Phenomenological model for 0^- mesons

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With the use of the spinor-spinor Bethe-Salpeter equation, a phenomenological vector-vector-type flat-bottom hadronic potential is assumed which is fully relativistic covariant. The 0^- meson wave functions in momentum space have been obtained. Then, the physical spacelike electromagnetic form factor of the 0^- meson is calculated directly from the Euclidean space 0^- meson wave functions. Numerical results show that the theoretical calculation gives a good fit to the experimental results by appropriately choosing the parameters in the phenomenological flat-bottom potential.

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I. INTRODUCTION

The quark hypothesis proposed by Gell-Mann and Zweig has led to a great deal of success in understanding the properties of hadrons. Most of these successes rely on nonrelativistic calculations, using simple assumptions such as the additivity of quark amplitudes, and simple approximations such as the instantaneous approximation method or null-plane approximation method. In the 1970s, Guth [1] studied the bound states of equal-mass quark-antiquark pairs, using the fully relativistic formalism of the Bethe-Salpeter equation with the phenomenological potential which incorporates Morpurgo's suggestion. He analyzed the symmetry of the bound states and obtained the numerical solutions of the bound states of quark-antiquark pairs, but the results were not as good as expected, because the radius of meson calculated by him was much smaller than the experimental result and the electromagnetic form factor of the meson was not calculated. Then, Wang *et al.* [2] assumed a phenomenological scalar-scalar-type flat-bottom potential which was fully relativistic covariant, and the difficulty in the calculation of the radius of meson was overcome by using the phenomenological potential; however, the result of electromagnetic form factor did not give a good fit to the experimental data. It was important that the scalar-scalar-type coupled phenomenological potential was used.

Recently, Gupta, Mitra, and Singh [3] stated that the interaction form of bound states of the quark-antiquark system should be a vector-vector-type structure $(\gamma_\mu^{(1)}\gamma_\mu^{(2)})$ and gave three advantages of it. The first advantage of the vector-vector form is that for a quark-antiquark system involving spin-$\frac{1}{2}$ constituents, the vector-vector form confinement, unlike a scalar-scalar form, simulates to a significant extent the operative aspects of gauge invariance that are usually sought to be incorporated through a standard phase integral structure involving a gluon field. The second advantage is that it satisfies chiral invariance. The third advantage is compatible with the possibility of a common origin for $q\bar{q}$ and qqq confinement while a scalar-scalar form is not. They also discussed the electromagnetic form factor of the pion, but their result only gave a good fit to the experimental result when k^2 was very small ($k^2 \leq 0.2$ GeV^2) [3]. They developed a kind of approximation method, the null-plane approximation method [3–8] which is similar to the instantaneous approximation method. In addition, many other problems can be discussed by using this approximation method.

In this paper, based on Wang's work and the discussion of Gupta *et al.*, first a vector-vector-type phenomenological flat-bottom potential is assumed which is fully relativistic covariant. The numerical solutions of the wave function for pseudoscalar bound states of equal-mass quark-antiquark pairs, the 0^- meson, are obtained, to the Bethe-Salpeter equation with the phenomenological flat-bottom potential. Then the physical spacelike electromagnetic form factor of the 0^- meson is calculated from the Euclidean wave functions. The calculated result fits well with the experimental one in a larger k^2 region without using the instantaneous approximation or null-plane approximation.

In Sec. II, first, the vector-vector coupled Bethe-Salpeter equation and the expansion method of Lorentz-invariant functions are introduced. It includes the calculation of the matrix elements H^{ij} which are given in Table I. Second, the vector-vector-type phenomenologi-

TABLE I. Values of $H^{ij}(p,k,Q)$.

$j \backslash i$	1	2	3
1	$4(p^2+Q^2+m^2)$	$8m$	0
2	$-4mQ^2$	$-2(Q^2-p^2+m^2)$	$-4(p\cdot Q)$
3	$-4m(k\cdot Q)$	$-4(k\cdot Q)-2D(m^2-p^2-Q^2)$ $\times[p^2(k\cdot Q)-(p\cdot Q)(k\cdot p)]$	$-4(k\cdot p)-2D(m^2-p^2-Q^2)$ $\times[Q^2(k\cdot p)-(p\cdot Q)(k\cdot Q)]$

TABLE II. Values of $K^{ij}_{nn'}$, where E_{ab} is

$$E_{ab}\equiv\int_0^{\pi}d\theta\frac{(\sin\theta)^a(\cos\theta)^b}{(m^2+p^2-Q^2)+4p^2Q^2\cos^2\theta}=\frac{1}{(m^2+p^2-Q^2)^2}\frac{\pi(-1)^{b/2}}{x^{(a+b)/2}}\left[(1+x)^{(a-1)/2}-\sum_{n=0}^{[(a+b)/2]-1}\frac{\Gamma[(a+1)/2]}{n!\Gamma\{[(a+1)/2]-n\}}x^n\right],$$

where $x\equiv 4p^2Q^2/[(m^2+p^2-Q^2)^2]$. (The expression holds only if a and b are even integrals, which includes all the necessary cases.)

i,n \ j,n'	1,0	1,2
1,0	$\frac{k^3}{\pi^3}(p^2+Q^2+m^2)L_0E_{20}$	$\frac{k^3}{\pi^3}(p^2+Q^2+m^2)\frac{1}{3}L_2(4E_{22}-E_{20})$
1,2	$\frac{k^3}{\pi^3}(p^2+Q^2+m^2)L_0(4E_{22}-E_{20})$	$\frac{k^3}{\pi^3}(p^2+Q^2+m^2)\frac{1}{3}L_2(16E_{24}-8E_{22}+E_{20})$
1,4	$\frac{k^3}{\pi^3}(p^2+Q^2+m^2)$ $\times L_0(16E_{24}-12E_{22}+E_{20})$	$\frac{k^3}{\pi^3}(p^2+Q^2+m^2)$ $\times\frac{1}{3}L_2(64E_{26}-64E_{24}+16E_{22}-E_{20})$
2,0	$\frac{2k^3}{\pi^3}mL_0E_{20}$	$\frac{2k^3}{\pi^3}m\frac{1}{3}L_2(4E_{22}-E_{20})$
2,2	$\frac{2k^3}{\pi^3}mL_0(4E_{22}-E_{20})$	$\frac{2k^3}{\pi^3}m\frac{1}{3}L_2(16E_{24}-8E_{22}+E_{20})$
2,4	$\frac{2k^3}{\pi^3}mL_0(16E_{24}-12E_{22}+E_{20})$	$\frac{2k^3}{\pi^3}m\frac{1}{3}L_2(64E_{26}-64E_{24}+16E_{22}-E_{20})$
3,1	0	0
3,3	0	0

