at which the others are valid. Still, the disagreement is never too large, which makes us hope that small course corrections in the running of the parameters allow most if not all of these relations to hold simultaneously at a unified scale. It is remarkable that for a top quark at the upper reaches of its allowed range, the long life of the bottom quark lends plausibility to the SO(10)-inspired relation (IV).

V. RUNNING THE RELATIONS IN THE SUPERSYMMETRIC STANDARD MODEL

A. Relations (I)–(IV)

In a previous publication [21], it was shown that low-energy supersymmetry allows relation (I) to be valid at the scale of gauge unification. In fact, if this relation is imposed at the unification scale, the top-quark mass is fixed (up to experimental errors in g_3), for a given value of the angle β. The reason is that the running of the bottom Yukawa coupling depends both on g_3 and on the top Yukawa coupling. We note that unlike the standard model case the top Yukawa coupling starts increasing at shorter length scales, providing us with an upper limit on the mass of the top quark. In addition, by relating the scalar quartic self-coupling below the supersymmetry breaking scale, to β, g_1, and g_2 above it, the mass of the lightest Higgs boson is fixed in terms of the mass of the top (or equivalently β). These results are displayed for two scales of supersymmetry breaking in Fig. 5. In the following, we shall take relation (I) as valid at $M_{\rm GUT}$ and assume the correspondence between β, M_t, and M_H depicted in these figures. We note, therefore, that this limits the top mass we can consider to be $\lesssim 200$ GeV. For $M_{\rm SUSY} = 1$ TeV, the gauge unification occurs between a low of 6.92×10^{15} GeV (low $M_{\rm GUT}$) and a high of 1.26×10^{16} GeV (high $M_{\rm GUT}$), corresponding to $g_3(M_Z) = 1.171$ and $g_3(M_Z) = 1.197$, respectively. The error bars in the strong coupling allow for a SUSY scale as high as ~ 10 TeV, with unification at 6.46×10^{15} GeV.

The strategy of the remaining part of this paper is to exploit the relations [(II)–(IV)] to constrain M_t and therefore β and M_H. For $M_{\rm SUSY} = 1$ TeV, we treat two cases, the first where unification takes place at its lowest value (low $M_{\rm GUT}$) and the second where it is at its highest value (high $M_{\rm GUT}$). In the following, we will not discuss our results for the supersymmetry-breaking scale of 10 TeV, since it adds nothing to our conclusions.

In the case of the mass relations among the light quark and lepton masses [relations (II)], we find that our plots do not depend on M_t, therefore we only display them for a representative value. Here we do not follow the strategy used in the standard model case (i.e., we do not vary m_s), although we still keep the ratios m_d/m_u and m_s/m_d fixed. Instead, we look for that value of m_s(1 GeV) which gives us the best agreement for relations (II) both for the low and high $M_{\rm GUT}$ (we cannot

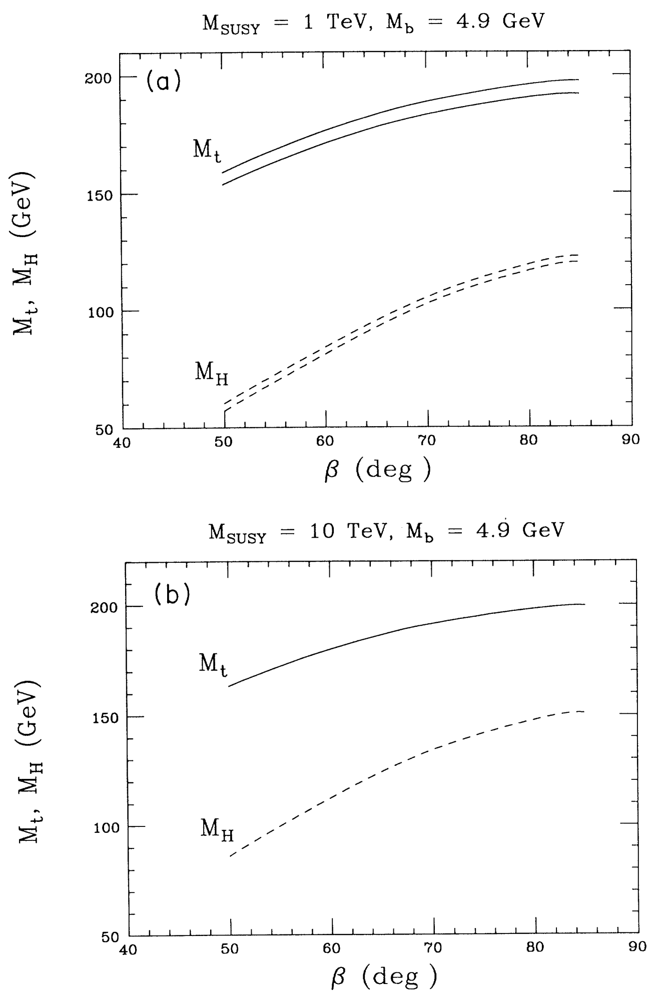

FIG. 5. Plot of M_t and M_H as a function of β. (a) For $M_{\rm SUSY} = 1$ TeV, the high curves (low curves) correspond to the highest (lowest) value of $\alpha_3(M_Z)$ consistent with unification. (b) For $M_{\rm SUSY} = 10$ TeV, the low and high curves coincide.

