Gluino-induced effects and precision measurements at the CERN e^+e^- collider LEP

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A gauge-invariant subset of minimal supersymmetry, driven by the quark-squark-gluino Lagrangian, is our concern in the context of precision measurements at the CERN e^+e^- collider LEP on Z decays. Shifts of the parameter $R=\Gamma_{\rm had}/\Gamma_{l\bar{l}}$, a precisely measured top-quark-free observable at LEP, due to vertex corrections induced by the above interaction, have been evaluated for different regions of the supersymmetric parameter space. Owing mainly to the stringent bounds on the masses of the squarks and the gluino from the direct search at the Fermilab Tevatron, the effects are not too large to cast an observable impact at the present stage of LEP precision.

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I. INTRODUCTION

Even at the end of the accumulation of the full 1990 data of the CERN e^+e^- collider LEP on the Z boson—a catch of around 550 000 Z decay events—no experimental signal contradicting the standard model (SM) prediction has come up. Nonetheless, the failure to detect the top quark and the Higgs boson remains a challenge. Our inability to pin down the top-quark mass (m_t) currently provides the largest source of uncertainty in predicting or limiting the parameters of much of the physics beyond the standard model which are either on the threshold of confirmation or in a position of being ruled out. Supersymmetry (SUSY) is one such extension of the standard model scenario crying out for verification. No concrete signal of SUSY has so far been on the scene—the unification of the strong and electroweak couplings might be providing a hint—but possibilities still seem to be there for its manifestation either through the direct production of superparticles at higher energies or through the radiative correction of observables which have been very precisely measured. An important criterion of those observables is that they should be free from uncertainties coming from the standard model itself which can mask a signal of physics beyond it. Our concern in this paper is the minimal version of the supersymmetric standard model which has a very strong theoretical motivation behind it. We calculate the supersymmetric effect on the parameter $R=\Gamma_{\rm had}/\Gamma_{l\bar{l}}$, which is a precisely measured variable in the current phase of LEP and, additionally, it does not suffer from the uncertainties coming from the top quark due to an accidental cancellation [1] between the oblique and the vertex correction contributions. The present experimental value [2] for R is 20.92 ± 0.11 whose standard model prediction is 20.75 ± 0.06. In this paper we limit our interest only to the supersymmetric effects on the $Zq\bar{q}$ vertices in the parameter R. Boulware and Finnel [3] have calculated the radiative correction to the $Zb\bar{b}$ vertex taking into account the triangle diagrams mediated by the gaugino and the Higgs sector of minimal SUSY. They showed that the effect would be detectable at the 1% level of experimental accuracy only in a limited region of supersymmetric parameter space. We explore a different sector of minimal SUSY driven by the quark-squark-gluino Lagrangian, which itself is a completely gauge-invariant subset of the whole theory and is a potentially promising one since it provides a strong correction to a weak vertex. We arrange this paper in the following way. In Sec. II we discuss the formalism for the calculation of the squark- and gluino-mediated triangle diagrams and comment briefly on the accidental cancellation of the top-quark contribution in the R parameter. In Sec. III we investigate the extent of the supersymmetric contribution to R for different SUSY parameter zones. Finally, in Sec. IV we guess about the possibilities of catching a hint of a supersymmetry signal from such indirect methods which in view of our results seem to be somewhat remote at this stage.