i,n \ j,n'	1,4	2,0
1,0	$\frac{k^3}{\pi^3}(p^2+Q^2+m^2)$ $\times\frac{1}{5}L_4(16E_{24}-12E_{22}+E_{20})$	$\frac{-k^3}{\pi^3}mQ^2L_0E_{20}$
1,2	$\frac{k^3}{\pi^3}(p^2+Q^2+m^2)$ $\times\frac{1}{5}L_4(64E_{26}-64E_{24}+16E_{22}-E_{20})$	$-\frac{k^3}{\pi^3}mQ^2L_0(4E_{22}-E_{20})$
1,4	$\frac{k^3}{\pi^3}(p^2+Q^2+m^2)\frac{1}{5}L_4$ $\times(256E_{28}-384E_{26}+176E_{24}-24E_{22}+E_{20})$	$-\frac{k^3}{\pi^3}mQ^2L_0(16E_{24}-12E_{22}+E_{20})$
2,0	$\frac{2k^3}{\pi^3}m\frac{1}{5}L_4(16E_{24}-12E_{22}+E_{20})$	$-\frac{k^3}{2\pi^3}(Q^2-p^2+m^2)L_0E_{20}$
2,2	$\frac{2k^3}{\pi^3}m\frac{1}{5}L_4(64E_{26}-64E_{24}+16E_{23}-E_{20})$	$-\frac{k^3}{2\pi^3}(Q^2-p^2+m^2)L_0(4E_{22}-E_{20})$
2,4	$\frac{2k^3}{\pi^3}m\frac{1}{5}L_4$ $\times(256E_{28}-384E_{26}+176E_{24}-24E_{22}+E_{20})$	$\frac{-k^3}{2\pi^3}(Q^2-p^2+m^2)L_0$ $\times(16E_{24}-12E_{22}+E_{20})$

TABLE II. (*Continued*).

i,n \ j,n'	1,4	2,0
3,1	0	$\frac{-2k^3}{\pi^3}pQL_0E_{22}$
3,3	0	$\frac{-4k^3}{\pi^3}pQL_0(2E_{24}-E_{22})$

i,n \ j,n'	2,2	2,4
1,0	$\frac{-k^3}{\pi^3}mQ^2\frac{1}{3}L_2(4E_{22}-E_{20})$	$-\frac{k^3}{\pi^3}mQ^2\frac{1}{5}L_4(16E_{24}-12E_{22}+E_{20})$
1,2	$-\frac{k^3}{\pi^3}mQ^2\frac{1}{3}L_2(16E_{24}-8E_{22}+E_{20})$	$-\frac{k^3}{\pi^3}mQ^2\frac{1}{5}L_4(64E_{26}-64E_{24}+16E_{22}+E_{20})$
1,4	$-\frac{k^3}{\pi^3}mQ^2\frac{1}{3}L_2$ $\times(64E_{26}-64E_{24}+16E_{22}-E_{20})$	$-\frac{k^3}{\pi^3}mQ^2\frac{1}{5}L_4$ $\times(256E_{28}-386E_{26}+176E_{24}-24E_{22}+E_{20})$
2,0	$-\frac{k^3}{2\pi^3}(Q^2-p^2+m^2)$ $\times\frac{1}{3}L_2(4E_{22}-E_{20})$	$-\frac{k^3}{2\pi^3}(Q^2-p^2+m^2)$ $\times\frac{1}{5}L_4(16E_{24}-12E_{22}+E_{20})$
2,2	$-\frac{k^3}{2\pi^3}(Q^2-p^2+m^2)$ $\times\frac{1}{3}L_2(16E_{24}-8E_{22}+E_{20})$	$-\frac{k^3}{2\pi^3}(Q^2-p^2+m^2)$ $\times\frac{1}{5}L_4(64E_{26}-64E_{24}+16E_{22}-E_{20})$
2,4	$-\frac{k^3}{2\pi^3}(Q^2-p^2+m^2)\frac{1}{3}L_2$ $\times(64E_{26}-64E_{24}+16E_{22}-E_{20})$	$-\frac{k^3}{2\pi^3}(Q^2-p^2+m^2)\frac{1}{5}L_4$ $\times(256E_{28}-384E_{26}+176E_{24}-24E_{22}+E_{20})$
3,1	$-\frac{2k^3}{3\pi^3}pQL_2(4E_{24}-E_{22})$	$-\frac{2k^3}{5\pi^3}pQL_4(16E_{26}-12E_{24}+E_{22})$
3,3	$-\frac{4k^3}{3\pi^3}pQL_2(8E_{26}-6E_{24}+E_{22})$	$-\frac{4k^3}{5\pi^3}pQL_4(32E_{28}-40E_{26}+14E_{24}-E_{22})$

i,n \ j,n'	3,1	3,3
1,0	$\frac{-k^4}{2\pi^3}mQ[L_0E_{20}+\frac{1}{3}L_2(4E_{22}-E_{20})]$	$-\frac{k^4}{2\pi^3}mQ\left[\frac{1}{3}L_2(4E_{22}-E_{20})+\frac{1}{5}L_4(16E_{24}-12E_{22}+E_{20})\right]$
1,2	$-\frac{k^4}{2\pi^3}mQ\left[L_0(4E_{22}-E_{20})+\frac{1}{3}L_2(16E_{24}-8E_{22}+E_{20})\right]$	$-\frac{k^4}{2\pi^3}mQ\left[\frac{1}{3}L_2(16E_{24}-8E_{22}+E_{20})+\frac{1}{5}L_4(64E_{26}-64E_{24}+16E_{22}-E_{20})\right]$
1,4	$-\frac{k^4}{2\pi^3}mQ\left[L_0(16E_{24}-12E_{22}+E_{20})+\frac{1}{3}L_2(64E_{26}-64E_{24}+16E_{22}-E_{20})\right]$	$-\frac{k^4}{2\pi^3}mQ\left[\frac{1}{3}L_2(64E_{26}-64E_{24}+16E_{22}-E_{20})+\frac{1}{5}L_4(256E_{28}-384E_{26}+176E_{24}-24E_{22}+E_{20})\right]$

