$N\pi$ decays of baryons in a relativized model

Simon Capstick
Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
*and Continuous Electron Beam Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606**

Winston Roberts
Department of Physics, Old Dominion University, Norfolk, Virginia 23529
and Continuous Electron Beam Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606
(Received 20 August 1992; revised manuscript received 9 December 1992)

We calculate the $N\pi$ decay amplitudes of baryon resonances in a semirelativistic version of the $^3P_0$ model of hadron decays. We use relativized wave functions for the baryons and mesons, and include an intuitive modification of the usual $^3P_0$ model. Our results are in reasonable agreement with the reported amplitudes for *all* known nonstrange resonances, and confirm a proposed solution to the mystery of the "missing" states.

PACS number(s): 13.30.Eg, 12.40.Qq, 14.20.Gk

I. INTRODUCTION

In the past 20 years, the nonrelativistic constituent quark model (NRQM) has experienced some measure of success. Much of this success has been in the area of spectroscopy. The interplay between quark model spectroscopy and experimental observation has contributed much to our understanding of low-energy phenomenology. In models of the type that concern us here, the relevant degrees of freedom are constituent quarks, and gluonic degrees of freedom are not excited.

Despite the success of these models, many problems, such as the significance and treatment of the relativistic motion of quarks within a hadron, still persist. Some of these problems are discussed in Sec. II. It is possible to correct the models by including some relativistic effects and other refinements, as has been done in Ref. [1] and as briefly described in Sec. II. However, none of these refinements is likely to offer a solution to the important problem of the "missing" baryon states, i.e., states that appear in the model but which have not been seen in πN partial-wave analyses.

One approach that has been used in dealing with the missing states is that of diminishing the number of effective degrees of freedom within the baryon. This is done by replacing the three-quark system with a quark-diquark system [2], with the result that the predicted spectrum contains fewer states. This approach raises the question of whether there is any diquark clustering within a baryon, and if so, to what extent. Indeed, potential model [3] studies and lattice simulations [4] show that there is little evidence for such clustering in baryons consisting of light quarks (unless they have large orbital angular momentum).

If the three-quark description of the baryon is retained, a possible solution to the question of missing baryon resonances is offered by considering the couplings of predicted states to formation channels. Koniuk and Isgur [5] find that the pattern of experimentally observed states matches that of states predicted to couple strongly to formation channels. These results indicate that consideration of spectroscopy alone is not enough in evaluating the utility of a model. Indeed, it is expected that in addition to the usual mixings (such as hyperfine, spin-orbit, etc.) observed in spectroscopic calculations, there should be mixings and mass shifts associated with couplings to decay channels [6].

This suggests that a model calculation of hadron spectra and strong couplings should be an iterative process. A successful model should provide at least a reasonable description of both the masses and couplings of the hadrons it claims to describe. Ideally, a calculation of a spectrum would automatically include some description of couplings, as these do affect the masses (and vice versa). Clearly, attempts to describe both sets of phenomena simultaneously are necessarily involved. The approach adopted here is to treat the problem as a step-by-step process. As an initial step, the wave functions from an existing model of the baryon spectrum are used to predict strong couplings. In this way, we hope to get an indication of the strengths and shortcomings of the model, which may provide us with insight for possible improvements.

This article is organized as follows. The rest of this section is devoted to general comments on models of strong hadron couplings, together with a brief synopsis of constituent quark models, with emphasis on the work of Isgur and Karl. In Sec. II we discuss the relativized model of baryon resonances used here. Section III describes the decay model, while our results are presented in Sec. IV. Section V contains our conclusions and an outlook, and some calculational details are relegated to an appendix.

*Current address.

A. Hadron transition models

While the study of hadron spectra is a well-developed field with many competing models, especially for the baryon spectrum, there are far fewer models for strong hadronic transitions. The Okubo-Zweig-Iizuka-(OZI-)allowed [7] strong decays of hadrons which we consider here have been examined in three classes of models described below.

The "hadrodynamic" models, illustrated in Fig. 1, in which all hadrons are treated as elementary pointlike objects, do not lend themselves easily to decay calculations of the kind we are carrying out here. This is understandable, since each transition is described in terms of an independent phenomenological coupling constant $g_{BB'M}$. While the use of SU(2) or SU(3) flavor symmetry arguments would give relationships among some of these coupling constants, the overall situation would nevertheless be largely unworkable.

A second class of models treats the baryons as objects with structure, but the decay takes place through elementary meson emission. Such an approach may be taken in bag models. Some potential model calculations have used a similar approach, as, for instance, the work of Koniuk and Isgur [5]. In these models, the mesons are emitted from quark lines (Fig. 2), and one replaces the set of $g_{BB'M}$ coupling constants with a smaller set of $g_{qq'M}$'s. In addition, one may use SU(2) or SU(3) flavor symmetry to relate the coupling constants for mesons within a single multiplet, as well as those for different quarks.

A third class of models may be referred to broadly as pair creation models. In such models both the baryons and mesons have some structure, and the decay of the baryon, say, is facilitated by the creation of a quark-antiquark pair somewhere in the hadronic medium. The created antiquark combines with one of the quarks from the decaying baryon to form the daughter meson, while the quark of the created pair becomes part of the daughter baryon. This is illustrated in Fig. 3.

There are several types of pair creation model. In the $^3P_0$ model popularized by LeYaouanc *et al.* [8], the quark-antiquark pair is created anywhere in space with the quantum numbers of the QCD vacuum, namely, 0^{++}. This corresponds to $^3P_0$, hence the name of the model. While the pair, in principle, may be created very far away from the decaying hadron, the wave-function overlaps required naturally suppress such contributions to the decay amplitude. This model has been quite popular in descriptions of hadron decays and has been applied to baryon decays [8], meson decays [8, 9], and even the decays of fictitious four-quark states [10].

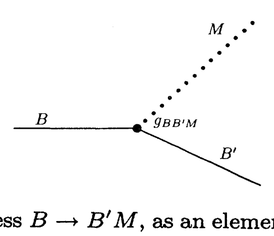

FIG. 1. Process $B \to B'M$, as an elementary meson emission from a pointlike baryon.

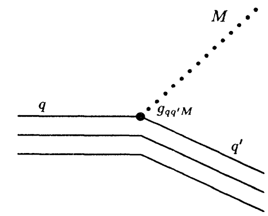

FIG. 2. Process $B \to B'M$, as an elementary meson emission from a quark.

Other pair creation models include the string-breaking models of Dosch and Gromes [11] and Alcock, Burfitt, and Cottingham [12]. In these models, the lines of color flux between quarks have collapsed into a string, and the pair is created when the string breaks. This is illustrated in Fig. 4. In the Dosch-Gromes version of this model, the created pair has the quantum numbers $^3P_0$, while in the version of Alcock, Burfitt, and Cottingham, the quantum numbers of the created pair are $^3S_1$.

Several authors have used similar ideas in describing decays of hadrons in flux-tube breaking models [13–16]. Here, the pair still has quantum numbers $^3P_0$, but is constrained to be created somewhere within the flux tube connecting quarks. The string-breaking picture arises in the zero-width limit of the flux tube.

B. Nonrelativistic quark model spectroscopy

The nonrelativistic quark model as applied to the baryon spectrum and decays owes its origins to many authors. We intend to focus on the model of Isgur and Karl [17, 18], which evolved from the pioneering work of others, and refer the reader to the literature for a discussion of the origins of the model [19].

The choice of dynamical degrees of freedom used to represent a baryon depends on momentum transfer; at low Q^2 they can be taken to be constituent quarks, which are dressed quarks with effective masses of 200–300 MeV for u and d. In this model the gluon fields affect the quark dynamics only by providing [20] a confining potential in which the quarks move; the effects of the quark motion on the gluon dynamics are neglected. At short distances one-gluon exchange provides the spin-dependent potential. This model will obviously only be applicable to "soft" (low-Q^2 or coarse-grid) aspects of hadron structure and is best applied to low-mass baryons where gluonic excitation is unlikely. It also ignores mass shifts [21, 6] and mixings from couplings to decay channels. One of the purposes of this paper is test the model's limit

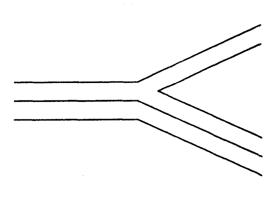

FIG. 3. OZI-allowed process $B \to B'M$, in a quark pair creation scenario.

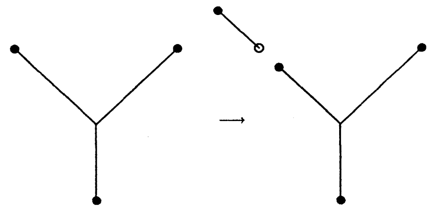

FIG. 4. Process $B \to B'M$ in the string-breaking picture.

of applicability by extending the calculation of spectra and strong decays to highly excited states where these approximations are expected to break down.

In the Isgur-Karl (IK) model [17, 18] the Schrödinger equation $H\Psi = E\Psi$ for the nonrelativistic three-valence-quark system is solved for baryon energies and wave functions. The Hamiltonian is

$$H = \sum_i \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) + \sum_{i<j} \left(V^{ij} + V^{ij}_{\mathrm{hyp}} \right), \qquad (1)$$

where the spin-independent potential V^{ij} has the form $V^{ij} = C_{qqq}/3 + br_{ij}/2 - 2\alpha_s/3r_{ij}$, with $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$. In practice, V^{ij} is written in terms of a harmonic-oscillator potential $Kr_{ij}^2/2$ plus an anharmonicity U_{ij}, which is treated as a perturbation. The hyperfine interaction V^{ij}_{hyp} is the sum

$$V^{ij}_{\mathrm{hyp}} = \frac{2\alpha_s}{3m_i m_j} \left\{ \frac{8\pi}{3} \mathbf{S}_i \cdot \mathbf{S}_j \delta^3(\mathbf{r}_{ij}) + \frac{1}{r_{ij}^3} \left[\frac{3(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j \right] \right\} \qquad (2)$$

of contact and tensor terms arising from the color-magnetic dipole-magnetic dipole interaction. Spin-orbit forces are neglected, as their inclusion spoils [17] the agreement with the spectrum (the resulting splittings tend to be too large). The relative strengths of the Coulomb, contact, and tensor terms are as determined from the Breit-Fermi limit of the one-gluon-exchange potential.

Nonstrange baryon states are then written as the product of a totally antisymmetric (under the exchange group S_3) color wave function C_A and a sum $\sum \psi\chi\phi$. The spatial (ψ) and flavor (ϕ) wave functions are chosen to represent S_3, and the usual quark-spin wave functions (χ_S, with $S = \frac{1}{2}, \frac{3}{2}$ from $\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2}$) automatically do so. The sum is arranged to be totally exchange symmetric and also implicitly includes Clebsch-Gordan coefficients for coupling the quark orbital angular momentum $\mathbf{L} = \mathbf{l}_\rho + \mathbf{l}_\lambda$ with the total quark spin $\mathbf{S}$. The spatial wave functions ψ are, in zeroth order in the perturbations U and H_{hyp}, the harmonic-oscillator eigenfunctions $\psi_{NLM}(\boldsymbol{\rho}, \boldsymbol{\lambda})$, where $\boldsymbol{\rho} = (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}$ and $\boldsymbol{\lambda} = (\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3)/\sqrt{6}$. Positive-parity ground states [such as $N(938)$ and $\Delta(1232)$] are described by wave functions with $N = 2(n_\rho + n_\lambda) + l_\rho + l_\lambda = 0$. The low-lying negative-parity excited resonances ("P waves") have $N = 1$ spatial wave functions with either $l_\rho = 1$ or $l_\lambda = 1$; the positive-parity excited resonances have $N = 2$ wave functions with radial excitations in one of the two oscillators or "orbital excitations" with $l_\rho + l_\lambda = 2$ and $L = 0$, 1, or 2.

