Magnitude of Higgs-boson-exchange *CP* violation in two-doublet models with large $\tan\beta$

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CP violation in neutral Higgs-boson exchange is studied in two-doublet models in an expansion in $1/\tan^2\beta$). The typical magnitude of various *CP*-violating quantities is found for large $\tan\beta$. In particular the electric dipole moment (EDM) of the electron and the coefficient c_S of the *CP*-violating electron-nucleon scalar-pseudoscalar operator are examined and it is found that in a simple class of two-doublet models c_S/d_e is typically $O(\tan^2\beta)$. Therefore c_S is more important than d_e for the EDM's of diamagnetic atoms and molecules (Hg, Xe, TlF) typically if $\tan\beta \gtrsim 5$, and for paramagnetic atoms (Cs, Tl) if $\tan\beta \gtrsim 15$. The dependence on $\tan\beta$ of the various contributions to the neutron EDM including the Weinberg three-gluon operator, and the dependence on $\tan\beta$ of the top-quark EDM are also discussed. Supersymmetric and three-doublet models are also considered.

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I. INTRODUCTION

It has been found that in models in which the exchange of neutral Higgs fields mediates *CP* violation the electric dipole moments of the neutron [1–3] and electron [2,4] are typically quite large—large enough to have a good chance of being seen with present methods. In practice, bounds on the electron electric dipole moment d_e are inferred from experimental bounds on the electric dipole moments of large atoms (such as Tl, Cs, Hg, and Xe) or diatomic molecules (such as TlF). Atomic electric dipole moments may also arise from neutron or proton electric dipole moments (EDM's) or from dimension-6 four-fermion operators that violate *P* and *T*. Of particular interest are dimension-6 electron-nucleon *T*- and *P*-odd operators, which, though negligible in most models, can be large enough to be observed and to compete with or dominate over d_e in models with Higgs-boson-mediated *CP* violation if $\tan\beta$ is large [5,6].

Moreover, various other *CP*-violating quantities that have been studied in the literature, such as the top-quark EDM, the Weinberg three-gluon operator, and the chromo-EDM's and EDM's of the light quarks, have different dependences on $\tan\beta$. It is therefore interesting to study *CP* violation of multi-Higgs-boson models in an expansion in $1/\tan^2\beta$. That is what is done in this paper.

There are three types of multi-Higgs-boson models that will be studied here, which for convenience will be called class I–III models. Class I models will be those two-doublet models in which the *CP*-violating phase in the Higgs potential, V_H, arises from the interplay of one non-Hermitian term quartic in the Higgs doublets, $(\phi_1^\dagger\phi_2)^2$, and one non-Hermitian term quadratic in the doublets, $(\phi_1^\dagger\phi_2)$ or $(\phi_1^\dagger\phi_2)\sigma$, for example. (Note that if only one of these terms were present the phase in its coefficient could be rotated away by redefining the relative phase of ϕ_1 and ϕ_2.) Class II models consist of those where *CP* violation arises in V_H from the presence of more than one term quadratic in the doublets, such as $(\phi_1^\dagger\phi_2)\sigma$ and $(\phi_1^\dagger\phi_2)\sigma'$. Important examples [7] arise in the context of supersymmetry, where typically the $(\phi_1^\dagger\phi_2)^2$ term is absent. Class III models are those in which *CP* violation arises in V_H due to the interplay of several quartic terms. This can happen in models with three or more doublets. Of course, in complicated models all three mechanisms could contribute.

In previous papers [8], *maximal* values of *CP*-violating quantities in multi-Higgs-boson models have been derived in terms of the parameter $\tan\beta$. Generally, these maximal values also give a reasonable order-of-magnitude estimate of the *typical* values of these parameters. One of the main points of this paper is that class I models are a counterexample to this. How this happens can be seen by contemplating the function $(a^2+b^2\cot^2\beta)^{-1/2}$, where $a\sim1$ and $b\sim1$. (Henceforth $\sim$ will mean "is of the same order of magnitude as.") Typically, this expression ~1 for $\tan\beta \gg 1$. But its maximal value is $b^{-1}\tan\beta$. However, to achieve values approaching this maximum, a^2 must be "fine-tuned" to be $O(1/\tan^2\beta)$.

An important consequence of this is that in class I models the dimension-6 electron-nucleon operators are enhanced relative to d_e by $O(\tan^2\beta)$ rather than by only $O(\tan\beta)$. Thus these operators may in class I models give the dominant effect in diamagnetic systems for $\tan\beta \gtrsim 5$ and in paramagnetic systems for $\tan\beta \gtrsim 15$ (rather than for $\tan\beta \gtrsim 25$ and $\tan\beta \gtrsim 250$, respectively, as given in Ref. [5]; the conclusions of Ref. [5] apply to general two-doublet models that are not of class I).

This paper is organized as follows. In Sec. II the results obtained in previous calculations of d_e and of the coefficients of the electron-nucleon operators are summarized and the relevant *CP*-violating parameters that will be estimated later are introduced. In Sec. III the simplest class I model, namely, two Higgs doublets with softly broken natural flavor conservation (NFC), is analyzed in an expansion in $1/\tan^2\beta$. In Sec. IV, the next simplest class I model, which has two doublets and a real Higgs singlet with NFC, is similarly analyzed and similar (and remarkably simple) results are found. In Sec. V other effects such as the Weinberg three-gluon operator and the EDM of the top quark are discussed and the typical values of the relevant *CP*-violating quantities are given. Section VI deals with class II models, specifically with supersymmetric models. Section VII deals with class III models, specifically a three-doublet model with NFC. Finally, Sec. VIII collects our conclusions.