TABLE II. (*Continued*).

i,n \ j,n'	3,1	3,3
2,0	$-\frac{k^4}{4\pi^3}\left[\frac{m^2-p^2+Q^2}{Q}\left[L_0-\frac{1}{3}L_2\right]E_{20}+\frac{8}{3}L_2QE_{22}\right]$	$-\frac{k^4}{4\pi^3}\left[\frac{m^2-p^2+Q^2}{Q}\left[\frac{1}{3}L_2-\frac{1}{5}L_4\right](6E_{22}-E_{20})-\frac{4}{3}L_2QE_{22}+\frac{4}{5}L_4Q(8E_{24}-3E_{22})\right]$
2,2	$-\frac{k^4}{4\pi^3}\left[\frac{m^2-p^2+Q^2}{Q}\left[L_0-\frac{1}{3}L_2\right]\times(4E_{22}-E_{20})+\frac{8}{3}L_2Q(4E_{24}-E_{22})\right]$	$-\frac{k^4}{4\pi^3}\left[\frac{m^2-p^2+Q^2}{Q}\left[\frac{1}{3}L_2-\frac{1}{5}L_4\right]\times(24E_{24}-10E_{22}+E_{20})-4Q\left[\frac{1}{3}L_2+\frac{1}{5}L_4\right](4E_{24}-E_{22})+\frac{8}{5}QL_4(16E_{26}-8E_{24}+E_{22})\right]$
2,4	$-\frac{k^4}{4\pi^3}\left[\frac{m^2-p^2+Q^2}{Q}\left[L_0-\frac{1}{3}L_2\right]\times(16E_{24}-12E_{22}+E_{20})+\frac{8}{3}L_2Q(16E_{26}-12E_{24}+E_{22})\right]$	$-\frac{k^4}{4\pi^3}\left[\frac{m^2-p^2+Q^2}{Q}\left[\frac{1}{3}L_2-\frac{1}{5}L_4\right]\times(96E_{26}-88E_{24}+18E_{22}-E_{20})-4Q\left[\frac{1}{3}L_2+\frac{1}{5}L_4\right](16E_{26}-12E_{24}+E_{22})+\frac{8}{5}L_4Q(64E_{28}-64E_{26}+16E_{24}-E_{22})\right]$
3,1	$-\frac{2k^4}{\pi^3}E_{22}\left[\frac{m^2+p^2-Q^2}{3p}L_2+\frac{1}{2}p\left[L_0-\frac{1}{3}L_2\right]\right]$	$-\frac{k^4}{\pi^3}\left\{p(L_2+L_4)(2E_{24}-E_{22})+\frac{m^2-p^2-Q^2}{p}\left[\frac{1}{5}L_4(8E_{24}-3E_{22})-\frac{1}{3}L_2E_{22}\right]\right\}$
3,3	$-\frac{4k^4}{\pi^3}\left[\frac{m^2-p^2-Q^2}{3p}L_2+\frac{1}{2}p(L_2+L_4)\right](2E_{24}-E_{22})$	$-\frac{k^4}{\pi^3}\left\{2p(L_2+L_4)(4E_{26}-4E_{24}+E_{22})+\frac{m^2-p^2-Q^2}{p}\left[\frac{2}{5}L_4(16E_{26}-12E_{24}+2E_{22}+E_{20})-\left[\frac{1}{3}L_2+\frac{1}{5}L_4\right](4E_{24}-2E_{22}+3E_{20})\right]\right\}$

cal flat-bottom potential and the condition of a flat bottom are assumed. Then the method of the expansion of O(4) eigenfunctions is introduced. It includes the calculation of kernel functions K_{nn}^{ij}, which are given in Table II. Finally, the 0^- meson wave functions are obtained by numerical calculation and are shown in Fig. 1. In Sec. III, first the matrix element of electromagnetic current in Euclidean space is reduced. Then the formula for the physical spacelike electromagnetic form factor of the 0^- meson is obtained. Finally, the result of the numerical calculation of the form factor is given. It conforms with the experimental one. The conclusion and discussion are given in the Sec. IV.

II. BETHE-SALPETER EQUATION OF VECTOR-VECTOR COUPLED AND THE PHENOMENOLOGICAL FLAT-BOTTOM POTENTIAL

A. Bethe-Salpeter equation

If the interaction between a quark and antiquark is a vector-vector-type structure ($\gamma_\mu^{(1)}\gamma_\mu^{(2)}$) and the mass of the quark and antiquark is same, the wave functions of bound states of the quark-antiquark pairs should be the solutions to Bethe-Salpeter equation which takes the form

$$X(p,Q)=-\frac{i\gamma(p+Q)-m}{(p+Q)^2+m^2}\int d^4k\,U(p-k)\gamma_\mu\delta_{\mu\nu}X(k,Q)\times\gamma_\nu\frac{i\gamma(p-Q)-m}{(p-q)^2+m^2}\,, \quad (1)$$

where $X(p,Q)$ is the Bethe-Salpeter wave function of the bound state; $2Q_\mu=(0,0,0,iM)$ is the four-momentum of the center of mass of the pion, 0^- meson; M is the mass of pion; p_μ is the relative four-momentum between the quark and antiquark in the pion; m is the mass of the quark; U is the phenomenological interaction potential; and γ_μ are Dirac matrices.