II. FORMALISM

In the SM the partial width $\Gamma(Z\to f\bar{f})$ can be written as

$$\Gamma(Z\to f\bar{f})=N_c^f(\sqrt{2}G_F m_Z^3/48\pi)(v_f^2+a_f^2)\ , \tag{1}$$

where G_F is the Fermi constant and v_f and a_f are the vector and the axial-vector couplings of Z to the fermion f:

$$v_f=\sqrt{\rho}(2t_3^f-4Q_f\sin^2\bar{\theta}_W)\ ,$$
$$a_f=\sqrt{\rho}2t_3^f\ , \tag{2}$$

where ρ is the ratio of the neutral- to charged-current cross sections at vanishing momentum transfer and $\sin^2\bar{\theta}_W$ is the effective $\sin^2\theta_W$ which takes into account γ-Z mixing at the Z pole:

$$\sin^2\bar{\theta}_W=\bar{s}_W^2\simeq s_0^2-\tfrac{3}{8}\delta\rho\ , \tag{3}$$

with

$$s_0^2=\tfrac{1}{2}-\tfrac{1}{2}(1-4\mu^2/m_Z^2)^{1/2}\ , \tag{4}$$

where $\mu^2=\pi\alpha(m_Z)/(\sqrt{2}G_F)$. The leading top dependence of the oblique correction $\delta\rho$ is given by [4]

$$\delta\rho=(3G_F m_t^2)/(8\pi^2\sqrt{2}) . \quad (5)$$

N_c^f is given by

$$N_c^f=1+(3\alpha/4\pi)Q_f^2 \text{ for leptons },$$
$$=3[1+(3\alpha/4\pi)Q_f^2][1+\alpha_s/\pi+1.405(\alpha_s/\pi)^2] \text{ for quarks }. \quad (6)$$

To take into consideration the extra top dependence coming from the $Zb\bar{b}$ vertex, one should use [5]

$$v_b=v_d-19\Delta_{bv}^{(t)}/30; \quad a_b=a_d-19\Delta_{bv}^{(t)}/30 \quad (7)$$

with

$$\Delta_{bv}^{(t)}=-(20\alpha/19\pi)[r_t^2+\tfrac{13}{6}\ln r_t^2], \quad r_t=m_t/m_z . \quad (8)$$

It should be noted that although the top dependences in $\delta\rho$ and $\Delta_{bv}^{(t)}$ have roughly the same magnitude, they have opposite signs, which plays a crucial role in making the quantities $R=\Gamma_{\rm had}/\Gamma_{l\bar{l}}$ and $\Gamma_{b\bar{b}}$ practically top insensitive.

In this paper we consider the leading gluino exchange contributions to the $Zq\bar{q}$ vertices from the diagrams shown in Fig. 1. The quark-squark-gluino Lagrangian, a supersymmetrization of the strong interaction vertex, is

$$\mathcal{L}_{q-\tilde{q}-\tilde{g}}=i\sqrt{2}g_s\tilde{q}_i^{\dagger a}\bar{\tilde{g}}_\alpha(\lambda_\alpha/2)_{ab} \times\left[\Gamma_L^{ip}\frac{1-\gamma_5}{2}+\Gamma_R^{ip}\frac{1+\gamma_5}{2}\right]q_p^b , \quad (9)$$

where for three generations the flavor indices $i=1-6$ (for each quark flavor there are two squark states), $p=1-3$ and the color indices $a,b=1-3$ and $\alpha=1-8$. The (6×3) matrices Γ_L and Γ_R are determined by the quark and squark mass matrices (see below).

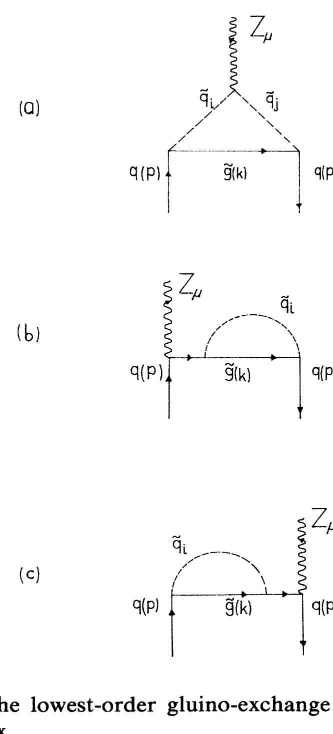

FIG. 1. The lowest-order gluino-exchange contributions to the $Zq\bar{q}$ vertex.