II. REVIEW OF d_e AND ELECTRON-NUCLEON *CP*-VIOLATING OPERATORS IN TWO-DOUBLET MODELS

Natural flavor conservation can be implemented in several ways. Here it will be assumed that the up quarks (u,c,t) derive their masses from one doublet of Higgs fields, ϕ_2, and the down quarks (d,s,b) and charged leptons (e,μ,τ) derive their masses from another ϕ_1. (This pattern is suggested both by low-energy supersymmetry and by the group theory of grand unification). Under this assumption the EDM of the electron arising from neutral Higgs-boson exchange, which has been computed by several groups [2,4] can be expressed in the form

$$\begin{aligned} d_e \cong & \sum_n A(M_W^2/m_n^2)(\sin^2\beta\,\mathrm{Im}Z_{0,n})/m_n^2 \\ & + \sum_n B(m_t^2/m_n^2)(\mathrm{Im}Z_{0,n})/m_n^2 \\ & + \sum_n C(m_t^2/m_n^2)(\mathrm{Im}\tilde{Z}_{0,n})/m_n^2 \ . \end{aligned} \tag{1}$$

The m_n are the mass eigenvalues of the neutral Higgs fields. A, B, and C are logarithmically varying functions, and $Z_{0,n}$ and $\tilde{Z}_{0,n}$ are defined by

$$\begin{aligned} \left\langle T\left\{\frac{\phi_2^0\phi_1^{0*}}{v_2 v_1^*}\right\}\right\rangle_q &= \sum_n \frac{\sqrt{2}G_G Z_{0,n}}{q^2+m_n^2} \ , \\ \left\langle T\left\{\frac{\phi_2^0\phi_1^0}{v_2 v_1}\right\}\right\rangle_q &= \sum_n \frac{\sqrt{2}G_G \tilde{Z}_{0,n}}{q^2+m_n^2} \ . \end{aligned} \tag{2}$$

The first term in Eq. (1) comes from a plethora of two-loop diagrams involving W bosons. The second and third terms come from a diagram with a top-quark loop, which contributes with opposite sign and smaller magnitude. For $z\to\infty$, $C(z)/B(z)\to 0$, and so C will be neglected. To leading order, then, in $1/\tan^2\beta$, d_e can be expressed as

$$d_e \cong \sum_n K\left[\frac{M_W^2}{m_n^2},\frac{m_t^2}{m_n^2}\right]\frac{1}{m_n^2}(\sin^2\beta\,\mathrm{Im}Z_{0,n}) \ . \tag{3}$$

Often, the simplifying assumption is made that the lightest Higgs eigenstate, denoted henceforth by index $n=H$, dominates the sum. Then one finds, numerically,

$$d_e \cong k(m_t^2,m_H^2)\sin^2\beta\,\mathrm{Im}Z_0(10^{-26}e\text{ cm}) \ , \tag{4}$$

where k is a dimensionless number of order 1. For $m_t=2M_W$, $k(m_H=120\text{ GeV})\simeq 1$, $k(m_H=160\text{ GeV})\simeq\frac{1}{2}$, and $k(m_H\geq 320\text{ GeV})\simeq\frac{1}{4}$. In this paper the somewhat less crude simplifying assumption will be made that m_n may be replaced by m_H in the function K in Eq. (3) (which is a logarithmically varying function), but the factor m_n^{-2} and the sum over n will be retained. Then Eq. (4) becomes

$$d_e \cong k(m_t^2,m_H^2)\sin^2\beta X_0(10^{-26}e\text{ cm}) \ , \tag{5}$$

where

$$X_0 \equiv \sum_n \left[\frac{m_H^2}{m_n^2}\right]\mathrm{Im}Z_{0,n} \ . \tag{6}$$

If the lightest Higgs boson $(n=H)$ dominates, it can be proved [7] simply that the maximal value X_0 can achieve in a two-doublet model is

$$X_{0(\max)} \cong (\mathrm{Im}Z_0)_{\max} = \tfrac{1}{4}\tan\beta/\sin^2\beta \ . \tag{7}$$

This parameter X_0 will be studied carefully in the next two sections where it will be found that in class I models it is, to leading order in $(1/\tan^2\beta)$, typically ~ 1 and not of $O(\tan\beta)$ and is, in fact, typically $\sim\frac{1}{4}$.