Now, consider the Bethe-Salpeter equation in the rest frame of the bound state; one can perform the Wick rotation [9] analytically continuing k and p into the Euclidean region where we will denote them by $\bar{k}$ and $\bar{p}$, respectively, and define

$$2\bar{Q}_\mu\equiv-2iQ_\mu\equiv(0,0,0,M)\;;$$

then the Bethe-Salpeter equation can be written as

$$X(\bar{p},\bar{Q})=-iF(\bar{p},\bar{Q})\int d^4\bar{k}\,S_-(\bar{p},\bar{Q})U(\bar{p}-\bar{k})\gamma_\mu\delta_{\mu\nu}\times X(\bar{k},\bar{Q})\gamma_\nu S_+(\bar{p},\bar{Q})\,, \quad (2)$$

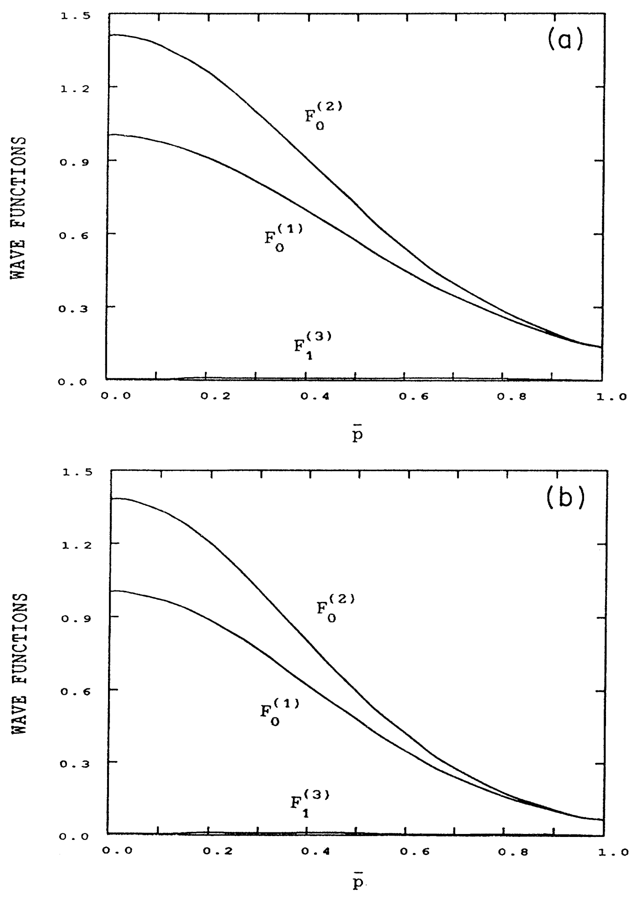

FIG. 1. Wave functions of bound states of the 0^- meson, with $B=0.07$, $\rho=2m$, and $N=0.4m$ in (a) (where m is the mass of the quark) and with $B=0.065$, $\rho=2m$, and $N=0.3m$ in (b).

where

$$S_\pm(\bar{p},\bar{Q})=\gamma\cdot(i\bar{p}\pm\bar{Q})-m\,,\qquad F(\bar{p},\bar{Q})=[(m^2+\bar{p}^2-\bar{Q}^2)^2+4(\bar{p}\cdot\bar{Q})^2]^{-1}\,, \quad (3)$$

and then the Bethe-Salpeter wave function $X(\bar{p},\bar{Q})$ of the bound state can be expanded as Lorentz-invariant functions [1]:

$$X(\bar{p},\bar{Q})=\sum_{i=1}^{4}X^{(i)}(\bar{p}^2,\bar{p}\cdot\bar{Q})M^{(i)}(\bar{p},\bar{Q})\,, \quad (4)$$

where

$$M^{(1)}=\gamma_5,\quad M^{(2)}=\bar{Q}\cdot\gamma\gamma_5\,,\qquad M^{(3)}=\bar{p}\cdot\gamma\gamma_5,\quad M^{(4)}=\epsilon_{\mu\nu\rho\lambda}\bar{Q}_\mu\bar{p}_\nu\sigma_{\rho\lambda}\,. \quad (5)$$

We then define the matrices $\tilde{M}^{(i)}(\bar{p},\bar{Q})$:

$$\tilde{M}^{(1)}=\tfrac{1}{4}\gamma_5,\quad \tilde{M}^{(2)}=\tfrac{1}{4}D[(\bar{p}\cdot\bar{Q})\bar{p}\cdot\gamma-\bar{p}^2\bar{Q}\cdot\gamma]\gamma_5\,,$$
$$\tilde{M}^{(3)}=\tfrac{1}{4}D[(\bar{p}\cdot\bar{Q})\bar{Q}\cdot\gamma-\bar{Q}^2\bar{p}\cdot\gamma]\gamma_5\,, \quad (6)$$
$$\tilde{M}^{(4)}=\tfrac{1}{16}D\epsilon_{\mu\nu\rho\lambda}\bar{Q}_\mu\bar{p}_\nu\sigma_{\rho\lambda}\,,$$

where

$$D=[\bar{p}^2\bar{Q}^2-(\bar{p}\cdot\bar{Q})^2]^{-1} \quad (7)$$

and the orthonormal relationships between $M^{(i)}$ and $\tilde{M}^{(j)}$ are

$$\mathrm{Tr}[\tilde{M}^{(i)}(\bar{p},\bar{Q})M^{(j)}(\bar{p},\bar{Q})]=\delta_{ij}\,. \quad (8)$$