The Lagrangian, Eq. (9), has been much studied particularly in the context of gluino-induced flavor-changing effects and CP violation [6,7]. We follow the notation of Ref. [7] where the SUSY effect on $Z\to t\bar{c}$ was calculated and refer to it for more details. The basic features are outlined below.

Each quark flavor has two chiralities, left and right, which correspond to two different squark states. Hence with three generations of quarks there are six up-type and six down-type squarks. Supersymmetry is assumed to be broken explicitly by soft terms generated from $N=1$ supergravity. We also note that the quark and squark mass matrices are not diagonal in the same basis.

In $N=1$ supergravity models, the $\tilde{d}$ mass squared matrix (in a basis in which the d-quark mass matrix is diagonal) is

$$M_{\tilde{d}}^2=\begin{bmatrix}\mu_L^2\mathbb{1}+\hat{M}_d^2+cK\hat{M}_u^2K^\dagger & Am_{3/2}\hat{M}_d\\ Am_{3/2}\hat{M}_d & \mu_R^2\mathbb{1}+\hat{M}_d^2\end{bmatrix} , \quad (10)$$

where μ_L and μ_R are flavor-blind SUSY-breaking terms for left- and right-type squarks, respectively. K is the standard Kobayashi-Maskawa matrix. $\hat{M}_u$ and $\hat{M}_d$ are diagonal $(3\times3)u$- and d-quark mass matrices. Quantum corrections affect the left- and the right-handed sectors differently. For example a d-type left squark couples to a u-type quark and a charged Higgsino. Such couplings are proportional to the quark mass at each vertex. Then it is obvious that through Higgsino loops the left-handed sector of the down-type squark mass matrix has a term proportional to $cK\hat{M}_u^2K^\dagger$. Owing to the fact that the b quark couples in full strength with the t quark via W exchange ($K_{bt}\approx1$), the radiative mass correction in the left-handed sector of the $M_{\tilde{d}}^2$ matrix must have one entry $\simeq {\rm c}m_t^2$. Thus one of the down-type squark mass eigenvalues is strongly dependent on the choice of the top-quark mass. The number c can be estimated from the renormalization-group equation of the mass parameters. In our calculation we treat c as a phenomenological input. The off-diagonal blocks in Eq. (10) correspond to the mixing of the left and right squarks which is induced by the hidden sector, $m_{3/2}$ being the gravitino mass and A is a parameter which contains the details of the hidden sector. Here we take A to be real. $\Gamma_{L,R}$ of Eq. (9) are given by $\Gamma_L=\tilde{U}\begin{bmatrix}\mathbb{1}\\0\end{bmatrix}$, $\Gamma_R=\tilde{U}\begin{bmatrix}0\\\mathbb{1}\end{bmatrix}$, where $\tilde{U}$ diagonalizes $M_{\tilde{d}}^2$ [Eq. (10)] and $\mathbb{1}$ is the (3×3) identity matrix.

The other ingredient necessary for this calculation is the Z coupling to squarks. For example, for d-type squarks,

$$\mathcal{L}_{Z-\tilde{d}-\tilde{d}}=i\frac{g_W}{4\cos\theta_W}S_{ij}\tilde{d}_i\overleftrightarrow{\partial}_\mu\tilde{d}_jZ^\mu . \quad (11)$$

Here $i,j=1-6$ and

$$S=\tilde{U}^\dagger\begin{bmatrix}(v_d+a_d)\mathbb{1} & 0\\ 0 & (v_d-a_d)\mathbb{1}\end{bmatrix}\tilde{U} , \quad (12)$$

where v_d and a_d are the Z couplings to the d-type quark [see Eq. (2)]. The above results for the d-type quarks and squarks can easily be extended to the u-type sector.

