

## Decay of the Z boson into scalar particles

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In extensions of the standard model, light scalar particles are often possible because of symmetry considerations. We study the decay of the Z boson into such particles. In particular, we consider for illustration the scalar sector of a recently proposed model of the 17-keV neutrino which satisfies all laboratory, astrophysical, and cosmological constraints.

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### I. INTRODUCTION

In the standard model, the Z boson may decay into the Higgs boson  $H$  and a fermion-antifermion pair. From the absence of such events, it has been deduced that  $m_H > 59$  GeV [1]. On the other hand, in extensions of the standard model with a richer scalar sector, there are usually many more scalar particles, some of which may be light enough to be decay products of the Z, but with perhaps different signatures. For example, the Z boson may now decay into two different scalar particles. Such a process is usually unsuppressed if kinematically allowed and serves to set stringent limits on the masses of the particles involved. There is, however, an important exception. If a certain scalar particle is coupled to the Z boson only in association with another heavier particle, then it may, in fact, be light and not be produced in a two-body decay. Three-body decays should then be considered and could well become a first hint for any new physics beyond the

standard model.

In Sec. II we present a specific example of an extended scalar sector which goes with a recently proposed model [2] of a possible 17-keV neutrino which satisfies all laboratory, astrophysical, and cosmological constraints [3]. In Sec. III we single out a particular scalar boson of this model and show how it can be light and still be consistent with all present experimental data. In Sec. IV we discuss the decay of the Z boson into this scalar particle as a possible means of discovering its existence. Finally, in Sec. V there are some concluding remarks.

### II. EXAMPLE OF AN EXTENDED SCALAR SECTOR

To be specific, consider the following extended scalar sector. Many features are common to other extensions of the standard model, so it will serve as a good example of what can be learned from the decay of the Z boson into scalar particles. Let

$$\begin{aligned}
 V = & \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\
 & + m_1^2 \bar{\chi}_1 \chi_1 + m_2^2 \bar{\chi}_2 \chi_2 + m_{12}^2 (\bar{\chi}_1 \chi_2 + \bar{\chi}_2 \chi_1) + \frac{1}{2} \eta_1 (\bar{\chi}_1 \chi_1)^2 + \frac{1}{2} \eta_2 (\bar{\chi}_2 \chi_2)^2 + \eta_3 (\bar{\chi}_1 \chi_1) (\bar{\chi}_2 \chi_2) + \sum_{i,j} f_{ij} (\Phi_i^\dagger \Phi_i) (\bar{\chi}_j \chi_j),
 \end{aligned}
 \tag{1}$$

where  $\Phi_i = (\phi_i^+, \phi_i^0)$  are doublets and  $\chi_i$  are singlets under the usual  $SU(2) \times U(1)$  electroweak gauge group. This is the specific Higgs potential which goes with a recently proposed model [2] of a possible 17-keV neutrino. There is an assumed  $Z_5$  discrete symmetry with elements 1,  $\omega$ ,  $\omega^2$ ,  $\omega^{-2}$ , and  $\omega^{-1}$ , with  $\omega^5 = 1$ , under which the three lepton families, each consisting of a left-handed doublet  $(\nu_i, l_i)_L$  and two right-handed singlets  $\nu_{iR}, l_{iR}$ , transform as  $\omega^{i-1}$  ( $i=1-3$ ). In addition,  $\Phi_1$  transform as 1,  $\Phi_2$  as  $\omega^{-2}$ ,  $\chi_1$  as  $\omega^2$ , and  $\chi_2$  as  $\omega$ . Furthermore,  $\nu_i$  and  $l_i$  have lepton number  $L=1$  and  $\chi_i$  have  $L=-2$ . Note also that the terms  $\mu_{12}^2$  and  $m_{12}^2$  in  $V$  break  $Z_5$  but only softly.

If  $\mu_{12}^2 = m_{12}^2 = 0$ , then  $V$  actually has four conserved additive quantum numbers corresponding to  $\Phi_1, \Phi_2, \chi_1$ , and  $\chi_2$  separately. As the vacuum expectation values  $\langle \phi_i^0 \rangle$ ,

$\langle \phi_2^0 \rangle$ ,  $\langle \chi_1 \rangle$ , and  $\langle \chi_2 \rangle$  become nonzero, four Goldstone bosons would appear. One of them gets absorbed by the Z boson because the gauge symmetry is also broken, but there would remain three physical massless Goldstone bosons. However, since  $\mu_{12}^2$  and  $m_{12}^2$  are nonzero,  $V$  has only two conserved additive quantum numbers. The one remaining physical massless Goldstone boson corresponds to the spontaneous breaking of lepton number and is called the Majoron [4].

