Remarks on flavor-changing neutral currents in walking technicolor

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We point out that since the running coupling $\bar{\alpha}(q^2)$ in walking technicolor (WTC) can be rather strong at the extended technicolor (ETC) scale $q^2 \sim \Lambda_{\rm ETC}^2$, the standard consideration of flavor-changing neutral currents (FCNCs) in WTC based on the lowest order in perturbation theory in α is not fully conclusive. We reanalyze this problem and conclude that FCNCs can indeed be suppressed in WTC *if* ETC interactions are chosen in an appropriate way. The crucial point is that the factor of enhancement of the masses of pseudo Goldstone bosons in WTC is just that which is sufficient to suppress FCNCs. FCNCs in the so-called strong ETC are also briefly discussed.

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Walking technicolor (WTC) [1] was proposed as a possible resolution of the flavor-changing neutral current (FCNC) problem [2] in the extended technicolor (ETC) scenario [3] for electroweak symmetry breaking. It has led to a recent resurgence of model building in ETC [4,5].

The dynamical toy model that WTC is based on is the quenched planar gauge theory with a nontrivial ultraviolet stable fixed point $\alpha=\alpha_c \sim 1$ and a large anomalous dimension $\gamma_m \approx 1$ of the composite operators $\bar{F}F$ and $\bar{F}\gamma_5 F$ ($\bar{F},F$ are technifermion fields) [6,7]. The conventional viewpoint is that such a large γ_m is responsible for an enhancement of the ordinary fermion masses without (and this is crucial) a simultaneous enhancement of FCNCs.

The purpose of the present note is a critical examination of this conventional wisdom. The point is the following. The conclusion about the absence of the enhancement of FCNCs in WTC is based on the analysis of the famous box diagrams leading to M^0-$\bar{M}^0$ mixing ($M^0=K^0$, D^0, or B^0 mesons) (see Fig. 1). However, these diagrams correspond only to the lowest order in perturbative theory in the technicolor (TC) coupling. While this approximation is justified in the case of QCD-like dynamics with a small running coupling $\bar{\alpha}(q^2)|_{q^2\sim\Lambda_{\rm ETC}^2} \ll 1$ ($\Lambda_{\rm ETC}$ is the ETC scale), it is unclear why it is also appropriate for WTC with the walking coupling $\bar{\alpha}(q^2)|_{q^2\sim\Lambda_{\rm ETC}^2}$ being rather strong: $\bar{\alpha}(q^2)|_{q^2\sim\Lambda_{\rm ETC}^2} \lesssim \alpha_c \sim 1$. To the best of our knowledge there has never been a fully conclusive analysis of this problem.

In the present work, we shall analyze this problem and the conditions under which the absence of the FCNC enhancement in WTC is guaranteed. In the low-energy region ($q^2 \ll \Lambda_{\rm ETC}^2$), the general form of ETC interactions is [3]

$$L = L_{LR} + L_{LL} + L_{RR}\ , \tag{1}$$

with

$$L_{LR} = -\frac{g_{\rm ETC}^2}{\Lambda_{\rm ETC}^2} \sum_{i,j=1}^{3} \sum_{r=1}^{n} \sum_{k=1}^{N_r} \Big\{ [\bar{u}_{iL}\gamma_\mu U_{kL}^r + \bar{d}_{iL}\gamma_\mu D_{kL}^r][\Gamma_{ij,r}^u \bar{U}_{kR}^r \gamma_\mu u_{jR} + \Gamma_{ij,r}^d \bar{D}_{kR}^r \gamma_\mu d_{jR}] + [\bar{\nu}_{iL}\gamma_\mu U_{kL}^r + \bar{e}_{iL}\gamma_\mu D_{kL}^r][\Gamma_{ij,r}^\nu \bar{U}_{kR}^r \gamma_\mu \nu_{jR} + \Gamma_{ij,r}^e \bar{D}_{kR}^r \gamma_\mu e_{jR}] + \text{H.c.} \Big\}\ , \tag{2}$$

