Calculation of the polarization tensors of $Z \to 3\gamma$ and $\gamma\gamma \to \gamma\gamma$ via W-boson loops in the standard model

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(Received 12 December 1991; revised manuscript received 15 September 1992)

The polarization tensors of $Z \to 3\gamma$ and $\gamma\gamma \to \gamma\gamma$ via W-boson loops are calculated in the standard model. The constrained equations for the tensors are deduced from crossing symmetry and gauge invariance. These equations are numerically checked by the calculated tensor of $\gamma\gamma \to \gamma\gamma$ to the first three lowest orders in its low-energy expansion.

PACS number(s): 12.15.Ji, 13.38.+c, 13.88.+e, 14.80.Er

The decay $Z \to 3\gamma$ is a rare decay of the neutral vector boson Z. The fermion-loop contribution to the decay has already been discussed [1]. But there is no calculation of the W-loop contribution [2]. Although these two contributions predicted by the standard model are very small and cannot be observed at the CERN e^+e^- collider LEP, there is still an interest in them, especially, in the W-loop contribution. It is concerned with the vertices ZWW, γWW, $Z\gamma WW$, and $\gamma\gamma WW$, which remain untouched experimentally, and it may become large in some composite model at an experimentally testable level [2].

The γ-γ scattering is intimately related to $Z \to 3\gamma$ decay in their calculations. The scattering is interesting because it is concerned with electromagnetic nonlinear interaction which is absent in the classical Maxwell theory. The fermion-loop contribution to the γ-γ scattering was calculated in the early 1950s when the renormalization scheme was available [3]. But the W-loop contribution to the scattering seems still not calculated in the standard model, perhaps because it is very small in comparison with the fermion-loop contribution at low energy.

In this paper, the polarization tensors of $Z \to 3\gamma$ and $\gamma\gamma \to \gamma\gamma$ via the W loop are calculated in the standard mode in the R_ξ renormalization gauge with $\xi = 1$. In Sec. I a general expression for the tensors with crossing symmetry and gauge symmetry is given. Because of the symmetries the A, B, and C coefficients of the tensors should satisfy certain equations which are different from those in the fermion-loop case [3]. These equations reduce 57 of the A, B, and C coefficients to 4 independent ones, simplifying the calculation, and they are also very useful in checking our final results. In Sec. II the A and B coefficients for both $Z \to 3\gamma$ and $\gamma\gamma \to \gamma\gamma$ are calculated. At low energy the A and B coefficients of $\gamma\gamma \to \gamma\gamma$ may expand in Taylor series. By using their expansions we have checked the constrained equations to the first three lowest orders. In Sec. III some discussions are given.

I. THE GENERAL FORMS OF THE POLARIZATION TENSORS OF $Z \to 3\gamma$ AND $\gamma\gamma \to \gamma\gamma$

Consider a Feynman diagram in a process with four external lines: three of them are photons, and the fourth one is a Z boson or a photon. Their four-momenta are k_i^μ $(i=1,2,3,4)$. For convenience all directions of the momenta are taken to be out going. Thus energy-momentum conservation law is written as

$$\sum_{i=1}^{4} k_i^\mu = 0 . \tag{1}$$

The amplitude for the process is

$$I = \epsilon_{\mu_1}(k_1,\lambda_1)\epsilon_{\mu_2}(k_2,\lambda_2)\epsilon_{\mu_3}(k_3,\lambda_3)\epsilon'_{\mu_4}(k_4,\lambda_4) \times G^{\mu_1\mu_2\mu_3\mu_4}(k_1,k_2,k_3,k_4) , \tag{2}$$

where $\epsilon_\mu(k,\lambda)$ is the photon polarization vector, and $\epsilon'_\mu(k,\lambda)$ is Z boson or photon polarization vector. These vectors should satisfy

$$\epsilon_\mu(k,\lambda)k^\mu = 0 \quad (\lambda=1,2) ,$$
$$\epsilon'_\mu(k,\lambda)k^\mu = 0$$
$$(\lambda=1,2,3, \text{ when } \epsilon'_\mu \text{ is the } Z\text{-boson polarization vector}) , \tag{3}$$

where λ are taken to be physical polarization degrees of freedom.

Because of Eqs. (3), the polarization tensor

$$G^{\mu_1\mu_2\mu_3\mu_4}(k_1,k_2,k_3,k_4) \equiv G^{1234}(1234)$$

in Eq. (2) cannot be determined exclusively. Polarization tensors are called equivalent when they give the same amplitude I in Eq. (2). Crossing symmetry prescribes that there is always a polarization tensor with S_3 (the symmetric group of rank 3) symmetry (in $\gamma\gamma\to\gamma\gamma$ case S_3 can be replaced by S_4):

$$G^{1234}(1234)=P_a\{G^{1234}(1234)\}\ , \qquad P_a\in S_3(a=1,2,3,4,5,6) \quad (4)$$

where P_a is an element of S_3. When P_a is applied to the G, its (k,μ) pairs should be permuted. For example, if $P_a=(123)=\binom{123}{231}$,

$$(123)\{G^{1234}(1234)\}=G^{2314}(2314)\ .$$

Suppose all Lorentz indices μ in the tensor G are pure vectors (not pseudovectors). Then G has the general expansion