Let  $\langle \phi_{1,2}^0 \rangle = v_{1,2}$  and  $\langle \chi_{1,2} \rangle = u_{1,2}$ , then the four equations of constraint are

$$\mu_1^2 + \lambda_1 v_1^2 + (\lambda_3 + \lambda_4) v_2^2 + f_{11} u_1^2 + f_{12} u_2^2 + \mu_{12}^2 v_2 / v_1 = 0,
 \tag{2}$$

$$\mu_2^2 + \lambda_2 v_2^2 + (\lambda_3 + \lambda_4) v_1^2 + f_{21} u_1^2 + f_{22} u_2^2 + \mu_{12}^2 v_1 / v_2 = 0, \quad (3)$$

$$m_1^2 + \eta_1 u_1^2 + \eta_3 u_2^2 + f_{11} v_1^2 + f_{21} v_2^2 + m_{12}^2 u_2 / u_1 = 0, \quad (4)$$

and

$$m_2^2 + \eta_2 u_2^2 + \eta_3 u_1^2 + f_{12} v_1^2 + f_{22} v_2^2 + m_{12}^2 u_1 / u_2 = 0. \quad (5)$$

As a result, there are only  $12 - 4 = 8$  massive degrees of freedom in the scalar sector. Two of these correspond to one charged particle:

$$h^\pm = \sin\beta \phi_1^\pm - \cos\beta \phi_2^\pm, \quad (6)$$

with the mass squared given by

$$m_\pm^2 = - \left[ \lambda_4 + \frac{\mu_{12}^2}{v_1 v_2} \right] (v_1^2 + v_2^2), \quad (7)$$

where the angle  $\beta$  is defined as

$$\tan\beta \equiv v_2 / v_1, \quad (8)$$

and

$$(v_1^2 + v_2^2)^{1/2} = (2\sqrt{2}G_F)^{-1/2} \simeq 174 \text{ GeV}. \quad (9)$$

Four others correspond to linear combinations of  $\text{Re}\phi_{1,2}^0$  and  $\text{Re}\chi_{1,2}$  forming a  $4 \times 4$  mass-squared matrix given by

$$M^2 = \begin{pmatrix} 2\lambda_1 v_1^2 - \mu_{12}^2 v_2 / v_1 & 2(\lambda_3 + \lambda_4) v_1 v_2 + \mu_{12}^2 & 2f_{11} v_1 u_1 & 2f_{12} v_1 u_2 \\ 2(\lambda_3 + \lambda_4) v_1 v_2 + \mu_{12}^2 & 2\lambda_2 v_2^2 - \mu_{12}^2 v_1 / v_2 & 2f_{21} v_2 u_1 & 2f_{22} v_2 u_2 \\ 2f_{11} v_1 u_1 & 2f_{21} v_2 u_1 & 2\eta_1 u_1^2 - m_{12}^2 u_2 / u_1 & 2\eta_3 u_1 u_2 + m_{12}^2 \\ 2f_{12} v_1 u_2 & 2f_{22} v_2 u_2 & 2\eta_3 u_1 u_2 + m_{12}^2 & 2\eta_2 u_2^2 - m_{12}^2 u_1 / u_2 \end{pmatrix}. \quad (10)$$

The two remaining massive scalar bosons are

$$h_5^0 = \sqrt{2}(\sin\beta \text{Im}\phi_1^0 - \cos\beta \text{Im}\phi_2^0) \quad (11)$$

and

$$h_6^0 = \sqrt{2}(\sin\gamma \text{Im}\chi_1 - \cos\gamma \text{Im}\chi_2), \quad (12)$$

where  $\tan\gamma \equiv u_2 / u_1$ , with masses squared given by  $-\mu_{12}^2(v_1^2 + v_2^2) / v_1 v_2$  and  $-m_{12}^2(u_1^2 + u_2^2) / u_1 u_2$ , respectively. This shows explicitly the role of the terms  $\mu_{12}^2$  and