To express the final results in a concise form it is convenient to define the following (3×3) matrices O_{mn} $(m,n=1-3)$:

$$(O^{ij}_{1,2})_{mn}=\tfrac{1}{2}[\Gamma_L^{\dagger mj}S_{ji}\Gamma_L^{in}\pm(L\leftrightarrow R)] , \tag{13}$$

$$(O^{ij}_{3,4})_{mn}=\tfrac{1}{2}[\Gamma_L^{\dagger mj}S_{ji}\Gamma_R^{in}\pm(L\leftrightarrow R)] , \tag{14}$$

$$(O^{i}_{5,6})_{mn}=\tfrac{1}{2}[\Gamma_L^{\dagger mi}\Gamma_L^{in}\pm(L\leftrightarrow R)] , \tag{15}$$

$$(O^{i}_{7,8})_{mn}=\tfrac{1}{2}[\Gamma_L^{\dagger mi}\Gamma_R^{in}\pm(L\leftrightarrow R)] . \tag{16}$$

In the above i,j are squark indices running from 1 to 6.

The amplitude following from Figs. 1(a), 1(b), and 1(c) for $Z\rightarrow q\bar{q}$ can now be written as

$$A_\mu^{(a)}=\left[\frac{g_s^2g_W}{24\cos\theta_W\pi^2}\right]\sum_{i,j}\bar{q}(p')\gamma_\mu\{2O_1[m^2(C_{11}+C_{21})-C_{24}]-2O_3mm_g(C_0+C_{11})+2O_2C_{24}\gamma_5\}q(p) , \tag{17}$$

$$A_\mu^{(b)}+A_\mu^{(c)}=-\left[\frac{g_s^2g_W}{24\cos\theta_W\pi^2}\right]\sum_{i}\bar{q}(p')\gamma_\mu\{[v_q(O_5F_1-O_7m_gF_2)+a_qO_6B_1]-(v_q\leftrightarrow a_q)\gamma_5\}q(p) , \tag{18}$$

where

$$F_1=\frac{d}{dm}(mB_1) \quad\text{and}\quad F_2=\frac{d}{dm}(B_0) .$$

m and m_g denote the quark and gluino masses, respectively. In writing Eq. (17) we have dropped form factors proportional to $(p-p')_\mu$, $(p-p')_\mu\gamma_5$, $(p-p')_\nu\sigma_{\mu\nu}$, and $(p-p')_\nu\sigma_{\mu\nu}\gamma_5$ which we have verified are negligible. The B and the C functions are defined in Appendix A. F_1 and F_2 can be expressed in terms of the C functions corresponding to the special case $i=j$. In our calculation the B and C functions have been evaluated numerically following the prescription of Ref. [8] and have been cross checked in several different ways [7].

It is rather easy to convince oneself that the total contribution from the diagrams is finite. For this purpose it is enough to note that of the B and C functions only B_0, B_1, and C_{24} are divergent and in the dimensional regularization scheme the divergent parts are related by

$$\text{div}B_0=-2\,\text{div}B_1=4\,\text{div}C_{24}=2/(d-4) . \tag{19}$$

Using the above relation and Eqs. (13)–(18) one can verify that the divergence cancels when the summation over the squark indices are carried out.

