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Possible connection between a longer τ lifetime and the τ-neutrino mass

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The experimental measurements of the τ-lepton lifetime suggest that it might be a few percent longer than the standard model prediction. One of the simplest ways of obtaining a slightly longer τ-lepton lifetime is to assume that the τ neutrino mixes with another neutral particle which is more massive than the τ lepton. We examine the possibility of implementing this simple idea by adding only a right-handed neutrino to the minimal standard model. We find that the resulting model has a range of parameters in which the lifetime is longer than the prediction of the standard model by more than 1%. We show that this range of parameters implies that the τ-neutrino mass must be greater than 10 MeV. Hence the model will be tested when the measurements of the mass of the τ neutrino are improved.

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The standard model relates the τ-lepton lifetime to the branching fraction

$$B(\tau \to e\nu_\tau\bar{\nu}_e) \equiv \frac{\Gamma(\tau \to e\nu_\tau\bar{\nu}_e)}{\Gamma(\tau \to \text{all})} \tag{1}$$

and the μ lifetime as follows:

$$\begin{aligned}\frac{1}{\Gamma(\tau \to \text{all})} &= \frac{\Gamma(\tau \to e\nu_\tau\bar{\nu}_e)}{\Gamma(\tau \to \text{all})}\frac{\Gamma(\mu \to e\nu_\mu\bar{\nu}_e)}{\Gamma(\tau \to e\nu_\tau\bar{\nu}_e)}\frac{1}{\Gamma(\mu \to e\nu_\mu\bar{\nu}_e)}\\ &= B(\tau \to e\nu_\tau\bar{\nu}_e)\,\frac{m_\mu^5}{m_\tau^5}\,\frac{1}{\Gamma(\mu \to e\nu_\mu\bar{\nu}_e)}.\end{aligned} \tag{2}$$

The experimental measurements of the τ-lepton lifetime suggest that it might be a few percent longer than the standard model prediction, Eq. (2) [1].

Motivated by a possible discrepancy between the theory prediction equation (2) and the direct measurements of the lifetime, there have been some ideas for modifying the standard model to obtain a longer lifetime [2–4]. Perhaps the simplest idea that we are aware of is the observation of Wirbel and others [3]. In this scenario, the standard model is modified by the addition of a fourth generation and the neutrino of the fourth generation is assumed to be heavier than the τ lepton. The weak eigenstate τ neutrino (ν_τ) may then be a Cabibbo-type mixture of the mass eigenstate τ neutrino (ν_τ^0) and the heavy fourth generation neutrino (ν_4^0):

$$\nu_\tau = \cos\phi\,\nu_\tau^0 + \sin\phi\,\nu_4^0. \tag{3}$$

If $m_{\nu_4^0} > m_\tau$ then on kinematic grounds the τ lepton cannot decay with ν_4^0 in the final state. This means that the decay rate of the τ lepton is smaller than in the standard model by a factor of $\cos^2\phi$. For the lifetime to be longer by 1%, it is required that $\sin\phi = 1/10$. (We will take this value for definiteness. Later in the paper we will consider longer τ lifetimes which are in fact favored by the data [1].) The idea that the weak eigenstate τ neutrino is a mixture of a heavy neutrino and a light one provides a simple physical explanation assuming that the τ-lepton lifetime is really slightly longer than the standard model prediction. The fourth generation model of Ref. [3] is not the only model with this mechanism. Any heavy neutral particle will do, as long as it mixes with the τ neutrino [4]. What we wish to examine is whether or not we can implement the idea by only minimally extending the standard model by adding only a right-handed neutrino (and not a fourth or exotic generation).

In addition to the motivation from a possible discrepancy with the τ-lepton lifetime, there are also other reasons to modify the lepton sector of the standard model. In particular, there are general arguments from electric-charge quantization which suggest that the lepton sector of the standard model should be modified [5]. Previous work [5] has shown that if the Lagrangian describing a gauge model contains gaugeable U(1) global symmetries, then electric charge is not quantized in general. The standard model has gaugeable global U(1) symmetries (associated with differences of the global lepton numbers) and thus there is no explanation for electric-charge quantization in the minimal standard model. The model that we will consider has in general no gaugeable U(1) symmetries, and electric charge is necessarily quantizated (correctly).