$m_{12}^2$  in breaking the would-be symmetries  $L_e - L_\tau$  and  $L_\mu - L_\tau$ . Of the four massless degrees of freedom, three are absorbed by the vector gauge bosons, and one remains as the Majoron:

$$\chi_0 = [2(u_1^2 + u_2^2)]^{1/2} \arctan \left[ \frac{\cos\gamma \text{Im}\chi_1 + \sin\gamma \text{Im}\chi_2}{\cos\gamma \text{Re}\chi_1 + \sin\gamma \text{Re}\chi_2} \right]. \quad (13)$$

Consider now the Yukawa interactions. The quarks couple only to  $\Phi_1$ , but the leptons interact with all the scalar bosons as follows [2]:

$$\begin{aligned} -\mathcal{L} = & \frac{a}{v_1} (\overline{v_1, l_1})_L l_{1R} \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} + \frac{b}{v_1} (\overline{v_2, l_2})_L l_{2R} \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} + \frac{c}{v_1} (\overline{v_3, l_3})_L l_{3R} \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \\ & + \frac{d}{v_2} (\overline{v_1, l_1})_L l_{3R} \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} + \frac{A}{v_1} (\overline{v_1, l_1})_L v_{1R} \begin{pmatrix} \overline{\phi_1^0} \\ -\phi_1^- \end{pmatrix} + \frac{B}{v_1} (\overline{v_2, l_2})_L v_{2R} \begin{pmatrix} \overline{\phi_1^0} \\ -\phi_1^- \end{pmatrix} \\ & + \frac{C}{v_1} (\overline{v_3, l_3})_L v_{3R} \begin{pmatrix} \overline{\phi_1^0} \\ -\phi_1^- \end{pmatrix} + \frac{D}{v_2} (\overline{v_3, l_3})_L v_{1R} \begin{pmatrix} \overline{\phi_2^0} \\ -\phi_2^- \end{pmatrix} + \frac{E}{2u_1} (\overline{v_{2R}^c} v_{3R} + \overline{v_{3R}^c} v_{2R}) \chi_1 + \frac{F}{2u_2} \overline{v_{3R}^c} v_{3R} \chi_2 + \text{H.c.} \end{aligned} \quad (14)$$

In the above,  $l_2$  is exactly  $\mu$ , but  $l_1$  and  $l_3$  mix to form  $e$  and  $\tau$ . Furthermore,  $a \simeq m_e$ ,  $b = m_\mu$ ,  $c \simeq m_\tau$ , and  $d/c$  is the mixing, which is perhaps of order 0.1. Let  $l_{1L} = c_L e_L - s_L \tau_L$ ,  $l_{3L} = s_L e_L + c_L \tau_L$ , and similarly for  $l_{1R}$  and  $l_{3R}$ , then  $s_L \simeq -d/c$ ,  $s_R \simeq s_L m_e/m_\tau$ , and  $c_{L,R} \equiv (1 - s_{L,R}^2)^{1/2} \simeq 1$ . Since we shall be interested in the scalar boson  $h_5^0$ , we note here its interaction with  $e$  and  $\tau$ :

$$\begin{aligned} -\mathcal{L} = & im_e \tan\beta (\sqrt{2}G_F)^{1/2} \left[ 1 + \frac{d^2}{c^2 \sin^2\beta} \right] \bar{e} \gamma_5 e h_5^0 \\ & + im_\tau \tan\beta (\sqrt{2}G_F)^{1/2} \left[ 1 - \frac{d^2}{c^2 \sin^2\beta} \right] \bar{\tau} \gamma_5 \tau h_5^0 \\ & + \frac{im_e (\sqrt{2}G_F)^{1/2}}{\sin\beta \cos\beta} \left[ \frac{d}{c} \right]^3 \bar{\tau}_L e_R h_5^0 \\ & - \frac{im_\tau (\sqrt{2}G_F)^{1/2}}{\sin\beta \cos\beta} \left[ \frac{d}{c} \right] \bar{e}_L \tau_R h_5^0. \end{aligned} \quad (15)$$