$$\begin{aligned}G^{1234}(123)=&\sum_{i,j,l,m}A_{ijlm}(123)\begin{bmatrix}1234\\ijlm\end{bmatrix}+\sum_{i,m}B^1_{im}(123)\,{}^{23}\begin{bmatrix}14\\im\end{bmatrix}+\sum_{j,m}B^2_{jm}(123)\,{}^{13}\begin{bmatrix}24\\jm\end{bmatrix}\\&+\sum_{l,m}B^3_{lm}(123)\,{}^{12}\begin{bmatrix}34\\lm\end{bmatrix}+\sum_{j,l}B^4_{jl}(123)\,{}^{14}\begin{bmatrix}23\\jl\end{bmatrix}+\sum_{i,l}B^5_{il}(123)\,{}^{24}\begin{bmatrix}13\\il\end{bmatrix}\\&+\sum_{i,j}B^6_{ij}(123)\,{}^{34}\begin{bmatrix}12\\ij\end{bmatrix}+C_1(123)g^{\mu_1\mu_2}g^{\mu_3\mu_4}+C_2(123)g^{\mu_1\mu_3}g^{\mu_2\mu_4}+C_3(123)g^{\mu_1\mu_4}g^{\mu_2\mu_3}\end{aligned}$$

$$(i=2,3;j=1,3;l=1,2;m=1,2,3)\ , \quad (5)$$

where

$$\begin{bmatrix}1234\\ijlm\end{bmatrix}\equiv k_i^{\mu_1}k_j^{\mu_2}k_l^{\mu_3}k_m^{\mu_4}\equiv(ijlm)\ ,$$

$${}^{23}\begin{bmatrix}14\\im\end{bmatrix}\equiv g^{\mu_2\mu_3}k_i^{\mu_1}k_m^{\mu_4},\ \text{etc.}$$

In expression (5) k_4 is already canceled by using Eq. (1) and all terms with $k_1^{\mu_1}$, $k_2^{\mu_2}$, or $k_3^{\mu_3}$ are discarded due to Eq. (3). The restriction on the summation indices i, j, l, and m is indicated in Eq. (5). All coefficients A, B, and C in Eq. (5) are Lorentz scalars. There are 24 A, 30 B, and 3 C coefficients. The base vectors $\{(ijlm),\ldots,g^{\mu_1\mu_4}g^{\mu_2\mu_3}\}$ in expansion (5) are so chosen that they are invariant under S_3; i.e., under the application of any element of S_3, they transform among themselves. Note that the index μ_4 in G^{1234} may not be a pure vector when μ_4 refers to the Z boson. But in our case only the vector part contributes to G.

Now suppose G in Eq. (5) is S_3 symmetric. We may further write G as

$$\begin{aligned}G^{1234}(123)=\sum_{a=1}^{6}P_a\Bigg\{&A_1(123)\begin{bmatrix}1234\\2111\end{bmatrix}+A_2(123)\begin{bmatrix}1234\\2121\end{bmatrix}+A_3(123)\begin{bmatrix}1234\\2123\end{bmatrix}+A_4(123)\begin{bmatrix}1234\\2311\end{bmatrix}\\&+B_1(123)\,{}^{12}\begin{bmatrix}34\\11\end{bmatrix}+B_2(123)\,{}^{12}\begin{bmatrix}34\\12\end{bmatrix}+B_3(123)\,{}^{12}\begin{bmatrix}34\\13\end{bmatrix}+B_5(123)\,{}^{14}\begin{bmatrix}23\\12\end{bmatrix}\\&+\frac{1}{2}\left[B_4(123)\,{}^{14}\begin{bmatrix}23\\11\end{bmatrix}+B_6(123)\,{}^{14}\begin{bmatrix}23\\32\end{bmatrix}+C_1(123)g^{\mu_1\mu_2}g^{\mu_3\mu_4}\right]\Bigg\}\ ,\end{aligned} \quad (6)$$

where $A_1\equiv A_{2111}$, $A_2\equiv A_{2121}$, $A_3\equiv A_{2123}$, $A_4\equiv A_{2311}$, $B_1\equiv B^3_{11}$, $B_2\equiv B^3_{12}$, $B_3\equiv B^3_{13}$, $B_4\equiv B^4_{11}$, $B_5\equiv B^4_{12}$, $B_6\equiv B^4_{32}$; and

$$B_4(123)=B_4(132),\quad B_6(123)=B_6(132)\ ,$$
$$C_1(123)=C_1(213)\ . \quad (7)$$

In $Z\to 3\gamma$ gauge invariance demands

$$k_{1\mu_1}\epsilon_{\mu_2}(k_2,\lambda_2)\epsilon_{\mu_3}(k_3,\lambda_3)\epsilon'_{\mu_4}(k_4,\lambda_4)\times G^{\mu_1\mu_2\mu_3\mu_4}(k_1,k_2,k_3,k_4)=0\ , \quad (8)$$

$$\epsilon_{\mu_1}(k_1,\lambda_1)k_{2\mu_2}\epsilon_{\mu_3}(k_3,\lambda_3)\epsilon'_{\mu_4}(k_4,\lambda_4)\times G^{\mu_1\mu_2\mu_3\mu_4}(k_1,k_2,k_3,k_4)=0\ ,$$
$$\epsilon_{\mu_1}(k_1,\lambda_1)\epsilon_{\mu_2}(k_2,\lambda_2)k_{3\mu_3}\epsilon'_{\mu_4}(k_4,\lambda_4)\times G^{\mu_1\mu_2\mu_3\mu_4}(k_1,k_2,k_3,k_4)=0\ , \quad (8')$$

where λ_i are physical polarization degrees of freedom. If G is S_3 symmetric, Eq. (8) guarantees Eqs. (8′). Usually from Eqs. (8), we cannot get

$$k_{1\mu_1}G^{\mu_1\mu_2\mu_3\mu_4}(k_1,k_2,k_3,k_4)=0 \ . \tag{9}$$

The tensor G of $\gamma\gamma\to\gamma\gamma$ from the fermion-loop contribution fortunately satisfies Eq. (9) [3]. Thus it simplifies the calculation a lot; i.e., only several coefficients A are needed for the calculation of G. But in the W-loop case the G, which is calculated from Feynman diagrams and is S_3 symmetric, does not satisfy Eq. (9). Still we can find a tensor G', which is equivalent to the G and satisfies Eq. (9). But, in general, G' is no longer S_3 symmetric.