There are six T- and P-odd electron-nucleon dimension-six operators that can contribute to the EDM's of atoms:

$$\begin{aligned} \mathcal{L}_{eN} = \sum_{i=p,n}\frac{G_F}{\sqrt{2}}\{ & c_S(i)\bar{N}_i N_i \bar{e}i\gamma_5 e + c_{P(i)}\bar{N}_i\gamma_5 N_i\bar{e}e \\ & + c_{T(i)}\bar{N}_i\sigma^{\mu\nu}N_i\bar{e}i\gamma_5\sigma_{\mu\nu}e\} \ . \end{aligned} \tag{8}$$

In Ref. [5], c_S, the coefficients of the scalar operators, were computed in two-doublet models of *CP* violation to be

$$\begin{aligned} c_{S(p)} & \simeq c_{S(n)} \\ & \simeq \frac{4}{29}m_e m_p \sum_n \frac{\mathrm{Im}Z_{1,n}-\tan^2\beta\,\mathrm{Im}Z_{2,n}}{m_n^2} \ , \end{aligned} \tag{9}$$

where $Z_{i,n}$ $(i=1,2)$ is defined [7] by

$$\left\langle T\left\{\frac{\phi_i^0\phi_i^0}{v_i^2}\right\}\right\rangle_q = \sum_n \frac{\sqrt{2}G_F Z_{i,n}}{q^2+m_n^2} \ . \tag{10}$$

Then

$$c_S \simeq (6.6\times 10^{-9})(m_{100})^{-2}Y \ , \tag{11}$$

where $m_{100}\equiv m_H/(100\text{ GeV})$ and

$$Y \equiv \sum_n \frac{m_H^2}{m_n^2}(\mathrm{Im}Z_{1,n}-\tan^2\beta\,\mathrm{Im}Z_{2,n}) \ . \tag{12}$$

If the sum over n is dominated by the lightest neutral Higgs boson $(n=H)$, it may be shown that the maximal value of Y is given in two-doublet models by

$$\begin{aligned} |Y_{\max} & \cong |\mathrm{Im}Z_1-\tan^2\beta\,\mathrm{Im}Z_2|_{\max} \\ & = \tfrac{1}{2}(\tan^2\beta+1) \ . \end{aligned} \tag{13}$$

It will be seen in Secs. III and IV that this indeed is also a good estimate of the *typical* size of Y in two doublet models. Therefore

$$|c_S| \sim (3.3\times 10^{-9})\frac{\tan^2\beta}{m_{100}^2} \ . \tag{14}$$

Thus, for $\tan\beta=10$ and $m_H=100$ GeV or for $\tan\beta=20$ and $m_H=200$ GeV, $|c_S|\sim 3.3\times 10^{-7}$. This should be compared to the experimental limit [9] from Tl of $c_S=(-2.7\times 8.3)\times 10^{-7}$.

In Ref. [5] the conditions required for the electron-nucleon operators to dominate over d_e in their effects in atoms were stated in terms of a parameter that was called there F. In terms of the parameters being studied here, $F=Y/2X_0$. Generally, one finds that $|F|\sim\tan\beta$. But for class I models, as will be seen in the next two sections, $|F|\sim\tan^2\beta$. In Ref. [5] it was found that for $|F|\gtrsim 15-25$ (depending on the atom) the effects of c_S are comparable or larger than those of d_e, for diamagnetic systems (Xe, Hg, and TlF), while for paramagnetic systems (Cs and Tl) $|F|$ would need to be $\gtrsim 250$.

III. CLASS I MODELS: TWO HIGGS DOUBLETS WITH SOFTLY BROKEN NFC

In order to study the properties of the Higgs sector in an expansion in $(1/\tan^2\beta)$, it is convenient to express the Higgs potential in terms not of masses and couplings, but of the vacuum expectation values and couplings, by expanding about the minimum of the potential. The most general two-doublet potential with softly broken NFC can be written as

$$\begin{aligned}V_H=&\tfrac{1}{2}g_1(\phi_1^+\phi_1-|v_1|^2)^2+\tfrac{1}{2}g_2(\phi_2^+\phi_2-|v_2|^2)^2\\&+g(\phi_1^+\phi_1-|v_1|^2)(\phi_2^+\phi_2-|v_2|^2)\\&+g'|\phi_1^+\phi_2-v_1^*v_2|^2+\mathrm{Re}[h(\phi_1^+\phi_2-v_1^*v_2)^2]\\&+\xi|v_2\phi_1-v_1\phi_2|^2\ .\end{aligned}\tag{15}$$

The sole *CP*-violating phase in V_H is $\arg(hv_1^{*2}v_2^2)\equiv 2\eta$.

In the unitary gauge, one can write the neutral components of the Higgs doublets as

$$\begin{aligned}\phi_1^0=&\frac{v_1}{\sqrt{2}|v_1|}\left[\Phi_1-i\frac{|v_2|}{v}\Phi_3\right],\\\phi_2^0=&\frac{v_2}{\sqrt{2}|v_2|}\left[\Phi_2+i\frac{|v_1|}{v}\Phi_3\right],\end{aligned}\tag{16}$$

where $v\equiv(|v_1|^2+|v_2|^2)^{1/2}$ and the Φ_i are real fields with $\langle\Phi_1\rangle=\sqrt{2}|v_1|$, $\langle\Phi_2\rangle=\sqrt{2}|v_2|$, and $\langle\Phi_3\rangle=0$. In the limit of vanishing *CP* violation, $\Phi_{1,2}$ are scalars and Φ_3 is a pseudoscalar. Denoting the mass eigenstates by $\tilde{\Phi}_n$, $\tilde{\Phi}_n=\sum_{m=1}^3 R_{nm}\Phi_m$. It can be shown straightforwardly that

$$\begin{aligned}X_0=&\tfrac{1}{2}(\sin\beta)^{-1}\sum_n\left[\frac{m_H^2}{m_n^2}\right][R_{n1}+\tan\beta R_{n2}]R_{n3}\ ,\\Y=&-(\tan^2\beta/\sin\beta)\sum_n\left[\frac{m_H^2}{m_n^2}\right][R_{n1}+\frac{1}{\tan\beta}R_{n2}]R_{n3}\ .\end{aligned}\tag{17}$$