### III. CONSTRAINTS ON THE $h_5^0$ MASS

Consider now the couplings of the  $W$  and  $Z$  bosons to the various scalar particles of the previous section. They are easily extracted from

$$\mathcal{L} = (D^\mu \Phi_1)^\dagger (D_\mu \Phi_1) + (D^\mu \Phi_2)^\dagger (D_\mu \Phi_2), \quad (16)$$

where the covariant derivative  $D_\mu$  is given by

$$\begin{aligned} D_\mu = & \partial_\mu + \frac{ie\sqrt{2}}{\sin\theta_W} (T^+ W_\mu^+ + T^- W_\mu^-) \\ & + \frac{ie}{\sin\theta_W \cos\theta_W} (T^3 - \sin^2\theta_W Q) Z_\mu + ieQ A_\mu. \end{aligned} \quad (17)$$

As a result, it can readily be verified that whereas  $Z_\mu Z^\mu \text{Re}\phi_{1,2}^0$  couplings exist, there is no  $Z_\mu Z^\mu h_5^0$  coupling. Hence the experimentally well-studied decay  $Z \rightarrow "Z" + H$ , where the virtual "Z" converts into a fermion-antifermion pair, is of no use in limiting the mass of  $h_5^0$ . Furthermore, whereas  $Z$  couples to  $h^+ h^-$ , it only couples to  $h_5^0$  in association with  $\text{Re}\phi_{1,2}^0$ . Similarly,  $W$  only couples to  $h_5^0$  in association with  $h^\pm$ . Since these other scalar particles may well be more massive than the  $Z$  and  $W$  bosons themselves, there is again no constraint on the mass of  $h_5^0$  from their two-body decays.

$$\begin{aligned} -\mathcal{L} = & v_1 \text{Re}\phi_{1,2}^0 [\lambda_1 \sin^2\beta + (\lambda_3 + \lambda_4) \cos^2\beta] (h_5^0)^2 + v_2 \text{Re}\phi_{1,2}^0 [\lambda_2 \cos^2\beta + (\lambda_3 + \lambda_4) \sin^2\beta] (h_5^0)^2 \\ & + u_1 \text{Re}\chi_1 (f_{11} \cos^2\beta + f_{21} \sin^2\beta) (h_5^0)^2 + u_2 \text{Re}\chi_2 (f_{12} \cos^2\beta + f_{22} \sin^2\beta) (h_5^0)^2 \\ & - (u_1 \text{Re}\chi_1 + u_2 \text{Re}\chi_2) \partial_\mu \chi_0 \partial^\mu \chi_0 / (u_1^2 + u_2^2), \end{aligned} \quad (20)$$

where the last term actually comes from the  $\partial_\mu \bar{\chi}_i \partial^\mu \chi_i$  piece of the Lagrangian. Now  $Z$  couples to  $h_5^0$  in association with  $\sqrt{2}(\sin\beta \text{Re}\phi_1^0 - \cos\beta \text{Re}\phi_2^0)$  with coupling strength  $e/(2\sin\theta_W \cos\theta_W)$  and  $\text{Re}\phi_{1,2}^0$  mixes with  $\text{Re}\chi_{1,2}$  through Eq. (10); hence, the decays

The decay  $\tau \rightarrow e h_5^0$  is possible if kinematically allowed. From Eq. (15), we find

$$\Gamma = \frac{\sqrt{2}G_F m_\tau^3}{32\pi \sin^2\beta \cos^2\beta} \left[ \frac{d}{c} \right]^2 \left[ 1 - \frac{m^2}{m_\tau^2} \right]^2, \quad (18)$$

where  $m$  is the mass of  $h_5^0$  and  $m_e$  has been neglected. For  $d/c=0.1$  and  $\sin^2\beta=\cos^2\beta=0.5$ , we have  $\Gamma=3.68 \times 10^{-8} (1 - m^2/m_\tau^2)^2$  GeV, which is some four orders of magnitude greater than the observed  $\tau$  decay rate, hence  $m > m_\tau$  is required. Since  $h_5^0$  also couples to  $\bar{e} \gamma_5 e$ , the decay  $\tau^\pm \rightarrow e^\pm e^\pm e^\mp$  is still possible and its amplitude is given by

$$\begin{aligned} \mathcal{A} = & \frac{m_e m_\tau G_F}{\sqrt{2} \cos^2\beta} \left[ \frac{d}{c} \right] \left[ 1 + \frac{d^2}{c^2 \sin^2\beta} \right] \\ & \times \frac{\bar{e}(k_2) \gamma_5 e(k_3) \bar{e}(k_1) (1 + \gamma_5) \tau(p)}{(p - k_1)^2 - m^2} - (k_{1,2} \rightarrow k_{2,1}). \end{aligned} \quad (19)$$