We have also repeated the whole calculation analytically by expanding in powers of the ratio of the quark mass to the minimum of the squark and gluino masses. The heaviest known quark is the b quark whose mass is around 4.6 GeV and we are exploring the squark and gluino masses in the region of 125 GeV onwards. As a matter of fact, in this second method of calculation we also had to expand in powers of the ratio of the Z mass ($\approx$91.2 GeV) to the minimum of the squark and gluino masses and retained only the leading term which goes as the square of this ratio. From a purist's standpoint this has to be taken with a grain of salt, as the expansion is valid only in the region $m_Z\ll\min(m_{\tilde{q}}$ or $m_g)$. But it turns out that the coefficient in front of this expansion parameter is small and even for 125 GeV squark and gluino masses the result obtained in this procedure with b quarks in the external legs agrees fairly well with the ones from the other method where no such expansion has been made. The motive behind this exercise lies in the following fact. In the former procedure the calculation of the self-energy diagrams with identical quarks in the external legs involve derivatives of the B functions which can be expressed as linear combinations of some of the C functions. These C functions are different from the ones required for the calculation of the triangle diagram only in the replacement of the Z mass as one of the input parameters by an extremely small artificial mass parameter to take care of some computational problems with two identical quarks in the external legs. Now, for the lighter quarks belonging to the first two generations, the numerical evaluation of C functions with three very small mass parameters cannot be done reliably. On the other hand, with the b quark in the external legs the aforesaid problem does not arise and the agreement between the two methods of calculation gives us confidence in the results obtained using the expansion technique which we have adopted to present our final results on the supersymmetric corrections to the R parameter.

In this method we write the amplitudes (only finite parts) of Figs. 1(a), 1(b), and 1(c) as

$$A_\mu^{(a)}=\left[\frac{g_s^2g_W}{24\cos\theta_W\pi^2}\right]\sum_{i,j}\bar{q}(p')\gamma_\mu\{-O_1[3r^2J_1(z)+r_z^2J_4(z)/6-J_0(z)]-O_3[2rJ_2(z)+rr_Z^2J_3(z)/3]$$
$$+O_2[r^2J_1(z)+r_Z^2J_4(z)/6-J_0(z)]\gamma_5\}q(p) , \tag{20}$$

$$A_\mu^{(b)}+A_\mu^{(c)}=\left[\frac{g_s^2g_W}{24\cos\theta_W\pi^2}\right]\sum_{i}\bar{q}(p')\gamma_\mu[(v_q\{O_5[-J_0(z)+3r^2J_1(z)]+2O_7rJ_2(z)\}$$
$$+a_qO_6[-J_0(z)+r^2J_1(z)])-(v_q\leftrightarrow a_q)\gamma_5]q(p) , \tag{21}$$

where $r = m/m_g$, $r_Z = m_Z/m_g$, and $z = m_i^2/m_g^2$, where m_i is the mass of the ith squark floating around the loops. The integrals J_0–J_4 as functions of z are listed in Appendix B.

III. RESULTS AND DISCUSSIONS

Before we enter into a discussion of the change in the R parameter due to supersymmetric vertex corrections, we stress again that due to the conspiring interplay between the quadratic top dependences coming from the effective weak angle $\bar{\theta}_W$ and from the $Zb\bar{b}$ vertex, R is practically stable against the top mass. So any change in the R parameter would be a hint of new physics. In Fig. 2 we have plotted δR against m_g. For all ranges of the parameter space scanned in our analysis δR is positive. In selecting the supersymmetric parameter space we have kept in mind the recent Fermilab Tevatron bounds on the squark and gluino masses. The c parameter has been fixed to the popular value -0.1. We have checked that the mixing between the left- and the right-handed squarks does not affect our results significantly and we prefer to set $A = 3$. As is observed from Fig. 2 δR is about 1.8% for $m_g = 125$ GeV and the squark mass parameter $\mu_L \simeq \mu_R = \mu = 150$ GeV. The contribution falls off with increasing gluino masses and for $m_g \simeq 400$ GeV becomes $\sim 1\%$. If we set $c = 0$, then for $\mu = 150$ GeV, $m_g = 125$ GeV, and $A = 3$, $\delta R = 1.7\%$. For nonzero c ($c = -0.1$) one of the $\tilde{b}$ squark masses ($\simeq 136$ GeV) is reduced by an amount $\simeq cm_t^2$ and thus the propagator suppression is less compared to the situation for $c = 0$. This is what is responsible for a smaller value of δR for $c = 0$. Increasing m_t from 100 to 200 GeV results in an increase of δR by 0.1%.

