Two-body decays of charm mesons and the role of exotic mesons

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Two-body decays of charm mesons (except for decays into the final states involving η or η') are studied from a perspective in which dynamical contributions of hadrons are explicitly taken into account. All the observed values of branching ratios for the two-body decays of charm mesons are reproduced well. In particular, a solution to the well-known puzzle, $\Gamma(D^0\to K^+K^-)/\Gamma(D^0\to\pi^+\pi^-)\simeq 2-3$, is presented. Contributions of exotic $(QQ)(\bar{Q}\bar{Q})$ mesons can explain the large violation of the charm counterpart of the $|\Delta I|=\frac{1}{2}$ rule in consistency with the small violation of the $|\Delta I|=\frac{1}{2}$ rule in the $K\to\pi\pi$ decays.

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The fact that the $D^0\to\bar{K}^0\phi$ decay [which is described by the so-called annihilation type of quark-line diagrams in the infinite weak-boson-mass limit $(m_W\to\infty)$] has a sizable rate [1] implies that long distance effects are important in nonleptonic weak decays of charm mesons. In the past, explicitly taking into account dynamical contributions of various hadrons, we have studied nonleptonic weak decays of K and charm mesons and have obtained good results on the approximate $|\Delta I|=\frac{1}{2}$ rule in the $K\to\pi\pi$ decays [2], the sizable rate [3] of $D^0\to\bar{K}^0\phi$, and the strong suppression [4] of $F^+\to\pi^+\rho^0$. In particular, we have demonstrated [5] that the observed large ratio [1],

$$\Gamma(D^0\to K^+K^-)/\Gamma(D^0\to\pi^+\pi^-)\simeq 2-3 ,$$

which is a long-standing puzzle, can be reproduced remarkably well by taking into account contributions of exotic four-quark mesons [6] to the intermediate states of the decay processes. However, in Ref. [5] we picked out only the contribution of the $[QQ][\bar{Q}\bar{Q}]$ but not that of the $(QQ)(\bar{Q}\bar{Q})$ mesons. Therefore, the predicted rate of the $D^+\to\pi^+\bar{K}^0$ decay was very small in disagreement with experiment [1]. In this paper, we study typical two-body decays of charm mesons by taking into account contributions of the $(QQ)(\bar{Q}\bar{Q})$ mesons and a glueball in addition to those of the ground-state $\{Q\bar{Q}\}_0$ and the exotic $[QQ][\bar{Q}\bar{Q}]$ mesons which were taken into account in Ref. [5] and demonstrate that all the observed values of the branching ratios for the two-body decays of charm mesons can be well reproduced. In particular, a solution to the well-known puzzle,

$$\Gamma(D^0\to K^+K^-)/\Gamma(D^0\to\pi^+\pi^-)\simeq 2-3 ,$$

is presented without contradiction to the other decays. The contributions of exotic $(QQ)(\bar{Q}\bar{Q})$ mesons can explain the large violation of the charm counterpart of the $|\Delta I|=\frac{1}{2}$ rule in consistency with the small violation of the $|\Delta I|=\frac{1}{2}$ rule in the $K\to\pi\pi$ decays.

We start from a very useful expression for the decay amplitude which manifests dynamical contributions of hadrons. The amplitude for a three-pseudoscalar (PS)-meson process, $P_1(\mathbf{p})\to P_2(\mathbf{k})+P_3(\mathbf{q})$, can be approximated in the form [2,7,8]

$$M\simeq M_{\rm ETC}+M_S , \tag{1}$$

which is realized by using the partially-conserved axial-vector current (PCAC) hypothesis and an extrapolation $\mathbf{q}\to 0$ in the light-cone frame [(LCF), i.e., $\mathbf{p}\to\infty$]. The above approximation can be considered as an innovation of the old soft-pion technique [9]. Here, the equal-time commutator (ETC) term,

$$M_{\rm ETC}(P_1\to P_2P_3)=-\left[\frac{i}{\sqrt{2}f_{P_3}}\langle P_2|[V_{\bar{P}_3},H_w^{(i)}]|P_1\rangle+(P_2\leftrightarrow P_3)\right] , \tag{2}$$

and the surface term M_S, which vanishes in the soft PS meson extrapolation but now survives,

$$M_S(P_1\to P_2P_3)=-\left\{\frac{i}{\sqrt{2}f_{P_3}}\left[\sum_n\left[\frac{m_2^2-m_1^2}{m_n^2-m_1^2}\right]\langle P_2|A_{\bar{P}_3}|n\rangle\langle n|H_w^{(i)}|P_1\rangle+\sum_l\left[\frac{m_2^2-m_1^2}{m_l^2-m_2^2}\right]\langle P_2|H_w^{(i)}|l\rangle\langle l|A_{\bar{P}_3}|P_1\rangle\right]+(P_2\leftrightarrow P_3)\right\} , \tag{3}$$

*Deceased.

have to be evaluated in the LCF, where $H_w^{(i)}$, $(i)=(\Delta C=-1,\ \Delta S=-1)$, $(\Delta C=-1,\ \Delta S=0)$, and $(\Delta C=0,\ \Delta S=-1)$, denote the effective weak Hamiltonians which are responsible for the Cabibbo-angle favored and suppressed decays of charm mesons and the nonleptonic weak decays of K mesons, respectively. V_α and A_α, with the flavor index α, are the vector and axial-vector charges, respectively, and their *asymptotic* matrix elements (matrix elements taken between single-hadron states with *infinite momentum*) will be parametrized by using the *asymptotic* flavor symmetry [10]. The summation in Eq. (3) is extended over all the possible *on-mass-shell* single-hadron (not only the ordinary $\{Q\bar{Q}\}$ but also hypothetical glue ball, hybrid, and multiquark meson) states with *infinite momentum*. Therefore, various kinds of hadrons can contribute to the decay amplitude through the intermediate states of M_S. The exotic hadrons could play an important role (although the overlapping of their wave functions with that of the ground-state $\{Q\bar{Q}\}_0$ meson is expected to be small) if their masses happen to be very close to those of the external hadrons, while contributions of the orbitally excited states will be small [5]. This will be possible in charm meson decays since some of the predicted mass values [11] of the four-quark mesons are very close to the parent charm meson masses, m_D and m_F. However, the mass of hybrid mesons with positive parity is expected to be considerably lower [12] than m_D and m_F, and therefore their contribution to two-body decays of charm mesons will be unimportant. In the $K\to\pi\pi$ decays, the ground-state meson contributions kinematically dominate among the intermediate states in Eq. (3) and the excited-state contribution which is most relevant is that of $(QQ)(\bar{Q}\bar{Q})$ mesons, since it explains naturally the *small* violation of the $|\Delta I|=\frac{1}{2}$ rule in the $K\to\pi\pi$ decays [2].