However, it is suppressed by a factor of at least  $m_e/m_\tau$  relative to the usual decay amplitudes of  $\tau$ , and can thus be safely ignored. Another possible decay is  $\tau \rightarrow e \gamma$ . Using Eq. (15) we find for  $m > m_\tau$  a branching fraction of less than  $10^{-6}$ , which is far below the present experimental upper limit of  $2 \times 10^{-4}$  [5]. Similarly, the anomalous magnetic moment of the electron receives from these interactions a maximum contribution of order  $10^{-15} e/2m_e$ , which is negligible compared to the present experimental error, which is of order  $10^{-11} e/2m_e$  [5]. Therefore, the only constraint on the mass of  $h_5^0$  from present data remains  $m > m_\tau$ .

### IV. Z DECAY INTO $h_5^0$

If  $h_5^0$  is indeed light, then perhaps we should consider  $Z \rightarrow h_5^0 h_5^0$  as a loop-induced decay. However, because of angular momentum conservation and Bose statistics, a vector particle is absolutely forbidden to decay into two identical scalar particles. We might try  $Z \rightarrow h_5^0 \chi_0$ , which is possible, but its rate is very much suppressed. Consider now  $Z \rightarrow h_5^0 + " \text{Re}\phi_{1,2}^0 "$ , where the virtual "Re $\phi_{1,2}^0$ " particle converts into a pair of  $h_5^0$ 's or  $\chi_0$ 's. For this possibility, we need to take a look at the triple scalar couplings contained in Eq. (1). We find

$$Z \rightarrow h_5^0 \chi_0 \chi_0 \quad (21)$$

and

$$Z \rightarrow h_5^0 h_5^0 h_5^0 \quad (22)$$

should have non-negligible rates comparable to that of  $Z \rightarrow Hf\bar{f}$  in the standard model.

Consider first  $Z \rightarrow h_5^0 \chi_0 \chi_0$ , as shown in Fig. 1. Assuming that the amplitude is dominated by one scalar intermediate state of mass  $M$ , we then have

$$\mathcal{A} = \frac{ef_{\text{eff}}}{\sin\theta_W \cos\theta_W [2(u_1^2 + u_2^2)]^{1/2}} \frac{\epsilon \cdot (k_1 - k_2 - k_3) k_2 \cdot k_3}{(k_2 + k_3)^2 - M^2}. \quad (23)$$

Using  $p = k_1 + k_2 + k_3$  and  $\epsilon \cdot p = 0$ , we find the spin-averaged amplitude squared in the center of mass to be given by

$$|\mathcal{A}|_{\text{av}}^2 = \frac{e^2 f_{\text{eff}}^2 |\mathbf{k}_1|^2}{6 \sin^2\theta_W \cos^2\theta_W (u_1^2 + u_2^2)} \times \left[ 1 + \frac{M^2}{m_Z^2 + m^2 - M^2 - 2m_Z E_1} \right]^2, \quad (24)$$

where  $m$  is the mass of  $h_5^0$ ,  $\mathbf{k}_1$  its momentum, and  $E_1$  its energy in the rest frame of the  $Z$  boson. Integrating over the invisible  $\chi_0$ 's, we obtain the decay energy spectrum

$$\frac{d\Gamma}{dE_1} = \frac{|\mathbf{k}_1|}{128\pi^3 m_Z} |\mathcal{A}|_{\text{av}}^2, \quad (25)$$

where  $E_1$  ranges from  $m$  to  $(m_Z^2 + m^2)/2m_Z$  and  $|\mathbf{k}_1| = (E_1^2 - m^2)^{1/2}$ . If  $m$  can be neglected, then