In Fig. 3 we have plotted δR against the squark mass parameter μ for fixed $m_g = 150$ GeV. c has been set to -0.1 and A to 3. For $\mu = 125$ GeV, δR is maximum, being slightly larger than 2%. The contribution falls off rapidly as we go from $\mu = 125$ to 450 GeV becoming $\simeq 0.5\%$. As has been pointed out before, the left-right mixing of squarks does not affect our results. We also mention that the change of m_t from 100 to 200 GeV does not make any perceptible shift of δR within the scale of our figure.

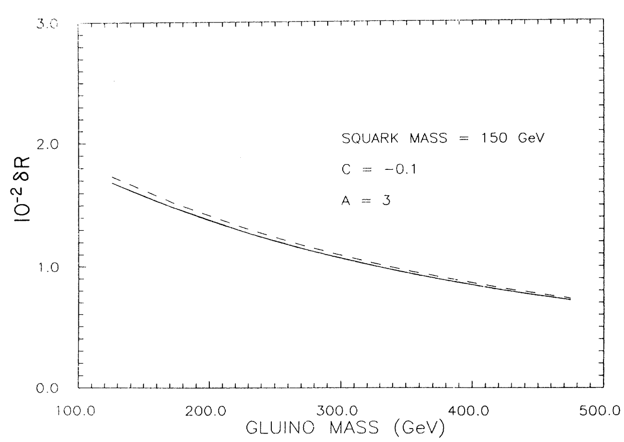

FIG. 2. The supersymmetric correction δR as a function of the gluino mass with the SUSY-breaking scale fixed to 150 GeV. The solid (dashed) line corresponds to $m_t = 100$ (200) GeV.

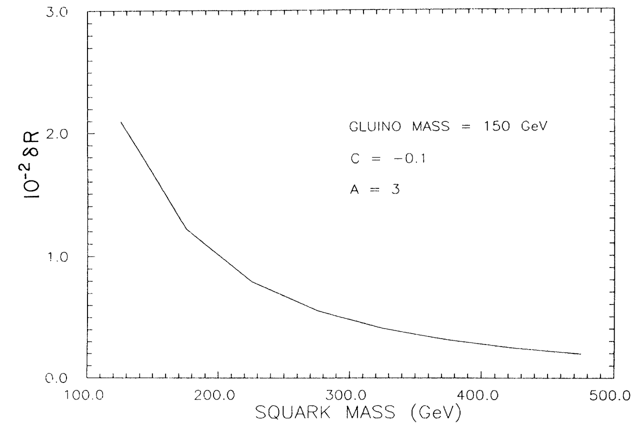

FIG. 3. The supersymmetric correction δR as a function of the average squark mass ($\simeq \mu$) for fixed gluino mass = 150 GeV and $m_t = 200$ GeV.

Though perhaps a minority point of view, the possibilities of existence of lighter gluino and squarks [9] cannot be strictly ruled out. Then, for example, for $m_g = 100$ GeV and $\mu = 120$ GeV (setting $c = 0$ to avoid any further reduction of $\tilde{b}$ masses), $\delta R = 2.6\%$. If, instead, the assignments of masses is $m_g = 120$ GeV and $\mu = 100$ GeV ($c = 0$), $\delta R = 3.2\%$. Of course, our estimate should not be taken too seriously in such parameter ranges as the expansion in m_Z^2/m_g^2 or m_Z^2/m_q^2 is no longer strictly valid.

It should be stressed that SUSY also entails contributions to the $Zq\bar{q}$ vertices from charged-Higgs-boson, Higgsino, and W-ino exchanges. But in this paper we have not taken into consideration this interaction, which is another gauge-invariant subset of minimal supersymmetry. It has been shown in Ref. [3] that this subset provides a positive correction to the parameter $R_b \equiv \Gamma_b/\Gamma_{\text{had}}$. We have checked that the corresponding correction from the gluino interactions is of the same sign. The gluino-induced effects are, however, small mainly in view of the fact that the bounds on masses of squarks and gluino are relatively stronger than the ones for gauginos causing larger propagator suppression. Typically, for $m_t = 200$ GeV, $m_g = 125$ GeV, $\mu = 150$ GeV, and $c = -0.1$, the gluino-induced $\delta R_b \simeq 4 \times 10^{-5}$.

