Properties of strange quark matter

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Assuming that the novel nuclei of $A=370$ amu and $Z=14$ recently observed in high-energy cosmic rays are really strange quark matter (SQM), the relationship between parameters describing SQM is discussed and the mass formula for SQM is derived by requiring that it yield the observed value of $A=370$ amu and $Z=14$.

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Strange quark matter (SQM) is the matter made of roughly equal numbers of up, down, and strange quarks [1]. It has been suggested that such matter is absolutely stable [2,3], partially because the increase in energy due to the strange quark mass could well be compensated by a decrease on the Fermi level of up and down quarks owing to the Pauli principle. If this is the case, SQM is the true ground state of QCD. One can imagine, in the course of the evolution of the Universe, that much of the baryon number of the Universe could condense into nuggets of SQM through the first-order quark-hadron phase transition. However, recent studies seem to indicate that SQM, even if made in the early Universe, has evaporated and may not remain today [4]. Another possibility for producing SQM would exist in the present era of our Universe as a result of certain catastrophic processes such as a collision of neutron stars or an explosion of strange stars [2,5]. The resulting small lumps of SQM thus produced could then be detected at the Earth. Recent observation has indicated the existence of novel nuclei with $A=370$ amu and $Z=14$ in high-energy cosmic rays [6].

In order to derive the mass formula for SQM which will be a guide for future searches for SQM [7], a bag model, in which the quarks are degenerate Fermi gas confined by a bag pressure, is taken to describe SQM. This Brief Report discusses the relationship between parameters of SQM such as the strange quark mass and quark chemical potentials, and mass formula on the basis of the new constraint that the observed values of $A=370$ amu and $Z=14$ be reproduced.

A nugget of SQM is approximated by a sphere with a radius R, in which u, d, and s quarks are confined within a volume V by a bag pressure B. The charges are assumed to be uniformly distributed in SQM and the "chemical" equilibrium between u, d, and s quarks and electrons is maintained by weak interactions, $d\to u+e+\bar{\nu}_e$, $u+e\to d+\nu_e$, $s\to u+e+\bar{\nu}_e$, $u+e\to s+\nu_e$, and $s+u\leftrightarrow u+d$. The state of SQM is then determined [8] by the thermodynamic potentials Ω_i ($i=u,d,s,e$) being functions of the chemical potentials μ_i and the strange quark mass m_s as well as by B and V. The number densities are simply given by $n_i=-\partial\Omega_i/\partial\mu_i$ and the baryon number density by $n_A=(n_u+n_d+n_s)/3$. Another important factor to determine properties of SQM is the gluon's effect due to quantum chromodynamics (QCD), which is partially included as the effect of the bag pressure. Although it is difficult to definitely describe the gluon's degree of freedom in the formation of SQM, the effect can be parametrized by α_c, if QCD is treated perturbatively. The first-order correction of α_c results in the replacement of $n_{u,d}\to(1-2\alpha_c/\pi)n_{u,d}$ for the u and d quarks, and similarly for the s quarks [8]. Fortunately, our main result turns out to be almost independent of this parametrization of QCD.

The SQM with $A\gtrsim10^7$ includes the electron degrees of freedom inside SQM. In this case, the "chemical" equilibrium among u, d, and s together with e, maintained by weak interactions, establishes $\mu_u=\mu-\mu_e$ and $\mu_d=\mu_s$ ($\equiv\mu$). The accompanying neutrinos will carry energy from SQM. Then, the chemical potential of neutrinos will be equal to zero. The total charge including electrons is set to be zero, $(2n_u-n_s)/3=n_e$. On the other hand, SQM with $A\lesssim10^7$ that might be a fragment from SQM with $A\gtrsim10^7$ becomes charged because of the absence of electrons in equilibrium. Weak equilibrium still maintains $\mu_d=\mu_s$. The total energy of SQM [3] with volume V can be described by analogy with the Bethe-Weizsäcker liquid-drop model for normal nuclei, which consists of three terms, a volume term, a surface term, and the Coulomb term:

$$E=\left[\sum_i^{u,d,s}(\mu_i n_i+\Omega_i)+B\right]V+4\pi\sigma_{\rm surf}R^2+\frac{3}{5}\alpha\frac{Z^2}{R}\ , \quad (1)$$

where $V=4\pi R^3/3$ and the $\sigma_{\rm surf}$ term takes care of the surface effect.