$$\Gamma = \frac{m_Z f_{\text{eff}}^2 M^4}{384\pi^3 v^2 u^2} \left[ -\frac{5}{16z} + \frac{19}{32} - \frac{7z}{24} + \frac{z^2}{64} - \frac{(1-z)^2(5-2z)}{16z^2} \ln(1-z) \right], \quad (26)$$

where  $v^2 \equiv v_1^2 + v_2^2$ ,  $u^2 \equiv u_1^2 + u_2^2$ , and  $z \equiv m_Z^2/M^2$ . As an illustration, let  $f_{\text{eff}}^2/4\pi = 0.01$  and  $u = v$ , then  $\Gamma = 3.8 \times 10^{-7}$  GeV for  $z = 1$ . Dividing by the total width [6]  $\Gamma_Z = 2.487 \pm 0.010$  GeV, this corresponds to a branching fraction of about  $1.5 \times 10^{-7}$ , which is an order of magnitude below the present experimental capability for detection.

Once  $h_5^0$  is produced, it will decay into a fermion-antifermion pair. Hence  $Z \rightarrow h_5^0 \chi_0 \chi_0$  has the signature of  $Z \rightarrow f\bar{f}$  + nothing, which can also be due to the standard-model process  $Z \rightarrow H + \text{“}Z\text{”}$  where  $H \rightarrow f\bar{f}$  and

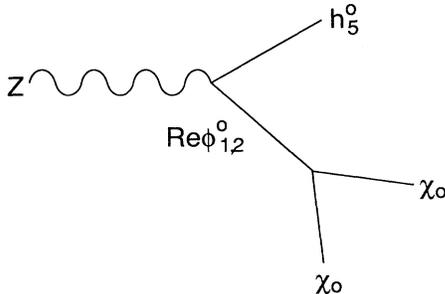


FIG. 1. Diagram for the decay  $Z \rightarrow h_5^0 \chi_0 \chi_0$ .

“ $Z$ ”  $\rightarrow \nu\bar{\nu}$ . The big difference is that the only unknown for the standard-model process is  $m_H$ , whereas here we have three unknowns:  $m$ ,  $M$ , and  $f_{\text{eff}}$ . However, it is reasonable to assume that  $M$  is of order  $m_Z$  and  $f_{\text{eff}}$  is not much less than unity, so if  $m^2 \ll m_Z^2$ , this process may be observable in the near future at the CERN  $e^+e^-$  collider LEP with more data. To tell it apart from  $Z \rightarrow H + \text{“}Z\text{”}$ , we note that the couplings of  $\text{Re}\phi_{1,2}^0$  to  $f\bar{f}$  are suppressed by  $m_f/v_{1,2}$  relative to those of the  $Z$  boson. Indeed,  $\chi_0 \chi_0$  may well be the dominant decay mode of  $\text{Re}\phi_{1,2}^0$ , so that even if the latter are produced, their decays would be invisible [7].

Consider now  $Z \rightarrow h_5^0 h_5^0 h_5^0$ . This proceeds as in Fig. 1, but with  $\chi_0$  replaced by  $h_5^0$ , and the amplitude is the sum of three terms, as it must be symmetric with respect to  $k_{1,2,3}$ :

$$\mathcal{A} = \frac{ev\lambda_{\text{eff}}}{\sin\theta_W \cos\theta_W} \left[ \frac{\epsilon \cdot k_1}{(p - k_1)^2 - M^2} + \frac{\epsilon \cdot k_2}{(p - k_2)^2 - M^2} + \frac{\epsilon \cdot k_3}{(p - k_3)^2 - M^2} \right]. \quad (27)$$

From the three-body kinematics, we find that, in the center of mass,

$$m < E_1 < \frac{m_Z^2 - 3m^2}{2m_Z}, \quad (28)$$

and, for a given  $E_1$ ,

$$E_2^{\text{max,min}} = \frac{m_Z - E_1}{2} \pm \frac{1}{2} |\mathbf{k}_1| \left[ 1 - \frac{4m^2}{m_Z^2 - 2m_Z E_1 + m^2} \right]^{1/2}. \quad (29)$$