The two-body decay amplitude, $M(P_1\to P_2P_3)$, is thus described solely by the *asymptotic* matrix elements of the effective weak Hamiltonians $H_w^{(i)}$. Therefore, our task is now to estimate them. To this end, we extend the quark-line argument on the *asymptotic* matrix elements of $H_w^{(0,-)}$ in Ref. [2] to those including $H_w^{(-,-)}$ and $H_w^{(-,0)}$. Our quark-line argument is applied only to the *asymptotic* matrix elements of H_w taken between single-hadron states but not to the whole amplitude.

The effective nonleptonic weak Hamiltonians, $H_w^{(i)}$, are usually written in the form [13]

$$H_w^{(i)}\simeq c_-^{(i)}O_-^{(i)}+c_+^{(i)}O_+^{(i)}+c_P^{(i)}O_P^{(i)}+\text{H.c.}\ , \tag{4}$$

where $c_P^{(-,-)}=0$. For example, the symmetric operator $O_+^{(0,-)}$ belongs to $\mathbf{27}\oplus\mathbf{8}_s$ of $SU_f(3)$ in its symmetry limit and can violate the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule, while the antisymmetric $O_-^{(0,-)}$ belongs to $\mathbf{8}_a$ and obeys the rule. The coefficients $c_P^{(i)}$ of the penguin operators are much smaller than $c_\pm^{(i)}$.

The normal-ordered operators $O_\pm^{(i)}$ can be expanded into a sum of products of (a) two annihilation and two creation operators, (b) one annihilation and three creation operators, (c) one creation and three annihilation operators, and (d) four annihilation or four creation operators of quarks and antiquarks. We associate these products of annihilation and creation operators with different types of weak vertices by requiring the usual connectedness of the quark lines. In this procedure, we have to be careful with the order of the quark(s) and antiquark(s). For (a), we utilize the two annihilation and the two creation operators to annihilate and create, respectively, the quarks and the antiquarks belonging to the meson states $|\{Q\bar{Q}\}\rangle$ and $\langle\{Q\bar{Q}\}|$ in the asymptotic matrix elements of $O_\pm^{(i)}$. However, in the cases (b) and (c), we now have to add a spectator quark or antiquark to reach the physical processes $\langle\{QQ\bar{Q}\bar{Q}\}|O_\pm^{(i)}|\{Q\bar{Q}\}\rangle$ and $\langle\{Q\bar{Q}\}|O_\pm^{(i)}|\{QQ\bar{Q}\bar{Q}\}\rangle$.

All the quarks and the antiquarks associated with the *external* mesons with *infinite momentum* move with *infinite momentum*. In this case, the spinors describing the quark and the antiquark are not independent of each other [14], $u_\pm(\mathbf{k})=-v_\mp(\mathbf{k})$ in the $|\mathbf{k}|\to\infty$ limit, where $\pm$ denote the helicities. Noting that the wave function of the $\{Q\bar{Q}\}_0$ meson is *antisymmetric* [15] under the exchange of its quark and antiquark, we obtain in the LCF,

$$\langle\{Q\bar{Q}\}_0|O_+^{(i)}|\{Q\bar{Q}\}_0\rangle=0\ . \tag{5}$$

In the same asymptotic limit,

$$\langle[QQ][\bar{Q}\bar{Q}]|O_+^{(i)}|\{Q\bar{Q}\}_0\rangle=\langle\{Q\bar{Q}\}_0|O_+^{(i)}|[QQ][\bar{Q}\bar{Q}]\rangle=0 \tag{6}$$

and

$$\langle(QQ)(\bar{Q}\bar{Q})|O_-^{(i)}|\{Q\bar{Q}\}_0\rangle=\langle\{Q\bar{Q}\}_0|O_-^{(i)}|(QQ)(\bar{Q}\bar{Q})\rangle=0 \tag{7}$$

can also be obtained. These are quite reasonable from the symmetry property of the wave functions of the $[QQ][\bar{Q}\bar{Q}]$ and $(QQ)(\bar{Q}\bar{Q})$ mesons under the exchange of the flavors of quarks and antiquarks. The above is a straightforward extension of the quark-line argument in Ref. [2] in which Eqs. (5)–(7) with $(i)=(0,-)$ have already been obtained. Equation (5) implies that the asymptotic ground-state meson matrix elements of $H_w^{(i)}$ satisfy the strict $|\Delta I|=\frac{1}{2}$ rule under the exact $SU_I(2)$ symmetry and its charm counterpart in the $SU_f(3)$ symmetry limit. The nonvanishing asymptotic matrix elements of $O_+^{(i)}$,

$$\langle(QQ)(\bar{Q}\bar{Q})|O_+^{(i)}|\{Q\bar{Q}\}_0\rangle$$

and

$$\langle\{Q\bar{Q}\}_0|O_+^{(i)}|(QQ)(\bar{Q}\bar{Q})\rangle\ ,$$

can give the natural origin of the *small* violation of the $|\Delta I|=\frac{1}{2}$ rule in the $K\to\pi\pi$ decays [2] and the *large* violation of its charm counterpart as will be seen later.

Asymptotic constraints on matrix elements of $H_w^{(i)}$ satisfying Eqs. (5) and (6) with $(i)=(0,-)$, $(-,-)$, and $(-,0)$ have already been derived by using an algebraic method in Ref. [5] and those satisfying Eqs. (5)–(7) with $(i)=(0,-)$ by using the quark-line argument in Ref. [2]. We can reproduce all the previous results on the asymptotic matrix elements of $H_w^{(i)}$ [under appropriate symmetries, i.e., asymptotic $SU_f(3)$ and asymptotic $SU_f(4)$, where the exact $SU_I(2)$ symmetry is always assumed] from the present quark-line arguments. We list, in Appendix A, only new constraints on the asymptotic matrix elements,

$$\langle (QQ)(\bar{Q}\bar{Q})|H_w^{(i)}|\{Q\bar{Q}\}_0\rangle$$

and

$$\langle \{Q\bar{Q}\}_0|H_w^{(i)}|(QQ)(\bar{Q}\bar{Q})\rangle ,$$

which satisfy Eq. (7) with $(i)=(-,-)$ and $(-,0)$. Since the QCD corrections to the operators $O_\pm^{(-,-)}$ and $O_\pm^{(-,0)}$ are equal to each other [13], we obtain

$$\langle \bar{K}^0|H_w^{(-,-)}|D^0\rangle = -\cot\theta_C\langle \pi^+|H_w^{(-,0)}|D^+\rangle ,$$

with θ_C the Cabibbo angle using the asymptotic $SU_f(3)$ rotations through $[O_\pm^{(-,-)},V_{K^0}]=O_\pm^{(-,0)}$. (Similar types of asymptotic matrix elements of $O_\pm^{(-,-)}$ and $O_\pm^{(-,0)}$ can be related to each other by using the asymptotic $SU_f(3)$ rotations as in Ref. [5].)