Finally, we comment on another often discussed variable (see, e.g., Ref. [1]) $T = (3R - 30\gamma)/59$, where $\gamma = 9\Gamma_{e\bar{e}}/[\alpha(M_Z)M_Z]$. In the SM $T \leq 0.530$. On account of the gluino-mediated interaction $\delta T = 3\delta R/59$ and the contribution is again too small, at least at this stage, to be detected.

IV. CONCLUSION

In conclusion, we have studied gluino-induced contributions to the variable R which has the attractive feature that it is free from potentially large top corrections. We have confined ourselves to a gauge-invariant subset of the minimal supersymmetry, the core of which is formed by the quark-squark-gluino Lagrangian. This is a potentially rich sector in the sense that it attributes a strong correction to a weak vertex. However, the effects are highly suppressed owing to large masses of the squarks and gluino, the lower bounds of which are pushed up to 150 GeV from the direct searches at the Tevatron. As a result, the present stage of experimental precision is unlikely to yield a positive signal for supersymmetry. Although we have not calculated the correction to the R parameter induced through gaugino- and Higgs-mediated processes, we have checked that the correction to $R_b=\Gamma_b/\Gamma_{\rm had}$ from the subset of our consideration is of the same sign to the corresponding correction as obtained in Ref. [3] from the other subset. The contributions to R_b from the two subsets are, however, different mainly because the present bounds on the gaugino masses are not as strong as the ones for squarks and gluino. However, one comment is worth mentioning. As has been discussed in Ref. [9] the Tevatron bounds on the squark and gluino masses stand on the assumption that the sparticles decay to a massless lightest supersymmetric particle (LSP). If the LSP becomes very massive there could be no bound on the squark and gluino masses at all. Moreover, the possibilities of cascade decays of squarks and the gluino can weaken their bounds to some extent. In that case, a precision measurement of R might turn out to be a sensible hunting ground of supersymmetry even if the top quark is not discovered soon.

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APPENDIX A

The B and C functions are defined as

$$\frac{1}{i\pi^2}\int d^4k\frac{(1,k_\mu)}{(k^2+m_{\tilde g}^2)[(k-p)^2+m_i^2]}=(B_0,-p_\mu B_1)\ , \tag{A1}$$

$$\frac{1}{i\pi^2}\int d^4k\frac{(1,k_\mu,k_\mu k_\nu)}{(k^2+m_{\tilde g}^2)[(k-p)^2+m_i^2][(k-p')^2+m_j^2]}$$
$$=\{C_0,-p_\mu C_{11}+(p-p')_\mu C_{12},p_\mu p_\nu C_{21}+(p-p')_\mu(p-p')_\nu C_{22}-[p_\mu(p-p')_\nu+p_\nu(p-p')_\mu]C_{23}+g_{\mu\nu}C_{24}\}\ . \tag{A2}$$

APPENDIX B

$$J_0(z)=\int_0^1 dx(1-x)\ln[x+(1-x)z]$$
$$=-1-\frac{z(2-z)}{2(1-z)^2}\ln z+\frac{1+3z^2-4z}{4(1-z)^2}\ ,$$

$$K_0(z)=\int_0^1 dx\frac{1}{x+(1-x)z}=\frac{-\ln z}{1-z}\ ,$$

$$K_1(z)=\int_0^1 dx\frac{x}{x+(1-x)z}=\frac{1}{(1-z)^2}(1-z+z\ln z)\ ,$$

$$K_2(z)=\int_0^1 dx\frac{x^2}{x+(1-x)z}$$
$$=\frac{1}{2(1-z)^3}(1-4z+3z^2-2z^2\ln z)\ ,$$