$$L_{LL} = -\frac{g_{\rm ETC}^2}{\Lambda_{\rm ETC_2}} \sum_{i,j=1}^{3} \sum_{r=1}^{n} \sum_{k=1}^{N_r} \Big\{ [\bar{u}_{iL}\gamma_\mu U_{kL}^r + \bar{d}_{iL}\gamma_\mu D_{kL}^r][\tilde{\Gamma}_{ij,r}^u \bar{U}_{kL}^r \gamma_\mu u_{jL} + \tilde{\Gamma}_{ij,r}^d \bar{D}_{kL}^r \gamma_\mu d_{jL}] + [\bar{\nu}_{iL}\gamma_\mu U_{kL}^r + \bar{e}_{iL}\gamma_\mu D_{kL}^r][\tilde{\Gamma}_{ij,r}^\nu \bar{U}_{kL}^r \gamma_\mu \nu_{jL} + \tilde{\Gamma}_{ij,r}^e \bar{D}_{kL}^r \gamma_\mu e_{jL}] + \text{H.c.} \Big\}\ , \tag{3}$$

and an analogous structure for L_{RR}. Here we consider three families of fermions, and n (N_r-dimensional) irreducible representations of the TC group. The matrices $\Gamma^{u,d,\nu,e}$, $\tilde{\Gamma}^{u,d,\nu,e}$ are determined by ETC couplings and by the mass matrix of ETC vector bosons.

Since the detailed structure of the matrices $\Gamma,\tilde{\Gamma}$ is not essential for our purposes, we will use the following schematic notations for L_{LR}, L_{LL}, and L_{RR}:

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$$L_{LR} \sim -\left[\frac{g_{\rm ETC}^2}{\Lambda_{\rm ETC}^2}\right][\bar{f}_L\gamma_\mu F_L \bar{F}_R\gamma_\mu f_R + \text{H.c.}]$$
$$\sim \left[\frac{g_{\rm ETC}^2}{\Lambda_{\rm ETC}^2}\right][\bar{f}f\bar{F}F - \bar{f}\gamma_5 f\bar{F}\gamma_5 F] , \quad (4)$$
$$L_{LL} \sim -\left[\frac{g_{\rm ETC}^2}{\Lambda_{\rm ETC}^2}\right]\bar{f}_L\gamma_\mu F_L \bar{F}_L\gamma_\mu f_L$$
$$\sim \left[\frac{g_{\rm ETC}^2}{\Lambda_{\rm ETC}^2}\right]\bar{f}_L\gamma_\mu f_L \bar{F}_L\gamma_\mu F_L$$
$$L_{RR} \sim -\left[\frac{g_{\rm ETC}^2}{\Lambda_{\rm ETC}^2}\right]\bar{f}_R\gamma_\mu F_R \bar{F}_R\gamma_\mu f_R \quad (5)$$
$$\sim \left[\frac{g_{\rm ETC}^2}{\Lambda_{\rm ETC}^2}\right]\bar{f}_R\gamma_\mu f_R \bar{F}_R\gamma_\mu F_R ,$$

In the second order in $g_{\rm ETC}^2$, one finds the amplitude leading to M^0-$\bar{M}^0$ mixing:

$$A = -\frac{1}{2}\chi\left[\frac{g_{\rm ETC}^2}{\Lambda_{\rm ETC}^2}\right]^2\left\langle p_1',p_2'\left|\int d^4x\, d^4y T\left\{\bar{f}(x)\gamma_a f(x)\bar{f}(y)\gamma_b f(y)\bar{F}(x)\gamma_a F(x)\bar{F}(y)\gamma_b F(y)\right\}\right|p_1,p_2\right\rangle , \quad (6)$$

where $a,b=S,P,V_L,V_R$ with $\gamma_S=1$, $\gamma_P=i\gamma_5$, $\gamma_{V_L}=\gamma_\mu(1-\gamma_5)/2$, $\gamma_{V_R}=\gamma_\mu(1+\gamma_5)/2$. The factor χ includes Cabibbo-like mixing angles.