When applying Eqs. (4) and (8) to the Bethe-Salpeter equation (2), the Bethe-Salpeter equation for Lorentz-invariant wave functions $X^{(i)}(\bar{p}^2,\bar{p}\cdot\bar{Q})$ is

$$X^{(i)}(\bar{p}^2,\bar{p}\cdot\bar{Q})=iF(\bar{p},\bar{Q})\sum_{j=1}^{3}\int d^4\bar{k}\,U(\bar{p}-\bar{k})H^{ij}(\bar{p},\bar{k},\bar{Q})\times X^{(j)}(\bar{k}^2,\bar{k}\cdot\bar{Q})\,, \quad (9)$$

where

$$H^{ij}(\bar{p},\bar{k},\bar{Q})=-\mathrm{Tr}[\tilde{M}^{(i)}(\bar{p},\bar{Q})S_-(\bar{p},\bar{Q})\gamma_\mu\delta_{\mu\nu}M^{(j)}(\bar{k},\bar{Q})\times\gamma_\nu S_+(\bar{p},\bar{Q})]\,. \quad (10)$$

Based on Ref. [10], we know that $X^{(4)}(\bar{p}^2,\bar{p}\cdot\bar{Q})$ does not couple with other components of the wave functions if the interaction is a vector-vector-type structure. So we can assume that $X^{(4)}(\bar{p}^2,\bar{p}\cdot\bar{Q})=0$. Feldman and coworkers have discussed the problem in detail in their work [10]. Of course, we are able to obtain the same conclusion from Eq. (10).

B. Phenomenological potential

Now we discuss the phenomenological potential in the Bethe-Salpeter equation. Considering that the Yukawa potential has the singular point when $r=0$, and that a quark seems to be free in the pion, and that the phenomenological potential should satisfy gauge invariance, chiral invariance, and fully relativistic covariance such as Gupta and co-workers have stated in Ref. [3], we assume a phenomenological vector-vector-type flat-bottom potential which is the sum of a set of Yukawa potentials. This phenomenological potential cannot only satisfy three conditions but also suppress the singular point which the Yukawa potential has. Actually the phenomenological flat-bottom potential is analogous to the exchange of a series of particles with different mass. And we assume that there is a relation among the exchange particle masses. The phenomenological vector-vector-type flat-bottom potential can be written as

$$U(p)=-\frac{iG^2}{(2\pi)^4}\sum_{j=0}^{n}\frac{a_j}{p^2+(N+j\rho)^2}\ , \tag{11}$$

where N is the minimum value of the mass of the exchange particles, ρ is the difference of the mass of the exchange particles, and a_j is the relative coupled constant of different particles; its value decides the relative strength of each exchange particle; its sign denotes the property of this term. The three-dimensional form of the phenomenological potential corresponding to the four-dimensional form of the phenomenological flat-bottom potential can be written as [2]

$$V(r)=-G^2\sum_{j=0}^{n}a_j\frac{e^{-(N+j\rho)r}}{r}\ . \tag{12}$$

Because the phenomenological potential has a flat bottom in its three-dimensional form and the singularity at the point $r=0$ has been suppressed we assume the phenomenological potentials satisfy the following conditions which are called the flat-bottom conditions:

$$\begin{gathered} V(0)=\text{const}\ ,\\ \frac{dV(0)}{dr}=\frac{d^2V(0)}{d^3r}=\cdots=\frac{d^{n-1}V(0)}{d^{n-1}r}=0 \end{gathered} \tag{13}$$

so $n+1$ relative coupled constants can be determined by the equations

$$\begin{gathered} \sum_{i=0}^{n}a_i=0\ ,\\ \sum_{i=0}^{n}a_i(N+i\rho)=\frac{V(0)}{G^2}\ ,\\ \sum_{i=0}^{n}a_j(N+i\rho)^2=0\ ,\\ \cdots\\ \sum_{i=0}^{n}a_i(N+i\rho)^n=0\ . \end{gathered} \tag{14}$$

From above, we know that if we choose the value of n [$n=9$ and $V(0)/G^2N=0.5$ in our work], there are only two variable parameters in the phenomenological flat-bottom potential.

C. Expansion of O(4) eigenfunctions

In order to obtain the numerical solutions of the wave functions from the integral equation (9), the Lorentz-invariant wave functions $X^{(i)}(\bar{p}^2,\bar{p}\cdot\bar{Q})$ are expanded in O(4) eigenfunctions or Gegenbauer functions. Considering the invariance of time reversal, we can write $X^{(i)}(\bar{p}^2,\bar{p}\cdot\bar{Q})$ as

$$X^{(i)}(\bar{p}^2,\bar{p}\cdot\bar{Q})=\sum_{n=0}^{\infty}X_n^{(i)}(\bar{p},\bar{Q})C_n^1(\cos\theta)$$

$$n=\begin{cases}\text{even} & \text{if } i=1,2\ ,\\ \text{odd} & \text{if } i=3\ ,\end{cases} \tag{15}$$

where the expression of Gegenbauer function is

$$C_n^1(\cos\theta)=\frac{\sin(n+1)\theta}{\sin\theta}\ , \tag{16}$$

where θ is the angle between $\bar{p}$ and $\bar{Q}$.

As above, the four-dimensional phenomenological flat-bottom potential can also be expanded in O(4) eigenfunctions or Gegenbauer functions:

$$U(\bar{p}-\bar{k})=-\frac{iG^2}{(2\pi)^4}\sum_{n=0}^{\infty}L_n(\bar{p},\bar{k})C_n^1(\cos\delta)\ , \tag{17}$$

$$L_n(\bar{p},\bar{k})=\sum_{i=0}^{n-1}\frac{1}{pk}(Z_i-\sqrt{Z_i^2-1})^{i+1}\ , \tag{18}$$

$$Z_i=\frac{1}{2pk}[\bar{p}^2+\bar{k}^2+(N+i\rho)^2]\ , \tag{19}$$

where δ is the angle between vectors $\bar{p}$ and $\bar{k}$.