$$K_3(z)=\int_0^1 dx\frac{x^3}{x+(1-x)z}$$
$$=\frac{1}{6(1-z)^4}(2-9z+18z^2-11z^3+6z^3\ln z)\ ,$$

$$\tilde K_1(z)=\int_0^1 dx\frac{x}{[x+(1-x)z]^2}=-\frac{1}{(1-z)^2}(1-z+\ln z)\ ,$$

$$\tilde K_2(z)=\int_0^1 dx\frac{x^2}{[x+(1-x)z]^2}=\frac{1}{(1-z)^3}(1-z^2+2z\ln z)\ ,$$

$$\tilde K_3(z)=\int_0^1 dx\frac{x^3}{[x+(1-x)z]^2}$$
$$=\frac{1}{2(1-z)^4}(1-6z+3z^2+2z^3-6z^2\ln z)\ ,$$

$$\tilde K_4(z)=\int_0^1 dx\frac{x^4}{[x+(1-x)z]^2}$$
$$=\frac{1}{3(1-z)^5}(1-6z+18z^2-10z^3-3z^4+12z^3\ln z)\ ,$$

$$J_1(z)=K_1(z)-2K_2(z)+K_3(z)\ ,$$

$$J_2(z)=K_1(z)-K_2(z)\ ,$$

$$J_3(z)=\tilde K_1(z)-3\tilde K_2(z)+3\tilde K_3(z)-\tilde K_4(z)\ ,$$

$$J_4(z)=K_0(z)-3K_1(z)+3K_2(z)-K_3(z)\ .$$

[1] G. Girardi, W. Hollik, and C. Verzegnassi, Phys. Lett B **240**, 492 (1990); A. Djouadi, G. Girardi, C. Verzegnassi, W. Hollik, and F. Renard, Nucl. Phys. **B349**, 48 (1991).

[2] J. R. Carter, in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics,* Geneva, Switzerland, 1991, edited by S. Hegarty, K. Potter, and E. Quercigh (World Scientific, Singapore, 1992).

[3] M. Boulware and D. Finnel, Phys. Rev. D **44**, 2054 (1991).

[4] M. Veltman, Nucl. Phys. **B123**, 89 (1977).

[5] A. A. Akhundov, D. Yu. Bardin, and T. Riemann, Nucl. Phys. **B276**, 1 (1986); W. Beenakker and W. Hollik, Z. Phys. C **40**, 141 (1988); J. Bernabeu, A. Pich, and A. Santamaria, Phys. Lett. B **200**, 569 (1988); J. H. Kuhn and P. M. Zerwas, in *Z Physics at LEP,* Proceedings of the Workshop, Geneva, Switzerland, 1989, edited by G. Altarelli, R. Kleiss, and C. Verzegnassi (CERN Yellow Report No. 89-08, Geneva, 1989), Vol. 1; F. Boudjema, A. Djouadi, and C. Verzegnassi, Phys. Lett. B **238**, 423 (1990).

[6] J. F. Donoghue, H. P. Nilles, and D. Wyler, Phys. Lett. **128B**, 55 (1983); J. M. Gerard *et al.*, Nucl. Phys. **B253**, 93 (1985); M. J. Duncan, *ibid.* **B221**, 285 (1983); M. Dugan *et al.*, *ibid.* **B255**, 413 (1985).

[7] Biswarup Mukhopadhyaya and Amitava Raychaudhuri, Phys. Rev. D **39**, 280 (1989).

[8] G. 't Hooft and M. Veltman, Nucl. Phys. **B153**, 365 (1979); G. Passarino and M. Veltman, *ibid.* **B160**, 151 (1979).

[9] H. Baer, X. Tata, and J. Woodside, Phys. Rev. D **44**, 207 (1991).