We have so far neglected contributions of the penguin terms with small coefficients [13]. However, they can induce matrix elements, $\langle G|O_P^{(i)}|\{Q\bar{Q}\}_0\rangle$, where G denotes a glueball. Therefore, the glueball could give a significant contribution to the Cabibbo-angle-suppressed decays if its mass is close to that of the parent charm meson mass. We here consider the contribution of the glueball with $J^{PC}=0^{++}$ to the intermediate state of M_S for the $D^0\to K\bar{K}$ and $\pi\pi$ decays. However, known glueball candidates seem to mix with ordinary $\{Q\bar{Q}\}$ mesons. Here we may consider $S^*(975)$, $\epsilon(1300)$, and $f_0(1770)$ as the candidates containing the glueball with $J^{PC}=0^{++}$, since this choice has explained [16] the mass relation and the decay properties of these scalar mesons reasonably well. The glueball contribution through the scalar mesons to $M_S(D^0\to P\bar{P})$ $(P=\pi,K)$ is given in the form

$$M_S^{(\text{glueball})}(D^0\to P\bar{P}) = -\left[\frac{i}{f_P}\right]\sum_S\left[\frac{m_P^2-m_D^2}{m_S^2-m_D^2}\right]\langle P|A_P|S\rangle \times\langle S|H_w^{(-,0)}|D^0\rangle , \quad (8)$$

in the present formalism, where S denotes the intermediate scalar mesons containing the glueball component, i.e., $S^*(975)$, $\epsilon(1300)$, and $f_0(1770)$. However, we neglect the contributions of the $S^*(975)$ and $\epsilon(1300)$ to the D^0 decays because of $(m_D^2-m_{S^*}^2)$, $(M_D^2-m_\epsilon^2) \gg (m_D^2-m_{f_0}^2)$. $f_0(1770)$ couples dominantly to $K\bar{K}$ (but very weakly to $\pi\pi$) [16]. Therefore, we here consider only the $f_0(1770)$ contribution to $M_S^{(\text{glueball})}(D^0\to K\bar{K})$.

Substituting the constraints on the diagonal and nondiagonal asymptotic matrix elements of $H_w^{(0,-)}$, $H_w^{(-,-)}$, and $H_w^{(-,0)}$ in Refs. [2,5] and Eqs. (A1)–(A6) in Appendix A into the amplitude, Eq. (1) with Eqs. (2) and (3), we can obtain the amplitudes for $K\to\pi\pi$, Cabibbo-angle favored and suppressed decays of charm mesons. They are now described in terms of the asymptotic ground-state meson matrix elements of $H_w^{(i)}$ $(\langle\pi^+|H_w^{(0,-)}|K^+\rangle$, $\langle\bar{K}^0|H_w^{(-,-)}|D^0\rangle$, and $\langle\pi^+|H_w^{(-,0)}|D^+\rangle)$, the phase factor which arises [17] from M_{ETC} relative to M_S, the invariant matrix element h of the axial-vector charge taken between two $\{Q\bar{Q}\}_0$ meson states with infinite momentum (which is estimated [7] to be $h\simeq 1.0$ from the observed decay rate [1] $\Gamma(K^*\to K\pi)_{\text{expt}}\simeq 50$ MeV), the parameters $f_a, f_a^*, f_a^{*'}$, and $f_s, f_s^*, f_s^{*'}$, and the glueball contribution to the $D^0\to K\bar{K}$ decays which is parametrized by f_g. The amplitudes for the two-body decays of charm mesons are listed in Appendix B. ($e^{i\delta}$ is the phase factor mentioned above. f_a and f_s are defined in Ref. [2]. f_a^* and $f_a^{*'}$ correspond to the fractions $-C_{\pi^0}$ and $-e_{\pi^0}$, respectively, in Ref. [5]. f_s^* and $f_s^{*'}$ are defined in Appendix A. $f_{a(s)}^*=f_{a(s)}^{*'}$ can be obtained by using the asymptotic $SU_f(3)$ rotations through $[O_\pm^{(-,-)},V_{K^0}]=O_\pm^{(-,0)}$. $\langle\bar{K}^0|H_w^{(-,-)}|D^0\rangle$ and $\langle\pi^+|H_w^{(-,0)}|D^+\rangle$ are related to each other by using the same asymptotic $SU_f(3)$ rotations.)

The amplitude for the $K^+\to\pi^+\pi^0$ decay, Eq. (14) of Ref. [2], arises from the contribution of the $(QQ)(\bar{Q}\bar{Q})$-type exotics to M_S since M_{ETC} and the contributions of the $\{Q\bar{Q}\}_0$ and $[QQ][\bar{Q}\bar{Q}]$ mesons to M_S vanish because of the asymptotic $|\Delta I|=\frac{1}{2}$ rules, Eqs. (8) and (12) in Ref. [2], which can be derived from the above Eqs. (5) and (6) with $(i)=(0,-)$. Therefore, it is, in fact, much smaller [because of $m_{\text{exotic}}^2\gg m_K^2$, m_π^2 and the small overlapping between the wave functions of the $\{Q\bar{Q}\}_0$ and $(QQ)(\bar{Q}\bar{Q})$ mesons] than the $K_S\to(\pi\pi)^0$ amplitudes of $|\Delta I|=\frac{1}{2}$ in which both of M_{ETC} and M_S survive. *(Small* violation of the $|\Delta I|=\frac{1}{2}$ rule. For details, see Ref. [2].)