We add a right-handed neutrino (i.e., gauge singlet Weyl fermion field) to the standard model and assume that it has both a Dirac and a Majorana mass term:

$$\mathcal{L}_{\text{mass}} = m\bar{\nu}_R\nu_L' + M\bar{\nu}_R(\nu_R)^c + \text{H.c.}, \tag{4}$$

where ν_L' is in general a linear combination of the three weak eigenstate neutrinos $(\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})$. We will assume that $\nu'_L \approx \nu_{\tau L}$ for definiteness [6]. Equation (4) implies the neutrino mass matrix:

$$\mathcal{L}_{\rm mass} = \bar{X}_L \mathcal{M} X_R, \tag{5}$$

where

$$X_L = \begin{pmatrix} \nu_{\tau L} \\ (\nu_R)^c \end{pmatrix}, \quad X_R = \begin{pmatrix} -(\nu_{\tau L})^c \\ \nu_R \end{pmatrix}, \tag{6}$$

and

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ -m & M \end{pmatrix}. \tag{7}$$

Diagonalizing this mass matrix, and expanding the eigenvalues in a power series assuming that $m < M$, then the mass eigenvalues are of the usual seesaw form

$$\begin{aligned} m_{\nu_\tau^0} &= \frac{m^2}{M}\left[1 + O\left(\frac{m^2}{M^2}\right)\right], \\ m_{\nu_4^0} &= M\left[1 + O\left(\frac{m^2}{M^2}\right)\right]. \end{aligned} \tag{8}$$

We can also obtain the weak eigenstate fields in terms of the mass eigenstate fields as follows:

$$\begin{aligned} \nu_{\tau L} &= \cos\phi \nu_{\tau L}^0 - \sin\phi (\nu_{4R}^0)^c, \\ (\nu_{4R})^c &= \sin\phi \nu_{\tau L}^0 + \cos\phi (\nu_{4R}^0)^c, \end{aligned} \tag{9}$$

where

$$\tan\phi = \frac{m}{M}. \tag{10}$$

If $m_{\nu_4^0} > m_\tau$ and $\tan\phi \approx 1/10$, then the τ-lepton lifetime will be longer than in the standard model by about 1% (as the decay rate is suppressed by $\cos^2\phi$). From experiment, we know that $m_{\nu_\tau^0} < 35$ MeV. Thus, from Eqs. (8) and (10) we have

$$m_{\nu_\tau^0} \simeq M \tan^2\phi < 35 \text{ MeV}. \tag{11}$$

Thus we have obtained an *upper bound* for $m_{\nu_4^0}$. For $\tan^2\phi \approx 1/100$, $M \simeq m_{\nu_4^0} < 3.5$ GeV.

What is the *lower bound* on $m_{\nu_4^0}$? For $\sin\phi = 1/10$, $m_{\nu_4^0} > m_\tau$ is sufficient to get a 1% increase in the τ-lepton lifetime. When $m_{\nu_4^0} < m_\tau$, the decays involving ν_4^0 in the final state can now contribute. To get a 1% increase in the τ-lepton lifetime, $\sin^2\phi$ will have to be bigger than 1/10. Including the effect of the mass of the ν_τ^0 and ν_4^0 neutrinos, then the ratio in Eq. (2) will be

$$\frac{\Gamma(\tau \to e\nu_\tau\bar{\nu}_e)}{\Gamma(\mu \to e\nu_\mu\bar{\nu}_e)} = \frac{m_\tau^5}{m_\mu^5}\left[\cos^2\phi F(\eta_1) + \sin^2\phi F(\eta_2)\right], \tag{12}$$

where

$$F(\eta) = 1 - 8\eta + 8\eta^3 - \eta^4 - 12\eta^2 \ln\eta, \tag{13}$$

and $\eta_1 = m_{\nu_\tau^0}^2/m_\tau^2$, $\eta_2 = m_{\nu_4^0}^2/m_\tau^2$. Note that if $m_{\nu_\tau^0} <$ 35 MeV, then $F(\eta_1) = 1$ to within 0.3%. Hence the kinematic effect of the τ-neutrino mass cannot be the source of any presently observable discrepancy [and we subsequently set $F(\eta_1) = 1$ in Eq. (12)]. The condition that the lifetime is longer by 1% implies from Eq. (12) that

$$\sin^2\phi\,[1 - F(\eta_2)] = \frac{1}{100}. \tag{14}$$

Using $\sin^2\phi \simeq m_{\nu_\tau^0}/m_{\nu_4^0}$, Eq. (14) implies that

$$\begin{aligned} m_{\nu_\tau^0} &= \frac{1}{100}\frac{m_{\nu_4^0}}{1 - F(\eta_2)}, \\ &= \frac{1}{100}\frac{m_\tau\sqrt{\eta_2}}{8\eta_2 - 8\eta_2^3 + \eta_2^4 + 12\eta_2^2\ln\eta_2}. \end{aligned} \tag{15}$$