The Schrödinger equation is then solved for the energies and compositions of the resonances by first-order perturbation theory in U and H_{hyp}. The anharmonicity is treated as a diagonal perturbation on the energies of the states and so is not allowed to mix the $N = 0$ and $N = 2$ band states. It cannot cause splittings within the $N = 0$ and $N = 1$ bands of nonstrange states, and in first order it splits up the $N = 2$ band states in a pattern which is independent of the exact form of the potential U. The hyperfine interaction is treated to first order in both the energies and wave functions. To a large degree it is the contact interaction (responsible, e.g., for the Δ-N and Σ-Λ splittings) and the anharmonic splitting in the $N = 2$ band which determine the coarse features of the spectrum.

The main features of the spectrum of the low-lying baryon resonances [17–19] are then quite convincingly described by this model. Just as importantly, the mixing of the states caused by the hyperfine interaction is crucial in explaining their observed strong and electromagnetic [5] decays. There are more states predicted by the model in the $N = 2$ band than exist in the partial-wave analyses; the Koniuk-Isgur [5] strong decay analysis established that the states whose hyperfine-mixed wave functions allow them to couple to the πN production channel largely correspond, in both energy and number, with the observed states.

II. RELATIVIZED-MODEL WAVE FUNCTIONS AND SPECTROSCOPY

Although successful, the above approach to baryon spectroscopy can be criticized on a number of grounds. In strongly bound systems such as the baryons, where $p/m \simeq 1$, the approximation of nonrelativistic kinematics and dynamics is not justified. For example, if one forms the one-gluon-exchange T-matrix element *without* performing a nonrelativistic reduction, factors of m_i in Eq. (2) are replaced, roughly, with factors of $E_i = \sqrt{\mathbf{p}_i^2 + m_i^2}$. In a potential model picture there should also be "kinematic" smearing of the interquark coordinate $\mathbf{r}_{ij}$ with a characteristic size given by the Compton wavelength of the quark $1/m_q$.

Neglect of the scale dependence of a cutoff field theory has resulted in nonfundamental values of parameters such as the quark mass, the string tension (implicit in the size of the anharmonic perturbations), and the strong coupling $\alpha_s \simeq 2$. A consistent theory with constituent quarks should give them a commensurate size, which would also smear out the interactions between the quarks. The model should use a string tension consistent with meson spectroscopy, and the relation between the anharmonicity and the meson string tension is unexplored. If there are genuine three-body forces in baryons, they are neglected. The neglect of spin-orbit interactions in the Hamiltonian is also inconsistent, independent of our choice of ansatz for the short-distance and confining physics. There is some evidence in the *observed* spectrum for spin-orbit splittings, e.g., that between the states $\Delta^*\frac{1}{2}^-(1620)$ and $\Delta^*\frac{3}{2}^-(1700)$.

The model also carries out a first-order perturbative evaluation of large perturbations. The contact term is formally infinite unless the above smearing is implemented. The size of the first-order anharmonic splitting of the $N = 2$ band must be larger than the zeroth-order harmonic splitting, to get the lightest $N = 2$ band nucleon [identified with the Roper resonance $N(1440)$] below the P-wave nonstrange states. This calls into question the usefulness of first-order perturbation theory. It also means that the wave functions of states such as the Roper resonance should have a large anharmonic *mixing* with the ground states. Some of these flaws of the nonrelativistic model are inessential and can be corrected. The relativized model [22, 1], briefly described below, puts the ideas of many authors together in an attempt to correct as many of these deficiencies as possible.

A. Details of the relativized model

The Schrödinger equation is once again solved in a Fock space made up of valence quarks, with a Hamiltonian now given by

$$H = \sum_i \sqrt{\mathbf{p}_i^2 + m_i^2} + V, \tag{3}$$

where V is a relative-position- and -momentum-dependent potential which tends in the nonrelativistic limit (*not* taken here) to

$$V \to V_{\text{string}} + V_{\text{Coul}} + V_{\text{hyp}} + V_{\text{so(cm)}} + V_{\text{so(Tp)}}. \tag{4}$$

Here V_{string} is the potential generated by adding the lengths of the gauge-invariant (Y-shaped) string configuration and multiplying by the meson string tension $\sqrt{\sigma}$. The string is assumed to adjust instantaneously to the motion of the quarks so that it is always in its minimum length configuration; this generates an adiabatic potential for the quarks [23] which includes genuine three-body forces. Here V_{Coul}, V_{hyp}, $V_{\text{so(cm)}}$, and $V_{\text{so(Tp)}}$ are color-Coulomb, color-hyperfine, color-magnetic spin-orbit, and Thomas-precession spin-orbit potentials, respectively. The color-Coulomb and hyperfine potentials are as in the Isgur-Karl model, except that the interquark coordinate $\mathbf{r}_{ij}$ is smeared out over mass-dependent distances, and the momentum dependence away from the $p/m \to 0$ limit is parametrized.

In practice this smearing is brought about by convoluting the potentials with a function

$$\rho_{ij}(\mathbf{r}_{ij}) = \frac{\sigma_{ij}^3}{\pi^{\frac{3}{2}}} e^{-\sigma_{ij}^2 r_{ij}^2}. \tag{5}$$

The σ_{ij} are chosen to smear the interquark coordinate over distances of $O(1/M_Q)$ for Q heavy and approximately 0.1 fm for light quarks. The potentials are made momentum dependent by introducing factors which replace m_i by, roughly, E_i. For example the contact part of V_{hyp} becomes $\sum_{i<j} V_{\text{cont}}^{ij}$, with

$$V_{\text{cont}}^{ij} = \left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon_{\text{cont}}} \frac{16\pi}{9} \alpha_s(r_{ij}) \frac{\mathbf{S}_i \cdot \mathbf{S}_j}{m_i m_j} \times \left[\frac{\sigma_{ij}^3}{\pi^{\frac{3}{2}}} e^{-\sigma_{ij}^2 r_{ij}^2}\right] \left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon_{\text{cont}}}. \tag{6}$$

Here ϵ_{cont} is a constant parameter, and $\alpha_s(r_{ij})$ is a running-coupling constant which runs according to the lowest-order QCD formula, saturating to 0.6 at $Q^2 = 0$.

The color-magnetic and Thomas-precession spin-orbit potentials are smeared and allowed to depend on momentum in a similar way; in the nonrelativistic limit, they also tend to the spin-orbit potentials which are calculated (but not included) in the Isgur-Karl model.

Nonstrange baryon states are then written as

$$\Psi = C_A \phi \sum \psi \chi, \tag{7}$$

where ϕ is one of uuu (Δ^{++}), uud (p or Δ^+), ddu (n or Δ^0), or ddd (Δ^-), and the sum is performed so that the result is only symmetric under exchange of quarks 1 and 2. The spatial wave functions ψ are made up of solutions of the two three-dimensional oscillators

$$\psi_{LMn_\rho l_\rho n_\lambda l_\lambda} = \sum_m C(l_\rho, l_\lambda, m, M-m; L, M) \times |n_\rho\, l_\rho\, m\rangle |n_\lambda\, l_\lambda\, M-m\rangle, \tag{8}$$

where

$$|n_\rho\, l_\rho\, m\rangle = \mathcal{N}_{n_\rho l_\rho} \alpha^{\frac{3}{2}} (\alpha\rho)^{l_\rho} e^{-\alpha^2\rho^2/2} L_{n_\rho}^{l_\rho+\frac{1}{2}}(\alpha^2\rho^2) Y_{l_\rho m}(\Omega_\rho), \tag{9}$$

and similarly for $|n_\lambda\, l_\lambda\, M-m\rangle$, where $\mathcal{N}_{nl} = \sqrt{2n!/\Gamma(n+l+\frac{3}{2})}$.

The wave functions of baryon states with given total spin J and parity P can be expanded in a basis of (implicitly L-S coupled) states $\psi\chi$; the energies and wave functions of the baryon states are then formed by diagonalizing the Hamiltonian H in this basis. Note that the basis mixes N ($I = \frac{1}{2}$) and Δ ($I = \frac{3}{2}$) states; the $m_u = m_d$ symmetry of H ensures that the eigenvectors are either Δ's, with linear combinations $\sum \psi\chi$ totally symmetric under S_3, or N's, with mixed-λ symmetry. The basis extends to at least $N = 6$ for positive-parity states [24] and $N = 7$ for negative-parity states, giving of the order of 100 substates for each J^P. Energies are minimized, state by state, by coarse variation of the oscillator size parameter α; however, in a calculation of transition amplitudes it is necessary to have all states expanded with the same α for orthogonality. A measure of the convergence of the expansion is the α dependence of the energies, which is relatively weak, and of these amplitudes, which will be discussed later.

The resulting spectroscopy is comparable to that of the Isgur-Karl model, with some improvements and some deterioration. This is a nontrivial test, as the model is much more tightly constrained; various quantities which were fit in the Isgur-Karl model (such as band centers of mass) are now predicted, and the same set of parameters [25] fits *all* mesons and baryons. Spin-orbit interactions are small but not neglected in this model; this is mainly due to the use of a smaller α_s, although there is, as expected, a partial cancellation of the color-magnetic and Thomas-precession spin-orbit terms, and the spin-orbit interactions are suppressed relative to the hyperfine contact term by the choice of $\epsilon_{\rm cont} < \epsilon_{\rm so}$. This smaller α_s yields the same contact splittings when the smeared contact interaction of Eq. (6) is evaluated without resorting to wave-function perturbation theory (apart from basis truncation beyond $N = 6$).

The wave functions which result from this process differ substantially from those of the Isgur-Karl model, due to the more realistic treatment of the spin-independent potential and inclusion of the configuration mixings that it causes. Since a nonsingular contact interaction with a smaller α_s is used, and all of the spin-dependent interactions are evaluated more precisely, we can also expect differences in the wave-function mixings due to the hyperfine interaction.

A convenient notation which we will use throughout this work is to label a quark model state with the oscillator band at which, by counting arguments, such a state first appears in the pure-oscillator spectrum. However, if we refer to an N=3 band state, say, we are not implying that the state has its wave function confined to the N=3 band of the oscillator; all oscillator states of a given flavor, spin, and parity mix with all others (up to some maximum N) in the model of Ref. [1]. It *is* likely that such a state has a large part of its wave function made up of N=3 band states, but this labeling should be thought of as merely a convenient way to visualize the counting of quark model states.

Problems which remain unsolved in the spectroscopy of the $N \leq 2$ nonstrange baryons are the overestimate of the masses of the Roper resonance $N\frac{1}{2}^+(1440)P_{11}$ and the P_{33} state $\Delta\frac{3}{2}^+(1600)$. Although the Roper resonance is naturally significantly lighter than the other states in its band, its mass is still overestimated by 60 MeV even if the N=2 band center of energy (predicted to be 40 MeV too heavy) is adjusted downwards. Richard [26] has shown that, in a broad class of models, it is impossible for the mass of this state to become less than that of the P-wave states. The situation for the two-star $\Delta\frac{3}{2}^+(1600)$ is worse, with the adjusted mass about 150 MeV too high. There are also discrepancies between the model predictions of the photocouplings of these states [27] and those extracted from the data. Models exist which describe one or both of these states as hybrids [28] (or with significant mixings with hybrid states). It is therefore of interest to examine their $N\pi$ couplings in a model with the above relativized-model wave functions.

B. Beyond the N=2 band

One of the advantages of the relativized model of Ref. [1] is that it can be extended to states which, by counting arguments, must correspond to states which first appear [29, 30] when the basis is extended beyond the N=2 band. The spectroscopy of some of these states was examined in Ref. [1], but comparison with the data was limited because an *ab initio* strong decay calculation was not performed. This meant that it was only possible to list the quark model states, but was not possible to make assignments of these states to those seen in the $N\pi$ (for nucleon and Δ states) or $\Lambda\bar{K}$ production channels. This deficiency is corrected here for the $N\pi$ states by calculating their production amplitudes in the $^3P_0$ model described below.

There are several well-established states which correspond to quark model states which first appear in the N=3 band. The lightest of these are [31] the three-star Δ states $\Delta\frac{1}{2}^-(1900)S_{31}$ [the notation is flavor/J^P/mass (MeV)/$N\pi$ partial wave] and $\Delta\frac{5}{2}^-(1930)D_{35}$. Quark model predictions for the masses of these states are consistently high [29, 30, 1, 32] by about 150–250 MeV. There are two four-star nucleon states which have no N=1 analogues, $N\frac{7}{2}^-(2190)G_{17}$ and $N\frac{9}{2}^-(2250)G_{19}$ whose masses seem to be quite well described in the spectroscopic models [29, 1]. There are also two two-star candidate nucleon resonances $N\frac{3}{2}^-(2080)D_{13}$ and $N\frac{5}{2}^-(2200)D_{15}$, which are N=3 band recurrences of the familiar light negative-parity nucleon resonances, and a two-star candidate Δ state $\Delta\frac{9}{2}^-(2400)G_{39}$.