We now study two-body decays of charm mesons. In M_S, the s-channel contribution of the exotic mesons will be important since some of the predicted masses [11] of the exotic mesons which can contribute to the s-channel intermediate states of the charm meson decays are very close to the masses of the parent charm mesons as was mentioned previously. In this case, the width of the intermediate meson *pole* would be important. Therefore, we here take into account the width Γ_{exotic} of the exotic mesons which are expected to be very broad. (If they were narrow, it might not be hard to observe them.) However, the contribution of exotic mesons through the u channel, in which only the mesons with charm can take part, is expected to be negligibly small since the kinematical mass-dependent factor $(m_2^2-m_1^2)/(m_l^2-m_2^2)$ in Eq. (3) will be small for $l=$ exotic mesons with charm. It implies that use of the asymptotic $SU_f(4)$ symmetry does not introduce very large errors in M_S. Then the unknown parameters involved are $\langle\bar{K}^0|H_w^{(-,-)}|D^0\rangle$, $e^{i\delta}$, f_a^*, f_s^*, Γ_{exotic}, and f_g. The phase factor $e^{i\delta}$ arises from M_{ETC} which now represents the contribution of nonresonant multihadron intermediate states. Therefore it is expected to have a mild energy dependence and to be less than 90°. (The corresponding phase in the $K\to\pi\pi$ decays was $\simeq 50°$ in Ref. [2].) f_a^* and f_s^* are introduced to parametrize the products of asymptotic matrix elements of A_α and H_w taken between the $\{Q\bar{Q}\}_0$ and the $[QQ][\bar{Q}\bar{Q}]$ and between the $\{Q\bar{Q}\}_0$ and the $(QQ)(\bar{Q}\bar{Q})$, i.e.,

$$\langle\{Q\bar{Q}\}_0|A_\alpha|[QQ][\bar{Q}\bar{Q}]\rangle\langle[QQ][\bar{Q}\bar{Q}]|H_w|\{Q\bar{Q}\}_0\rangle$$

and

$$\langle \{Q\bar{Q}\}_0 | A_\alpha | (QQ)(\bar{Q}\bar{Q}) \rangle \langle (QQ)(\bar{Q}\bar{Q}) | H_w | \{Q\bar{Q}\}_0 \rangle \ ,$$

respectively, as shown in Appendix A. Since the overlappings between wave functions of the $\{Q\bar{Q}\}_0$ and the $[QQ][\bar{Q}\bar{Q}]$ and between the $\{Q\bar{Q}\}_0$ and the $(QQ)(\bar{Q}\bar{Q})$ are expected to be small, f_a^* and f_s^* are constrained to be much less than unity. $\Gamma_{\rm exotic}$ is expected to be much larger than those of the usual resonant states as stated above. Our calculated branching ratios are not very sensitive to the value of $\Gamma_{\rm exotic}$ as long as it is taken to be very large ($\Gamma_{\rm exotic} \simeq 0.3-0.5$ GeV). Therefore, we fix the values of these parameters as follows (they are still tentative); $\delta \simeq 50°$, $\Gamma_{\rm exotic} \simeq 0.4$ GeV, $f_a^* \simeq f_s^* \simeq 0.05$. Substituting these values of the parameters into the amplitudes in Appendix B, we obtain the branching ratios listed in (iii) of Table I, where $\langle \bar{K}^0 | H_w^{(-,-)} | D^0 \rangle$ and f_g have been fixed by using the observed values of $B(D^0 \to K^- \pi^+)$ and $B(D^0 \to K^0 \bar{K}^0)$ as the input data. We can see that the contribution of the $(QQ)(\bar{Q}\bar{Q})$ mesons is crucial for the decays $D^+ \to \pi^+ \bar{K}^0$ and $D^+ \to \pi^+ \pi^0$ (the direct charm counterpart of $K^+ \to \pi^+ \pi^0$), which were strongly suppressed in Ref. [5], as was the case with the $K^+ \to \pi^+ \pi^0$ decay and now leads to a large violation of the charm counterpart of the $|\Delta I| = \frac{1}{2}$ rule. In Table I, the present result (iii), which reproduces well the observed values, is compared with those of the other extensive studies, for example, (i) by Bauer, Stech, and Wirbel [18] based on the factorization prescription supplemented by final-state interactions and (ii) by Blok and Shifman [19] from the perspective of the QCD sum rule. (Charm meson decays were also studied extensively by Chau and Cheng [20] by using the conventional quark-line arguments, and by Gibilisco and Preparata [21] by using their anisotropic chromodynamic theory. However, in these analyses, the most interesting decay rates, $\Gamma(D^0 \to K^+ K^-)$ and $\Gamma(D^0 \to \pi^+ \pi^-)$, were not predicted but their observed values were used as the input data.) The above choice of the values of the parameters involved leads to the phase difference

$$\Delta_{\pi\bar{K}} = (\delta_{1/2} - \delta_{3/2})_{\pi\bar{K}} \simeq 80°$$

between the amplitudes for the $D \to \pi\bar{K}$ decays into the $I = \frac{1}{2}$ and $\frac{3}{2}$ final states, which is consistent with the result $\Delta_{\pi\bar{K}} = (79 \pm 11)°$ from the phenomenological analysis [20]. The phases of the charm meson decay amplitudes are very sensitive to the widths of the exotic mesons which take part in the s-channel intermediate states.

The $D^0 \to K^0 \bar{K}^0$ decay was predicted to be strongly suppressed in Ref. [5] since contributions of the $(QQ)(\bar{Q}\bar{Q})$ mesons and the glueball with $J^{PC} = 0^{++}$ were not taken into account. However, the $(QQ)(\bar{Q}\bar{Q})$ mesons contribute to this decay only through the u channel and their contributions are suppressed kinematically. Therefore, its dominant part comes from the glueball contribution possibly through the $f_0(1770)$. We have fixed the size of the glueball contribution to the Cabibbo-angle suppressed decays by using the observed branching ratio [1],

$$B(D^0 \to K^0 \bar{K}^0)_{\rm expt} \simeq 0.1\% \ ,$$

and its sign has been chosen so that $M_S^{(\rm glueball)}(D^0 \to K^+ K^-)$ interferes constructively with the ground-state meson contribution $M_S^{(L=0)}(D^0 \to K^+ K^-)$ in the narrow-width limit.