expect exact agreement at one scale, since m_s/m_d does not run). For instance, we can get the same value of m_s(1 GeV) in two different ways, either by demanding that at low $M_{\rm GUT}$ $m_d/3m_e = 1.1$ and $3m_s/m_\mu = 1$ or at high $M_{\rm GUT}$ that $m_d/3m_e = 1$ and $3m_s/m_\mu = 0.9$. In both cases, the masses of the lightest quarks at 1 GeV are $m_u = 3.80$ MeV, $m_d = 6.67$ MeV, and $m_s = 141$ MeV. These results are summarized in Fig. 6.

In the above, our philosophy has been to take the known low-energy data, and using the renormalization group, derive its implications at high energy. As we originally did for relation (I) [21], we could impose both of relations (II) at one unification scale (low or high $M_{\rm GUT}$). This would fix m_s/m_d at this scale, and since m_s/m_d and m_μ/m_e do not run, that would yield $m_s/m_d = 23.6$, a value that is only 12% larger than the value in Refs. [27, 20]. Furthermore, the results of our runs yield the masses at 1 GeV of the down and strange quarks to be $m_d = 5.86$ MeV, $m_s = 138$ MeV, in the low $M_{\rm GUT}$ case, and $m_d = 6.49$ MeV, $m_s = 153$ MeV, in the high $M_{\rm GUT}$ case. We note that this approach has also been taken by the authors of Ref. [18].

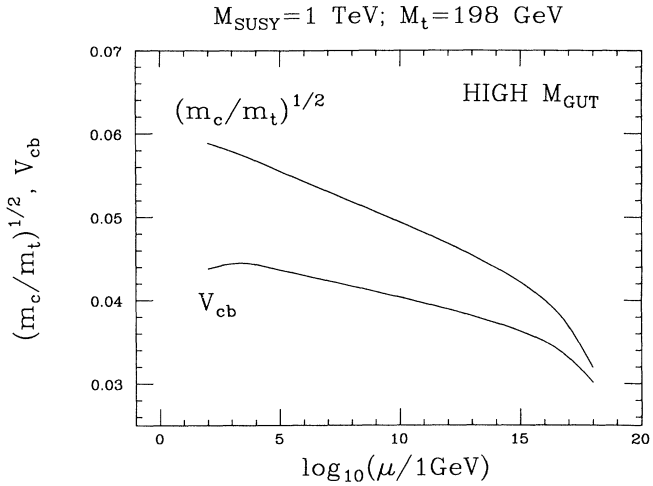

FIG. 7. Plot of V_{cb} and $\sqrt{m_c/m_t}$ as a function of scale in the SUSY case with $M_{\rm SUSY} = 1$ TeV for $M_t = 198$ GeV and for $V_{cb}(M_Z) = 0.043$.