If the lightest model state in each flavor and J^P sector is assigned to these experimental states, a roughly consistent picture of their spectroscopy emerges. In some cases, as for the two $J^P = \frac{9}{2}^-$ states, this exhausts the model states in this band, making such an assignment natural. In other cases there are many light model states which could correspond to the observed states and it is necessary (if we are to determine which states are conventional three-quark states and which are not) to determine which states couple to $N\pi$.

This necessity becomes even stronger for the states which first occur in the N=4 band (and above); there are three nucleon states (with a two-star rating or better) with $J^P = \frac{9}{2}^+$, $\frac{11}{2}^-$, and $\frac{13}{2}^+$ which cannot be $N \leq 3$ band states since they must have $L \geq 4$. Similarly there are four such Δ states with $J^P = \frac{9}{2}^+$, $\frac{11}{2}^+$, $\frac{13}{2}^-$, and $\frac{15}{2}^+$. In Ref. [1] the spectroscopy was limited to states with $J \leq \frac{11}{2}$; here the masses and wave functions of states with $\frac{13}{2} \leq J \leq \frac{15}{2}$ are estimated in this model, and assignments of quark model states to these experimental states and candidates are made. At this level of excitation it is likely that gluon dynamics, and decay-channel mixings and mass shifts play an essential role in the spectroscopy of these states. This study, which neglects these effects, may allow us to pinpoint states which are not easily explained with simple three-quark model assignments.

III. DECAY MODEL

A. The model

Our starting point in modeling the $N\pi$ transitions of the baryons is the ansatz that the operator T responsible for the transition is

$$T = -3\gamma \sum_{i,j} \int d\mathbf{p}_i d\mathbf{p}_j \delta(\mathbf{p}_i + \mathbf{p}_j) C_{ij} F_{ij} e^{-\lambda^2(\mathbf{p}_i - \mathbf{p}_j)^2/2} \times \sum_m \langle 1, m; 1, -m | 0, 0\rangle \times \chi_{ij}^m \mathcal{Y}_1^{-m}(\mathbf{p}_i - \mathbf{p}_j) b_i^\dagger(\mathbf{p}_i) d_j^\dagger(\mathbf{p}_j). \quad (10)$$

Here, C_{ij} and F_{ij} are the color and flavor wave functions of the created pair, both assumed to be singlet, χ_{ij} is the spin-triplet wave function of the pair, and $\mathcal{Y}_1(\mathbf{p}_i - \mathbf{p}_j)$ is the vector harmonic indicating that the pair is in a relative p wave.

For the transition $A \to BC$, we are interested in evaluating the transition amplitude M, given by

$$M = \langle BC|T|A\rangle. \quad (11)$$

The wave functions of the states involved must be written in comparable second-quantized form in order to evaluate M. All of the details of this calculation are given elsewhere [33]. Our full transition amplitude is given in the Appendix. In arriving at this form, we use the notation illustrated in Fig. 5, and we denote $L_{\lambda_a} = \ell_a$, with a similar definition for baryon B. We have also assumed that the decaying baryon is at rest and that the final baryon has three-momentum $\mathbf{k}$.

There are two phenomenological parameters in our decay model. These are γ, the usual $^3P_0$ coupling strength, and λ, which is a new parameter that we have introduced. In the usual version of this model, λ is zero. We have introduced the exponential factor to serve primarily as a cutoff that can be used to regulate the higher-momentum components of the created pair's wave function. In principle, one may take the Gaussian parameter α of the wave

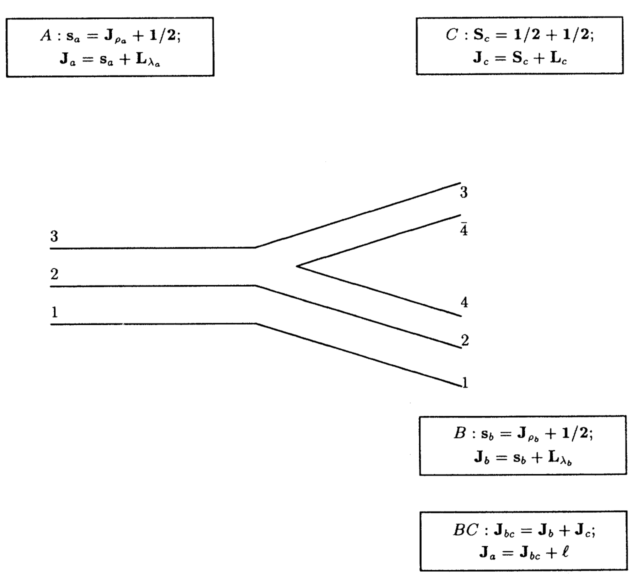

FIG. 5. Schematic diagram of the decay $B \to B'M$ in the $^3P_0$ model. The angular momentum notation is shown. The decay proceeds through $B(123) \to 12(4\bar{4})3 \to B'(124)M(\bar{4}3)$.

functions as a parameter as well, but if our expansion of the wave functions has converged, our results should be independent of this choice, which amounts to a choice of basis. We will demonstrate this insensitivity below.

B. Phase space

To obtain a decay width from the amplitude we have evaluated, we use

$$\Gamma(A \to BC) = \sum_{J_{bc},\ell} |M_{A\to BC}(J_{bc}, \ell, k_0)|^2 \, \Phi(ABC), \quad (12)$$

where Φ is the phase space for the decay. Here, a number of options are available to us. The usual prescription is to use

$$\Phi(ABC) = 2\pi \frac{E_b(k_0) E_c(k_0) k_0}{m_a}, \quad (13)$$

with $E_b(k_0) = \sqrt{k_0^2 + m_b^2}$, $E_c(k_0) = \sqrt{k_0^2 + m_c^2}$. This is a "semirelativistic" prescription, since it is usually used with a matrix element calculated nonrelativistically, while E_b and E_c have been calculated relativistically. A fully nonrelativistic prescription consists in using

$$\Phi(ABC) = 2\pi \frac{m_b m_c k_0}{m_a}. \quad (14)$$

In their calculation of meson decay widths, Kokoski and Isgur [14] use the prescription

$$\Phi(ABC) = 2\pi \frac{\tilde{m}_b \tilde{m}_c k_0}{\tilde{m}_a}, \quad (15)$$

where the $\tilde{m}$'s are effective meson masses, evaluated with the spin-independent interaction. They argue that this is valid in the weak-binding limit, where ρ and π are degenerate and $\tilde{m}_\pi = 5.1 m_\pi$.

In our calculation of the baryon decay widths, there are some features that are similar to the Kokoski-Isgur calculation of the meson decay widths: (i) The baryon wave functions we are using [1] were obtained in the same spirit as the Godfrey-Isgur [22] meson wave functions used in the Kokoski-Isgur calculation, and in fact, many of the parameters of both spectroscopic calculations were chosen to be the same or similar; (ii) we use a single-Gaussian wave function for the pion with a size parameter [34] $\beta_\pi = 0.4$ GeV, which is one of two values used by Kokoski and Isgur. We would therefore argue that it makes sense for us to use Eq. (15) in our calculation of the decay widths. For the decays $R \to N\pi$, we take $\tilde{m}_N = 1.1$ GeV, $\tilde{m}_\pi$=0.72 GeV, consistent with Kokoski and Isgur, and $\tilde{m}_R = m_R$.

IV. RESULTS AND DISCUSSION

Our approach is to fit the two parameters γ and λ of our decay model to the $N\pi$ decay amplitudes of the nonstrange resonances with Particle Data Group [31] (PDG) ratings of two stars or better. We include only the low-lying states in the fit, i.e., those with quark model analogues in the N=0, 1, or 2 bands, where both experiment and the model are the most trustworthy. The reader is reminded that in the relativized model the labeling of states by oscillator bands is a matter of notation and does not imply that their wave functions are restricted to those bands. In order to test for sensitivity to the harmonic-oscillator size parameter, we have performed fits for $\alpha = 0.5$ and 0.6 GeV. The fit is strongly sensitive to γ since all amplitudes are simply proportional to this parameter. With the value of β_π adopted above, our best fit gives $\lambda \simeq 0$. The optimal value of λ is, however, correlated to the pion size. For example, using the Godfrey-Isgur effective $\beta_\pi = 0.63$ GeV requires $\lambda \simeq 2.0$ GeV^{-1} for the best fit; for this value of β_π the momentum spread in the pion wave function is large and a nonzero λ is required to cut off the high-momentum components.

Once these parameters have been fixed, the model that results is used to calculate the $N\pi$ amplitudes of all other model states for which there are experimental candidates. Since there are already many states in the N=2 band which are "missing" in the $N\pi$ analyses, it is not interesting to do a complete survey of N=4 band recurrences of the positive-parity states with $J \leq 7/2$, although there are some exceptions as noted below. However, *all* N=1 band nonstrange states predicted by the quark model are well established in $N\pi$, and as we have seen above, there are several states in the N=3 band which are given good ratings. Accordingly, for each $J \leq 9/2$, we have calculated the $N\pi$ amplitudes for all negative-parity N=3 band model states, up to a given cutoff mass. This mass is chosen to exceed the mass of the heaviest resonance of this J^P reported by the PDG. In some cases there are a small number of states of a given J in the N=3 band, and so we simply calculate all of their $N\pi$ amplitudes.

A similar procedure is used to limit the calculation of amplitudes to a workable number of model states in the case of the $N \geq 4$ band states. For all resonances reported by the PDG we have, as we shall argue below, assigned reasonable model analogues based on both the masses and the predicted $N\pi$ amplitudes of the model states.

Although the signs of these $N\pi$ amplitudes are not experimentally accessible in pion production exeriments, the combined signs of the $N\pi$ vertices and those of the $N\gamma$ vertices in single-pion photoproduction are. These photoproduction amplitude signs are examined in the relativized model in Refs. [35, 36], using the signs of the $N\pi$ amplitudes calculated here. In most cases the *calculated* signs from our $^3P_0$ model agree with the signs *fit* to those of the single-pion photoproduction amplitudes (by their choice of the signs of reduced $N\pi$ amplitudes) by Koniuk and Isgur [14]. There are some differences, however, and the $N\pi$ signs of some states are sensitive to mixings; for details see Refs. [35, 36].

A. $g_{NN\pi}$

Since we are attempting to describe the couplings of baryons to mesons in our model, we should be able to reproduce a reasonable value for $g_{NN\pi}$. In defining this quantity, however, we have to be careful since the usual definition arises from a completely relativistic treatment of the nucleon and pion, while the description we have may best be described as "relativized." This is crucial when we consider how the states should be normalized.

In essence, our calculation resembles that of Miller [37]. Our ansatz is to evaluate the amplitude for scattering of an on-shell pion and an on-shell nucleon into an off-shell nucleon, both in our model and using the usual hadrodynamic prescription. In order to account for differences in normalizations in the two calculations, we evaluate a "decay rate" for this process, since the choice of phase space is dictated by the normalization of our states. Equating the two decay rates and defining the amplitude calculated in our model as

$$A_{NN\pi} \equiv A_0(k_0)k_0, \tag{16}$$

where k_0 is the three-momentum of one of the nucleons in the rest frame of the other nucleon, we obtain

$$g_{NN\pi} = \frac{4}{\sqrt{3}}\pi^{3/2}\sqrt{M\tilde{m}_N\tilde{m}_\pi}A_0(0). \tag{17}$$

Here $M = m_N + m_\pi$, and $\tilde{m}_N$ and $\tilde{m}_\pi$ have the values noted in Sec. III.

With this ansatz, we obtain a value of 17.3 for $g_{NN\pi}$, in reasonable agreement with the accepted value of 13.4 and consistent with the deviation that we expect in our model.