Neglecting $SU(3)_c\times SU(2)_L\times U(1)_Y$ interactions, the amplitude A can be rewritten as

$$A=\frac{1}{2i}\chi\left[\frac{g_{\rm ETC}^2}{\Lambda_{\rm ETC}^2}\right]^2(2\pi)^{-2}\delta^4(p_1'+p_2'-p_1-p_2)$$
$$\times\bar{u}^+(p_1')\gamma_a u^+(p_2')\bar{u}^-(p_2)\gamma_b u^-(p_1)\Delta_{ab}(p_1+p_2) , \quad (7)$$

where

$$\Delta_{ab}(q)=\frac{1}{i}\int d^4x e^{iqx}\langle 0|T\{\bar{F}(x)\gamma_a F(x)\bar{F}(0)\gamma_b F(0)\}|0\rangle , \quad (8)$$

and $\bar{u}^+(u^-)$, $u^+(\bar{u}^-)$ are wave functions of outgoing (incoming) quarks and antiquarks, respectively.

Thus, the problem is reduced to studying the propagator of the composite operators $\bar{F}\gamma_a F$ in TC theory. The general method for calculating Green's functions of composite operators was elaborated on in Ref. [8]. We shall use the technique discussed in this reference in the following.

One can show that the equation for $\Delta_{ab}(q)$ takes the form [8]

$$\Delta_{ab}(q)=\frac{1}{2i}\int\frac{d^4k}{(2\pi)^4}\text{Tr}[G(k)\Gamma_a(k,k+q)G(k+q)\gamma_b]$$
$$+(q\to -q, a\leftrightarrow b) , \quad (9)$$

where $G(k)=i[A(k^2)\gamma\cdot k-\Sigma(k^2)]^{-1}$ is the technifermion propagator and Γ_a is the proper vertex connecting the composite operator $\bar{F}\gamma_a F$ with $\bar{F}$ and F:

$$i\langle 0|T\{F(x)\bar{F}(y)\bar{F}(z)\gamma_a F(z)\}|0\rangle=\frac{1}{(2\pi)^8}\int d^4k\, d^4p\, e^{-ik(x-z)+ip(y-z)}G(k)\Gamma_a(k,p)G(p) . \quad (10)$$

The graphic representation of Eq. (9) is Fig. 2. The vertex $\Gamma_a(k,k+q)$ satisfies the Bethe-Salpeter (BS) equation:

$$(\Gamma_a)_{\alpha\beta}(k,k+q)=i(\gamma_a)_{\alpha\beta}+\int\frac{d^2r}{(2\pi^4)}K_{\alpha\beta;\gamma\delta}(k,k+q;r)[G(r)\Gamma_a(r,r+q)G(r+q)]_{\gamma\delta} , \quad (11)$$

where $K_{\alpha\beta;\gamma\delta}$ is the BS kernel.

Since $q^2=(p_1+p_2)^2\ll\Lambda_{\rm TC}^2\ll\Lambda_{\rm ETC}^2$, one can use the approximation $\Delta_{ab}(q)\approx\Delta_{ab}(0)$.

The crucial point for FCNCs is that the Green's function Δ_{ab} in Eq. (9) is quadratically divergent in a free field theory. Then $\Delta_{ab}\sim\Lambda_{\rm ETC}^2$ and therefore the amplitude A (7) is $A\sim 1/\Lambda_{\rm ETC}^2$ (and not $A\sim 1/\Lambda_{\rm ETC}^4$), which is the well-known result for the box diagram in perturbation theory. This result leads to the usual bound $\Lambda_{\rm ETC}\gtrsim 100$ TeV for fermions in the first two families. Notice that if, naively, one were to consider the effect of WTC as an enhancement factor $(\Lambda_{\rm ETC}/\Lambda_{\rm TC})^2$ on top of the estimate for a free field theory, the result would be disastrous since then $A\sim 1/\Lambda_{\rm TC}^2$ and FCNC operators would be unacceptably large. As we shall see this is not the case.