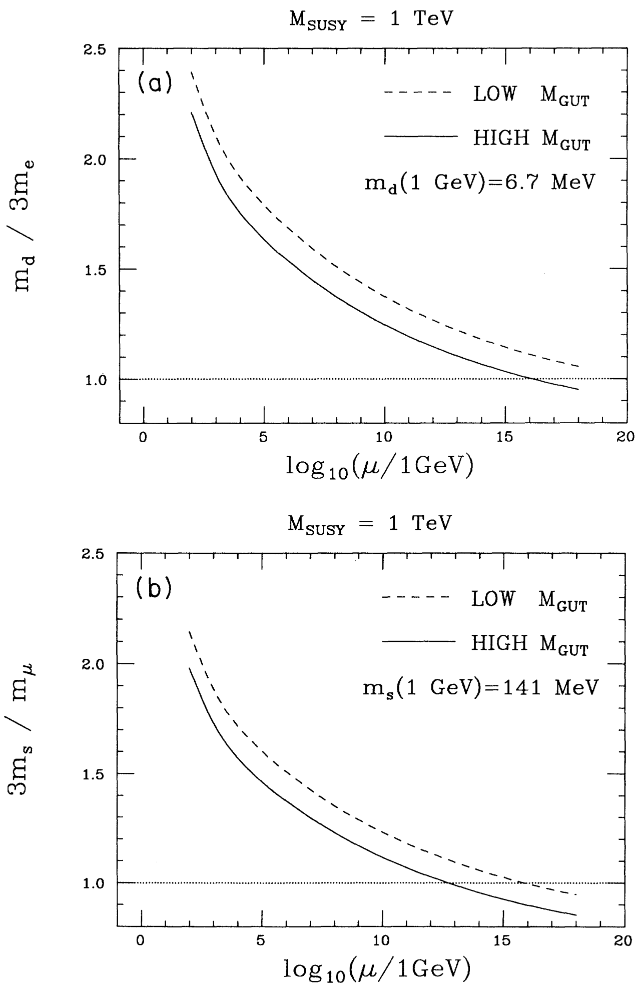

FIG. 6. Plots of (a) $m_d/3m_e$ and (b) $3m_s/m_\mu$ as a function of scale in the SUSY case with $M_{\rm SUSY} = 1$ TeV, for the low and high unification scales (no appreciable dependence on M_t was found).

Before discussing relation (IV), let us note that, with supersymmetry, relation (III) is again well satisfied at all scales. We now turn to relation (IV). As we did in the standard model case we display our results both for the central value of V_{cb} (0.043) and for its upper value (0.050). Then we look for values of M_t which give us agreement at the unification scale (low or high $M_{\rm GUT}$). Using the central value for V_{cb}, we find no agreement at the unification scale. However, this relation is satisfied at the Planck scale, if we use both a high $M_{\rm GUT}$ and $M_t = 198$ GeV [the highest possible value consistent with relation (I)], as displayed in Fig. 7. We have also made

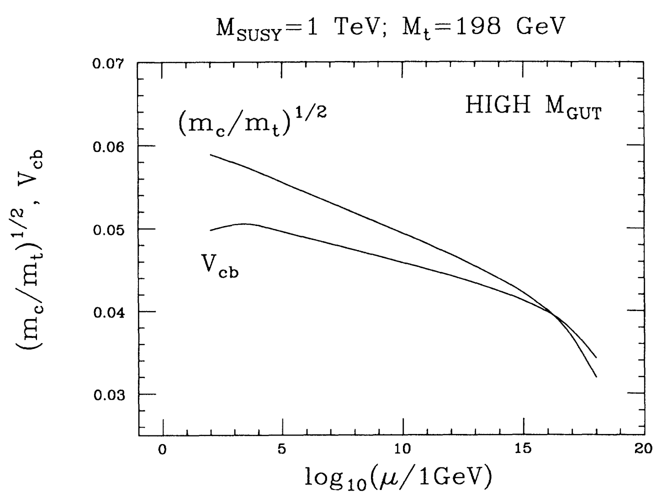

FIG. 8. Same as Fig. 7 with $V_{cb}(M_Z) = 0.050$ and $M_t = 198$ GeV.

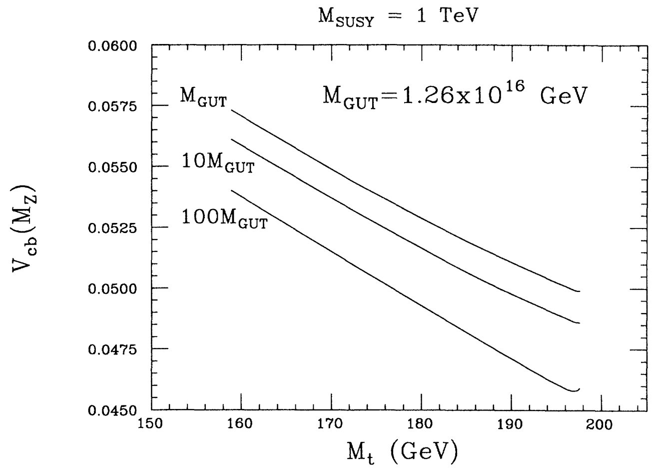

FIG. 9. Plot of $V_{cb}(M_Z)$ as a function of M_t assuming relation (IV) holds at various scales $\gtrsim M_{\rm GUT}$.

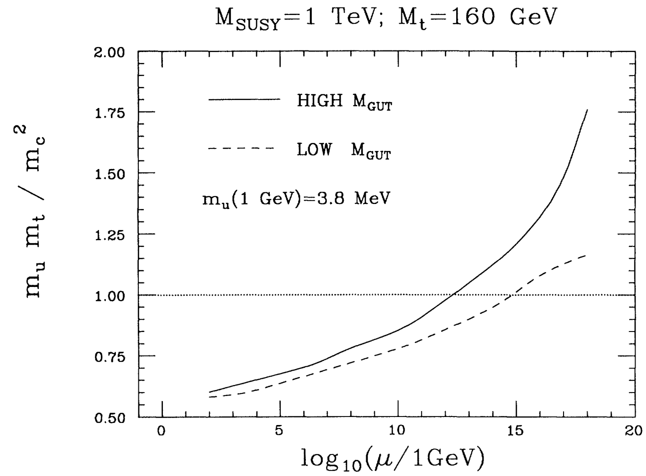

FIG. 10. Plot of $m_u m_t/m_c^2$ as a function of the scale for the highest value of $\alpha_3(M_Z)$ (high curve) and the lowest value of $\alpha_3(M_Z)$ (low curve) and for $M_t = 160$ GeV.