How to get G' from the G in Eq. (5)? For simplicity, suppose k_1 refers to the photon and k_2,k_3,k_4 refer to massive vector bosons. For a polarization vector $\epsilon_\mu(k,\lambda)$ of the massive vector boson with mass M,

$$\sum_{\lambda=1}^{3}\epsilon_\mu(k,\lambda)\epsilon_\nu(k,\lambda)=-g_{\mu\nu}+\frac{k_\mu k_\nu}{M^2} \ , \tag{10}$$

where $\lambda=1,2,3$ are the physical polarization degrees of freedom. By using Eq. (10), Eq. (8) can be turned into

$$\begin{aligned}-k_{1\mu_1}G^{\mu_1\nu_2\nu_3\nu_4}(1234)&+\frac{1}{M^2}k_{1\mu_1}[k_{2\mu_2}G^{\mu_1\mu_2\nu_3\nu_4}(1234)k_2^{\nu_2}+k_{3\mu_3}G^{\mu_1\nu_2\mu_3\nu_4}(1234)k_3^{\nu_3}+k_{4\mu_4}G^{\mu_1\nu_2\nu_3\mu_4}(1234)k_4^{\nu_4}]\\&-\frac{1}{M^4}k_{1\mu_1}[k_{2\mu_2}k_{3\mu_3}G^{\mu_1\mu_2\mu_3\nu_4}(1234)k_2^{\nu_2}k_3^{\nu_3}+k_{2\mu_2}k_{4\mu_4}G^{\mu_1\mu_2\nu_3\mu_4}(1234)k_2^{\nu_2}k_4^{\nu_4}+k_{3\mu_3}k_{4\mu_4}G^{\mu_1\nu_2\mu_3\mu_4}(1234)k_3^{\nu_3}k_4^{\nu_4}]\\&+\frac{1}{M^6}k_{1\mu_1}k_{2\mu_2}k_{3\mu_3}k_{4\mu_4}G^{\mu_1\mu_2\mu_3\mu_4}(1234)k_2^{\nu_2}k_3^{\nu_3}k_4^{\nu_4}=0 \ . \end{aligned} \tag{11}$$

The left-hand side of Eq. (11) is a tensor of rank 3 with vector indices. So it may expand according to base vectors:

$$\{k_j^{\nu_2}k_l^{\nu_3}k_m^{\nu_4},g^{\nu_2\nu_3}k_m^{\nu_4},g^{\nu_2\nu_4}k_l^{\nu_3},g^{\nu_3\nu_4}k_j^{\nu_2}\} \ , \tag{12}$$

where, due to Eq. (1), each j,l,m can take three values out of 1, 2, 3, and 4. Noting that $k_2^{\nu_2},k_3^{\nu_3},k_4^{\nu_4}$ already appear in Eq. (11), we may take $j=1,2,3$, $l=1,2,3$, and $m=1,2,4$, for example. Substituting the G of Eq. (5) into Eq. (11), we find that in Eq. (5) $m=1,2,3$, but in Eq. (11) m was just taken to be 1,2,4. So for consistency within the same base, we should substitute $k_3^{\nu_4}=-(k_1^{\nu_4}+k_2^{\nu_4}+k_4^{\nu_4})$ into G of Eq. (5):

$$G^{1234}(1234)=G'^{1234}(1234)+K^{1234}(1234) \ , \tag{13}$$

where all terms containing $k_4^{\nu_4}$ belong to K, so G' consists of all terms in G which do not contain $k_4^{\nu_4}$. After substituting G of Eq. (13) into Eq. (11), the left hand of Eq. (11) has two parts: the first part is $k_{1\mu_1}G'^{\mu_1\nu_2\nu_3\nu_4}$, which does not contain any of $k_2^{\nu_2}$, $k_3^{\nu_3}$, and $k_4^{\nu_4}$, and in the other part every term at least contains one of $k_2^{\nu_2}$, $k_3^{\nu_3}$, and $k_4^{\nu_4}$. In this way the left hand of Eq. (11) has expanded according to base vectors (12) with $j=1,2,3$, $l=1,2,3$, and $m=1,2,4$. So the two parts in Eq. (11) should vanish, respectively; thus,

$$k_{1\mu_1}G'^{1234}(1234)=0 \ . \tag{14}$$

So one may easily get G' from the G of Eq. (5), substitute $k_3^{\mu_4}=-(k_1^{\mu_4}+k_2^{\mu_4}+k_4^{\mu_4})$ into the G of Eq. (5), and discard all terms in G containing $k_4^{\mu_4}$, the remaining part of G is just G'. According to Eqs. (3), G' is equivalent to G. If G is S_3 symmetric as in Eq. (6), we get