It is a simple matter to expand Eq. (15) in terms of $\overline{\Phi}_i\equiv\Phi_i-\langle\Phi_i\rangle$, extract the $(\text{mass})^2$ matrix, and compute R_{nm} and m_n^2 to leading order in $(1/\tan^2\beta)$. It is then found after laborious but straightforward algebra that

$$\begin{aligned}X_0=&\tfrac{1}{4}\sin 2\eta\frac{(m_1^2-m_3^2)m_H^2}{m_1^2m_3^2}(1-\Delta^2/m_2^2)+O\left[\frac{1}{\tan^2\beta}\right],\\Y=&-\tfrac{1}{2}(\tan^2\beta)\sin 2\eta\frac{(m_1^2-m_3^2)m_H^2}{m_1^2m_3^2}+\text{const}\ ,\\F=&\frac{Y}{2X_0}=-\tan^2\beta(1-\Delta^2/m_2^2)+\text{const}\ ,\end{aligned}\tag{18}$$

where $\Delta^2\equiv 2(g-\xi)v^2$ and $m_2^2\cong 2g_2v^2$. Δ^2 can be either sign. One expects that typically Δ^2/m_2^2 is of order 1.

From Eq. (17) it can be seen where Eq. (7) comes from. By orthogonality, $\sum_{m=1}^3(R_{Hm})^2=1$. If the sum is replaced by the contribution of the lightest Higgs boson $(n=H)$,

$$X_0=\tfrac{1}{2}(\sin\beta)^{-1}(R_{H1}+\tan\beta R_{H2})R_{H3}\ ,$$

and this is clearly maximized by

$$(R_{H1},R_{H2},R_{H3})=\left[\frac{\cos\beta}{\sqrt{2}},\frac{\sin\beta}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$$

to give $X_{0(\max)}=\frac{1}{4}(\tan\beta/\sin^2\beta)$. However, from the actual $(\text{mass})^2$ matrix derived from Eq. (15), one finds that, in an expansion in $(1/\tan^2\beta)$, R_{12} and R_{23} are $O(1/\tan\beta)$, while the remaining R_{mn} are ~ 1. Thus $(\tan\beta R_{H2}R_{H3})$ is always ~ 1, whichever is the lightest Higgs boson. The maximum value of X_0 is achieved only if some ratio of the dimensionless parameters (g_1,g_2,g,g',h,ξ) is suitably tuned to be $O(1/\tan\beta)$.

In order to gain confidence that this result is not peculiar to the specific potential of Eq. (15), another class I model where NFC is broken spontaneously rather than softly is studied in the next section.

IV. CLASS I MODELS: TWO DOUBLETS PLUS A SINGLET WITH NFC

Natural flavor conservation in this case will mean that V_H is even under $\phi_0\rightarrow-\phi_1$, $\phi_2\rightarrow+\phi_2$, and $\sigma\rightarrow-\sigma$ (and/or under $\phi_1\rightarrow+\phi_1$, $\phi_2\rightarrow-\phi_2$, and $\sigma\rightarrow-\sigma$). σ is a real singlet field. There is a slight technical difficulty in writing V_H in terms of vacuum expectation values as in Eq. (7) while preserving the NFC symmetry. For example, the term $m_{12}\phi_1^+\phi_2\sigma$ would lead to the term $m_{12}(\phi_1^+\phi_2-v_1^*v_2)(\sigma-w)$, which has cross terms σ and $\phi_1^+\phi_2$ that both violate NFC and are linear in $\overline{\Phi}_i$ and $\overline{\sigma}$ $(w\equiv\langle\sigma\rangle)$. This problem is easily solved in this case by introducing redundant terms into V_H to cancel the offending σ and $\phi_1^+\phi_2$. The coefficients of the redundant terms are therefore solvable in terms of the other parameters of the potential. The most general potential can then be written as

$$V_H=\frac{1}{2}g_1(\phi_1^+\phi_1-|v_1|^2)^2+\frac{1}{2}g_2(\phi_2^+\phi_2-|v_2|^2)^2+g(\phi_1^+\phi_1-|v_1|^2)(\phi_2^+\phi_2-|v_2|^2)$$
$$+g'|\phi_1^+\phi_2-v_1^*v_2|^2+g''(\phi_1^+\phi_2\phi_2^+\phi_1-\phi_1^+\phi_1\phi_2^+\phi_2)+\mathrm{Re}[h(\phi_1^+\phi_2-v_1^*v_2)^2]$$
$$+a(\sigma^2-w^2)+\frac{1}{2}\mu^2(\sigma-w)^2+\mathrm{Re}[m_{12}(\phi_1^+\phi_2-v_1^*v_2)(\sigma-w)]$$
$$+k_1(\phi_1^+\phi_1-|v_1|^2)(\sigma^2-w^2)+k_2(\phi_2^+\phi_2-|v_2|^2)(\sigma^2-w^2)\ , \tag{19}$$

where the redundant parameters μ^2 and g' are given by

$$\mu^2 w+\mathrm{Re}[m_{12}v_1^*v_2]=0\ ,$$
$$g'|v_1v_2|^2+h(v_1^{*2}v_2^2)+\frac{1}{2}m_{12}(v_1^*v_2)w=0\ . \tag{20}$$