B. $N \leq 2$ states and our fit

With relativized-model wave functions expanded in bases with α=0.5 GeV, and with $\gamma = 2.6$ and $\lambda = 0.0$ GeV^{-1}, we obtain the fit to the two-, three-, and four-star nonstrange resonances up to the N=2 band illustrated in Tables I and II and Figs. 6 and 7. The data for the amplitudes are obtained from the total widths and $N\pi$ branching fractions quoted by the PDG with the exception of those of two P_{11} resonances, the Roper resonance, and $N\frac{1}{2}^+(1710)P_{11}$. In a recent reanalysis of two different sets of P_{11} partial-wave data, Cutkosky and Wang [38] have reported total widths for the Roper resonance of roughly 550–650 MeV, with at least a 30% uncertainty in their estimate. This is considerably larger than the 200 MeV "best guess" quoted by the PDG [31]. We have adopted their $N\pi$ partial widths for these two states, taking an average of their two fits. In addition, the partial-wave analysis of $N\pi\pi$ carried out by Manley and Saleski [39] supports this picture of a broad Roper resonance.

The resulting χ^2 is roughly 76 for 18 degrees of freedom. Note, however, that a large part of χ^2 arises from our overestimate of the amplitude of $N\frac{3}{2}^+(1720)P_{13}$. With this state excluded, the same parameters give a considerably smaller χ^2 of 46 for 17 degrees of freedom. Following Forsyth and Cutkosky [29], a simple measure of the "theoretical error" of our model is the value of τ which gives a χ^2 per degree of freedom of 1.0, when added in quadrature with the experimental errors for each state i. For our problem, τ is defined by

$$\chi^2(\tau) = \sum_{i=1}^{N} \frac{(A_i - E_i)^2}{\sigma_i^2 + \tau^2} = N - 2, \quad (18)$$

where $\{A_i\}$ are the theoretical predictions and $\{E_i \pm \sigma_i\}$ are the measured amplitudes. If $N\frac{3}{2}^+(1720)P_{13}$ is left out of the fit, the theoretical error is 1.9 $\text{MeV}^{\frac{1}{2}}$; with this state included the error increases to 2.8 $\text{MeV}^{\frac{1}{2}}$.

In our fit and the predictions that follow, we have used the wave functions expanded in bases with a harmonic-oscillator size parameter α=0.5 GeV. A measure of the convergence of the expansion of the wave functions is the sensitivity of the predictions to α. When the amplitudes are calculated with α=0.6 GeV (and γ and λ are refitted), the theoretical error increases slightly to 2.1 $\text{MeV}^{\frac{1}{2}}$ when $N\frac{3}{2}^+(1720)P_{13}$ is omitted from the fit and to 3.0 $\text{MeV}^{\frac{1}{2}}$ when this state is included.

In all of the tables of results that we show, we have included an uncertainty with each of the amplitudes. This is completely distinct from the theoretical error we have estimated above and arises from taking into account the uncertainties in the masses of the decaying states. For two-, three-, and four-star states, the uncertainty in mass is that quoted by the PDG [31]. For one-star states and missing states, we use an uncertainty in mass of 150 MeV. It is gratifying to note that with the exception of a very few cases, the amplitudes that we present are largely independent of the masses we use, at least within the range of masses we have mentioned above.

From the pattern of the sizes of these amplitudes a simple picture of the contrast between states which are seen in the $N\pi$ partial-wave analyses and those which are not (the "missing" states) emerges. The pattern we observe is similar to that reported by Koniuk and Isgur [5]. In all cases the states which are missing have smaller amplitudes than the (usually lighter) states with the same isospin and J^P which are seen. In the case of the two-star state $N\frac{5}{2}^+(2000)F_{15}$, we have made a rather arbitrary assignment to the lighter quark model state which has a slightly larger $N\pi$ amplitude. States such as these which are close in mass and which have similar couplings to the $N\pi$ production channel in our simple model are likely to

TABLE I. Absolute values of the $N\pi$ amplitudes for *all* N^* resonances in the N=1 and N=2 bands. Notation for model states is $\Phi[J^P]_n$(mass [MeV]), where Φ is the flavor, J^P are the spin and parity, and n is the principal quantum number. States from the partial-wave [31] analyses are listed (along with their overall rating) in the same row as our model state assignment. "Missing" states are those with no experimental analogues. Data sources and theoretical errors are discussed in the text.

Model state	$\lvert A_{N\pi}\rvert$ ($\text{MeV}^{\frac{1}{2}}$)	$N\pi$ state assignment	Rating	$\sqrt{\Gamma_{\text{tot}}}(\text{BR})_{N\pi}$ ($\text{MeV}^{\frac{1}{2}}$)
$[N\frac{1}{2}^-]_1(1460)$	14.7 ± 0.5	$N\frac{1}{2}^-(1535)$	****	8.0±2.8
$[N\frac{1}{2}^-]_2(1535)$	12.2 ± 0.8	$N\frac{1}{2}^-(1650)$	****	8.7±1.9
$[N\frac{3}{2}^-]_1(1495)$	8.6 ± 0.3	$N\frac{3}{2}^-(1520)$	****	8.3±0.9
$[N\frac{3}{2}^-]_2(1625)$	5.8 ± 0.6	$N\frac{3}{2}^-(1700)$	***	3.2±1.3
$[N\frac{5}{2}^-]_1(1630)$	5.3 ± 0.1	$N\frac{5}{2}^-(1675)$	****	7.7±0.7
$[N\frac{1}{2}^+]_2(1540)$	$20.3^{+0.8}_{-0.9}$	$N\frac{1}{2}^+(1440)$	****	19.9±3.0
$[N\frac{1}{2}^+]_3(1770)$	4.2 ± 0.1	$N\frac{1}{2}^+(1710)$	***	4.7±1.2
$[N\frac{1}{2}^+]_4(1880)$	$2.7^{+0.6}_{-0.9}$			
$[N\frac{1}{2}^+]_5(1975)$	$2.0^{+0.2}_{-0.3}$			
$[N\frac{3}{2}^+]_1(1795)$	14.1 ± 0.1	$N\frac{3}{2}^+(1720)$	****	5.5±1.6
$[N\frac{3}{2}^+]_2(1870)$	$6.1^{+0.6}_{-1.2}$			
$[N\frac{3}{2}^+]_3(1910)$	$1.0^{+0.1}_{-0.2}$			
$[N\frac{3}{2}^+]_4(1950)$	$4.1^{+0.4}_{-0.7}$			
$[N\frac{3}{2}^+]_5(2030)$	1.8 ± 0.2			
$[N\frac{5}{2}^+]_1(1770)$	6.6 ± 0.2	$N\frac{5}{2}^+(1680)$	****	8.7±0.9
$[N\frac{5}{2}^+]_2(1980)$	1.3 ± 0.3	$N\frac{5}{2}^+(2000)$	**	2.0±1.2
$[N\frac{5}{2}^+]_3(1995)$	0.9 ± 0.2			
$[N\frac{7}{2}^+]_1(2000)$	2.4 ± 0.4	$N\frac{7}{2}^+(1990)$	**	4.6±1.9

TABLE II. Absolute values of the $N\pi$ amplitudes for *all* Δ resonances in the N=0, 1, and 2 bands. Notation as in Table I.

Model state	$\lvert A_{N\pi}\rvert$ ($\mathrm{MeV}^{\frac{1}{2}}$)	$N\pi$ state assignment	Rating	$\sqrt{\Gamma_{\rm tot}}(\mathrm{BR})_{N\pi}$ ($\mathrm{MeV}^{\frac{1}{2}}$)
$[\Delta\frac{1}{2}^-]_1(1555)$	5.1 ± 0.7	$\Delta\frac{1}{2}^-(1620)$	****	6.5±1.0
$[\Delta\frac{3}{2}^-]_1(1620)$	4.9 ± 0.7	$\Delta\frac{3}{2}^-(1700)$	****	6.5±2.0
$[\Delta\frac{1}{2}^+]_1(1835)$	$3.9^{+0.4}_{-0.7}$			
$[\Delta\frac{1}{2}^+]_2(1875)$	9.4 ± 0.4	$\Delta\frac{1}{2}^+(1910)$	****	6.6±1.6
$[\Delta\frac{3}{2}^+]_1(1230)$	10.4 ± 0.1	$\Delta\frac{3}{2}^+(1232)$	****	10.7±0.3
$[\Delta\frac{3}{2}^+]_2(1795)$	8.7 ± 0.2	$\Delta\frac{3}{2}^+(1600)$	**	7.6±2.3
$[\Delta\frac{3}{2}^+]_3(1915)$	4.2 ± 0.3	$\Delta\frac{3}{2}^+(1920)$	***	7.7±2.3
$[\Delta\frac{3}{2}^+]_4(1985)$	$3.3^{+0.8}_{-1.1}$			
$[\Delta\frac{5}{2}^+]_1(1910)$	3.4 ± 0.3	$\Delta\frac{5}{2}^+(1905)$	****	5.5±2.7
$[\Delta\frac{5}{2}^+]_2(1990)$	1.2 ± 0.3	$\Delta\frac{5}{2}^+(2000)$	**	5.3±2.3
$[\Delta\frac{7}{2}^+]_1(1940)$	7.1 ± 0.1	$\Delta\frac{7}{2}^+(1950)$	****	9.8±2.7

mix in the presence of many open decay channels. Such mixings could easily make one state more likely and one less likely to be produced (and hence missing), which would correspond to the results of the partial-wave analyses. A similar mechanism may be in effect in the case of the third and fourth P_{33} model states, for which there is one experimental analogue $\Delta\frac{3}{2}^+(1920)P_{33}$ with underestimated couplings (see Fig. 7).

For the majority of the resonances in Figs. 6 and 7, the model gives a reasonably good quantitative fit to the production amplitudes. For example the relative sizes of the $N\pi$ amplitudes of the $\Delta\frac{3}{2}^+(1232)P_{33}$ and its predominantly radially excited partner $\Delta\frac{3}{2}^+(1600)P_{33}$ are quite well explained. Our model gives a large amplitude for the Roper resonance $N\frac{1}{2}^+(1440)P_{11}$, in keeping with results of the recent analyses mentioned above. A large part of χ^2 arises from our overestimate, by a factor of about 2.5, of the amplitude of $N\frac{3}{2}^+(1720)P_{13}$. In order to establish the source of the large amplitudes for these states, we have examined their $N\pi$ amplitudes in the $^3P_0$ model with pure-oscillator and hyperfine-mixed Isgur-Karl model wave functions [40, 35]. The Roper resonance amplitude goes from 10 to 16 $\mathrm{MeV}^{\frac{1}{2}}$ when the initial and final states are mixed [the overall strength γ is fixed by normalizing to the $\Delta(1232)$ amplitude] in the manner outlined in Ref. [35]. This result is insensitive to the choice of smearing parameter λ. This amplitude increases further to 20 $\mathrm{MeV}^{\frac{1}{2}}$ when the relativized model wave functions expanded with α=0.5 GeV are used.

The $N\pi$ amplitude for $N\frac{3}{2}^+(1720)P_{13}$ is quite sensitive to mixing, going from 14 to 11 $\mathrm{MeV}^{\frac{1}{2}}$ when the Isgur-Karl model wave functions are hyperfine mixed and back to 14 $\mathrm{MeV}^{\frac{1}{2}}$ when calculated with the relativized-model wave functions. In all cases the $^3P_0$ model overestimates this amplitude by a factor of at least 2. This overestimate seems to persist for the other P_{13} N^* model states, two of which are predicted to have widths similar to that of the observed width of the $N\frac{3}{2}^+(1720)P_{13}$, meaning that they should have been seen in the analyses. Similar results for both the Roper resonance and $N\frac{3}{2}^+(1720)P_{13}$ are found by Stancu and Stassart [16] in the version of their model which most closely resembles ours.