The question we want to answer is what is the behavior

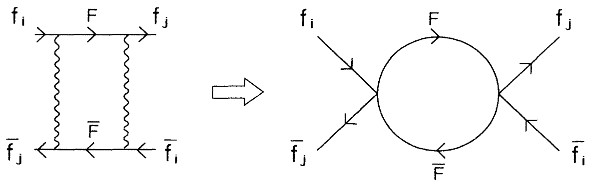

FIG. 1. The box diagrams leading to M^0-$\overline{M}^0$ mixing. The wavy lines correspond to ETC vector bosons. F stands for technifermion and f for ordinary fermion.

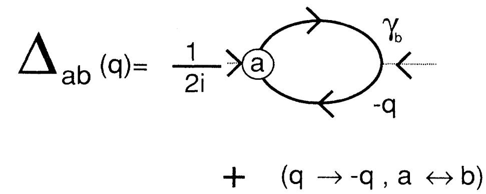

FIG. 2. Graphic representation of Eq. (8). See text.

of Δ_{ab} in WTC? Let us start the analysis by considering Eqs. (9) and (11) in QCD-like TC dynamics. Since we are interested in finding the leading divergence in Δ_{ab} which appears from the uv region $|x|^{-1} \gg \Lambda_{TC}$, one needs to determine the uv behavior $(k^2 \gg \Lambda_{TC}^2)$ of the vertex $\Gamma_a(k,k)$.

Let us first consider the Green's functions $\Delta_{ab}(0)$ with $a,b=S,P$. Assuming that TC interactions are vectorial, one finds that $\Delta_{ab}(0)=\delta_{ab}\Delta_{aa}$. At $k^2 \gg \Lambda_{TC}^2$, the kernel $K_{\alpha\beta;\gamma\delta}$ takes the following simple form in the Landau gauge [9,10]:

$$K_{\alpha\beta;\gamma\delta} \approx \frac{iC(F)}{(2\pi)^4}\bar{g}^2((k-r)^2)(\gamma_\nu)_{\alpha_\gamma}(\gamma_\lambda)_{\delta\beta} d_{\nu\lambda}(k-r) , \tag{12}$$

where $\bar{g}$ is the running coupling $\bar{g}^2(k^2)=[(b/2)\ln(k^2/\Lambda_{TC}^2)]^{-1}$, b is the first coefficient of the technicolor β function, $C(F)$ is the value of the Casimir operator of technifermions, and $d_{\nu\lambda}$ is the technigluon propagator $d_{\nu\lambda}(p)=[g_{\nu\lambda}-(p_\nu p_\lambda/p^2)]/p^2$. In the leading logarithmic approximation [10] one can take $\bar{g}^2((p-k)^2)$ as

$$\bar{g}^2((p-k)^2)=\bar{g}^2(p^2)\theta(p^2-k^2)+\bar{g}^2(k^2)\theta(k^2-p^2).$$

This is the so-called improved ladder approximation (see Fig. 3).

The solutions of Eq. (11) with cutoff $\Lambda \equiv \Lambda_{ETC}$ and $a=S,P$ then are[1] [8]

$$\Gamma_S(k,k)=iC_S(k^2) , \quad \Gamma_P(k,k)=i\gamma_5 C_P(k^2) , \tag{13}$$

with

$$C_S(k^2) \sim Z_m^{-1}\left[\ln\frac{k^2}{\Lambda_{TC}^2}\right]^{-[3C(F)/8\pi^2 b]} \quad \text{for } k^2 \gg \Lambda_{TC}^2 , \tag{14}$$