Note that there is only one CP-violating phase $2\eta\equiv\arg[h(v_1^{*2}v_2^2)]$, the phase of $m_{12}v_1^*v_2$ being fixed by Eq. (20). As in the last section, the (mass)^2 matrix can be diagonalized and R_{mn} and m_n^2 found to leading order in $1/\tan^2\beta$. In spite of the greater number of free parameters (13 versus 9) and the larger matrices (4×4 versus 3×3), the final results are remarkably simple and similar in form to those found in the last section:

$$X_0=\frac{1}{4}\sin2\eta\frac{(m_1^2-m_3^2)m_H^2}{m_1^2m_3^2}\{1-\delta\}+O(1/\tan^2\beta)\ ,$$
$$Y=-\frac{1}{2}\tan^2\beta\sin2\eta\frac{(m_1^2-m_3^2)m_H^2}{m_1^2m_3^2}+\mathrm{const}\ ,$$
$$F=-\tan^2\beta\{1-\delta\}+\mathrm{const}\ ,$$
$$\delta\equiv\cos^2\zeta\frac{\Delta^2}{m_2^2}+\sin^2\zeta\frac{\Delta^2}{m_4^2}+2\sin\zeta\cos\zeta\left[\frac{\Delta'^2}{m_2^2}-\frac{\Delta'^2}{m_4^2}\right]\ . \tag{21}$$

Note that for $\sin\zeta\to0$ these reduce to the forms given in Eqs. (18). here m_n are the mass eigenvalues, $\overline{\Phi}_4\equiv\sigma-w$, $\Delta^2\equiv2gv^2$, $\Delta'^2\equiv\sqrt{2}k_1(vw)$, ζ is the $\overline{\Phi}_2-\overline{\Phi}_4$ mixing angle, and η is the CP-violating phase as well as the $\overline{\Phi}_1-\overline{\Phi}_3$ mixing angle. One expects, as in the simpler example of the last section, that the factors $(m_1^2-m_3^2)m_H^2/m_1^2m_3^2$ and $\Delta^2/m_2^2\cong g/g_2$ will be of order unity typically. Here, however, there is a feature that did not arise in the previous example. The vacuum expectation value w of the singlet field σ is not necessarily of the order of the weak scale. And since $v_1/v_2\equiv1/\tan\beta$ is being assumed to be small, it is not clear what the "typical" value of w should be assumed to be. However, it turns out not to matter. If $w\gg v$, $\Delta^2/m_4^2\sim(v/w)^2$, $\Delta'^2/m_4^2\sim(v/w)$, $\Delta'^2/m_2^2\sim(w/v)$, and $\sin\zeta\sim(v/w)$, so that δ remains of order unity. The same holds if $w\ll v$. Thus the conclusion of the previous section remains true as well here: Unless some ratio of the dimensionless couplings is tuned to be $O(1/\tan\beta)$, one has the results $X_0\sim\frac{1}{4}\sin2\eta$, $Y\sim\frac{1}{2}\tan^2\beta\sin2\eta$, $F\sim\tan^2\beta$.

V. TYPICAL MAGNITUDES OF OTHER CP-VIOLATING EFFECTS AND PARAMETERS IN TWO-DOUBLET MODELS

In several recent papers [10,11], it has been suggested that the top-quark EDM may be large enough in two-doublet models to be measurable at future colliders such as the Superconducting Super Collider (SSC), CERN Large Hadron Collider (LHC), and Next Linear Collider (NLC). The largest contribution to the top-quark EDM in these models arises from the one-loop diagram. The EDM is given in Ref. [10] to be

$$d_t(q^2)=\frac{2\sqrt{2}}{3(4\pi)^2}G_Fm_te\sum_n\mathrm{Im}Z_{2,n}f\left[\frac{m_n^2}{m_t^2},\frac{q^2}{m_t^2}\right] \tag{22}$$

in the notation of this paper. (Note the labels of ϕ_1 and ϕ_2 are interchanged in Ref. [10] with respect to this paper.) f is a function which for small q^2/m_t^2 and large m_n^2/m_t^2 goes as

$$\frac{m_t^2}{m_n^2}\left[\ln\frac{m_n^2}{m_t^2}-\frac{3}{2}\right]\ .$$

One can obtain a reasonable estimate of $d_t(q^2)$ by replacing

$$f\left[\frac{m_n^2}{m_t^2},\frac{q^2}{m_t^2}\right]$$

by

$$\frac{m_H^2}{m_n^2}f\left[\frac{m_H^2}{m_t^2},\frac{q^2}{m_t^2}\right]\ ;$$

then

$$d_t(q^2)\simeq\frac{2\sqrt{2}}{3(4\pi)^2}G_Fm_tef\left[\frac{m_H^2}{m_t^2},\frac{q^2}{m_t^2}\right]X_2\ , \tag{23}$$