As developed by Berger and Jaffe [9], we treated the energy E of Eq. (1) as an ideal case, in which the Coulomb and surface effects are neglected, and then corrections were added to the energy E_0 of the ideal case [10]. Minimizing E with respect to the volume and charge, $\partial E/\partial V = \partial E/\partial Z = 0$, one gets the minimal value of Z, $Z_{\rm min}$, which yields the most energetically favorable value of E, $E_{\rm min}$:

$$Z_{\rm min} = -\frac{\mu_Z}{(\delta_Z/A) + (\delta_{\rm Coul}/A^{1/3})}, \quad (2)$$

$$E_{\rm min} = \epsilon_0 A + \frac{1}{3}\delta_{\rm surf} A^{2/3} - \left[\frac{\delta_Z}{2A} + \frac{\delta_{\rm Coul}}{3A^{1/3}}\right] Z_{\rm min}^2, \quad (3)$$

where $\epsilon_0 = E_0/A$, $\mu_Z = \mu_u - \mu_d$, $\delta_{\rm surf} = 4\pi\sigma_{\rm surf} a_0^2$ for $a_0 = (3/4\pi n_A)^{1/3}$, $\delta_{\rm Coul} = 6\alpha/5a_0$, and $\delta_Z = A\,\partial\mu_Z/\partial Z|_0$. It should be noted that the A-Z relation does not involve the surface term, which is rather ambiguous to be estimated. For a reference, one can use the estimation due to Ref. [9], which contains the positive surface tension from s quarks only that results in the modification of the density of states $dn(k)/dk = g(k^2V/2\pi^2) \rightarrow g\{k^2V/2\pi^2 - (k/2)R^2[1-(2/\pi)\arctan(k/m_s)]\}$. Therefore, the energy $E_{\rm min}$ increases slightly for small A due to Coulomb and surface energies.

The A-Z relation given by Eq. (2) originally described by six parameters, the chemical potentials μ and μ_u, the s-quark mass m_s, the bag pressure B, the volume V, and the QCD coupling α_c, now contains only four independent parameters: ϵ_0, m_s, and A as well as α_c. Other parameters, B, δ's, and $Z_{\rm min}$, are expressed by functions of those four parameters. The A-Z relation for SQM is thus determined by the three parameters ϵ_0, m_s, and A and by the additional QCD coupling α_c. The mass of SQM given by Eq. (3) further depends on the surface tension. The relationship among ϵ_0, m_s, and α_c and the relation of A and Z are obtained by assuming that SQM allows the state with the observed values of $A = 370$ amu and $Z = 14$.

Shown in Fig. 1 is the relationship between m_s and ϵ_0 for different values of $\alpha_c = 0.0$, 0.3, 0.6, and 1.0. It shows that the increase of α_c requires the increase of m_s to maintain $(A,Z) = (370,14)$. The feature is the result of Eq. (3), where n_A remains almost constant and where an increase in α_c, which reduces Z through μ_Z due to the factor $(1-2\alpha_c/\pi)$, is compensated by an appropriate increase in Z through m_s, which reduces δ_Z (see Ref. [9] for the m_s dependence of δ_Z). (A,Z) is thus kept fixed by the interplay between α_c and m_s. Numerically, we found the following results: The chemical potential μ, which varies from $\sim$300 MeV to $\sim$315 MeV as m_s increases, is almost independent of α_c and μ_u is approximately given as $\mu - 10$. The number density n_A, which determines $n_Z = n_A Z/A$, varies from $\sim$(100 MeV)^3 to $\sim$(140 MeV)^3 as α_c decreases from 1.0 to 0. Since A is fixed, this dependence of n_A is directly translated into that of the radius of SQM, R. The deviations of these parameters due to the variations of α_c and m_s become small because α_c and m_s are tuned to yield $(A,Z) = (370,14)$.