As we know, Gegenbauer functions satisfy the general orthogonality relation

$$\frac{2}{\pi}\int_0^{\pi}\sin^2\theta C_n^1(\cos\theta)C_m^1(\cos\theta)d\theta=\delta_{nm}\ . \tag{20}$$

From above, the Bethe-Salpeter equation can be reduced to the following infinite set of one-dimensional coupled integral equations:

$$X_n^{(i)}(\bar{p},\bar{Q})=\sum_{j=1}^{3}\sum_{n'=0}^{\infty}G^2\int d\bar{k}\,K_{nn'}^{ij}X_{n'}^{(j)}(\bar{k},\bar{Q})\ ,$$

$$n,n'=\begin{cases}\text{even} & \text{if } i=1,2\ ,\\ \text{odd} & \text{if } i=3\ ,\end{cases} \tag{21}$$

$$\begin{aligned} K_{nn'}^{ij}=\frac{2}{\pi}\frac{\bar{k}^3}{(2\pi)^4}\sum_{m=0}^{\infty}&\int_0^{\pi}\sin^2\theta_p C_n^1(\cos\theta_p)F(\bar{p},\bar{Q})d\theta_p\\ &\times\int d^4\Omega_k L_m(\bar{p},\bar{k})C_m^1(\cos\delta)H^{ij}\\ &\times C_{n'}^1(\cos\theta_k)\ , \end{aligned} \tag{22}$$

where $d\Omega_k$ is the four-dimensional angle of Euclidean vector $\bar{k}$,

$$d\Omega=\sin^2\theta\sin\psi\,d\theta\,d\psi\,d\varphi$$

and H^{ij} and $K_{nn'}^{ij}$ are given in Tables I and II.

D. Numerical solutions of Bethe-Salpeter equation

Before we solve the integral equations (21), it is important to change the vectors $\bar{p}$, $\bar{k}$, and $\bar{Q}$ into dimensionless quantities by the change

$$\bar{p}\to\frac{\bar{p}}{m},\quad \bar{k}\to\frac{\bar{k}}{m},\quad B\to\frac{\bar{Q}}{m} \tag{23}$$

and to make the wave functions $X^{(i)}(\bar{p},\bar{Q})$ with the same dimension by the change

$$F_n^{(1)}=X_n^{(1)},\quad F_n^{(2)}=mX_n^{(2)},\quad F_n^{(3)}=mX_n^{(3)} . \qquad (24)$$

Actually, it is impossible to solve the infinite set of one-dimensional coupled integral equations (21); therefore the first step in solving Eqs. (21) is to truncate the infinite sum over n'. The computer calculations have been carried out using only the lowest nonvanishing term ($n=0$ or $n=1$) in the expansion of each Lorentz-invariant function. With this truncation, it is possible to calculate many properties of the bound states with accuracy. That our theoretical calculation result gives a good fit to the experimental result is a good example. In the lowest-order approximation, Bethe-Salpeter wave function can be written as

$$X=F_0^{(1)}\gamma_5+F_0^{(2)}B\gamma_4\gamma_5+F_1^{(3)}\gamma\cdot p\gamma_5 C_1^1(\cos\theta) . \qquad (25)$$

In the lowest-order approximation, Eq. (21) can be written as

$$\begin{aligned} F_0^{(1)}&=G^2\int_0^\infty d\bar{k}(K_{00}^{11}F_0^{(1)}+K_{00}^{12}F_0^{(2)}+K_{01}^{13}F_1^{(3)}) ,\\ F_0^{(2)}&=G^2\int_0^\infty d\bar{k}(K_{00}^{21}F_0^{(1)}+K_{00}^{22}F_0^{(2)}+K_{01}^{23}F_1^{(3)}) ,\\ F_1^{(3)}&=G^2\int_0^\infty d\bar{k}(K_{10}^{31}F_0^{(1)}+K_{10}^{32}F_0^{(2)}+K_{11}^{33}F_1^{(3)}) . \end{aligned} \qquad (26)$$

The numerical solutions of the wave functions, $F_0^{(1)}$, $F_0^{(2)}$, and $F_1^{(3)}$ are obtained by using a Vax 8700 computer. The numerical solutions of the wave functions of the bound state, $F_0^{(1)}$, $F_0^{(2)}$, and $F_1^{(3)}$ are shown in Fig. 1.

III. ELECTROMAGNETIC FORM FACTOR

A. Matrix element of electromagnetic current

Recently, many experiments [12] of the pion electromagnetic form factor have been done, which not only give information on the distribution of the charge of the hadron but also on the radius of the hadron. When one discusses the pion phenomenological model with a vector-vector-type interaction, it is an effective test to see whether the model can obtain an electromagnetic form factor which gives a good fit to the experimental result. For this reason, we calculate the electromagnetic form factor of the pion with our model.

As we know, the relationship between the electromagnetic form factor of the bound states of a quark-antiquark system $F(k^2)$ and the matrix element of the electromagnetic current between two bound states is

$$\begin{aligned} F(k^2)(P_i+P_f)_\mu&=\langle P_f|J_\mu(0)|P_i\rangle ,\\ 2k&=P_i-P_f , \end{aligned} \qquad (27)$$

where J_μ is the operator of the electromagnetic current, $|p_i\rangle$ and $|p_f\rangle$ are two bound states, and

$$F(k^2)=-\frac{1}{2(k^2+M^2)}P_{i\mu}\langle P_f|J_\mu(0)|P_i\rangle . \qquad (28)$$

We know, in the lowest order, the matrix element of electromagnetic current between two bound states is given by [11]

$$\begin{aligned} \langle P_f|J_\mu(z)|P_i\rangle &=\int d^4u\,\mathrm{Tr}\left[\bar{\phi}^f(u,z)\Delta_\mu\phi^i(z,u)\left[\frac{\hat{\partial}}{\partial u}+m\right]\right.\\ &\qquad\left.+\bar{\phi}^f(z,u)\left[\frac{\hat{\partial}}{\partial u}+m\right]\phi^i(u,z)\Delta_\mu\right] , \end{aligned} \qquad (29)$$

where

$$\begin{aligned} J_\mu(z)&=i\bar{\psi}(z)\Delta_\mu\psi(z) ,\\ \Delta_\mu&=e\gamma_\mu ,\\ \phi^f(z,u)&=\langle 0|T\psi(z)\bar{\psi}(u)|P_f\rangle ,\\ \bar{\phi}^f(u,z)&=\langle P_f|T\bar{\psi}(u)\psi(z)|0\rangle . \end{aligned} \qquad (30)$$

$\psi(z)$ is the operator of the quark field and e is the charge matrix element of the SU(3) group of the quark. Based on Ref. [2], Eq. (28) can be written as

$$\begin{aligned} \langle P_f|J_\mu(0)|P_i\rangle &=\frac{e}{(2\pi)^4}\int d^4q\,\mathrm{Tr}\left\{\bar{X}\left[q-k,\frac{1}{2}P_f\right]\gamma_\mu X\left[q,\frac{1}{2}P_i\right]\right.\\ &\qquad\left.\times\left[i\left[q-\frac{1}{2}P_i\right]\gamma+m\right]\right\} . \end{aligned} \qquad (31)$$

The Feynman figures corresponding to the matrix element of the electromagnetic current in the lowest order are shown in Fig. 2.