$$X_2\equiv\sum_n\left[\frac{m_H^2}{m_n^2}\right]\mathrm{Im}Z_{2,n}\ , \tag{24}$$

in analogy with the definition of X_0 given in Eq. (6). One finds

$$X_2=\frac{1}{\tan\beta\sin\beta}\sum_n\left[\frac{m_N^2}{m_n^2}\right]R_{n2}R_{n3}\ ,$$

so that, if the lightest Higgs boson dominates the sum over n,

$$|X_2|_{(\max)}\cong\frac{1}{2\tan\beta\sin\beta}\ . \tag{25}$$

Thus, typically, in multi-Higgs-boson models, one expects that $X_2\sim\frac{1}{2}(\tan\beta)^{-1}$. However, in the models of Secs. III and IV,

$$X_2=\frac{-1}{2\tan^2\beta}\sin 2\eta\frac{(m_1^2-m_3^2)m_H^2}{m_1^2m_3^2}\delta+\text{const}\ , \qquad (26)$$

where δ is given by Δ^2/m_2^2 and by Eq. (21), respectively, for the two models. So that, typically, in class I models, in contrast to the general case, one expects

$$X_2\sim\frac{1}{2\tan^2\beta}\ . \qquad (27)$$

Another quantity of great importance is the neutron EDM. There are three kinds of operators that are expected to give the largest contributions to d_n: the Weinberg three-gluon operator [1], the EDM's [2,3] of the u and d quarks, and the chromo-EDM's [2,3] of the u,d and s quarks.

A. Three-gluon operator

The largest contribution in two-doublet models to the Weinberg three-gluon operator comes from a diagram with a top-quark loop. It is easily seen that in the kind of approximation used above the coefficient of this operator is proportional to X_2 and therefore by Eq. (25) is generally of $O(1/\tan\beta)$, but by Eqs. (26) and (27) is of $O(1/\tan^2\beta)$ in class I models.

B. EDM's of the light quarks

The EDM of the d quark has the same dependence of the Higgs parameters as does d_e and so goes as $\sin^2\beta X_0\cong X_0$ [see Eqs. (4)–(6)], which is $O(\tan\beta)$ generally, but in class I models is typically ~ 1 by Eqs. (18) and (21). The EDM of the u quark, on the other hand, goes as $\cos^2\beta X_0\cong(1/\tan^2\beta)X_0$, which is typically of $O(1/\tan\beta)$, but in class I models is $O(1/\tan^2\beta)$.

C. Chromo-EDM's of the light quarks

The chromo-EDM's of the d and s quarks have the same dependence as the top-quark-loop contribution to the electron EDM and so, for large m_H, are proportional to X_0. The chromo-EDM of the u quark, on the other hand, is proportional to X_2.

For large $\tan\beta$, then, both in the general case and in class I models, the dominant contributions to d_n come from the EDM and chromo-EDM operators of the down quarks. The other contributions including the Weinberg three-gluon operator are down by $O(1/\tan^2\beta)$.

For completenesses, one can define

$$X_1\equiv\sum_n\left[\frac{m_H^2}{m_n^2}\right]\text{Im}Z_{1,n}\ ,$$
$$\tilde{X}_0\equiv\sum_n\left[\frac{m_H^2}{m_n^2}\right]\text{Im}\tilde{Z}_{0,n}\ . \qquad (28)$$

One can show that

$$X_1=-\frac{\tan^2\beta}{\sin\beta}\sum_n\left[\frac{m_H^2}{m_n^2}\right]R_{n1}R_{n3}\ ,$$
$$\tilde{X}_0=\frac{1}{2\sin\beta}\sum_n\left[\frac{m_H^2}{m_n^2}\right][R_{1n}-\tan\beta R_{2n}]R_{3n}\ .$$

So that if the lightest Higgs field dominates the sum over n,

$$|X_1|_{(\max)}\cong+\frac{1}{2}\frac{\tan^2\beta}{2\sin\beta}\ ,$$
$$|\tilde{X}_0|_{(\max)}\cong\frac{1}{4}\frac{\tan\beta}{\sin^2\beta}\ . \qquad (29)$$

So that generally, $X_1\sim\frac{1}{2}\tan^2\beta$ and $\tilde{X}_0\sim\frac{1}{4}\tan\beta$. In the models of Secs. III and IV, however,

$$X_1=-\frac{1}{2}\tan^2\beta\sin 2\eta\frac{(m_1^2-m_3^2)m_H^2}{m_1^2m_3^2}+\text{const}\ ,$$
$$\tilde{X}_0=\frac{1}{4}\sin 2\eta\frac{(m_1^2-m_3^2)m_H^2}{m_1^2m_3^2}\{1+\delta\}+O(1/\tan^2\beta)\ , \qquad (30)$$

where δ is given by Δ^2/m_2^2 or Eq. (21), respectively, for the two models, so that in class I models typically, for $\tan\beta\gg 1$,

$$X_1\sim\tfrac{1}{2}\tan^2\beta\ ,$$
$$\tilde{X}_0\sim\tfrac{1}{4}\ . \qquad (31)$$

Note that, in general, $X_0+\tilde{X}_0=-\cot^2\beta X_1$ and $X_0-\tilde{X}_0=\tan^2\beta X_2$, as can be verified by Eqs. (18), (26), and (30).