The relation of A and Z, i.e., the mass formula for SQM, which are free from the ambiguity of the surface tension, is shown in Fig. 2. The solid curve in Fig. 2 indicates the mass formula and the dashed curve indicates the relation of normal nuclei. These curves are almost degenerated since the parameters in Eq. (3), μ_Z, δ_Z, and $\delta_{\rm Coul}$, are tuned to be almost constant owing to the compensation of the drastic change of α_c by the appropriate change of m_s to reproduce $(A,Z) = (370,14)$. The corresponding values of m_s can be read off from Fig. 1. Note that our relation is useful in regions of $A \gtrsim 100$ because of the possible importance of shell effects [11] in regions of $A \lesssim 100$.

One may consider that our process is merely reducing the number of free parameters by one, from four to three. However, these parameters are found to be strongly correlated to each other, once one experimental point of A and Z is known. The dependence of m_s on ϵ_0 is fairly weak and only a tiny range of α_c is allowed for a given value of m_s. For instance, if you take $\alpha_c = 0.6$, then m_s is constrained to lie between 180 and 190 MeV. This less dependence seems to originate from the requirement that all the parameters are set to produce SQM with $A = 370$ amu and $Z = 14$. Since the mass is almost independent of ϵ_0, the surface tension of Berger and Jaffe also exhibits the similar behavior that the surface tension is almost constant once α_c is given.

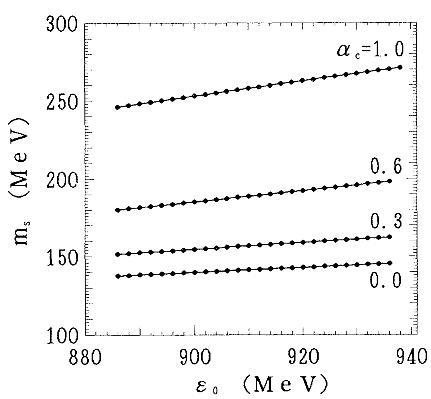

FIG. 1. The relationship between m_s and ϵ_0 ($\equiv E_0/A$) for SQM for $\alpha_c = 0.0$, 0.3, 0.6, and 1.0.

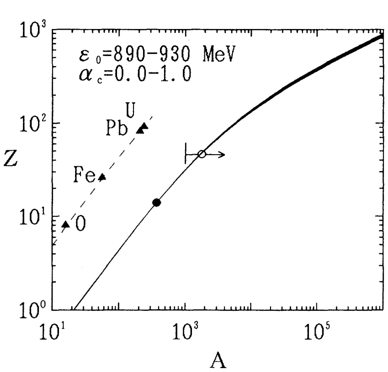

FIG. 2. The mass formula for SQM for ranges of $\epsilon_0 = 880-930$ MeV and $\alpha_c = 0.0-1.0$.

We close our discussions by stressing again that (1) parameters describing SQM are fairly tightly constrained once the observed values of $A = 370$ amu and $Z = 14$ are adopted and (2) the A-Z curves are nearly degenerate for the different values of α_c as well as of ϵ_0. When nuggets of SQM will be detected in a future experiment [7], they might scatter around this curve. Since Z grows slowly with A, we will find many stable isotopes for each value of Z. Recently, Price recalled to us [12] a special event which was found in their reassessment of a monopole candidate [13]. One possible interpretation for the event is as a massive particle of $Z = 46$ and $A \gtrsim 1000$ amu. The open circle in Fig. 2 shows this event on our curve ($A \cong 1800$ amu). Although our prediction locates close to the lower bound of Price's event, it still seems consistent with the A-Z relation derived on the basis of the observed value of $A = 370$ amu and $Z = 14$.

Is there really SQM described by our mass formula? The answer to that question can have an enormous impact on our understanding of the effects of QCD in nuclei.

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