$$\begin{aligned}G'^{1234}(123)=&A_{13}(123)(2111)+A_{13}(213)(2122)+A_{12}(132)(3111)+A_{12}(231)(2322)+A_{23}(123)(2121)\\&+A_{23}(213)(2112)-A_{12}(312)(3311)-A_{12}(321)(3322)-A_{13}(312)(3312)-A_{23}(321)(2321)\\&-A_{13}(321)(3321)-A_{23}(312)(3112)-A'_4(312)(2311)-A'_4(321)(3122)+A'_4(132)(3121)\\&+A'_4(231)(2312)+B_{13}(123)\,{}^{12}\begin{bmatrix}34\\11\end{bmatrix}+B_{13}(213)\,{}^{12}\begin{bmatrix}34\\22\end{bmatrix}+B_{12}(132)\,{}^{13}\begin{bmatrix}24\\11\end{bmatrix}\\&+B_{12}(231)\,{}^{23}\begin{bmatrix}14\\22\end{bmatrix}+B_{23}(123)\,{}^{12}\begin{bmatrix}34\\12\end{bmatrix}+B_{23}(213)\,{}^{12}\begin{bmatrix}34\\21\end{bmatrix}-B_{12}(321)\,{}^{23}\begin{bmatrix}14\\32\end{bmatrix}\\&-B_{12}(312)\,{}^{13}\begin{bmatrix}24\\31\end{bmatrix}-B_{23}(132)\,{}^{13}\begin{bmatrix}24\\12\end{bmatrix}-B_{13}(321)\,{}^{23}\begin{bmatrix}14\\31\end{bmatrix}-B_{23}(231)\,{}^{23}\begin{bmatrix}14\\21\end{bmatrix}\\&-B_{13}(312)\,{}^{13}\begin{bmatrix}24\\32\end{bmatrix}+B_5(123)\,{}^{14}\begin{bmatrix}23\\12\end{bmatrix}+B_5(213)\,{}^{24}\begin{bmatrix}13\\21\end{bmatrix}+B_5(132)\,{}^{14}\begin{bmatrix}23\\31\end{bmatrix}\\&+B_5(321)\,{}^{34}\begin{bmatrix}12\\23\end{bmatrix}+B_5(231)\,{}^{24}\begin{bmatrix}13\\32\end{bmatrix}+B_5(312)\,{}^{34}\begin{bmatrix}12\\31\end{bmatrix}+B_4(123)\,{}^{14}\begin{bmatrix}23\\11\end{bmatrix}\\&+B_4(213)\,{}^{24}\begin{bmatrix}13\\22\end{bmatrix}+B_4(321)\,{}^{34}\begin{bmatrix}12\\33\end{bmatrix}+B_6(123)\,{}^{14}\begin{bmatrix}23\\32\end{bmatrix}+B_6(213)\,{}^{24}\begin{bmatrix}13\\31\end{bmatrix}\\&+B_6(321)\,{}^{34}\begin{bmatrix}12\\21\end{bmatrix}+C_1(123)g^{\mu_1\mu_2}g^{\mu_3\mu_4}+C_1(132)g^{\mu_1\mu_3}g^{\mu_2\mu_4}+C_1(321)g^{\mu_2\mu_3}g^{\mu_1\mu_4} \ ,\end{aligned} \tag{15}$$

where

$$(2121)=k_2^{\mu_1}k_1^{\mu_2}k_2^{\mu_3}k_1^{\mu_4}\ ,$$

$$^{13}\begin{bmatrix}24\\12\end{bmatrix}=g^{\mu_1\mu_3}k_1^{\mu_2}k_2^{\mu_4},\quad \text{etc.}$$

and

$$A_{12}(123)\equiv A_1(123)-A_2(213)\ ,$$

$$A_{13}(123)\equiv A_1(123)-A_3(213)\ ,$$

$$A_{23}(123)\equiv A_2(123)-A_3(123)\ ,$$

$$B_{ij}(123)\equiv B_i(123)-B_j(123)\ ,$$

$$(i,j=1,2,3),\quad A_4'(123)\equiv A_4(123)-A_4(231)\ .$$

Note that

$$A_{12}(123)+A_{23}(213)=A_{13}(123)\ ,$$

$$B_{12}(123)+B_{23}(123)+B_{31}(123)=0\ ,$$

so there are only 9 independent A, and B, and C coefficients in Eq. (15). Now substitute the G' of Eq. (15) into Eq. (14), which already expands in terms of the base (12) with $j=1,3$, $l=1,2$, and $m=1,2$. Thus we obtain eleven equations:

$$\begin{aligned}
&[12]A_{13}(123)+[13]A_{12}(132)+B_{13}(123)\\
&\qquad\qquad+B_{12}(132)+B_4(123)=0\ ,\\
&[12]A_{23}(213)-[13]A_{23}(312)+B_{23}(123)-B_{23}(132)=0\ ,\\
&[12]A_{23}(123)+[13]A_4'(132)+B_{23}(213)+B_5(123)=0\ ,\\
&[12]A_{13}(123)-[23]A_4'(312)+B_{13}(123)=0\ ,\\
&B_5(132)-[13]A_{12}(312)-[12]A_4'(312)-B_{12}(312)=0\ ,\\
&B_6(123)-[12]A_{23}(321)-[13]A_{13}(321)=0\ ,\\
&[12]A_{12}(231)-[13]A_{12}(321)=0\ ,\\
&[12]B_{12}(231)-[13]B_{12}(321)=0\ ,\\
&[12]B_4(213)+[13]B_5(231)=0\ ,\\
&C_1(321)-[12]B_{23}(231)-[13]B_{13}(321)=0\ ,\\
&C_1(123)+[12]B_6(321)+[13]B_5(312)=0\ ,
\end{aligned}\tag{16}$$

where $[ij]\equiv k_i\cdot k_j$ $(i,j=1,2,3)$. From Eqs. (16) and (7), we obtain

$$\begin{aligned}
&B_{12}(123)=B_5(213)-[12]A_{12}(123)-[23]A_4'(123)\ ,\\
&B_{23}(123)=-B_5(213)-[12]A_{23}(213)-[23]A_4'(231)\ ,\\
&B_{13}(123)=[23]A_4'(312)-[12]A_{13}(123)\ ,\\
&B_4(123)=[23]\{A_4'(132)-A_4'(312)\}-B_5(312)\ ,\\
&B_6(123)=[12]A_{23}(321)+[13]A_{13}(321)\ ,\\
&C_1(123)=-[13]B_5(312)\\
&\qquad-[12]\{[23]A_{23}(123)+[13]A_{13}(123)\}
\end{aligned}\tag{17}$$

and

$$\begin{aligned}
&[13]A_{12}(123)-[23]A_{12}(213)=0\ ,\\
&[12]B_5(123)-[13]B_5(132)=0\ ,\\
&B_5(123)-B_5(321)=[13]\{A_4'(312)-A_4'(132)\}\ .
\end{aligned}\tag{18}$$