VI. CLASS II MODELS: SUPERSYMMETRY

The simplest supersymmetric extension of the standard model that can have *CP* violation in the Higgs sector is discussed in Ref. [7] (where, however, *CP* was imposed). In addition to the two doublets H_1 and H_2 (which will be called here ϕ_1 and $\tilde{\phi}_2$), there is a singlet superfield N. The superpotential is chosen to be purely cubic and contains H_1H_2N and N^3 terms. This gives rise in the ordinary potential to $(\phi_1^\dagger\phi_2N)$ and $\phi_1^\dagger\phi_2(N^*)^2$ terms as well as an N^2 term. This gives three complex coefficients. Two phases can be rotated away by rephasing ϕ_1, ϕ_2, and N. But one phase remains. There is no $(\phi_1^\dagger\phi^2)^2$ term in V_H. This then is an example of a class II model.

This model can be compared to the class I model of Sec. IV, which also had a singlet field σ that mixed with the two doublets. There the mixing of $\overline{\Phi}_2\equiv\sigma-\langle\sigma\rangle$ with the pseudoscalar Φ_3 arose from the term in Eq. (19),

$$\text{Re}[m_{12}(\phi_1^\dagger\phi_2-v_1^*v_2)(\sigma-w)]\ .$$

This term gives

$$-\frac{v}{\sqrt{2}}[\text{Im}(m_{12}v_1^*v_2)|v_1v_2|]\{\Phi_3\overline{\Phi}_4\}\ .$$

However, the phase of $(m_{12}v_1^*v_2)$ could be rotated away were it not for the presence of the quartic term

$\text{Re}[h(\phi_1^\dagger\phi_2 - v_1^\dagger v_2)^2]$. Thus it happens that there is a relation

$$\text{Im}(m_{12}v_1^* v_2) = -2\,\text{Im}[h(v_1^* v_2)^2]/w \ , \qquad (32)$$

as can be seen from Eq. (20). Therefore the mixing of $\Phi_3\overline{\Phi}_4$ is given by

$$\sqrt{2}\left[\frac{v}{w}\right]\{\text{Im}[h(v_1^* v_2)^2]/|v_1 v_2|\}\{\Phi_3\overline{\Phi}_4\}$$

and is $O(v_1)$, which is to say $O(1/\tan\beta)$. Now consider the expression for X_2:

$$X_2 = \frac{1}{\tan\beta\sin\beta}\sum_n\left[\frac{m_H}{m_n}\right]^2 R_{n2}R_{n3} \ . \qquad (33)$$

The term with $n=4$ contributes $O(1/\tan^2\beta)$ since $R_{43}=O(1/\tan\beta)$, as was just shown. [The other terms also contribute $O(1/\tan\beta)$ since R_{12} and R_{23} are also $O(1/\tan\beta)$.]

Now consider the supersymmetric model. Calling the scalar part of N also by $\overline{\Phi}_4$, one sees that the $\overline{\Phi}_4-\Phi_3$ mixing arises from both the terms $\phi_1^\dagger\phi_2 N$ and $\phi_1^\dagger\phi_2(N^*)^2$. Calling the coefficients of these terms m_{12} and m'_{12}, this mixing, as in the class I model, goes as

$$[\text{Im}(m_{12}v_1^* v_2)/|v_1 v_2|][(\Phi_3\overline{\Phi}_4)$$

or similarly with m'_{12}. The crucial difference is that the phases $\text{Im}(m_{12}v_1^* v_2)$ and $\text{Im}(m'_{1,2}v_1^* v_2)$ are related *to each other* rather than to a term higher order in v_1 as in Eq. (32). Thus $R_{43}\sim 1$ and $X_2=O(1/\tan\beta)$ rather than $O(1/\tan^2\beta)$.

For class II models, which include the generality of supersymmetric models, the typical values of the *CP*-violating parameters X_i are of the same order in $\tan\beta$ as the maximum values stated in Eqs. (7), (25), and (29).

VII. CLASS III MODELS: THREE-DOUBLET MODELS

Consider a three-Higgs-doublet model where NFC is enforced by a discrete symmetry which ensures that V_H is even in each of the fields ϕ_1, ϕ_2, ϕ_3. There are then three terms in V_H that have complex coefficients: $c_{ij}(\phi_i^\dagger\phi_j)^2+\text{H.c.}$, where $i\neq j$. Since the two relative phases of the ϕ_i can be redefined, there is one genuine *CP*-violating phase in V_H. Expanding

$$\phi_i = \frac{1}{\sqrt{2}}\frac{v_i}{|v_i|}(\Phi_i + i\Pi_i) \ ,$$

where $\langle\Phi_i\rangle=\sqrt{2}|v_i|$ and $\langle\Pi_i\rangle=0$, one finds that there are linear terms for the Π_i of the form

$$-\sum_{i\neq j}\sqrt{2}(\text{Im}c_{ij})v_i v_j(v_i\Pi_j - v_j\Pi_i) \ ,$$

in the basis where the v_i are real. In that basis it must therefore be that

$$\text{Im}(c_{ij}) = \sin\eta\,\epsilon_{ijk}(v_k)^2 \ . \qquad (34)$$

This implies that for $v_2/v_1\gg 1$, $\text{Im}c_{23}\sim(1/\tan^2\beta)\text{Im}c_{31}$. If one were to assume that all of the quartic couplings, including all of the $\text{Im}c_{ij}$, were of the same order, then the conclusions with regard to the order in $1/\tan^2\beta$ of the quantities X_0, $\tilde{X}_0$, X_1, and X_2 would be the same as stated in the preceding sections for two-doublet models. However, the dependence of the phases on $\tan^2\beta$ given in Eq. (34) vitiates those earlier conclusions for three-doublet models.