$$C_P(k^2) \sim Z_m^{-1}\left[\left[\ln\frac{k^2}{\Lambda_{TC}^2}\right]^{-[3C(F)/8\pi^2 b]} + \sum_i \frac{3C(F)}{N_F b}\frac{(\langle\bar{F}F\rangle_{\rm ren})^2}{F_{TC}^2 m_{\pi_i}^2}\frac{1}{k^2}\left[\ln\frac{k^2}{\Lambda_{TC}^2}\right]^{-1+[3C(F)/8\pi^2 b]}\right] \quad \text{for } k^2 \gg \Lambda_{TC}^2 , \tag{15}$$

where N_F is the dimension of the representation of the TC group for F, $Z_m \sim (\ln\Lambda_{ETC}^2/\Lambda_{TC}^2)^{-[3C(f)/8\pi^2 b]}$ is the renormalization constant for the composite operator $\bar{F}F$, and $\langle\bar{F}F\rangle_{\rm ren} \sim \Lambda_{TC}^3$ is the condensate relating to the scale $\mu=\Lambda_{TC}$: $\langle\bar{F}F\rangle_{\rm ren}=Z_m\langle\bar{F}F\rangle$; $\sum_i$ means summation over all pseudo Goldstone bosons.

The physical meaning of expressions (14) and (15) is clear. C_P contains two parts: one corresponds to the perturbative contribution of technifermions and the other to the nonperturbative contribution of the lightest technihadrons (the contribution of heavier technihadrons is omitted). Since no light technihadrons are expected in the s-channel, we retained only the perturbative piece in $C_S(k^2)$.

Substituting $\Gamma_a(k,k)$ in Eq. (9), and noticing that in this approximation $A(p)=1$ and $\Sigma(p)$ is a monotonically decreasing function[2] of p, one can see that it is just this perturbative (slowly decreasing) piece in Γ_a that leads to the quadratic divergence in Δ_{aa} in QCD-like theories. Expression (15) for Γ_P also implies that the contribution of pseudo Goldstone bosons in Δ_{PP} is suppressed (with respect to the perturbative one) by the factor $\Lambda_{TC}^4/m_{\pi_i}^2\Lambda_{ETC}^2$. This, in turn, means that the condition $m_{\pi_i}^2 \gtrsim \Lambda_{TC}^4/\Lambda_{ETC}^2$ is sufficient to suppress FCNCs con-

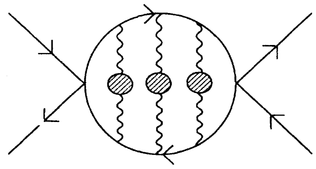

FIG. 3. A typical diagram summed up in the improved ladder approximation. The blobs represent the loop contribution in the technigluon propagator.

[1]The simplest way of obtaining these solutions is to use the relations [8] $C_s(k^2)=\partial\Sigma(k^2)/\partial m^{(0)}$, $C_p(k^2)=\Sigma(k^2)/m^{(0)}$, where $\Sigma(k^2)$ is the mass function of technifermions and $m^{(0)}$ is an auxiliary bare mass of technifermions imitating explicit chiral symmetry breaking so that $m_\pi^2=-2\langle\bar{F}F\rangle m^{(0)}/F_{TC}^2(F_{TC}\sim\Lambda_{TC}$ is the decay constant of pseudo Goldstone bosons).

[2]See Refs. [9,10] for the exact expression.

nected with the exchange of pseudo Goldstone bosons. Since $\Lambda_{\rm ETC}^2 \gg \Lambda_{\rm TC}^2$, this restriction for $m_{\pi_i}^2$ is rather mild.