The physical quantities of Eq. (31) are in Minkowski space. Before we calculate the pion electromagnetic form factor by using the Bethe-Salpeter bound-state wave functions which have been obtained in the Euclidean region, we must perform the Wick rotation analytically continuing k and q into the Euclidean region so that the bound-state wave functions which have been obtained in the Euclidean region can be used directly.

Under the spacelike condition $k^2>0$, because of Lorentz invariance, it is convenient to consider the following choice of kinematic variables:

$$\begin{aligned} k&=(k,0,0,0) ,\\ P_i&=(k,0,0,i\sqrt{k^2+M^2}) ,\\ P_f&=(-k,0,0,i\sqrt{k^2+M^2}) . \end{aligned} \qquad (32)$$

As a result, Eq. (28) can be written as

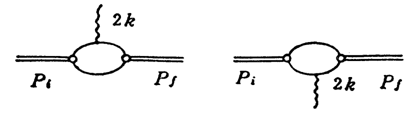

FIG. 2. The Feynman figure of the matrix element of the electromagnetic current to lowest order.

$$F(k^2)=-\frac{1}{2(k^2+M^2)}\left[k\langle P_f|J_1(0)|P_i\rangle\right.$$
$$\left.+i\sqrt{k^2+M^2}\langle P_f|J_4(0)|P_i\rangle\right]\,. \tag{33}$$

$\bar X$ can be related to X by analytic continuation in p_0 [11]:

$$\bar X(p,Q)=\gamma_4 X^{\dagger}(\mathbf{p},p_0^*,Q)\gamma_4\,. \tag{34}$$

When applying Eq. (4) to Eq. (34), it can be written as

$$\bar X(p,Q)=-X^{(1)}(p^2,p\cdot Q)\gamma_5+iX^{(2)}(p^2,p\cdot Q)\gamma_5\gamma\cdot Q$$
$$+X^{(3)}(p^2,p\cdot Q)\gamma_5(\boldsymbol{\gamma}\cdot\mathbf{p}-\gamma_4 p_4)\,. \tag{35}$$

When we apply Eq. (35) to Eq. (31) and perform the Wick rotation analytically continuing q into the Euclidean region, we obtain the matrix element of the electromagnetic current:

$$\begin{aligned}
\langle P_f|J_\mu(0)|P_i\rangle=&\frac{4ie}{(2\pi)^4}\int d^4\bar q\,\mathrm{Tr}\left\{\bar X\left[\bar q-\bar k,\frac{1}{2}\bar P_f\right]\gamma_\mu X\left[\bar q,\frac{1}{2}\bar P_i\right]\left[i\left[\bar q-\frac{1}{2}P_i\right]\gamma+m\right]\right\}\\
=&\frac{4ie}{(2\pi)^4}\int d^4\bar q\Bigg\{\bar X^{(1)}X^{(1)}\left[i\bar q_\mu+\frac{1}{2}\bar P_{i\mu}\right]-\frac{1}{2}\bar X^{(1)}X^{(2)}m\bar P_{i\mu}-\frac{1}{2}\bar X^{(2)}X^{(1)}m\bar P_{f\mu}-\bar X^{(1)}X^{(3)}m\bar q_\mu\\
&+\bar X^{(3)}X^{(1)}m(\bar q-\bar k)_\mu+\frac{1}{4}\bar X^{(2)}X^{(2)}\Bigg[\bar P_{f\mu}\bar P_i\cdot\left[i\bar q+\frac{1}{2}\bar P_i\right]\\
&\qquad-(\bar P_f\cdot\bar P_i)\left[i\bar q_\mu+\frac{1}{2}\bar P_{i\mu}\right]+\bar P_{i\mu}\bar P_f\cdot\left[i\bar q+\frac{1}{2}\bar P_i\right]\Bigg]\\
&+\frac{1}{2}\bar X^{(2)}X^{(3)}\Bigg[\bar P_{f\mu}\bar q\cdot\left[i\bar q+\frac{1}{2}\bar P_i\right]-(\bar P_f\cdot\bar q)\left[i\bar q_\mu+\frac{1}{2}\bar P_{i\mu}\right]+\bar q_\mu\bar P_f\cdot\left[i\bar q+\frac{1}{2}\bar P_i\right]\Bigg]\\
&-\frac{1}{2}\bar X^{(3)}X^{(2)}\Bigg[(\bar q-\bar k)_\mu\bar P_i\cdot\left[i\bar q+\frac{1}{2}\bar P_i\right]-(\bar q-\bar k)\cdot\bar P_i\left[i\bar q_\mu+\frac{1}{2}\bar P_{i\mu}\right]\\
&\qquad+\bar P_{i\mu}(\bar q-\bar k)\cdot\left[i\bar q+\frac{1}{2}\bar P_i\right]\Bigg]\\
&-\bar X^{(3)}X^{(3)}\Bigg[(\bar q-\bar k)_\mu\bar q\cdot\left[i\bar q+\frac{1}{2}\bar P_i\right]-\left[i\bar q_\mu+\frac{1}{2}\bar P_{i\mu}\right](\bar q-\bar k)\cdot\bar q\\
&\qquad+\bar q_\mu(\bar q-\bar k)\cdot\left[i\bar q+\frac{1}{2}\bar P_i\right]\Bigg]\Bigg\}\,,
\end{aligned}\tag{36}$$

where

$$\begin{aligned}
&\bar X^{(i)}\equiv X^{(i)}((\bar q-\bar k)^2,(\bar q-\bar k)\cdot\tfrac{1}{2}\bar P_f^*)\,,\\
&X^{(i)}\equiv X^{(i)}(\bar q^2,\bar q\cdot\tfrac{1}{2}\bar P_i)\,,\\
&(\bar q-\bar k)\equiv(q_1-k,q_2,q_3,-q_0)\,,\\
&\bar P_i\equiv(-ik,0,0,\sqrt{M^2+k^2})\,,\\
&\bar P_f\equiv(ik,0,0,\sqrt{M^2+k^2})\,.
\end{aligned}\tag{37}$$