Thus we express all A, B, and C coefficients and so G' in terms of A_{12}, A_{23}, A_4', and B_5 which are further constrained by Eqs. (18).

Note that k_2, k_3, and k_4 may refer to photons or massive vector bosons and Eq. (14) still holds.

II. CALCULATION OF A AND B COEFFICIENTS

Although by using Eqs. (17) it is enough to calculate A_{12}, A_{23}, A_4', and B_5 in order to get G', still we have calculated all A and B coefficients in Eq. (6), which gives our freedom to use Eqs. (16) to check our calculation.

There are the following Feynman diagrams for $Z\to 3\gamma$ or $\gamma\gamma\to\gamma\gamma$ via the W loop (see Fig. 1). In Fig. 1 the internal lines (curly lines) are ghost lines, the dashed lines are $\phi^{\pm}$ lines, the wiggly lines are W lines. The four external lines are photons in $\gamma\gamma\to\gamma\gamma$, and three photons, one Z boson in $Z\to 3\gamma$, where we refer (μ_4,k_4) to Z. The calculation is carried out in the renormalization gauge with the parameter $\xi=1$. The 9 diagrams [(1)–(3) and (8)–(13)] are divergent. Their divergences are canceled among themselves. In the $\xi=1$ gauge, only the diagrams of (1), (2), and (3) contribute to A coefficients and only the diagrams of (1)–(9) contribute to B coefficients. But all diagrams contribute to C coefficients. In a unitary gauge only diagrams (3), (9), and (12) remain. The most complicated calculation comes from diagram (3).

First we calculated the tensor G in $Z\to 3\gamma$, which has S_3 symmetry and expands in Eq. (5). Through a complicated and tedious calculation we obtain all A and B coefficients. They can be expressed as

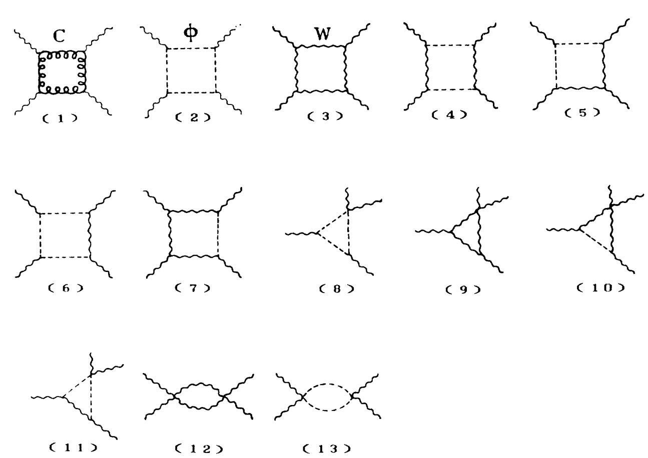

FIG. 1. Feynman diagrams of $Z\to 3\gamma$ and $\gamma\gamma\to\gamma\gamma$.

$$A_j^Z(123)=\sum_{s=1}^{3}\int DX\frac{2}{M_s^4}(\alpha_1 R_{js}^Z+\alpha_2 T_{js}^Z) \quad (j=1,2,3,4) ,$$

$$B_i^Z(123)=\sum_{s=1}^{3}\int DX\left[\frac{1}{M_s^2}(\alpha_1 X_{is}^Z-2\alpha_2 Y_{is}^Z)+\frac{2}{M_s^4}\sum_{m=1}^{3}d_m(\alpha_1 X_{ism}^Z-2\alpha_2 Y_{ism}^Z)\right] \quad (i=1,2,\ldots,6) \quad (19)$$

where $\alpha_1\equiv e^3 e_1/(4\pi)^2$, $\alpha_2\equiv e^3 e_2/(4\pi)^2$, and $e_1\equiv -g_2 C_W$, $e_2\equiv(g_1^2-g_2^2)/2(g_1^2+g_2^2)^{1/2}$, $e=g_1 g_2/(g_1^2+g_2^2)^{1/2}$, $C_W\equiv\cos\theta_W=g_2/(g_1^2+g_2^2)^{1/2}$; g_1 and g_2 are the coupling constants of U(1) and SU(2), respectively, in the standard SU(2)×U(1) model. $d_1\equiv[23]=k_2k_3$, $d_2\equiv[13]=k_1k_3$, $d_3\equiv[12]=k_1k_2$. The X, Y, R, and T coefficients are independent of the external momenta and only depend on the parameters of Feynman integrations. These coefficients are available upon request. In the $\xi=1$ gauge there is a factor

$$\frac{1}{(l^2-M_W^2)[(l-k_1)^2-M_W^2][(l-k_1-k_2)^2-M_W^2][(l-k_1-k_2-k_3)^2-M_W^2]} , \quad (20)$$

which comes from the four propagators of diagrams (1)–(7) and may be expressed in Feynman parameter integration by different choices of Feynman parameters:

$$\int DX\frac{6}{[(l-t_1)^2-M_1^2]^4} ,$$

where $t_1=ak_1+bk_2+ck_3$; by using the on-shell condition $k_1^2=k_2^2=k_3^2=0$,

$$M_1^2=M_W^2+2db[12]+2dc[13]+2(b-1)c[23] ,$$

$d\equiv a-1$, $\int DX=\frac{1}{6}$, M_W is the mass of the W boson. One of the choices of the integration parameters is

$$a=xz+1-z,\quad b=xz+(1-z)y,\quad c=xz ;$$

$$\int DX=\int_0^1 dx\int_0^1 dy\int_0^1 z(1-z)dz .$$

M_2^2 and M_3^2 come from M_1^2 through permutation of k_1,k_2 and k_2,k_3, respectively:

$$M_2^2=M_W^2+2db[12]+2(b-1)c[13]+2dc[23] ,$$

$$M_3^2=M_W^2+2dc[12]+2db[13]+2(b-1)c[23] .$$

In the calculation of B coefficients, there are only three propagators in the diagrams (8)–(10). But we still manage to get the factor of the expression (20) so that the final expressions of the A and B coefficients are simple.

It is easy to get the A and B coefficients of $\gamma\gamma\to\gamma\gamma$ from those of $Z\to 3\gamma$, just by setting $e_1=e_2=e$ or $\alpha_1=\alpha_2=\alpha^2=e^4/(4\pi)^2$, where α is the fine-structure constant of electromagnetic interaction. So A and B coefficients of $\gamma\gamma\to\gamma\gamma$ are

$$A_j^\gamma(123)=2\alpha^2\sum_{s=1}^{3}\int DX\frac{1}{M_s^4}R_{js}^\gamma \quad (j=1,2,3,4) ,$$

$$B_i^\gamma(123)=\alpha^2\sum_{s=1}^{3}\int DX\left[\frac{1}{M_s^2}X_{is}^\gamma+\frac{2}{M_s^4}\sum_{m=1}^{3}d_m X_{ism}^\gamma\right] \quad (i=1,2,\ldots,6) , \quad (21)$$

$$R_{js}^\gamma=R_{js}^Z+T_{js}^Z,\quad X_{is}^\gamma=X_{is}^Z-2Y_{is}^Z ,$$

$$X_{ism}^\gamma=X_{ism}^Z-2Y_{ism}^Z .$$

All R_{js}^γ, X_{is}^γ, and X_{ism}^γ are available upon request. It is enough for us to know the following A,B coefficients:

$$A_1^Z(123)=2\int DX\left[\frac{1}{M_1^4}[\alpha_1 d(32d^2b+16db+9b+5d-2)+8\alpha_2 d^2 b(2d+1)]+\frac{1}{M_2^4}[\alpha_1(32db^3-48db^2+26db+4b^2-5d-6b+2)+8\alpha_2 db(b-1)(2b-1)]+\frac{1}{M_3^4}[\alpha_1 d(32d^2c+16dc+9c+5d-2)+8\alpha_2 d^2 c(2d+1)]\right] ,$$

$$A_2^Z(123)=2\int DX\left[\frac{1}{M_1^4}[\alpha_1(32d^2b^2-32d^2b+16db^2-6db+4b^2-5d-6b+2)+8\alpha_2 db(2d+1)(b-1)]+\frac{1}{M_2^4}[\alpha_1 d(32db^2-16db+5d+9b-2)+8\alpha_2 d^2 b(2b-1)]+\frac{1}{M_3^4}[\alpha_1 c(32d^2c+16dc+5c+9d+2)+8\alpha_2 dc^2(2d+1)]\right] ,$$

$$A_3^Z(123)=2\int DX\left\{\frac{1}{M_1^4}[\alpha_1(32db^2c-16db^2-32dbc+4b^2-6db+32bc-6b-5d-32c+2)+8\alpha_2 db(b-1)(2c-1)]+\frac{1}{M_2^4}[\alpha_1 d(32dbc-16db+5d-23b+32c-2)+8\alpha_2 d^2 b(2c-1)]+\frac{1}{M_3^4}[\alpha_1 c(32dbc-16dc+5c-23d+32b-30)+8\alpha_2 dc^2(2b-1)]\right\},$$

$$A_4^Z(123)=2\int DX\left\{\frac{1}{M_1^4}[\alpha_1(32d^2bc+16dbc+4db-27dc+5bc+4d+2b-4c-4)+8\alpha_2 dbc(2d+1)]+\frac{1}{M_2^4}[\alpha_1(32db^2c-48dbc+21dc-28bc+5db+24c-d-2b-2)+8\alpha_2 dc(b-1)(2b-1)]+\frac{1}{M_3^4}[\alpha_1(32d^2bc-32d^2c+16dbc-27db-11dc+4bc-d-2b-8c-2)+8\alpha_2 dc(2d+1)(b-1)]\right\},$$