C. N=3 band states

The results of applying this model with the fitted parameters to the $N\pi$ amplitudes of the N=3 band negative-parity baryons are shown in Figs. 8 and 9 and Tables III and IV. A striking pattern in the sizes of the amplitudes emerges in the predictions for the N=3 band N^* states; the lightest model state of a given J almost always couples the most strongly to $N\pi$, and there is rapid falloff of the amplitudes as the model masses of the states increase. If, as in Fig. 8, we assign these lightest model states to the experimental states, we obtain good agreement between the $N\pi$ widths extracted from the data and the model. In all but one case there is a clean separation, in both mass and the size of the $N\pi$ amplitude, of the lightest state and the next heaviest. In the case of the two lightest $N\frac{5}{2}^-$ model states at 2080 and 2095 MeV, it seems unlikely that these states will be resolved in the partial-wave analyses given their proximity in mass and similar couplings. There is the possibility that decay-channel mixings of these nearby states cause one state to couple significantly more strongly than its neighbor. These assignments demonstrate that the relativized-model predictions for the masses of these states are too low by roughly 100 MeV.

The situation for the Δ states at this level of excitation

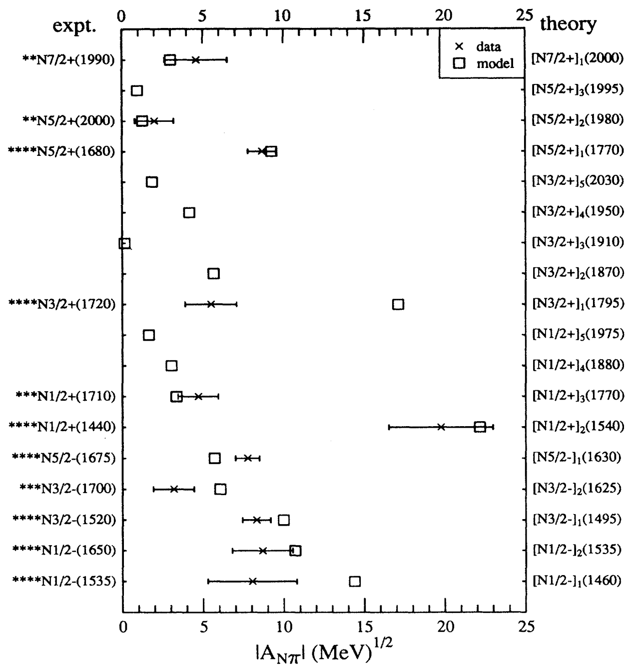

FIG. 6. Absolute values of the $N\pi$ amplitudes for *all* N^* resonances in the N=1 and N=2 bands. For each model state the nominal model mass is listed along with its total spin, parity, and principal quantum number on the right axis. States from the partial-wave [31] analyses are shown on the left axis (along with their overall rating from Ref. [31]) aligned with our model assignment, and the extracted experimental and theoretical amplitudes are plotted along a line parallel with the bottom axis for each such state. "Missing" states are those with no experimental analogues.

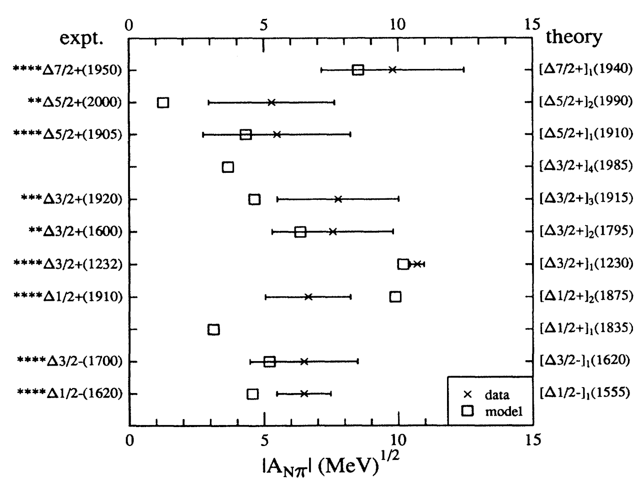

FIG. 7. Absolute values of the $N\pi$ amplitudes for *all* Δ resonances in the N=0, 1, and 2 bands. Legend as in Fig. 6.

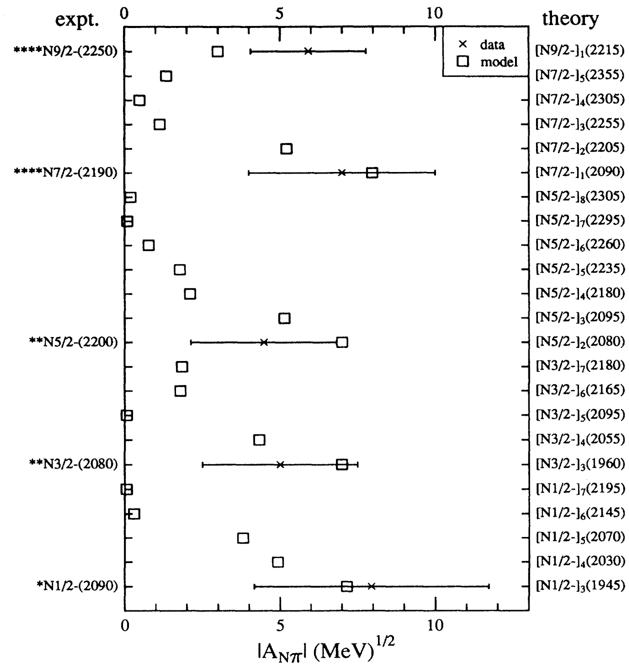

FIG. 8. Absolute values of the $N\pi$ amplitudes for the lightest few negative-parity nucleon resonances of each J in the N=3 band. Legend as in Fig. 6.

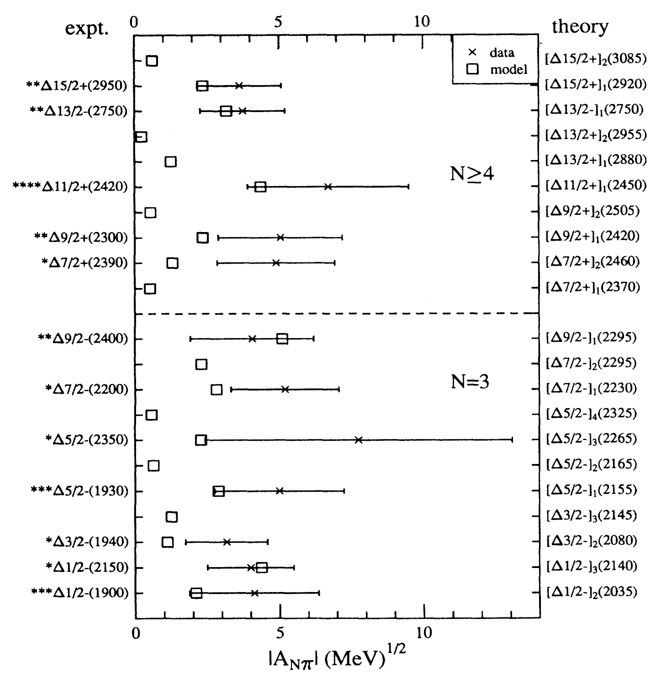

FIG. 9. Absolute values of the $N\pi$ amplitudes for the lightest few negative-parity Δ resonances of each J in the N=3 band and for the lightest few Δ resonances for J^P values which first appear in the N=4, 5, and 6 bands. Legend as in Fig. 6.

is more complicated, and the data are less certain. For the states given two- or three-star ratings by the PDG, our model predictions for the $N\pi$ amplitudes are somewhat underestimated, but again the lightest state of each J is the one which couples. In the case of the $\Delta\frac{3}{2}^-$ and $\Delta\frac{7}{2}^-$ states, the two lightest N=3 band states have similar (underestimated) couplings in our model, but once again only one state is resolved in the partial-wave analyses. The consequences for spectroscopy of the model-state assignments illustrated in Fig. 9 are interesting; for the lightest two well-established states $\Delta\frac{1}{2}^-(1900)S_{31}$ and $\Delta\frac{5}{2}^-(1930)D_{35}$ the model masses are too *high* by 135 and 225 MeV [41], respectively, in contrast with the 100-MeV *underestimate* of the N=3 band N^* masses. The overestimate of the mass of $\Delta\frac{5}{2}^-(1930)D_{35}$ in either the nonrelativistic or relativized models has led to suggestions that it might be a hybrid baryon (a state with excited glue) [42]. Although in nonrelativistic models the [56,1$^-$] SU(6) supermultiplet to which this state belongs is lighter than the other N=3 band supermultiplets [43], suggesting a conventional interpretation of this state, it is not a particularly light state when a more realistic potential and a more sophisticated treatment of the wave functions are used [30, 1], and this problem persists [32].

D. More massive states

Figure 10 and Table V illustrate the result of applying this model to N^* model states which have $L \geq 4$ and so must first appear when the oscillator basis is extended to the N=4 to N=6 bands. The pattern established in the N=3 band is repeated, and we see that the lightest states in each J^P sector couple the most strongly to $N\pi$, with a rapid falloff in coupling strength as the masses increase. There is a remarkable agreement, given the necessarily approximate nature of the model for such highly excited

TABLE III. Absolute values of the $N\pi$ amplitudes for the lightest few negative-parity nucleon resonances of each J in the N=3 band. Notation as in Table I.

Model state	$\vert A_{N\pi}\vert$ (MeV$^{\frac{1}{2}}$)	$N\pi$ state assignment	Rating	$\sqrt{\Gamma_{tot}}(\mathrm{BR})_{N\pi}$ (MeV$^{\frac{1}{2}}$)
$[N\frac{1}{2}^-]_3(1945)$	$5.7^{+0.5}_{-1.6}$	$N\frac{1}{2}^-(2090)$	*	7.9±3.8
$[N\frac{1}{2}^-]_4(2030)$	$3.7^{+0.5}_{-1.1}$			
$[N\frac{1}{2}^-]_5(2070)$	$2.1^{+0.8}_{-1.5}$			
$[N\frac{1}{2}^-]_6(2145)$	0.4 ± 0.1			
$[N\frac{1}{2}^-]_7(2195)$	0.1 ± 0.1			
$[N\frac{3}{2}^-]_3(1960)$	$8.2^{+0.7}_{-1.7}$	$N\frac{3}{2}^-(2080)$	**	5.0±2.5
$[N\frac{3}{2}^-]_4(2055)$	$6.2^{+0.1}_{-0.6}$			
$[N\frac{3}{2}^-]_5(2095)$	$0.2^{+0.1}_{-0.2}$			
$[N\frac{3}{2}^-]_6(2165)$	$1.5^{+0.1}_{-0.2}$			
$[N\frac{3}{2}^-]_7(2180)$	$1.7^{+0.1}_{-0.2}$			
$[N\frac{5}{2}^-]_2(2080)$	$5.1^{+0.2}_{-0.8}$			
$[N\frac{5}{2}^-]_3(2095)$	$5.2^{+0.4}_{-1.0}$	$N\frac{5}{2}^-(2200)$N	**	4.5±2.3
$[N\frac{5}{2}^-]_4(2180)$	$1.9^{+0.1}_{-0.3}$			
$[N\frac{5}{2}^-]_5(2235)$	$2.0^{+0.1}_{-0.3}$			
$[N\frac{5}{2}^-]_6(2260)$	0.4 ± 0.1			
$[N\frac{5}{2}^-]_7(2295)$	0.2 ± 0.1			
$[N\frac{5}{2}^-]_8(2305)$	0.3 ± 0.1			
$[N\frac{7}{2}^-]_1(2090)$	6.9 ± 1.3	$N\frac{7}{2}^-(2190)$	****	7.0±3.0
$[N\frac{7}{2}^-]_2(2205)$	4.0 ± 1.1			
$[N\frac{7}{2}^-]_3(2255)$	0.8 ± 0.2			
$[N\frac{7}{2}^-]_4(2305)$	0.4 ± 0.1			
$[N\frac{7}{2}^-]_5(2355)$	1.1 ± 0.3			
$[N\frac{9}{2}^-]_1(2215)$	2.5 ± 0.3	$N\frac{9}{2}^-(2250)$	****	5.9±1.9

states, between the predicted couplings for the lightest $N\frac{9}{2}^+$, $N\frac{11}{2}^-$, and $N\frac{13}{2}^+$ states and those extracted from the partial-wave analyses for resonances with these spins and parities. The model predictions of the masses of the positive-parity states are only approximately correct (to within roughly 100 MeV) given these assignments.