In the enhancement of the $D^0 \to K^+ K^-$ decay relative to $D^0 \to \pi^+ \pi^-$, the $\hat{\sigma}^s$ contribution to the $D^0 \to K^+ K^-$ decay is crucial. It interferes constructively with both of $M_{\rm ETC}$ and the ground-state meson contribution $M_S^{(L=0)}$ to M_S under the present choice of the values of the parameters and its mass $m_{\hat{\sigma}^s}$ is much closer to m_D than the mass $m_{\hat{\sigma}}$ of $\hat{\sigma}$ which contributes to the $D^0 \to \pi^+ \pi^-$ decay. The contributions of the $(QQ)(\bar{Q}\bar{Q})$ mesons and the glueball to the $D^0 \to K^+ K^-$ decay are very different from those to $D^0 \to \pi^+ \pi^-$. The above is the main reason for the large $SU_f(3)$ symmetry breaking in the amplitudes for the $D^0 \to K^+ K^-$ and $D^0 \to \pi^+ \pi^-$ decays in the present perspective.

TABLE I. Branching ratios (%) for typical two-body decays of charm mesons. The results (i) and (ii) are given by Bauer, Stech, and Wirbel in Ref. [18] and by Blok and Shifman in Ref. [19], respectively. (iii) is the result of the present theory which contains the contributions of the $\{Q\bar{Q}\}_0$, the exotic $[QQ][\bar{Q}\bar{Q}]$, and the $(QQ)(\bar{Q}\bar{Q})$ mesons ($f_a^* = f_s^* = 0.05$) and the glueball. The values with an * are used as the input data. The data values are taken from Ref. [1] except for that with a (‡) which is taken from Ref. [22].

Decays	(i) BSW	(ii) BS	(iii)	Experiments (%)
$D^+ \to \pi^+ \bar{K}^0$	3.6	6.3	2.1	2.6±0.4
$D^0 \to \pi^+ K^-$	5.8	6.4	3.7*	3.65±0.21
$D^0 \to \pi^0 \bar{K}^0$	2.5	1.5	1.6	2.1±0.6
$F^+ \to K^+ \bar{K}^0$	1.4	0.9	3.5	2.8±0.7
$D^0 \to K^+ K^-$	0.56	0.3	0.49	0.41±0.04
$D^0 \to \pi^+ \pi^-$	0.39	0.28	0.16	0.163±0.019
$D^0 \to \pi^0 \pi^0$		0.15	0.17	0.09±0.02±0.02(‡)
$D^0 \to K^0 \bar{K}^0$		0.0	0.10*	0.11±0.04
$D^+ \to \pi^+ \pi^0$	0.14	0.1	0.08	<0.53 (90 % C.L.)
$D^+ \to K^+ \bar{K}^0$	1.18	0.5	0.37	0.73±0.18
$F^+ \to K^0 \pi^+$	0.26		0.09	<0.6 (90 % C.L.)
$F^+ \to K^+ \pi^0$	0.02		0.09	

In summary, we have studied dynamical contributions of various hadrons to two-body decays of the K and charm mesons. We have written the decay amplitudes in terms of the asymptotic matrix elements of $H_w^{(i)}$ taken between single-meson (the ordinary $\{Q\bar{Q}\}$, glueball, $\{QQ\bar{Q}\bar{Q}\}$, hybrid $\{Q\bar{Q}g\}$) states. We have evaluated these contributions to the K and charm meson decays and have found that the small violation of the $|\Delta I|=\frac{1}{2}$ rule in the $K\to\pi\pi$ decays and the large violation of its charm counterpart can be explained simultaneously by the $(QQ)(\bar{Q}\bar{Q})$ contributions. A solution to the long-standing puzzle,

$$\Gamma(D^0\to K^+K^-)/\Gamma(D^0\to\pi^+\pi^-)\simeq 2-3 ,$$

has also been obtained in consistency with the $K\to\pi\pi$ and the Cabibbo-angle favored decays in addition to the other Cabibbo-angle suppressed decays. We have predicted branching ratios for some decays which have not yet been observed. However, the present result is still crude. We need much more information about the exotic mesons and their properties for more accurate discussions. Nevertheless, it is interesting to see that nonleptonic weak decays are intimately related to hadron spectroscopy.

APPENDIX A: CONSTRAINTS ON ASYMPTOTIC MATRIX ELEMENTS OF $H_w^{(i)}$

We list constraints on the asymptotic matrix elements of $H_w^{(i)}$,

$$\langle (QQ)(\bar{Q}\bar{Q})|H_w^{(i)}|\{Q\bar{Q}\}_0\rangle \quad \text{and} \quad \langle \{Q\bar{Q}\}_0|H_w^{(i)}|(QQ)(\bar{Q}\bar{Q})\rangle ,$$

which satisfy Eq. (7) in the text,

$$\langle (QQ)(\bar{Q}\bar{Q})|O_-^{(i)}|\{Q\bar{Q}\}_0\rangle=\langle \{Q\bar{Q}\}_0|O_-^{(i)}|(QQ)(\bar{Q}\bar{Q})\rangle=0 ,$$

with $(i)=(-,-)$ and $(-,0)$.

$$\sqrt{3/2}\langle E^{*+}_{\pi\bar{K}}|H_w^{(-,-)}|D^+\rangle=(3/\sqrt{2})\langle E^{*0}_{\pi\bar{K}}|H_w^{(-,-)}|D^0\rangle=3\langle C^{*0}_{\bar{K}}|H_w^{(-,-)}|D^0\rangle=\sqrt{3}\langle C^{s*+}_{\pi}|H_w^{(-,-)}|F^+\rangle$$
$$=(f_s^*/g^*)\langle \bar{K}^0|H_w^{(-,-)}|D^0\rangle , \tag{A1}$$

$$-\sqrt{3/2}\langle \pi^+|H_w^{(-,-)}|E^{*+}_{\pi F}\rangle=\sqrt{3/2}\langle \pi^0|H_w^{(-,-)}|E^{*0}_{\pi F}\rangle=-(3/\sqrt{2})\langle \bar{K}^0|H_w^{(-,-)}|E^{*0}_{\pi D}\rangle=\sqrt{3/2}\langle K^-|H_w^{(-,-)}|E^{*-}_{\pi D}\rangle$$
$$=3\langle \bar{K}^0|H_w^{(-,-)}|C^{*0}_{D}\rangle=-\sqrt{3}\langle \bar{K}^0|H_w^{(-,-)}|C^{s*0}_{D}\rangle=\sqrt{3/2}\langle K^+|H_w^{(-,-)}|E^{*+}_{KF}\rangle$$
$$=(\tilde{f}_s^*/g^*)\langle \bar{K}^0|H_w^{(-,-)}|D^0\rangle , \tag{A2}$$