$$B_5^Z(123)=\int DX\left\{\frac{1}{M_1^2}[\alpha_1(38db-29d+24b-70)+4\alpha_2(db+d+4b-8)]+\frac{1}{M_2^2}[\alpha_1(44db-46d+31b+10)-16\alpha_2 d(2b+1)]+\frac{1}{M_3^2}[\alpha_1(38dc+23d+16c+38)+4\alpha_2 c(d+6)]\right.$$
$$+\frac{2}{M_1^4}[\alpha_1 d_1(-40db^2c-3db^2+69dbc+2db+26bc-6b^2-29dc-4b+d-10c+2)-4\alpha_2 d_1(-db^2c+4dbc+2b^2c-8bc-3dc+6c)+\alpha_1 d_2(-40d^2bc-3d^2b-16dbc+42d^2c+16db-12bc+9d^2+46dc+4b-4d+20c+4)-4\alpha_2 d_2 d(-dbc+4bc+dc-8c)+\alpha_1 d_3(-40d^2b^2+39d^2b-16db^2+62db+2d^2-12b^2-26d+24b-12)-4\alpha_2 d_3 db(-db+d+4b-8)]$$
$$+\frac{2}{M_2^4}[\alpha_1 d_1 d(-40dbc+3db-17bc+12dc+b-6d-7c+6)+16\alpha_2 d_1 d^2 c(b+1)+\alpha_1 d_2(-40db^2c+3db^2+46dbc-14b^2c+15db+9b^2-10dc+5b-2d+6c+2)+16\alpha_2 d_2 dc(b^2-1)+\alpha_1 d_3 d(-40db^2+18db-17b^2+18b-12d-6)+16\alpha_2 d_3 d^2 b(b+1)]$$
$$\left.+\frac{2}{M_3^4}[\alpha_1 d_1 c(-40dbc-13db+27dc+8bc+5d-22b-14c+10)-4\alpha_2 d_1 c^2(-db+3d+4b-4)+\alpha_1 d_2(-40d^2bc-3d^2c-8dbc-10d^2b+20dc-12bc+7d^2-26db+8c-8b+22d+16)+4\alpha_2 d_2 dbc(d-6)+\alpha_1 d_3 c(-40d^2c-13d^2-8dc-6d-12c)+4\alpha_2 d_3 dc^2(d-6)]\right\}. \tag{22}$$

By using A_j^Z, B_i^Z and A_j^γ, B_i^γ, one may check the Eqs. (16). We have not found the general way to do so. But in $\gamma\gamma\to\gamma\gamma$, when the external momenta k_i are small in comparison with M_W, one may check Eqs. (16) order by order by expanding $1/M_W^{2i}$ in a Taylor series. We have checked those equations in Eqs. (16), which contain only A and B coefficients, to orders of $1/M_W^2$, $1/M_W^4$, and $1/M_W^6$ for $\gamma\gamma\to\gamma\gamma$. The correct check gives credit both to the Eqs. (16) and to the calculation of A and B coefficients.

The A_j^γ and B_i^γ can be expanded in terms of $1/M_W^{2i}$:

$$\begin{aligned} A_j^\gamma(123) &= \sum_{n=0}^{\infty} A_j^{\gamma n}(123) , \\ B_i^\gamma(123) &= \sum_{n=0}^{\infty} B_i^{\gamma n}(123) , \end{aligned} \tag{23}$$

where

$$A_j^{\gamma 0} = \frac{2\alpha^2}{M_W^4} \int DX \sum_{s=1}^{3} R_{js}^\gamma ,$$

$$B_i^{\gamma 0} = \frac{\alpha^2}{M_W^2} \int DX \sum_{s=1}^{3} X_{is}^\gamma ,$$

and

$$B_i^{\gamma 1}(123) = \frac{2\alpha^2}{M_W^4} \int DX \sum_{s=1}^{3} \sum_{m=1}^{3} (X_{ism}^\gamma - t_{sm} X_{is}^\gamma) d_m ,$$

with

$$\frac{1}{M_s^2} = \frac{1}{M_W^2} \left[1 - \frac{2}{M_W^2} \sum_{m=1}^{3} t_{sm} d_m + \cdots \right] ,$$

$$M_s^2 = M_W^2 + 2 \sum_{m=1}^{3} d_m t_{sm} ,$$

$$t_{11} = t_{22} = t_{31} = (b-1)c , \quad t_{12} = t_{21} = t_{33} = dc ,$$

$$t_{13} = t_{23} = t_{32} = db .$$

Note that $A_j^{\gamma 0}$ and $B_i^{\gamma 0}$ are independent of k_i.

We can obtain

$$\begin{aligned} & A_1^{\gamma 0} = \frac{11}{60} \frac{2\alpha^2}{M_W^4}, \quad A_2^{\gamma 0} = \frac{11}{60} \frac{2\alpha^2}{M_W^4} , \\ & A_3^{\gamma 0} = -\frac{1}{60} \cdot \frac{2\alpha^2}{M_W^4}, \quad A_4^{\gamma 0} = -\frac{23}{20} \frac{2\alpha^2}{M_W^4} , \\ & B_1^{\gamma 0} = B_2^{\gamma 0} = B_3^{\gamma 0} = -\frac{2}{3} \frac{\alpha^2}{M_W^2} , \\ & B_4^{\gamma 0} = B_5^{\gamma 0} = B_6^{\gamma 0} = 0 , \\ & B_1^{\gamma 1}(123) = \frac{2\alpha^2}{M_W^4} \left(2d_1 - \tfrac{11}{60} d_2 - \tfrac{5}{12} d_3\right) , \\ & B_2^{\gamma 1}(123) = \frac{2\alpha^2}{M_W^4} \left(-2\tfrac{11}{30} d_1 + \tfrac{17}{20} d_2 + \tfrac{37}{60} d_3\right) , \\ & B_3^{\gamma 1}(123) = \frac{2\alpha^2}{M_W^4} \left(\tfrac{66}{60} d_1 - \tfrac{13}{12} d_2 - \tfrac{67}{60} d_3\right) , \\ & B_4^{\gamma 1}(123) = \frac{2\alpha^2}{M_W^4} \left(-5\tfrac{2}{15} d_1 + \tfrac{4}{15} d_2 + \tfrac{4}{15} d_3\right) , \\ & B_5^{\gamma 1}(123) = \frac{2\alpha^2}{M_W^4} \left(-\tfrac{31}{30} d_1 + 4\tfrac{11}{30} d_2 - \tfrac{31}{30} d_3\right) , \\ & B_6^{\gamma 1}(123) = \frac{2\alpha^2}{M_W^4} \left(\tfrac{2}{5} d_1 + \tfrac{3}{5} d_2 + \tfrac{3}{5} d_3\right) . \end{aligned} \tag{24}$$