Because $F^{(3)}\ll 10^{-3}$, we can ignore it. By combining Eq. (33) with Eqs. (36) and a null dimension treatment, we obtain the formula of the electromagnetic form factor of the pion in the lowest-order approximation. It can be shown that

$$F(k^2)=\frac{2m^4e}{(2\pi)^4}\int d^4\bar q[\bar F_0^{(1)}F_0^{(1)}-2\bar F_0^{(1)}F_0^{(2)}+B^2\bar F_0^{(2)}F_0^{(2)}]\,, \tag{38}$$

where

$$\begin{aligned}
&\bar F^{(i)}\equiv F^{(i)}[|(q_1-k)^2+q_2^2+q_3^2+q_0^2|]\,,\\
&F^{(i)}\equiv F^{(i)}(|q_1^2+q_2^2+q_3^2+q_0^2|)\,.
\end{aligned}\tag{39}$$

Because q_2, q_3, and q_0 are completely symmetric in the integral (38), we can simplify the integral by the following change of integration variables:

$$Q^2\equiv q_2^2+q_3^2+q_0^2\,.$$

Then the form factor can be written as

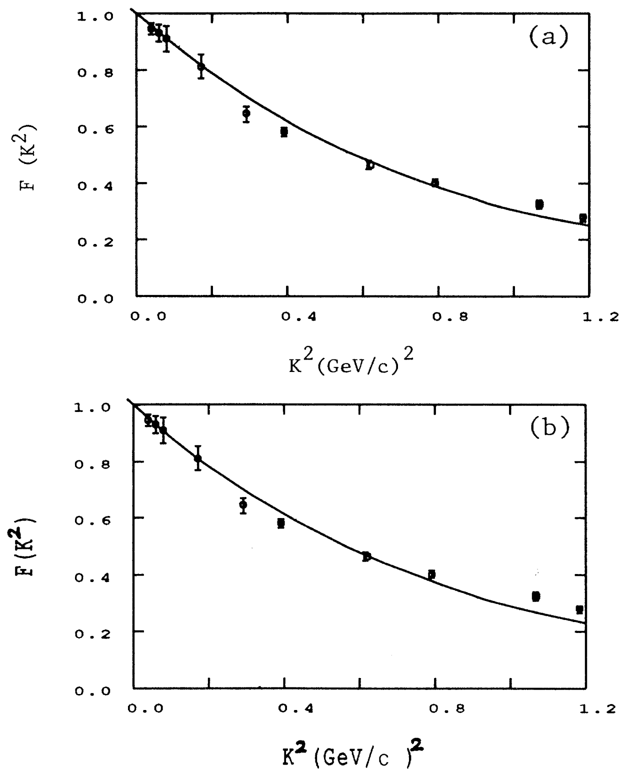

FIG. 3. The calculated result of the electromagnetic form factor of the pion corresponding to the wave function (a) and (b), respectively. The parameters are the same as Figs. 1(a) and 1(b), respectively. The experimental data of Ref. [12] are used.

$$F(k^2)=\frac{4\pi}{N}\int_{-\infty}^{\infty}dq\int_0^{\infty}dQ\,Q^2[\bar{F}_0^{(1)}F_0^{(1)}-2\bar{F}_0^{(1)}F_0^{(2)}+B^2\bar{F}_0^{(2)}F_0^{(2)}]\ , \qquad (40)$$

where N is the constant of normalization:

$$N=2\pi^2\int_0^{\infty}dq\,q^3[F_0^{(1)^2}-2F_0^{(1)}F_0^{(2)}+B^2F_0^{(2)^2}]\ , \qquad (41)$$

where

$$q^2\equiv q_1^2+q_2^2+q_2^2+q_0^2\ . \qquad (42)$$

Now, based on Eq. (40), Eq. (41), and the wave functions of bound states, we can obtain the electromagnetic form factor of 0^- meson by using the computer Vax 8700. It is shown in Fig. 3. Some experimental data of Ref. [12] are used in Fig. 3. This result denotes that the calculated result can give a good fit to the experimental result in a large k^2 region if we choose the appropriate parameters in the phenomenological vector-vector-type flat-bottom potential.

IV. CONCLUSION AND DISCUSSION

In order to find a better phenomenological model, first the phenomenological vector-vector-type flat-bottom hadronic potential which satisfies relativistic covariance, gauge invariance, and chiral invariance is constructed. Then, the numerical solutions of the wave functions of the bound state of the 0^- meson in Euclidean momentum space are obtained by solving the Bethe-Salpeter equation in the phenomenological model without using the instantaneous approximation or null-plane approximation method. Finally, in order to check the phenomenological model, the physical spacelike electromagnetic form factor of the 0^- meson, which is in very good accord with the experimental result in a large k^2 region, is calculated by using the wave functions of the bound state in Euclidean momentum space. From above, we regard that the phenomenological vector-vector-type flat-bottom hadronic potential model is successful in studying the system of bound states of equal-mass quark-antiquark pairs. Actually, it is very important to find a better phenomenological hadronic model to study the system and its properties while we do not know what the real strong interaction is. It is not only an important method for studying a system with strong interaction and its properties but also a powerful tool for solving actual questions; we can therefore research some other systems and their properties by using the phenomenological model, for example, the $Q\bar{q}$ system and qqq system, etc.

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