We have included in this comparison the lightest two N=4 band $N\frac{1}{2}^+$ states (the sixth and seventh P_{11} states), since there is some weak evidence for a $N\frac{1}{2}^+(2100)P_{11}$ state with a mass considerably higher than the model predictions for the missing states in the N=2 band. Our prediction for the coupling strength of the sixth P_{11} model state $N\frac{1}{2}^+(2065)$ is considerably larger than those of the N=2 band missing states, making this a natural assignment for this candidate resonance. Our results for the lightest N=4 band $N\frac{7}{2}^+$ states indicate that there should be a state at roughly 2400 MeV in the F_{17} partial wave which couples relatively strongly to $N\pi$. The model also has a natural explanation (in contrast with the situation for the Δ states) for the absence of N^* resonances with $J^P = \frac{11}{2}^+$, $\frac{13}{2}^-$, and $\frac{15}{2}^+$, the lightest of which all have quite weak coupling to $N\pi$.

Figure 9 and Table IV also show our results for the lightest few model Δ states for J values which first appear in these higher bands. The lightest $\Delta\frac{11}{2}^+$ model state (the next lowest state is considerably more massive) has approximately the right mass and coupling to be assigned to the well-established $\Delta\frac{11}{2}^+(2420)H_{3\,11}$, and the same is true for the lightest $\Delta\frac{13}{2}^-$ and $\Delta\frac{15}{2}^+$ model states and the two-star candidate resonances $\Delta\frac{13}{2}^-(2750)I_{3\,13}$ and $\Delta\frac{15}{2}^+(2950)K_{3\,15}$. The situation is less certain for the two-star $\Delta\frac{9}{2}^+(2300)H_{39}$, although once again the lightest $\Delta\frac{9}{2}^+$ model state couples the most strongly. The two lightest model $\Delta\frac{7}{2}^+$ states above the N=2 band couple quite weakly to $N\pi$ and so our assignment to the one-star candidate in this sector is quite arbitrary. In

TABLE IV. Absolute values of the $N\pi$ amplitudes for the lightest few negative-parity Δ resonances of each J in the N=3 band and for the lightest few Δ resonances for J^P values which first appear in the N=4, 5, and 6 bands. Notation as in Table I.

Model state	$\lvert A_{N\pi}\rvert$ (MeV$^{\frac{1}{2}}$)	$N\pi$ state assignment	Rating	$\sqrt{\Gamma_{tot}}(\mathrm{BR})_{N\pi}$ (MeV$^{\frac{1}{2}}$)
$[\Delta\frac{1}{2}^-]_2(2035)$	1.2 ± 0.2	$\Delta\frac{1}{2}^-(1900)$	***	4.1±2.2
$[\Delta\frac{1}{2}^-]_3(2140)$	$3.1^{+0.4}_{-1.1}$	$\Delta\frac{1}{2}^-(2150)$	*	4.0±1.5
$[\Delta\frac{3}{2}^-]_2(2080)$	$2.2^{+0.1}_{-0.2}$	$\Delta\frac{3}{2}^-(1940)$	*	3.2±1.4
$[\Delta\frac{3}{2}^-]_3(2145)$	$2.2^{+0.1}_{-0.3}$			
$[\Delta\frac{5}{2}^-]_1(2155)$	5.2 ± 0.1	$\Delta\frac{5}{2}^-(1930)$	***	5.0±2.3
$[\Delta\frac{5}{2}^-]_2(2165)$	0.6 ± 0.1			
$[\Delta\frac{5}{2}^-]_3(2265)$	$2.4^{+0.5}_{-0.7}$	$\Delta\frac{5}{2}^-(2350)$	*	7.7±5.3
$[\Delta\frac{5}{2}^-]_4(2325)$	0.1 ± 0.1			
$[\Delta\frac{7}{2}^-]_1(2230)$	2.1 ± 0.6	$\Delta\frac{7}{2}^-(2200)$	*	5.2±1.9
$[\Delta\frac{7}{2}^-]_2(2295)$	1.8 ± 0.4			
$[\Delta\frac{9}{2}^-]_1(2295)$	4.8 ± 0.9	$\Delta\frac{9}{2}^-(2400)$	**	4.1±2.1
$[\Delta\frac{7}{2}^+]_2(2370)$	$1.5^{+0.6}_{-0.9}$	$\Delta\frac{7}{2}^+(2390)$	*	4.9±2.0
$[\Delta\frac{7}{2}^+]_3(2460)$	1.1 ± 0.1			
$[\Delta\frac{9}{2}^+]_1(2420)$	$1.2^{+0.5}_{-0.4}$	$\Delta\frac{9}{2}^+(2300)$	**	5.1±2.2
$[\Delta\frac{9}{2}^+]_2(2505)$	0.4 ± 0.1			
$[\Delta\frac{11}{2}^+]_1(2450)$	2.9 ± 0.7	$\Delta\frac{11}{2}^+(2420)$	****	6.7±2.8
$[\Delta\frac{13}{2}^+]_1(2880)$	0.8 ± 0.2			
$[\Delta\frac{13}{2}^+]_2(2955)$	0.2 ± 0.1			
$[\Delta\frac{13}{2}^-]_1(2750)$	2.2 ± 0.4	$\Delta\frac{13}{2}^-(2750)$	**	3.7±1.5
$[\Delta\frac{15}{2}^+]_1(2920)$	1.6 ± 0.3	$\Delta\frac{15}{2}^+(2950)$	**	3.6±1.5
$[\Delta\frac{15}{2}^+]_2(3085)$	0.4 ± 0.1			

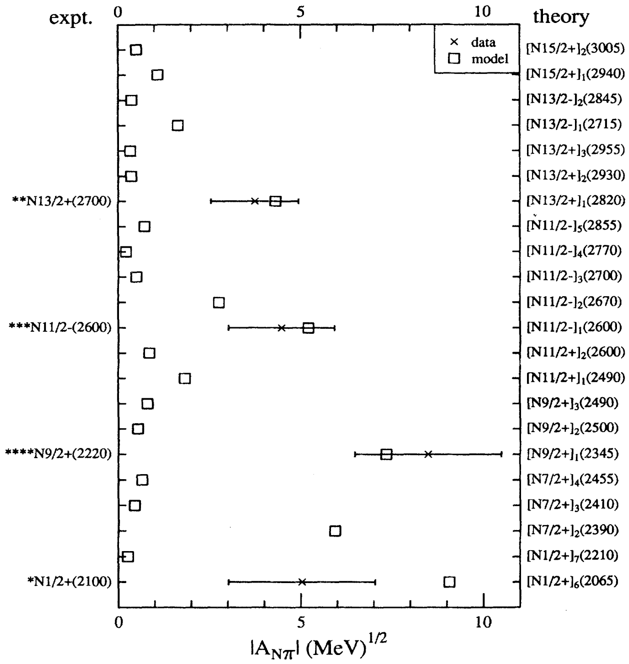

FIG. 10. Absolute values of the $N\pi$ amplitudes for the lightest few nucleon resonances for J^P values which first appear in the N=4, 5, and 6 bands. Legend as in Fig. 6.

this sector our model appears to consistently underestimate the couplings of known states to $N\pi$, a pattern that was already seen in the $N = 3$ band.

V. CONCLUSIONS AND OUTLOOK

The results described above show in many cases a remarkable agreement with the data, given the simplicity of the model. Most noticeably absent from our analysis is a treatment of coupled-channel effects in the spectroscopy and decay-channel couplings of these states. These can be expected to cause both mass shifts and mixings between the states. As mentioned above, there are sectors where states which are close in mass and which have similar couplings to the $N\pi$ production channel in our simple model are likely to mix in the presence of many open decay channels. Such mixings could easily make one state more likely and one less likely to be produced, which would help explain why some states with appreciable couplings in our model remain unseen. We have seen several examples where such a mechanism could be operating. On the other hand, our model can be considered reliable for states which are appreciably separated in mass and coupling strength from other states with the same quantum numbers and similar energies.

It should be stressed that our results for the $N\pi$ couplings of states which first appear when the bases are extended beyond the N=2 band are predictions based on the fit to the lower-lying states. The generally good agreement of these predictions with the couplings extracted from the partial-wave analyses makes it rather hard to decide, on this basis alone, between a conventional three-quark or hybrid explanation for these states. However, the discrepancy noted by many authors between the model predictions and the *mass* of the well-established $\Delta\frac{5}{2}^-(1930)D_{35}$ remains. A remarkable pattern emerges for many J^P sectors of a given flavor; the lightest state in a given oscillator band couples most strongly to $N\pi$ with the coupling strengths falling off rapidly as the masses of the states increase. This suggests a mechanism which correlates a lower expectation value of the spectral Hamiltonian with a stronger coupling to this production channel.

Our fit to the states below the N=3 band confirms the results of the recent reanalysis of the P_{11} partial wave by Cutkosky and Wang [38] for the Roper resonance with a conventional three-quark description of this state. On the basis of the $N\pi$ couplings alone, the same conventional explanation suffices for $\Delta\frac{3}{2}^+(1600)P_{33}$, although the mass of this state is considerably too high in quark potential models. Our model is unable to explain the small width of the well-established resonance $N\frac{3}{2}^+(1720)P_{13}$ which appears in the partial-wave analyses; there are suggestions in the literature [44] that these quantum numbers (along with P_{11} and P_{33}) are natural for a low-lying hybrid state. Our results indicate that this possibility deserves further study.

We close this article by noting that there is much that may yet be done in the framework of our model. Decays of strange baryons as well as decays of nonstrange baryons to strange final states are of interest, especially in the light of experiments that are proposed for CEBAF. In addition, multipion final states demand some attention, as they provide large branching fractions for many resonances. In the model we have described, such decays could be treated as cascade processes. For a two-pion final state, for example, one would assume that the resonance of interest first decayed to something such as $\Delta\pi$ or $N\rho$, followed by the strong decay of the Δ or the ρ. Such analyses have been carried out [5, 45] using the elementary meson emission model. We intend to examine these issues within the framework of our model.

ACKNOWLEDGMENTS

We would like to thank Professor Richard E. Cutkosky and Professor Nathan Isgur for engaging in helpful discussions while this work was in progress. This work was supported by the U.S. Department of Energy under Contract No. DE-AC02-76ER03066 (S.C.) and Contract No. DE-AC05-84ER40150 (S.C. and W.R.). The hospitality of the Institute for Nuclear Theory at the University of Washington (S.C., W.R.) and of Harvard University, Institut für Kernphysik, Darmstadt, Germany, and Institut des Sciences Nucléaires, Grenoble, France (W.R.), are gratefully acknowledged.

APPENDIX: TRANSITION AMPLITUDE

To begin, we note that momentum conservation yields a factor $\delta(\mathbf{K}_0)$ in the amplitude, and we write

$$\langle BC|T|A\rangle = \delta(\mathbf{K}_0)M_{A\to BC}. \tag{A1}$$

The final form we obtain is

$$M_{A\to BC}=\frac{6\gamma}{3\sqrt{3}}(-1)^{J_a+J_b+\ell_a+\ell_b-1}$$
$$\times\sum_{J_\rho,s_a,s_b}\hat{J}_\rho^2\hat{s}_a\hat{S}_a\hat{L}_a\hat{s}_b\hat{S}_b\hat{L}_b\begin{Bmatrix} S_a & L_\rho & s_a\\ \ell_a & J_a & L_a\end{Bmatrix}\begin{Bmatrix} L_\rho & S_\rho & J_\rho\\ \frac{1}{2} & s_a & S_a\end{Bmatrix}\begin{Bmatrix} S_b & L_\rho & s_b\\ \ell_b & J_b & L_b\end{Bmatrix}\begin{Bmatrix} L_\rho & S_\rho & J_\rho\\ \frac{1}{2} & s_b & S_b\end{Bmatrix}$$
$$\times(-1)^{\ell+\ell_a+J_c-L_c-S_c}\mathcal{F}(ABC)\mathcal{R}(ABC)$$
$$\times\sum_{S_{bc}}(-1)^{s_a-S_{bc}}\begin{bmatrix} J_\rho & 1/2 & s_b\\ 1/2 & 1/2 & S_c\\ s_a & 1 & S_{bc}\end{bmatrix}\sum_{L_{bc}}(-1)^{L_{bc}}\begin{bmatrix} s_b & \ell_b & J_b\\ S_c & L_c & J_c\\ S_{bc} & L_{bc} & J_{bc}\end{bmatrix}$$
$$\times\sum_L\hat{L}^2\begin{Bmatrix} s_a & \ell_a & J_a\\ L & S_{bc} & 1\end{Bmatrix}\begin{Bmatrix} S_{bc} & L_{bc} & J_{bc}\\ \ell & J_a & L\end{Bmatrix}\varepsilon(\ell_b,L_c,L_{bc},\ell,\ell_a,L,k_0),\qquad\text{(A2)}$$

Here we have written

TABLE V. Absolute values of the $N\pi$ amplitudes for the lightest few nucleon resonances for J^P values which first appear in the N=4, 5, and 6 bands. Notation as in Table I.