To the first two orders $1/M_W^2$, $1/M_W^4$ of Eqs. (16), we obtain

$$\begin{aligned} & A_1^{\gamma 0} = A_2^{\gamma 0} , \\ & B_1^{\gamma 0} = B_2^{\gamma 0} = B_3^{\gamma 0}, \quad B_4^{\gamma 0} = B_5^{\gamma 0} = B_6^{\gamma 0} = 0 , \\ & B_6^{\gamma 1}(123) - d_2 A_{13}^{\gamma 0} - d_3 A_{23}^{\gamma 0} = 0 , \\ & d_3 A_{13}^{\gamma 0} + B_{13}^{\gamma 1}(123) = 0 , \\ & d_3 A_{23}^{\gamma 0} + B_{23}^{\gamma 1}(213) + B_5^{\gamma 1}(123) = 0 , \\ & d_3 A_{23}^{\gamma 0} - d_2 A_{23}^{\gamma 0} + B_{23}^{\gamma 1}(123) - B_{23}^{\gamma 1}(132) = 0 , \\ & d_3 A_{13}^{\gamma 0} + B_{13}^{\gamma 1}(123) + B_{12}^{\gamma 1}(132) + B_4^{\gamma 1}(123) = 0 , \\ & B_5^{\gamma 1}(132) - B_{12}^{\gamma 1}(312) = 0 . \end{aligned} \tag{25}$$

By using the numerical values of Eqs. (24), the Eqs. (25) are checked correctly. Note that in the $\gamma\gamma\to\gamma\gamma$ case, $d_1 + d_2 + d_3 = 0$. We also checked the Eqs. (16) to the order of $1/M_W^6$ in the $\gamma\gamma\to\gamma\gamma$ case. Therefore, all A and B coefficients are checked.

III. DISCUSSION

In our discussion above we have confined ourselves to the S_3 symmetry of the polarization tensor G. But we know the tensor may be written in S_4 symmetry form in $\gamma\gamma\to\gamma\gamma$ as the manifestation of the crossing symmetry [3]. This can also be done for $Z\to 3\gamma$ in the fermion-loop case. How about $Z\to 3\gamma$ in the W-loop case? In the unitary gauge only three diagrams (3), (9), and (12) of Fig. 1 contribute to the tensor G. It is easy to see the contributions from diagrams (3) and (12) of $Z\to 3\gamma$ can be written in S_4 symmetry form. As to the diagram (10), there are two terms corresponding to Z boson at triple vertex and quadruple vertex, respectively. For each term exists only S_3 symmetry. But the sum of them still can be written in S_4 symmetric form. So there is also a tensor G with S_4 symmetry in $Z\to 3\gamma$ for the W-loop case.

We already mentioned that the polarization tensor G with S_3 symmetry in $Z\to 3\gamma$ for the W-loop case does not satisfy Eq. (9), but the counterpart of fermion-loop case satisfies Eq. (9). Suppose the G in Eq. (6) satisfies Eq. (9), we can obtain equations as follows:

$$[12]A_1(123) + [13]A_1(132) + B_1(123) + B_1(132) + B_4(123) = 0 ,$$

$$[12]A_2(213) + [13]A_3(312) + B_2(123) + B_3(132) = 0 ,$$

$$[12]A_2(123) + [13]A_4(132) + B_5(123) + B_2(213) = 0 ,$$

$$[12]A_1(213) + [13]A_4(213) + B_1(213) = 0 ,$$

$$\begin{aligned}
&[12]A_3(123)+[13]A_4(321)+B_3(213)=0\ ,\\
&[12]A_3(321)+[13]A_3(231)+B_6(123)=0\ ,\\
&[12]A_1(231)+[13]A_2(231)=0\ ,\\
&[12]B_2(231)+[13]B_1(321)=0\ ,\\
&[12]B_3(231)+[13]B_3(321)+C_1(231)=0\ ,\\
&[12]B_6(312)+[13]B_5(312)+C_1(123)=0\ ,\\
&[12]B_4(231)+[13]B_5(231)=0\ .
\end{aligned} \tag{26}$$

It is not difficult to see that Eqs. (16) can be deduced from Eqs. (26), but one cannot get Eqs. (26) from Eqs. (16). In $\gamma\gamma \rightarrow \gamma\gamma$, at lowest order, we can obtain, from Eqs. (26), for example,

$$A_1^{\gamma 0}=A_2^{\gamma 0}=0,\quad B_1^{\gamma 0}=B_2^{\gamma 0}=0\ . \tag{27}$$

The numerical values of $A_j^{\gamma 0}$ and $B_j^{\gamma 0}$ in Eqs. (24) do not satisfy Eqs. (27), which in turn indicates that the G with S_3 symmetry in the W-loop case does not satisfy Eq. (9).

ACKNOWLEDGMENTS

We would like to thank Professor D.-S. Du and Professor F.-X. Dong for beneficial discussions and Dr. Boudjema for his communication concerning his contribution to the γ-γ scattering via the W loops [Phys. Lett. B **187**, 362 (1987)]. This work was supported in part by the National Natural Science Foundation of China, No. 19075040 and by the Grant LWTZ-1298 of Chinese Academy of Science.

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