$$\langle \pi^+|H_w^{(-,-)}|C^{*+}_F\rangle=0 , \tag{A3}$$

$$\sqrt{3}\langle E^{*+}_{\pi\pi}H_w^{(-,0)}|D^+\rangle=(3/\sqrt{2})\langle E^{*0}_{\pi\pi}|H_w^{(-,0)}|D^0\rangle=-\sqrt{3}\langle C^{*+}_{\pi}|H_w^{(-,0)}|D^+\rangle=\sqrt{3}\langle C^{s*+}_{\pi}|H_w^{(-,0)}|D^+\rangle$$
$$=\sqrt{6}\langle C^{s*0}_{\pi}|H_w^{(-,0)}|D^0\rangle=-3\langle C^{*}|H_w^{(-,0)}|D^0\rangle=\sqrt{6}\langle C^{s*}|H_w^{(-,0)}|D^0\rangle$$
$$=-(3/\sqrt{2})\langle E^{*+}_{\pi K}|H_w^{(-,0)}|F^+\rangle=-3\langle C^{*+}_{K}|H_w^{(-,0)}|F^+\rangle=(f_s^{*\prime}/g^*)\langle \pi^+|H_w^{(-,0)}|D^+\rangle , \tag{A4}$$

$$(3/2\sqrt{2})\langle \pi^+|H_w^{(-,0)}|E^{*+}_{\pi D}\rangle=3\langle \pi^0|H_w^{(-,0)}|E^{*0}_{\pi D}\rangle=-\sqrt{3/2}\langle \pi^-|H_w^{(-,0)}|E^{*-}_{\pi D}\rangle=-3\langle \pi^+|H_w^{(-,0)}|C^{*+}_{D}\rangle$$
$$=-3\sqrt{2}\langle \pi^0|H_w^{(-,0)}|C^{*0}_{D}\rangle=\sqrt{3}\langle \pi^+|H_w^{(-,0)}|C^{s*+}_{D}\rangle=\sqrt{6}\langle \pi^0|H_w^{(-,0)}|C^{s*0}_{D}\rangle$$
$$=-(\sqrt{3}/2)\langle K^0|H_w^{(-,0)}|E^{*0}_{\pi F}\rangle=-\sqrt{3/2}\langle K^+|H_w^{(-,0)}|C^{*+}_{F}\rangle=\sqrt{3}\langle \bar{K}^0|H_w^{(-,0)}|E^{*0}_{D\bar{K}}\rangle$$
$$=\sqrt{3/2}\langle K^-|H_w^{(-,0)}|E^{*-}_{D\bar{K}}\rangle=(\tilde{f}_s^{*\prime}/g^*)\langle \pi^+|H_w^{(-,0)}|D^+\rangle , \tag{A5}$$

$$\langle C^{*0}_{\pi}|H_w^{(-,0)}|D^0\rangle=\langle K^+|H_w^{(-,0)}|E^{*+}_{\pi F}\rangle=0 , \tag{A6}$$

in the LCF, where contributions of the penguin terms with small coefficients have been neglected. g^* is the asymptotic invariant matrix element of the axial-vector charge A_α taken between the $\{Q\bar{Q}\}_0$ and $(QQ)(\bar{Q}\bar{Q})$ meson states and is defined by $g^*=\langle C^{*+}_{K}|A_{\pi^+}|K^0\rangle$. f_s^*, $\tilde{f}_s^*$, $f_s^{*\prime}$, and $\tilde{f}_s^{*\prime}$ denote the parameters introduced and correspond to f_s in Ref. [2]. They are related to the overlapping between the wave functions of the $\{Q\bar{Q}\}_0$ and $(QQ)(\bar{Q}\bar{Q})$ mesons and are expected to be small. Use of the commutation relation, $[[H_w^{(-,-)},V_{F^-}],V_{F^-}]=[[H_w^{(-,-)},V_{\pi^-}],V_{\pi^-}]$, with the asymptotic $SU_f(4)$ symmetry leads to $f_s^{*}(')=\tilde{f}_s^{*}(')$. Equations (A4)–(A6) can be derived also from Eqs. (A1)–(A3) by using the asymptotic $SU_f(3)$ rotations through $[O_\pm^{(-,-)},V_{K^0}]=O_\pm^{(-,0)}$.

APPENDIX B: TWO-BODY DECAY AMPLITUDES

We list the amplitudes for two-body decays of charm mesons which can be obtained by substituting the constraints on the asymptotic matrix elements of $H_w^{(-,-)}$ and $H_w^{(-,0)}$ in Ref. [5] and Eqs. (A1)–(A6) in Appendix A into the general form of the decay amplitude, Eq. (1) with Eqs. (2) and (3).

(i) Cabibbo-angle favored decays:

$$
\begin{aligned}
M(D^+\to\pi^+\bar{K}^0)\simeq -\frac{i}{\sqrt{2}f_\pi}\langle\bar{K}^0|H_w^{(-,-)}|D^0\rangle \Bigg\{ & \left[1-\frac{f_\pi}{f_K}\right]e^{i\delta}+\left[\left(\frac{m_D^2-m_K^2}{m_{D^*}^2-m_K^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_{F^*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]\sqrt{\tfrac{1}{2}}h \\
&+\left[\left(\frac{m_D^2-m_K^2}{m_{\hat{D}^*}^2-m_K^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_{\hat{F}_1^*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]f_a^* \\
&+\Bigg[2\left(\frac{m_D^2-m_K^2}{m_D^2-m_{E^*_{\pi K}}^2}\right)+2\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{E^*_{\pi K}}^2}\right)\left(\frac{f_\pi}{f_K}\right) \\
&\quad-\left(\frac{m_D^2-m_K^2}{m_{E^*_{\pi D}}^2-m_K^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_{E^*_{\pi F}}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\Bigg]f_s^*\Bigg\}\,,
\end{aligned}
\tag{B1}
$$