The situation with the Green's functions $\Delta_{V_L V_L}$ and $\Delta_{V_R V_R}$ is even simpler: since the anomalous dimension of vector and axial currents is equal to zero, their dependence on $\Lambda_{\rm ETC}^2$ should be similar to that in free theory (quadratically divergent). Let us consider now FCNCs in WTC. The main difference from the previous case is the behavior of the running coupling $\bar{g}^2(k^2)$: now it is a slowly changing ("walking") function of the momentum. The toy model [6,7] of such dynamics is the ladder approximation for the kernel:

$$K_{\alpha\beta;\gamma\delta} \approx \frac{i}{4\pi^3}\alpha(\gamma^\nu)_{\alpha\gamma}(\gamma^\lambda)_{\delta\beta} d_{\nu\lambda}(k-r) , \tag{16}$$

with $\alpha = g^2 C(F)/4\pi$ being close to the critical value $\alpha_c = \pi/3$ (for a justification of this approximation see Ref. 11). Such a large value of α leads to a large anomalous dimension $\gamma_m \approx 1$ of the composite operators $\bar{F}F$ and $\bar{F}\gamma_5 F$ [6,7]. It will be, however, useful to consider all values of α from $\alpha=0$ to $\alpha=\alpha_c=\pi/3$, when $\gamma_m = 1-(1-3\alpha/\pi)^{1/2}$ is changing from zero to one. First of all, we note that the anomalous dimension manifests itself only in the Green's functions Δ_{SS} and Δ_{PP}; the behavior of $\Delta_{V_L V_L}$ and $\Delta_{V_R V_R}$ in WTC should be nearly the same as in the case of QCD-like dynamics.

Let us consider the Green's functions Δ_{SS} and Δ_{PP}. The solutions of Eq. (11) with the kernel (16) are [compare with Eqs. (14) and (15)]

$$C_S(k^2) \sim Z_m^{-1} \left[\frac{k^2}{\Lambda_{\rm TC}^2}\right]^{-\gamma_m/2} \quad \text{for } k^2 \gg \Lambda_{\rm TC}^2 , \tag{17}$$

$$C_P(k^2) \sim Z_m^{-1} \left[\left[\frac{k^2}{\Lambda_{\rm TC}^2}\right]^{-\gamma_m/2} + \sum_i \frac{4\pi^2 \gamma_m (\langle \bar{F}F \rangle_{\rm ren})^2}{F_{\rm TC}^2 m_{\pi_i}^2 N_F} \frac{1}{\Lambda_{\rm TC}^2} \left[\frac{k^2}{\Lambda_{\rm TC}^2}\right]^{(\gamma_m-2)/2} \right] \quad \text{for } k^2 \gg \Lambda_{\rm TC}^2 , \tag{18}$$

with $Z_m^{-1} \sim (\Lambda_{\rm ETC}/\Lambda_{\rm TC})^{\gamma_m}$ and $\gamma_m = 1-(1-3\alpha/\pi)^{1/2}$.

Note that, unlike Eqs. (14) and (15), there are no logarithmic factors in expressions (17) and (18). This is, of course, due to the present approximation with a constant α.

Concerning the perturbative piece $(k^2/\Lambda_{\rm TC}^2)^{-\gamma_m/2}$ in Eqs. (17) and (18), one can see that its contribution to Δ_{aa} exactly compensates the enhancement factor Z_m^{-1} yielding a final $1/\Lambda_{\rm ETC}^2$ dependence as in the QCD-like case. However, the contribution of pseudo Goldstone bosons in Δ_{PP} depends on $\Lambda_{\rm ETC}$ in an essentially different way from that case. In fact, comparing this contribution with that in QCD-like TC (QTC), one finds that the suppression of FCNC's in WTC is sufficient if

$$m_{\pi_i}^2({\rm WTC}) \gtrsim (\Lambda_{\rm ETC}^2/\Lambda_{\rm TC}^2)^{\gamma_m} m_{\pi_i}^2({\rm QTC}) , \tag{19}$$

where m_{π_i} (WTC) and m_{π_i} (QTC) relate to WTC and QTC, respectively.[3,4] It is crucial that this enhancement factor in m_{π_i} (WTC) exactly coincides with the one that appears in WTC as the result of the action of ETC four-fermion operators leading to the explicit breakdown of chiral symmetry of technifermions (and therefore to the generation of m_{π_i}). [1] This enhancement mechanism was used before to avoid problems with unobservable light pseudo Goldstone bosons. What we have shown is that just the same enhancement of $m_{\pi_i}^2$ is sufficient to avoid the problem with FCNCs in WTC.