This means that  $d\Gamma/dE_1$  has a kinematical zero not only at  $E_1 = m$  but also at  $E_1 = E_1^{\text{max}}$  for which  $E_2^{\text{max}} = E_2^{\text{min}} = (m_Z^2 + 3m^2)/4m_Z$ . From Eq. (27) we see also that  $\mathcal{A}$  is zero to order  $M^{-2}$  because  $\epsilon \cdot (k_1 + k_2 + k_3) = 0$ ; hence, we expect in general a significant suppression of the  $Z \rightarrow h_5^0 h_5^0 h_5^0$  rate. Assuming that  $m_Z^2 \ll M^2$ , we then have

$$|\mathcal{A}|_{\text{av}}^2 \simeq \frac{8\lambda_{\text{eff}}^2 m_Z^4}{3M^8} |E_1 \mathbf{k}_1 + E_2 \mathbf{k}_2 + E_3 \mathbf{k}_3|^2. \quad (30)$$

If we also assume that  $m^2 \ll m_Z^2$ , then

$$\frac{d\Gamma}{dE_1} \simeq \frac{\lambda_{\text{eff}}^2 m_Z^3 E_1^3}{48\pi^3 M^8} (m_Z^2 - \frac{14}{3} m_Z E_1 + \frac{28}{5} E_1^2). \quad (31)$$

This distribution has an interesting shape because there is a local maximum at  $2E_1/m_Z = 0.54$  and a local minimum at  $2E_1/m_Z = 0.79$ . This qualitative feature remains even if we go away from the limit  $m^2 \ll m_Z^2 \ll M^2$ . We plot this in Fig. 2 for  $m = 10$  GeV and  $M = 100$  GeV. Note that the kinematical zero at  $E_1 = E_1^{\text{max}}$  for  $m \neq 0$  forces the rising  $d\Gamma/dE_1$  to turn over near the end. Integrating over  $E_1$  for  $m = 0$  and dividing by 6 for the three identical particles in the final state, we find

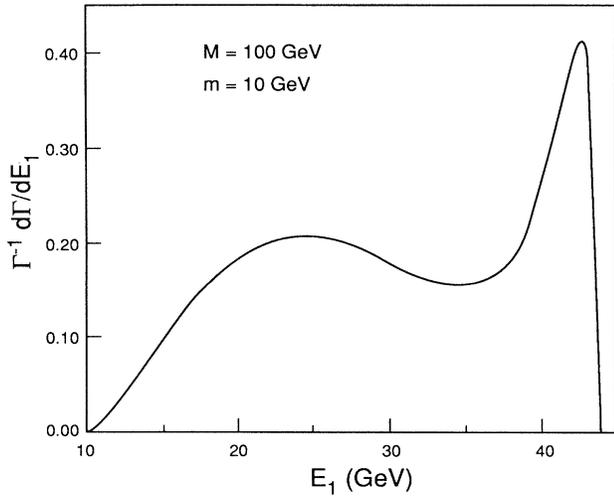


FIG. 2. The  $\Gamma^{-1}d\Gamma/dE_1$  distribution for  $Z \rightarrow h_5^0 h_5^0 h_5^0$  with  $m = 10$  GeV and  $M = 100$  GeV.

$$\Gamma \simeq \frac{m_Z \lambda_{\text{eff}}^2}{192\pi^3} \frac{1}{1440} \left[ \frac{m_Z}{M} \right]^8. \quad (32)$$

For comparison, let  $\lambda_{\text{eff}}^2/4\pi = 0.01$  and  $M = 2m_Z$ , then  $\Gamma \simeq 5.22 \times 10^{-9}$  GeV, which shows clearly that unless

$M \approx m_Z$ , this rate would be much too small to be of any practical value.

## V. CONCLUSION

In summary, we have demonstrated how the existence of a light scalar boson can be consistent with all present experimental data in a specific extension of the standard model. It has the potential of being discovered in the future as a decay product of the  $Z$  boson at the level of  $10^{-7}$  in the branching fraction. This result is based on the study of the three-body decays  $Z \rightarrow h_5^0 \chi_0 \chi_0$  and  $Z \rightarrow h_5^0 h_5^0 h_5^0$ , where  $h_5^0$  can be as light as the  $\tau$  lepton and  $\chi_0$  is the Majoron which is massless. Although we have used a specific model [2] for our analysis, such decays of the  $Z$  boson are generally present in extensions of the standard model with two or more scalar doublets and possibly some singlets. They may provide the first glimpse of new physics that is just beyond the reach of present high-energy accelerators.

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