Solving this equation numerically assuming that $m_{\nu_\tau^0} <$ 35 MeV, we find that $m_{\nu_4^0} > 120$ MeV. Thus we conclude that we can obtain a 1% increase of the τ-lepton lifetime for $m_{\nu_4^0}$ in the range

$$120 \text{ MeV} < m_{\nu_4^0} < 3.5 \text{ GeV}. \tag{16}$$

Having established the range on $m_{\nu_4^0}$ to obtain a 1% increase in the τ-lepton lifetime, we now examine the corresponding range for the τ-neutrino mass $m_{\nu_\tau^0}$. As $m_{\nu_4^0}$ ranges from 3.5 GeV to $m_\tau \simeq 1.8$ GeV, $m_{\nu_\tau^0}$ ranges from 35 to 18 MeV. For $m_{\nu_4^0} < m_\tau$, $m_{\nu_\tau^0}$ is related to $m_{\nu_4^0}$ through Eq. (15). Numerically solving this equation, we find that as $m_{\nu_4^0}$ ranges between its allowed range 120 MeV $< m_{\nu_4^0} < m_\tau$, the minimum value of $m_{\nu_\tau^0}$ is 10 MeV. If there is a greater than 1% discrepancy in the length of the τ lifetime then the minimum value of $m_{\nu_\tau^0}$ must be *greater* than this value. Thus we find that

$$m_{\nu_\tau^0} > 10 \text{ MeV}, \tag{17}$$

to have a τ-lepton lifetime longer than 1% in the model. This should be a testable prediction of the model.

It might be that the τ lepton is longer lived than 1%. In Table I, we show the allowed ranges of the physical parameters to have the τ lepton longer lived by 2% and 3%. If the τ lepton is longer lived by more than 3.5% of the standard model prediction, then our model is ruled out from the 35 MeV limit on the τ-neutrino mass.

To fully test the model the new neutrino ν_4^0 will have to be discovered. Its dominant decay mode is expected to be via a flavor-nondiagonal coupling to a virtual Z boson [7]:

$$\nu_4^0 \to \nu_\tau^0 + Z^*. \tag{18}$$

Note that the flavor-changing vertex exists because of the absence of a Glashow-Iliopoulos-Maiani mechanism. When the virtual Z^* goes to a e^+e^- or $\mu^+\mu^-$ pair then this should lead to an observable signature. However the ν_4^0 neutrino tends to have a relatively long lifetime so that it will not decay inside a detector unless $m_{\nu_4^0} \gtrsim 1$ GeV (assuming that the ν_4^0 is relativistic). Thus, there is only a limited range of the mass of the ν_4^0 which is accessible to collider experiments.

In conclusion, we have examined the possibility that the τ-lepton lifetime is slightly longer than the standard

TABLE I. Table showing the allowed range for the mass of the τ neutrino (ν_τ^0) and the mass of the right-handed neutrino (ν_4^0) in the model to obtain a 1%, 2%, or 3% increase in the standard model prediction of the τ lepton lifetime.

$\Delta\tau$ (%)	$m_{\nu_4^0}$	$m_{\nu_\tau^0}$
1% increase	120 MeV $< m_{\nu_4^0} <$ 3.5 GeV	$10 < m_{\nu_\tau^0} <$ 35 MeV
2% increase	260 MeV $< m_{\nu_4^0} <$ 1.8 GeV	$20 < m_{\nu_\tau^0} <$ 35 MeV
3% increase	480 MeV $< m_{\nu_4^0} <$ 1.1 GeV	$30 < m_{\nu_\tau^0} <$ 35 MeV

model prediction. We have argued that the simplest way to achieve a slightly longer τ lifetime is to add one right-handed neutrino to the minimal standard model. We then analyzed the resulting model and showed that to get a lifetime increase of greater than 1% in the model implies that the τ-neutrino must have a mass greater than 10 MeV. The results for our model are summarized in Table I. The experimental upper limit on the τ-neutrino mass is 35 MeV so that the model will be tested when measurements of the τ neutrino mass improve. Finally, we note that for a significant range of parameters of this model, the lifetimes of the massive neutrinos are compatible with the bounds imposed by the standard cosmological model [8].