Using experimental information on FCNCs in the K^0-$\bar{K}^0$ system one can get a bound on the pseudo Goldstone boson mass, $m_\pi \gtrsim \theta 5$ TeV, where θ is a Cabibbo-type mixing angle. With $\theta \sim 0.2$, the pseudo Goldstone boson mass is of order a typical technihadron mass. Such a heavy mass for a pseudo Goldstone boson raises serious doubts as to the validity of the chiral-Lagrangian approach for describing its dynamics in WTC.

Let us note that the case with $\gamma_m = 1 (\alpha=\alpha_c)$ is rather special. In this case, the power of k^2 in the perturbative and nonperturbative terms in $C_P(k^2)$ (18) is the same [12,13]. This implies that one cannot clearly separate perturbative and nonperturbative physics in this case. The reason for this is the following: while in QCD-like TC the dynamics forming technihadrons is connected with the infrared region wherein $k^2 \sim \Lambda_{\rm TC}^2$, in WTC (with $\gamma_m = 1$) both infrared and ultraviolet regions with strong-coupling dynamics are responsible for the formation of technihadrons.

From a physical viewpoint it means that Δ_{PP} can be accurately approximated by the contribution of the lightest technihadrons in WTC. From a formal viewpoint, this implies the multiplicative renormalizability of Green's functions of the local composite operators $\bar{F}F$ and $\bar{F}\gamma_5 F$ [12]:

$$\Delta_{PP}^{(\rm ren)} = Z_m^2 \Delta_{PP} , \quad \Delta_{SS}^{(\rm ren)} = Z_m^2 \Delta_{SS} . \tag{20}$$

[as is well known [14], there is no multiplicative renor-

[3]Strictly speaking one is not really allowed to use partially conserved axial-vector current (PCAC) relations, as we did, to derive Eq. (18) when Eq. (19) makes the pseudo Goldstone boson too heavy. Our analysis only shows under what circumstances one does not run immediately into trouble with FCNCs.

[4]The contribution of the pseudo Goldstone bosons to FCNC processes can be simply estimated by assuming they couple to light fermions with a Yukawa coupling of order $m_f/\Lambda_{\rm TC}$ and using the relation $m_f = \langle \bar{F}F \rangle / \Lambda_{\rm ETC}^2 (\Lambda_{\rm ETC}/\Lambda_{\rm TC})^{\gamma_m}$.

malizability (MR) of these Green's function either in free theory with $Z_m = 1$ or in QCD-like dynamics. In both cases, MR takes place only for Green's functions containing one composite operator and any number of elementary ones].

This point can be relevant for FCNCs in the so-called strong-ETC scenario [15]. Since the coupling $g^2_{\rm ETC}$ is strong in that case, the present consideration treating the ETC interactions perturbatively cannot be directly applied to such dynamics. The toy model of strong ETC is the gauged Nambu–Jona-Lasinio (NJL) model [7] considered in the so-called ladder-bubble approximation wherein the TC gauge interactions are treated in the ladder approximation and the four-fermion interactions (corresponding to ETC) are treated in the bubble approximation (see Fig. 4).

For our purposes one needs to know that in the gauged NJL model, as in WTC, the contribution of the lightest technihadrons dominates the corresponding Green's function in the low-energy region (see the second paper in Ref. [12]). This, in turn, leads us to the conjecture that, as in WTC, the enhancement factor in $m^2_{\pi_i}$ will also be sufficient to suppress FCNCs in strong ETC.