$$
\begin{aligned}
M(D^0\to\pi^+K^-)\simeq \frac{i}{\sqrt{2}f_\pi}\langle\bar{K}^0|H_w^{(-,-)}|D^0\rangle & \\
\times\Bigg\{ & e^{i\delta}+\left[\left(\frac{m_D^2-m_K^2}{m_D^2-m_{K^*}^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{K^*}^2}\right)\left(\frac{f_\pi}{f_K}\right)+\left(\frac{m_D^2-m_\pi^2}{m_{F^*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]\sqrt{\tfrac{1}{2}}h \\
&+\left[\left(\frac{m_D^2-m_K^2}{m_D^2-m_{\hat{K}^*}^2}\right)+\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{\hat{K}^*}^2}\right)\left(\frac{f_\pi}{f_K}\right)-\left(\frac{m_D^2-m_\pi^2}{m_{\hat{F}_I^*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]f_a^* \\
&+\Bigg[\left(\frac{m_D^2-m_K^2}{m_{E^*_{\pi K}}^2-m_D^2}\right)+\left(\frac{m_D^2-m_\pi^2}{m_{E^*_{\pi K}}^2-m_D^2}\right)\left(\frac{f_\pi}{f_K}\right) \\
&\quad+2\left(\frac{m_D^2-m_K^2}{m_{E^*_{\pi D}}^2-m_K^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_{E^*_{\pi F}}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\Bigg]f_s^*\Bigg\}\,,
\end{aligned}
\tag{B2}
$$

$$
\begin{aligned}
M(D^0\to\pi^0\bar{K}^0)\simeq -\frac{i}{\sqrt{2}f_\pi}\langle\bar{K}^0|H_w^{(-,-)}|D^0\rangle & \\
\times\Bigg\{ & \left[2-\frac{f_\pi}{f_K}\right]e^{i\delta}+\left[\left(\frac{m_D^2-m_K^2}{m_D^2-m_{K^*}^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{K^*}^2}\right)\left(\frac{f_\pi}{f_K}\right)+\left(\frac{m_D^2-m_K^2}{m_{D^*}^2-m_K^2}\right)\right]\sqrt{\tfrac{1}{2}}h \\
&+\left[\left(\frac{m_D^2-m_K^2}{m_D^2-m_{\hat{K}^*}^2}\right)+\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{\hat{K}^*}^2}\right)\left(\frac{f_\pi}{f_K}\right)+\left(\frac{m_D^2-m_K^2}{m_{\hat{D}^*}^2-m_K^2}\right)-2\left(\frac{m_D^2-m_\pi^2}{m_{\hat{F}_1^*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]f_a^* \\
&+\Bigg[\left(\frac{m_D^2-m_K^2}{m_D^2-m_{E^*_{\pi K}}^2}\right)+\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{E^*_{\pi K}}^2}\right)\left(\frac{f_\pi}{f_K}\right)+\left(\frac{m_D^2-m_K^2}{m_{E^*_{\pi D}}^2-m_K^2}\right) \\
&\quad-2\left(\frac{m_D^2-m_\pi^2}{m_{E^*_{\pi F}}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\Bigg]f_s^*\Bigg\}\,,
\end{aligned}
\tag{B3}
$$

$$M(F^+ \to K^+ \overline{K}^0) \simeq -\frac{i}{\sqrt{2} f_K} \langle \overline{K}^0 | H_w^{(-,-)} | D^0 \rangle$$

$$\times \left\{ e^{i\delta} + \left(\frac{m_F^2 - m_K^2}{m_{D^*}^2 - m_K^2} \right) \sqrt{\tfrac{1}{2}} h + \left[2 \left(\frac{m_F^2 - m_K^2}{m_F^2 - m_{\delta^{s*}}^2} \right) - \left(\frac{m_F^2 - m_K^2}{m_{\hat{D}^{s*}}^2 - m_K^2} \right) \right] f_a^* \right.$$

$$\left. - \left[\left(\frac{m_F^2 - m_K^2}{m_{C_\pi^{s*}}^2 - m_F^2} \right) + \left(\frac{m_F^2 - m_K^2}{m_{E_{KF}^*}^2 - m_K^2} \right) \right] f_s^* \right\} . \quad \text{(B4)}$$

(ii) Cabibbo-angle suppressed decays:

$$M(D^0 \to K^0 \overline{K}^0) \simeq \frac{i}{\sqrt{2} f_K} \langle \pi^+ | H_w^{(-,0)} | D^+ \rangle \left\{ \left[\left(\frac{m_D^2 - m_K^2}{m_{E_{\overline{K}D}^*}^2 - m_K^2} \right) - 2 \left(\frac{m_D^2 - m_K^2}{m_{E_{\pi F}^*} - m_K^2} \right) \right] f_s^* + 2 \left(\frac{m_D^2 - m_K^2}{m_D^2 - m_{f_0}^2} \right) f_g \right\} , \quad \text{(B5)}$$

$$M(D^0 \to K^+ K^-) \simeq \frac{i}{\sqrt{2} f_K} \langle \pi^+ | H_w^{(-,0)} | D^+ \rangle$$

$$\times \left\{ e^{i\delta} + \left[\left(\frac{m_D^2 - m_K^2}{m_{F^*}^2 - m_K^2} \right) \right] \sqrt{\tfrac{1}{2}} h + \left[\left(\frac{m_D^2 - m_K^2}{m_D^2 - m_{\hat{\sigma}^{s*}}^2} \right) + \left(\frac{m_D^2 - m_K^2}{m_D^2 - m_{\hat{\delta}^{s*}}^2} \right) - \left(\frac{m_D^2 - m_K^2}{m_{\hat{F}_I^*}^2 - m_K^2} \right) \right] f_a^* \right.$$

$$+ \left[\left(\frac{m_D^2 - m_K^2}{m_{C^{s*}}^2 - m_D^2} \right) + \left(\frac{m_D^2 - m_K^2}{m_{C_\pi^{s*}}^2 - m_K^2} \right) + 2 \left(\frac{m_D^2 - m_K^2}{m_{E_{\overline{K}D}^*}^2 - m_K^2} \right) - \left(\frac{m_D^2 - m_K^2}{m_{C_F^*}^2 - m_K^2} \right) \right] f_s^*$$

$$\left. + 2 \left(\frac{m_D^2 - m_K^2}{m_D^2 - m_{f_0}^2} \right) f_g \right\} , \quad \text{(B6)}$$

$$M(D^0 \to \pi^+ \pi^-) \simeq -\frac{i}{\sqrt{2} f_\pi} \langle \pi^+ | H_w^{(-,0)} | D^+ \rangle$$

$$\times \left\{ e^{i\delta} + \left(\frac{m_D^2 - m_\pi^2}{m_{D^*}^2 - m_\pi^2} \right) \sqrt{\tfrac{1}{2}} h + \left[2 \left(\frac{m_D^2 - m_\pi^2}{m_D^2 - m_{\hat{\sigma}^*}^2} \right) - \left(\frac{m_D^2 - m_\pi^2}{m_{\hat{D}^*}^2 - m_\pi^2} \right) \right] f_a^* \right.$$