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[1] For a recent discussion on the possible τ-lepton lifetime discrepancy, see, for example, W. J. Marciano, in *The Vancouver Meeting—Particles and Fields '91*, Proceedings of the Joint Meeting of the Division of Particles and Fields of the American Physical Society and the Particle Physics Division of the Canadian Association of Physicists, Vancouver, 1991, edited by D. Axen, D. Bryman, and M. Comyn (World Scientific, Singapore, 1992), p. 461; Phys. Rev. D **45**, R721 (1992).

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[5] For a review of electric charge quantization in the standard model and its extensions, see R. Foot, G. C. Joshi, H. Lew, and R. R. Volkas, Mod. Phys. Lett. A **5**, 2721 (1990).

[6] Phenomenology will require that $\nu_L' \approx \nu_{\tau L}$ since e-μ universality in weak interactions is known to hold quite well. Note that if ν_L' is exactly equal to $\nu_{\tau L}$ then both electron- and μ-lepton numbers will be unbroken and hence the model would have a gaugeable U(1) global symmetry (which is generated by the difference between the electron-lepton number and the μ-lepton number). Thus, if this is the case, then the Lagrangian of the model does not have electric charge quantization. However, if $\nu_L' \approx \nu_{\tau_L}$ then the electron- and μ-lepton numbers are broken, and charge quantization results.

[7] If $m_{\nu_4^0} > m_\tau$ then the decay via virtual W boson $\nu_4^0 \to \tau + W^*$ is kinematically allowed and can also be significant.

[8] For neutrinos with masses in the range between 100 eV and 2 GeV there exists constraints on the neutrino lifetime from the standard cosmological model. In the model considered here, the τ neutrino can decay via the charged-current interaction, i.e.,

$$\nu_\tau \;\to\; e^- \;+\; W^* \;\to\; e^- \;+\; e^+ \;+\; \nu_{e,\mu}.$$

The lifetime is given by

$$\tau_{\nu_\tau} \simeq \left(\frac{3\times 10^{-3}}{\alpha}\right)^2 \left(\frac{10\ \mathrm{MeV}}{m_{\nu_\tau}}\right)^5 \times 3\times 10^4\ \mathrm{sec},$$

where α is a mixing parameter of the leptonic charged-current interaction matrix which is the analogue of the Kobayashi-Maskawa matrix for the quark sector. This is obtained by diagonalizing the full neutrino mass matrix and so on. These details will be given elsewhere. From neutrino experiments [see Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992)],

$$\alpha \lesssim\ 0.05 \text{ for } m_{\nu_\tau} < 20\ \mathrm{MeV},$$

$$\alpha \lesssim\ 0.003 \text{ for } m_{\nu_\tau} > 20\ \mathrm{MeV}.$$

Thus, the τ neutrino with a mass between 10 and 35 MeV can have a lifetime which is compatible with the cosmological bound of 10^4 sec [see A. D. Dolgov and Ya. B. Zeldovich, Rev. Mod. Phys. **53**, 1 (1981); S. Sarkar and A. M. Cooper, Phys. Lett. **148B**, 347 (1984)]. For the case of the ν_4 neutrino its dominant decay mode is via the neutral-current interaction (provided $m_{\nu_4} < m_\tau$), i.e.,

$$\nu_4 \;\to\; \nu_\tau \;+\; Z^* \;\to\; \nu_\tau \;+\; f \;+\; \bar{f},$$

where f denotes light fermions. The lifetime of ν_4 is given by

$$\tau_{\nu_4} \simeq \frac{2\times 10^{-11}}{\sin^2\phi}\left(\frac{\mathrm{GeV}}{m_{\nu_4}}\right)^5\ \mathrm{sec}.$$

Hence, ν_4 has a lifetime which is easily compatible with the standard cosmology model.