In unitary gauge one can write

$$\phi_1^0 = \frac{1}{\sqrt{2}}\left[\Phi_1 - i\frac{v_1 v_3}{v v_0}\Phi_5\right] \ ,$$
$$\phi_2^0 = \frac{1}{\sqrt{2}}\left[\Phi_2 + i\frac{v_1}{v_0}\Phi_4 + i\frac{v_2 v_3}{v v_0}\Phi_5\right] \ , \qquad (35)$$
$$\phi_3^0 = \frac{1}{\sqrt{2}}\left[\Phi_3 - i\frac{v}{v_0}\Phi_5\right] \ ,$$

in the basis where the v_i are real. $v\equiv(v_1^2+v_2^2+v_3^2)^{1/2}$, $v_0\equiv(v_1^2+v_2^2)^{1/2}$. Then one can show that

$$c_S \propto \frac{v^2}{v_1^2 v_2}\sum_n(v_1 R_{n2}+v_2 R_{n1})\left[\frac{-v_2}{v_0}R_{n4}+\frac{v_1 v_3}{v v_0}R_{n5}\right] \ . \qquad (36)$$

The scalar-pseudoscalar mixing is given by

$$\sum_{ij}2(\text{Im}c_{ij})v_i v_j(\Pi_i\overline{\Phi}_j) \propto (\sin\eta)v_1 v_2 v_3\epsilon_{ijk}(\Pi_i\overline{\Phi}_j v_k) \ ,$$

where Π_i are the imaginary parts of ϕ_i given in Eq. (35) and are linear combinations of Φ_4 and Φ_5. From these equations it can be seen straightforwardly that $c_S\sim(v_1)^0\sim(\tan\beta)^0$. This is in contrast with the two-doublet models in general and class I models in particular where $c_S\sim\tan^2\beta$.

VIII. CONCLUSIONS

In general two-doublet models, one has, for large $\tan\beta$,

$$|X_0|\sim\tfrac{1}{4}(\tan\beta)^{+1} \ ,$$
$$|\tilde{X}_0|\sim\tfrac{1}{4}(\tan\beta)^{+1} \ ,$$
$$|X_1|\sim\tfrac{1}{2}(\tan\beta)^2 \ , \qquad (37)$$
$$|X_2|\sim\tfrac{1}{2}(\tan\beta)^{-1} \ ,$$
$$|Y| = |X_1 - \tan^2\beta X_2| \sim\tfrac{1}{2}(\tan\beta)^2 \ ,$$

where for class I models, in particular,

$$|X_0|\sim\tfrac{1}{4}(\tan\beta)^0 \ ,$$
$$|\tilde{X}_0|\sim\tfrac{1}{4}(\tan\beta)^0 \ ,$$
$$|X_1|\sim\tfrac{1}{2}(\tan\beta)^2 \ , \qquad (38)$$
$$|X_2|\sim\tfrac{1}{2}(\tan\beta)^{-2} \ ,$$
$$|Y|\sim\tfrac{1}{2}(\tan\beta)^2 \ .$$

The EDM of the electron can be written

$$d_e \simeq k(m_t^2, m_H^2)X_0(10^{-26}e\text{ cm}) \ , \qquad (39)$$

where for $m_t \simeq 2M_W$, $k(m_H=120\text{ GeV}) \simeq 1$, $k(m_H=160\text{ GeV}) \simeq \frac{1}{2}$, and $k(m_H \gtrsim 320\text{ GeV}) \simeq \frac{1}{4}$. [See discussion after Eq. (4).] The scalar $e-N$ operator has a coefficient given by

$$c_S \simeq (6.6\times 10^{-9})(m_{100})^{-2} Y \ , \tag{40}$$

$m_{100} = m_H/100$ GeV, so that

$$c_S \sim (3.3\times 10^{-9})(\tan\beta/m_{100})^2 \ . \tag{41}$$

Finally, in two-doublet models, whether or not class I, the dominant contributions to the neutron EDM are the EDM and chromo-EDM operators of down-type quarks. The other contributions including the three-gluon operator are relatively suppressed by $O(1/\tan^2\beta)$. The top-quark EDM, which is roughly proportional to X_2, is suppressed by $O(1/\tan\beta)$ generally and by $O(1/\tan^2\beta)$ in class I models.

The most significant result is that in the simple type of models that have been called here class I, the $e-N$, T- and P-odd operators are enhanced relative to d_e by $O(\tan^2\beta)$ and can therefore give the dominant contribution in atoms for $\tan\beta \gtrsim 5$ (diamagnetic) or $\tan\beta \gtrsim 15$ (paramagnetic).

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