Model state	$\lvert A_{N\pi}\rvert$ ($\mathrm{MeV}^{\frac{1}{2}}$)	$N\pi$ state assignment	Rating	$\sqrt{\Gamma_{\rm tot}}(\mathrm{BR})_{N\pi}$ ($\mathrm{MeV}^{\frac{1}{2}}$)
$[N\frac{1}{2}^+]_6(2065)$	$7.7^{+2.4}_{-2.9}$	$N\frac{1}{2}^+(2100)$	*	5.0±2.0
$[N\frac{1}{2}^+]_7(2210)$	$0.3^{+0.2}_{-0.1}$			
$[N\frac{7}{2}^+]_2(2390)$	$4.9^{+0.1}_{-0.4}$			
$[N\frac{7}{2}^+]_3(2410)$	$0.4^{+0.2}_{-0.4}$			
$[N\frac{7}{2}^+]_4(2455)$	0.5 ± 0.1			
$[N\frac{9}{2}^+]_1(2345)$	$3.6^{+0.9}_{-0.8}$	$N\frac{9}{2}^+(2220)$	****	8.5±2.0
$[N\frac{9}{2}^+]_2(2500)$	0.4 ± 0.1			
$[N\frac{9}{2}^+]_3(2490)$	0.6 ± 0.2			
$[N\frac{11}{2}^+]_1(2490)$	1.3 ± 0.4			
$[N\frac{11}{2}^+]_2(2600)$	0.7 ± 0.2			
$[N\frac{11}{2}^-]_1(2600)$	$3.3^{+1.1}_{-0.9}$	$N\frac{11}{2}^-(2600)$	***	4.5±1.5
$[N\frac{11}{2}^-]_2(2670)$	1.8 ± 0.5			
$[N\frac{11}{2}^-]_3(2700)$	0.3 ± 0.1			
$[N\frac{11}{2}^-]_4(2770)$	0.2 ± 0.1			
$[N\frac{11}{2}^-]_5(2855)$	0.6 ± 0.1			
$[N\frac{13}{2}^+]_1(2820)$	$2.0^{+1.0}_{-0.8}$	$N\frac{13}{2}^+(2700)$	**	3.7±1.2
$[N\frac{13}{2}^+]_2(2930)$	0.2 ± 0.1			
$[N\frac{13}{2}^+]_3(2955)$	0.2 ± 0.1			
$[N\frac{13}{2}^-]_1(2715)$	1.1 ± 0.3			
$[N\frac{13}{2}^-]_2(2845)$	0.2 ± 0.1			
$[N\frac{15}{2}^+]_1(2940)$	0.7 ± 0.1			
$[N\frac{15}{2}^+]_2(3005)$	0.4 ± 0.1			

$$\mathbf{J}_a = \mathbf{L}_a + \mathbf{S}_a = \boldsymbol{\ell}_a + \mathbf{s}_a, \tag{A3}$$

with

$$\mathbf{L}_a = \mathbf{L}_{\lambda_a} + \mathbf{L}_{\rho_a} \equiv \boldsymbol{\ell}_a + \mathbf{L}_{\rho_a}, \quad \mathbf{S}_a = \mathbf{S}_{\rho_a} + \mathbf{1/2}, \tag{A4}$$

and

$$\mathbf{s}_a = \mathbf{J}_{\rho_a} + \mathbf{1/2} = \mathbf{L}_{\rho_a} + \mathbf{S}_{\rho_a} + \mathbf{1/2}, \tag{A5}$$

with similar definitions for B. The first four 6-j symbols of Eq. (A2) are necessary for transforming from the usual angular momentum basis for the baryons, given by Eq. (A4), to the basis of Eq. (A5), which is the more convenient one for evaluating the transition amplitude. L, L_{bc}, and S_{bc} are internal summation variables, and $\mathcal{F}(ABC)$ is the flavor overlap for the decay.

The purely "spatial" part of the transition amplitude is

$$\begin{aligned}\varepsilon(\ell_b, L_c, L_{bc}, \ell, \ell_a, L, k_0) = {} & \mathcal{J}(A)(-1)^{L_{bc}} \frac{1}{2} \frac{\exp(-F^2 k_0^2)}{G^{\ell_a+\ell_b+L_c+4}} N_a N_b N_c \\ & \times \sum_{\ell_1,\ell_2,\ell_3,\ell_4} C_{\ell_1}^{\ell_b} C_{\ell_2}^{L_c} C_{\ell_3}^{1} C_{\ell_4}^{\ell_a} (x-\omega_1)^{\ell_1} (x-\omega_2)^{\ell_2} (x-1)^{\ell_3} x^{\ell_4} \\ & \times \sum_{\ell_{12},\ell_5,\ell_6,\ell_7,\ell_8} (-1)^{\ell_{12}+\ell_6} \frac{\hat{\ell}_5}{\hat{L}} \begin{bmatrix} \ell_1 & \ell_1' & \ell_b \\ \ell_2 & \ell_2' & L_c \\ \ell_{12} & \ell_6 & L_{bc} \end{bmatrix} \begin{bmatrix} \ell_3 & \ell_3' & 1 \\ \ell_4 & \ell_4' & \ell_a \\ \ell_7 & \ell_8 & L \end{bmatrix} \\ & \times \begin{Bmatrix} \ell & \ell_{12} & \ell_5 \\ \ell_6 & L & L_{bc} \end{Bmatrix} B_{\ell_1\ell_2}^{\ell_{12}} B_{\ell\ell_{12}}^{\ell_5} B_{\ell_1'\ell_2'}^{\ell_6} B_{\ell_3\ell_4}^{\ell_7} B_{\ell_3'\ell_4'}^{\ell_8} \\ & \times \sum_{\lambda,\mu,\nu} D_{\lambda\mu\nu}(\omega_1,\omega_2,x) I_\nu(\ell_5,\ell_6,\ell_7,\ell_8;L) \left(\frac{\ell_1'+\ell_2'+\ell_3'+\ell_4'+2\mu+\nu+1}{2}\right)! \\ & \times k_0^{\ell_1+\ell_2+\ell_3+\ell_4+2\lambda+\nu} / G^{2\mu+\nu-\ell_1-\ell_2-\ell_3-\ell_4}. \end{aligned} \tag{A6}$$

In this expression, N_a is a normalization coefficient that results from writing a single component of the wave function of A as

$$\begin{aligned}\Psi_{LMn_\rho\ell_\rho n_\lambda\ell_\lambda}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3) = {} & \eta(AA')^{3/2} \sum_m \langle \ell_\rho \ell_\lambda m M-m | LM \rangle \mathcal{N}_{n_\rho\ell_\rho} (A'p_\rho)^{\ell_\rho} e^{-\frac{A'^2 p_\rho^2}{2}} L_{n_\rho}^{\ell_\rho+1/2}(A'p_\rho) Y_{\ell_\rho m}(\mathbf{\Omega}_\rho) \\ & \times \mathcal{N}_{n_\lambda\ell_\lambda}(Ap_\lambda)^{\ell_\lambda} e^{-\frac{A^2 p_\lambda^2}{2}} L_{n_\lambda}^{\ell_\lambda+1/2}(Ap_\lambda) Y_{\ell_\lambda m}(\mathbf{\Omega}_\lambda). \end{aligned} \tag{A7}$$

For proper exchange symmetry among the quarks, $A' = \frac{2}{\sqrt{3}}A$ and

$$\mathbf{p}_\rho = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2), \ \mathbf{p}_\lambda = \frac{1}{3}(\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_3). \tag{A8}$$

η is a phase factor that arises from calculating the Fourier transform of the configuration space wave functions and has the value

$$\eta = (-i)^{2n_\rho + 2n_\lambda + \ell_\rho + \ell_\lambda}. \tag{A9}$$

With these definitions, $N_a = A^{\ell_\lambda + 3/2} \mathcal{N}_{n_\lambda \ell_\lambda}$, with

$$\mathcal{N}_{n\ell} = \left[\frac{2n!}{\Gamma\left(n+\ell+\frac{3}{2}\right)}\right]^{\frac{1}{2}}. \tag{A10}$$

The $L_n^{\ell+1/2}$ are the associated Laguerre polynomials

$$L_n^{\ell+1/2}(x) = \sum_{m=0}^{n} (-1)^m \binom{n+\ell+\frac{1}{2}}{n-m} \frac{x^{2m}}{m!}, \tag{A11}$$

while the $Y_{\ell m}$ are the usual spherical harmonics.

$\mathcal{J}$ is a Jacobian factor needed to convert from the basis used in evaluating the space factor ε in Ref. [33], to the basis used in the evaluation of the wave functions we are using for explicit calculation of the decay amplitudes. The wave functions of Ref. [1] use

$$\mathbf{p}_\rho' = \frac{1}{\sqrt{2}}(\mathbf{p}_1 - \mathbf{p}_2), \ \mathbf{p}_\lambda' = \frac{1}{\sqrt{6}}(\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_3), \tag{A12}$$

so that both the Jacobian factor mentioned above as well as a redefinition of the Gaussian parameters of the wave functions are required in order to use the wave functions of Ref. [1] with the above expression for the decay amplitude.

The factor $\mathcal{R}$ of Eq. (A2) is obtained as the overlap of the wave functions in the ρ coordinates in the initial and final baryon. Since we are using a model in which quarks 1 and 2 are spectators ($\ell_{\rho_a} = \ell_{\rho_b}$, $S_{\rho_a} = S_{\rho_b}$, $J_{\rho_a} = J_{\rho_b}$) and our basis is fully orthogonalized (α is the same in the initial and final baryons, so that $n_{\rho_a} = n_{\rho_b}$), this overlap is always unity. In addition, this means that the Jacobian discussed above is only necessary for the transformation in $\mathbf{p}_\lambda$.

The $\sum_{\lambda,\mu,\nu} D_{\lambda\mu\nu}(\omega_1,\omega_2,x)I_\nu(\ell_5,\ell_6,\ell_7,\ell_8;L)$ term arises from writing (we define $\mathbf{q}_a \equiv \mathbf{p}_{\lambda_a}$, with a similar definition for the daughter baryon)

$$L^{\ell_a}_{n_{\lambda_a}} e^{-A^2q_a^2/2} L^{\ell_b}_{n_{\lambda_b}} e^{-B^2q_b^2/2} L^{L_c}_{n_c} e^{-C^2q_c^2/2} \equiv \sum_{\lambda,\mu,\nu} D_{\lambda\mu\nu}(\omega_1,\omega_2,x) e^{-A^2q_a^2/2} e^{-B^2q_b^2/2} e^{-C^2q_c^2/2}. \tag{A13}$$

When the substitutions $\mathbf{q}_a = x\mathbf{k}+\mathbf{q}$, $\mathbf{q}_b = (x-\omega_1)\mathbf{k}+\mathbf{q}$, $\mathbf{q}_c = (x-\omega_2)\mathbf{k}+\mathbf{q}$ are made, and the integrals over $\mathbf{k}$ and $\mathbf{q}$ are evaluated, the expression above results. We note that it is not particularly enlightening to write out explicitly the full form of the $D_{\lambda\mu\nu}$ that we have obtained.

In Eqs. (A2) and (A6),

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \hat{c}\hat{f}\hat{g}\hat{h}\hat{i} \begin{Bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{Bmatrix}, \tag{A14}$$

where

$$\begin{Bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{Bmatrix}$$

is the 9-j symbol and $\hat{J} = \sqrt{2J+1}$.