We conclude that the FCNCs problem can be solved in WTC if $\Lambda_{\rm ETC} \gtrsim 100$ TeV (for the first two families) and the ETC interactions contain four-fermion operators leading to large enough masses for pseudo Goldstone bosons. The WTC dynamics provides just such an enhancement factor in $m^2_{\pi_i}$ which is sufficient to suppress FCNCs. We have serious doubts whether chiral-Lagrangian analyses can be applied to such pseudo Goldstone bosons. Although these conclusions are not new [1,2], we feel they are now based on a self-consistent analysis of a simple model of WTC.

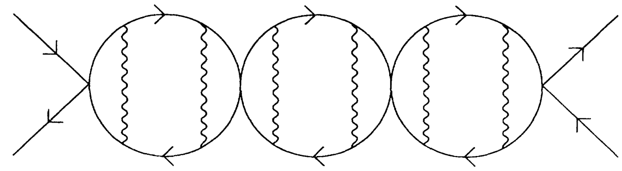

FIG. 4. A typical diagram corresponding to the ladder-bubble approximation.

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[1] B. Holdom, Phys. Rev. D **24**, 1441 (1981); Phys. Lett. B **150**, 301 (1985); K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986); T. Akiba and T. Yanagida, Phys. Lett. B **169**, 432 (1986); T. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986).
[2] S. Dimopoulos and J. Ellis, Nucl. Phys. **B182**, 505 (1981).
[3] S. Dimopoulos and L. Susskind, Nucl. Phys. **B155**, 237 (1979); E. Eichten and K. Lane, Phys. Lett. **90B**, 125 (1980).
[4] G. F. Giudice and S. Raby, Nucl. Phys. **B368**, 221 (1992).
[5] B. Holdom, Phys. Lett. B **246**, 169 (1990); S. King and S. Mannan, Nucl Phys. **B369**, 119 (1992); M. Einhorn and D. Nash, ITP Report No. NSF-ITP-91-91 (unpublished); T. Appelquist and J. Terning, Yale Report No. YCTP-P50-91, 1991 (unpublished); R. Sundrum, Berkeley Report No. LBL-32107, 1992 (unpublished).
[6] V. A. Miransky, Nuovo Cimento A **90**, 149 (1985).
[7] W. A. Bardeen, C. N. Leung, and S. T. Love, Phys. Rev. Lett. **56**, 1335 (1986); C. N. Leung, S. T. Love, and W. A. Bardeen, Nucl. Phys. **B273**, 649 (1986).
[8] V. P Gusynin, V. A. Kushnir, and V. A. Miransky, Phys. Rev. D **39**, 2355 (1989).
[9] K. Lane, Phys. Rev. D **10**, 2605 (1974).
[10] V. A. Miransky, Yad. Fiz. **38**, 468 (1983) [Sov. J. Nucl. Phys. **38**, 280 (1983)]; K. Higashijima, Phys. Rev. D **29**, 1228 (1984).
[11] T. Appelquist, K. Lane, and U. Mahanta, Phys. Rev. Lett. **61**, 1553 (1988); B. Holdom, *ibid.* **62**, 997 (1989); U. Mahanta, Phys. Lett. B **225**, 181 (1989).
[12] V. P. Gusynin and V. A. Miransky, Phys. Lett. B **198**, 79 (1987); V. A. Miransky, Int. J. Mod. Phys. A (to be published).
[13] A. G. Cohen and H. Georgi, Nucl. Phys. **B314**, 7 (1989).
[14] J. C. Collins, *Renormalization* (Cambridge University Press, Cambridge, England, 1984).
[15] T. Appelquist, M. Einhorn, T. Takeuchi, and L. C. R. Wijerwardhana, Phys. Lett. B **220**, 223 (1989); V. A. Miransky and K. Yamawaki, Mod. Phys. Lett. A **4**, 129 (1989); K. Matumoto, Prog. Theor. Phys. **81**, 277 (1989).