several runs with a higher value V_{cb}. There, the relation can actually be satisfied provided that we use the high $M_{\rm GUT}$ scale and $M_t = 198$ GeV as shown in Fig. 8. In the low $M_{\rm GUT}$ case, the two curves meet closer to the Planck scale. In fact, theory does not dictate to us the exact scale at which the SO(10)-inspired relation is valid; it could be much higher than the scale of unification of the standard model's gauge couplings. To account for this, we now plot, in Fig. 9, V_{cb} as a function of M_t, assuming that relation (IV) is valid at $M_{\rm GUT}$, $10M_{\rm GUT}$, and $100M_{\rm GUT}$, and using the higher value of $g_3(M_Z)$. Given an initial value of V_{cb} at M_Z, Fig. 9 can be used to determine the needed M_t (and hence β) to satisfy relation (IV) at $M_{\rm GUT}$, $10M_{\rm GUT}$, or $100M_{\rm GUT}$. We can see from this figure that as long as V_{cb} is larger than its central value, then relation (IV) can be satisfied above the SU(5) GUT scale and still allow for a lower value of M_t.

B. Other possible mass relations

The relation in Sec. IV E involving the determinants of the charge $-\frac{1}{3}$ and charge -1 fermion mass matrices holds in the minimal supersymmetric model at 10^{14} GeV in the high $M_{\rm GUT}$ case and at 10^{18} GeV in the low $M_{\rm GUT}$ case. In both cases, this relation holds within $\sim 10\%$ at 10^{16} GeV. These results are true for the light-quark masses chosen in Sec. V A and for all $160 \leq M_t \leq 198$ GeV.

A priori one might naively assume that $m_u m_t = m_c^2$ could be easily satisfied because of the uncertainty in m_t. However, two facts make the relation viable in the supersymmetric case. First, the value predicted for the top mass is within the range allowed by experiment and the ρ-parameter bound. Second, and most remarkable, is the fact that this top mass value is compatible with relations (I)–(III) given the choice of light-quark masses in Sec. V A. In Fig. 10, we display the running of the ratio $m_u m_t/m_c^2$ for the low and high $M_{\rm GUT}$ cases. We show the curves representing the lower M_t value of 160 GeV for which the relation is best satisfied at $M_{\rm GUT}$. This relation is incompatible with relation (IV) however, since the latter favors a higher top mass. We note that m_c affects these two relations in an "inverse" manner. A lower experimental value for the charm-quark mass favors relation (IV) whereas a higher experimental value favors the geometric mean relation.

One may also consider the geometric mean relation in the down sector. We find however that this relation fails to hold in the supersymmetric case. Other relations among the Yukawa couplings have been considered in the literature. Theoretical bias or numerology can lead to still other relations valid at some unifying scale. In all cases, a thorough renormalization group analysis will be required in investigations of a possible deeper structure.

VI. CONCLUSION

The aim of this paper has been to explore physics beyond the standard model by studying mass and mixing-angle patterns suggested by grand unified theories, first within the standard model context and second in its minimal supersymmetric extension. We have reduced some of the parameter space by constraining the ratios m_d/m_u and m_s/m_d, which are better known than the masses themselves.

In the standard model case, there are many unsatisfactory features, not the least of which is the failure of the gauge couplings to unify within experimental error, forming a GUT triangle. The simplest of the SU(5) relations, $m_b = m_\tau$, can only be satisfied some ten orders of magnitude from the scale of the GUT triangle. The other relations $m_d = 3m_e$, $3m_s = m_\mu$, $\tan\theta_c = \sqrt{m_d/m_s}$, and $V_{cb} = \sqrt{m_c/m_t}$, could be satisfied at 10^{16} GeV. The geometric-mean relations we considered can also be simultaneously satisfied, but this requires a top-quark mass greater than 200 GeV.

In the SUSY case, for which the GUT triangle collapses, we achieved a striking agreement for the four main relations considered. But, for this to be true, several things must occur: first V_{cb} must be larger than its presently measured value; second the top-quark mass must be around 190 GeV (if it is a bit lighter, then agreement dictates that V_{cb} should be larger still); third the Higgs-boson mass should hover around 120 GeV. These conclusions are qualitatively correct if one demands maximum agreement. An analysis which recently appeared in the literature has reached similar conclusions [18]. However, it is difficult to arrive at more definite numbers without an exhaustive analysis of the parameter space.

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