In Eq. (A6)

$$\begin{aligned} x &= \left(B^2\omega_1 + C^2\omega_2\right)\left(A^2+B^2+C^2\right)^{-1}, \\ F^2 &= \frac{1}{2}\left[A^2x^2 + B^2(x-\omega_1)^2 + C^2(x-\omega_2)^2\right], \\ G^2 &= \frac{1}{2}(A^2+B^2+C^2). \end{aligned} \tag{A15}$$

ω_1 and ω_2 are ratios of various linear combinations of quark masses. In general,

$$\omega_1 = \frac{m_1+m_2}{m_1+m_2+m_4},\ \omega_2 = \frac{m_3}{m_3+m_4}, \tag{A16}$$

where the subscripts refer to the quark labels shown in Fig. 5. For the decays that we are considering, $\omega_1 = \frac{2}{3}$ and $\omega_2 = \frac{1}{2}$. In addition,

$$\begin{aligned} C^\ell_{\ell_1} &= \sqrt{\frac{4\pi(2\ell+1)!}{(2\ell_1+1)![2(\ell-\ell_1)+1]!}}, \\ B^\ell_{\ell_1\ell_2} &= \frac{(-1)^\ell}{\sqrt{4\pi}}\hat{\ell}_1\hat{\ell}_2 \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ 0 & 0 & 0 \end{pmatrix}, \end{aligned} \tag{A17}$$

and $\ell_1' = L_b - \ell_1$, $\ell_2' = L_c - \ell_2$, $\ell_3' = 1 - \ell_3$, $\ell_4' = L_a - \ell_4$, and the geometric factor I_ν is

$$\begin{aligned} I_{2p}(\ell_5,\ell_6,\ell_7,\ell_8;L) &= (-1)^L(2p)!\hat{\ell}_5\hat{\ell}_6\hat{\ell}_7\hat{\ell}_8 \\ &\quad\times \sum_{\lambda=0}^{p} \frac{4^\lambda(4\lambda+1)(p+\lambda)!}{(2p+2\lambda+1)!(p-\lambda)!} \begin{pmatrix} 2\lambda & \ell_5 & \ell_7 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2\lambda & \ell_6 & \ell_8 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \ell_5 & \ell_6 & L \\ \ell_8 & \ell_7 & 2\lambda \end{Bmatrix}, \\ I_{2p+1}(\ell_5,\ell_6,\ell_7,\ell_8;L) &= 2(-1)^{L+1}(2p+1)!\hat{\ell}_5\hat{\ell}_6\hat{\ell}_7\hat{\ell}_8 \sum_{\lambda=0}^{p} \frac{4^\lambda(4\lambda+3)(p+\lambda+1)!}{(2p+2\lambda+3)!(p-\lambda)!} \begin{pmatrix} 2\lambda+1 & \ell_5 & \ell_7 \\ 0 & 0 & 0 \end{pmatrix} \\ &\quad\times \begin{pmatrix} 2\lambda+1 & \ell_6 & \ell_8 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \ell_5 & \ell_6 & L \\ \ell_8 & \ell_7 & 2\lambda+1 \end{Bmatrix}. \end{aligned} \tag{A18}$$

[1] S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986).

[2] See, for example J. W. Norbury, Can. J. Phys. **67**, 876 (1989), and references therein.

[3] S. Fleck, B. Silvestre-Brac, and J.-M. Richard, Phys. Rev. D **38**, 1519 (1988).

[4] D. B. Leinweber, R. M. Woloshyn, and T. Draper, Phys. Rev. D **43**, 1659 (1991); D. B. Leinweber, presented at Baryons '92, New Haven, Connecticut, 1992 (unpublished).

[5] R. Koniuk and N. Isgur, Phys. Rev. D **21**, 1868 (1980).

[6] D. Gromes and N. Marinescu, Z. Phys. C **2**, 301 (1979); W. N. Cottingham, K. Tsu, and J. M. Richard, Nucl. Phys. **B179**, 541 (1981); A. K. A. Maciel and J. E. Paton, *ibid.* **B181**, 277 (1981); A. W. Thomas, S. Théberge, and G. A. Miller, Phys. Rev. D **24**, 216 (1981); P. Żenczykowski, Z. Phys. C **26**, 441 (1984); Ann. Phys. (N.Y.) **169**, 453 (1986); N. A. Törnqvist and P. Zenczykowski, Z. Phys. C **30**, 83 (1986); M. Brack and R. K. Bhaduri, Phys. Rev. D **35**, 3451 (1987), and references therein; W. H. Blask, M. G. Huber, and B. C. Metsch, Z. Phys. A **326**, 413 (1987); W. H. Blask, S. Furui, R. Kaiser, B. C. Metsch, and M. G. Huber, *ibid.* **337**, 451 (1990); S. Kumano, Phys. Rev. D **41**, 195 (1990); B. Silvestre-Brac and C. Gignoux, *ibid.* **43**, 3699 (1991).

[7] S. Okubo, Phys. Lett. **5**, 165 (1963); G. Zweig, CERN report No. TH-401, 1964 (unpublished); CERN Report No. TH-402 1965 (unpublished); in *Symmetries in Elementary Particle Physics*, Proceedings of the International School of Physics, "Ettore Majorana," Erice, Italy, 1964, edited by A. Zichichi (Academic, New York, 1965); J. Iizuka, Prog. Theor. Phys. Suppl. **37–38**, 21 (1966).

[8] See, for example, A. Le Yaouanc, L. Oliver, O. Pène, and

J. C. Raynal, *Hadron Transitions In The Quark Model* (Gordon and Breach, New York, 1988). This work contains many references to hadron decay models. Other references to hadron transition models are Phys. Rev. D **8**, 2223 (1973); **9**, 1415 (1974); **11**, 680 (1975); **11**, 1272 (1975); E. Eichten, K. Gottfried, T. Kinoshita, K. Lane, and T. M. Yan, *ibid.* **17**, 3090 (1978).

[9] See, for example, L. Micu, Nucl. Phys. **B10**, 521 (1969); A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Lett. **71B**, 397 (1977).

[10] M. Barbour and J. P. Gilchrist, Z. Phys. C **7**, 225 (1981); **8**, 282(E) (1981); J. P. Ader, B. Bonnier, and S. Sood, Nuovo Cimento **68**, 1 (1982); Phys. Lett. **84B**, 488 (1979); W. Roberts, B. Silvestre-Brac, and C. Gignoux, Phys. Rev. D **41**, 182 (1990).

[11] H. Dosch and D. Gromes, Phys. Rev. D **33**, 1378 (1986).

[12] J. W. Alcock, M. J. Burfitt, and W. N. Cottingham, Z. Phys. C **25**, 161 (1984).

[13] N. Isgur and J. Paton, Phys. Lett. **124B**, 247 (1983); Phys. Rev. D **31**, 2910 (1985).

[14] R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987).

[15] S. Kumano and V. R. Pandharipande, Phys. Rev. D **38**, 146 (1988).

[16] Fl. Stancu and P. Stassart, Phys. Rev. D **38**, 233 (1988); **39**, 343 (1989); **41**, 916 (1990); **42**, 1521 (1990).

[17] N. Isgur and G. Karl, Phys. Lett. **72B**, 109 (1977); **74B** 353 (1978); Phys. Rev. D **18**, 4187 (1978).

[18] N. Isgur and G. Karl, Phys. Rev. D **19**, 2653 (1979).

[19] Ref. [1] contains an extensive list of the references for the interested reader.

[20] N. Isgur and J. Paton, Phys. Lett. **124B**, 247 (1983); Phys. Rev. D **31**, 2910 (1985); J. Merlin and J. Paton, J. Phys. G **11**, 439 (1985).

[21] P. Geiger and N. Isgur, Phys. Rev. D **41**, 1595 (1990); **44**, 799 (1991).

[22] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[23] H. G. Dosch and V. Müller, Nucl. Phys. **B116**, 470 (1976); R. E. Cutkosky and R. E. Hendrick, Phys. Rev. D **16**, 786 (1977); **16**, 793 (1977); J. Carlson, J. B. Kogut, and V. R. Pandharipande, *ibid.* **27**, 233 (1983); **28**, 2807 (1983).

[24] In Ref. [1] the $J^P = \frac{1}{2}^+$ states were expanded to $N = 8$; however, in decay calculations it is preferable to have the initial and final states expanded to the same level, and there is little change in the energies of these states in going from $N = 6$ to $N = 8$. For highly excited states which first appear in the $N = 6$ band, the basis was extended to $N = 8$.

[25] A good fit to the baryon spectrum was obtained in Ref. [1] with all of the parameters the same as in the meson physics calculation of Ref. [22]. Our *best* fit used a string tension which was reduced 15% from the meson value.

[26] J.-M. Richard, Phys. Rep. **212**, 1 (1992).

[27] See Capstick in Ref. [35], and references therein.

[28] T. Barnes and F. E. Close, Phys. Lett. **158B**, 89 (1983); E. Golowich, E. Haqq, and G. Karl, Phys. Rev. D **28**, 160 (1983); F. E. Close, Rep. Prog. Phys. **51**, 833 (1988); Z.-P. Li, Phys. Rev. D **44**, 2841 (1991); Z.-P. Li, V. Burkert, and Z.-J. Li, *ibid.* **46**, 70 (1992).

[29] C. P. Forsyth and R. E. Cutkosky, Phys. Rev. Lett. **46**, 576 (1981); Z. Phys. C **18**, 219 (1983). C.P. Forsyth, Ph.D. thesis, Carnegie Mellon University, 1981.

[30] Fl. Stancu and P. Stassart, Phys. Lett. B **269**, 243 (1991).

[31] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).

[32] R. E. Cutkosky, talk presented at Hadron '91, College Park, Maryland, 1991 (unpublished).

[33] W. Roberts and B. Silvestre-Brac, Few Body Syst. **11**, 171 (1992).

[34] A value of β_π smaller than that which results from fitting a single Gaussian to the Godfrey-Isgur pion wave function [22] gives the best results in decay calculations. See Refs. [14] and [21]; also N. Isgur (private communication).

[35] S. Capstick, Phys. Rev. D **46**, 1965 (1992).

[36] S. Capstick, Phys. Rev. D **46**, 2864 (1992).

[37] G. A. Miller, Phys. Rev. C **39**, 1563 (1989).

[38] R. E. Cutkosky and S. Wang, Phys. Rev. D **42**, 235 (1990).

[39] D. M. Manley and E. M. Saleski, Phys. Rev. D **45**, 4002 (1992).

[40] N. Isgur, G. Karl, and R. Koniuk, Phys. Rev. Lett. **41**, 1269 (1978); Phys. Rev. D **25**, 2394 (1982).

[41] Table IV in Ref. [1] gives the correct prediction of the model for the lowest-mass $\Delta\frac{5}{2}^-$ state. The discussion in the text uses an erroneous value from a previous fit. We would like to thank Fl. Stancu and P. Stassart for bringing this to our attention.

[42] R. E. Cutkosky, in *Proceedings of the Topical Conference on Baryon Resonances*, Oxford, England, 1976, edited by R. T. Ross and D. H. Saxon (Rutherford Laboratory, Chilton, Didcot, England, 1976); R. E. Cutkosky *et al.*, Nucl. Phys. **B102**, 139 (1976); R. E. Cutkosky and R. E. Hendrick, Phys. Rev. D **16**, 786 (1977); A. J. G. Hey and R. Kelly, Phys. Rep. **96**, 71 (1983).

[43] K. C. Bowler *et al.*, Phys. Rev. Lett. **45**, 97 (1980); P. J. Corvi, J. Phys. G **7**, 255 (1977); A. J. G. Hey and R. Kelly, in *Proceedings of the Topical Conference on Baryon Resonances* [42].

[44] Ref. [28]; also F. E. Close, in *Proceedings of the Topical Workshop on Excited Baryons*, Troy, New York, 1988, edited by G. Adams, N. C. Mukhopadhyay, and P. Stoler (World Scientific, Singapore, 1989), p. 67.

[45] R. Koniuk, Nucl. Phys. **B195**, 452 (1982); N. Isgur, in *Proceedings of the CEBAF/SURA 1984 Summer Workshop*, Newport News, Virginia, 1984, edited by F. Gross and R. R. Whitney (CEBAF, Newport News, 1984).