$$\left. + \left[2 \left(\frac{m_D^2 - m_\pi^2}{m_{C^*}^2 - m_D^2} \right) + \left(\frac{m_D^2 - m_\pi^2}{m_{E_{\pi D}^*}^2 - m_\pi^2} \right) \right] f_s^* \right\} , \quad \text{(B7)}$$

$$M(D^0 \to \pi^0 \pi^0) \simeq -\frac{i}{2 f_\pi} \langle \pi^+ | H_w^{(-,0)} | D^+ \rangle$$

$$\times \left\{ e^{i\delta} + \left(\frac{m_D^2 - m_\pi^2}{m_{D^*}^2 - m_\pi^2} \right) \sqrt{\tfrac{1}{2}} h + \left[2 \left(\frac{m_D^2 - m_\pi^2}{m_D^2 - m_{\hat{\sigma}^*}^2} \right) - \left(\frac{m_D^2 - m_\pi^2}{m_{\hat{D}^*}^2 - m_\pi^2} \right) \right] f_a^* \right.$$

$$\left. + \left[2 \left(\frac{m_D^2 - m_\pi^2}{m_D^2 - m_{C^*}^2} \right) - \left(\frac{m_D^2 - m_\pi^2}{m_{E_{\pi D}^*}^2 - m_\pi^2} \right) \right] f_s^* \right\} , \quad \text{(B8)}$$

$$M(D^+\to\pi^+\pi^0)\simeq-\frac{i}{\sqrt{2}f_\pi}\sqrt{2}\langle\pi^+|H_w^{(-,0)}|D^+\rangle\left[2\left[\frac{m_D^2-m_\pi^2}{m_D^2-m_{E^*_{\pi\pi}}^2}\right]-\left[\frac{m_D^2-m_\pi^2}{m_{E^*_{\pi D}}^2-m_\pi^2}\right]\right]f_s^*\,, \tag{B9}$$

$$\begin{aligned}M(D^+\to K^+\bar{K}^0)\simeq{}&\frac{i}{\sqrt{2}f_K}\langle\pi^+|H_w^{(-,0)}|D^+\rangle\\&\times\left\{e^{i\delta}+\left[\left[\frac{m_D^2-m_K^2}{m_{F^*}^2-m_K^2}\right]\right]\sqrt{\tfrac{1}{2}}h+\left[2\left[\frac{m_D^2-m_K^2}{m_D^2-m_{\hat{\delta}^{s*}}^2}\right]-2\left[\frac{m_D^2-m_K^2}{m_{\hat{F}^{0*}}^2-m_K^2}\right]+\left[\frac{m_D^2-m_K^2}{m_{\hat{F}_I^*}^2-m_K^2}\right]\right]f_a^*\right.\\&\left.-\left[2\left[\frac{m_D^2-m_K^2}{m_{C^{s*}_\pi}^2-m_D^2}\right]+\left[\frac{m_D^2-m_K^2}{m_{E^*_{\bar{K}D}}^2-m_K^2}\right]-\left[\frac{m_D^2-m_K^2}{m_{C^*_F}^2-m_K^2}\right]\right]f_s^*\right\},\end{aligned} \tag{B10}$$

$$\begin{aligned}M(F^+\to\pi^+K^0)\simeq{}&-\frac{i}{\sqrt{2}f_\pi}\langle\pi^+|H_w^{(-,0)}|D^+\rangle\\&\times\left\{\left[2\left[\frac{f_\pi}{f_K}\right]-1\right]e^{i\delta}-\left[\left[\frac{m_F^2-m_K^2}{m_F^2-m_{K^*}^2}\right]-\left[\frac{m_F^2-m_\pi^2}{m_F^2-m_{K^*}^2}\right]\left[\frac{f_\pi}{f_K}\right]-\left[\frac{m_F^2-m_\pi^2}{m_{D^*}^2-m_\pi^2}\right]\left[\frac{f_\pi}{f_K}\right]\right]\sqrt{\tfrac{1}{2}}h\right.\\&+\left[\left[\frac{m_F^2-m_K^2}{m_F^2-m_{\hat{K}^*}^2}\right]+\left[\frac{m_F^2-m_\pi^2}{m_F^2-m_{\hat{K}^*}^2}\right]\left[\frac{f_\pi}{f_K}\right]-2\left[\frac{m_F^2-m_K^2}{m_{\hat{F}_I^*}^2-m_K^2}\right]+\left[\frac{m_F^2-m_\pi^2}{m_{D^{s*}}^2-m_\pi^2}\right]\left[\frac{f_\pi}{f_K}\right]\right]f_a^*\\&-\left[\left[\frac{m_F^2-m_K^2}{m_F^2-m_{E^*_{\pi K}}^2}\right]+\left[\frac{m_F^2-m_\pi^2}{m_F^2-m_{E^*_{\pi K}}^2}\right]\left[\frac{f_\pi}{f_K}\right]-2\left[\frac{m_F^2-m_K^2}{m_{E^*_{\pi F}}^2-m_K^2}\right]\right.\\&\left.\left.-\left[\frac{m_F^2-m_\pi^2}{m_{C^{s*}_D}^2-m_\pi^2}\right]\left[\frac{f_\pi}{f_K}\right]\right]f_s^*\right\},\end{aligned} \tag{B11}$$

$$\begin{aligned}M(F^+\to\pi^0K^+)\simeq{}&-\frac{i}{\sqrt{2}f_\pi}\sqrt{\tfrac{1}{2}}\langle\pi^+|H_w^{(-,0)}|D^+\rangle\\&\times\left\{\left[2\left[\frac{f_\pi}{f_K}\right]-1\right]e^{i\delta}-\left[\left[\frac{m_F^2-m_K^2}{m_F^2-m_{K^*}^2}\right]-\left[\frac{m_F^2-m_\pi^2}{m_F^2-m_{K^*}^2}\right]\left[\frac{f_\pi}{f_K}\right]-\left[\frac{m_F^2-m_\pi^2}{m_{D^*}^2-m_\pi^2}\right]\left[\frac{f_\pi}{f_K}\right]\right]\sqrt{\tfrac{1}{2}}h\right.\\&+\left[\left[\frac{m_F^2-m_K^2}{m_F^2-m_{\hat{K}^*}^2}\right]+\left[\frac{m_F^2-m_\pi^2}{m_F^2-m_{\hat{K}^*}^2}\right]\left[\frac{f_\pi}{f_K}\right]-2\left[\frac{m_F^2-m_K^2}{m_{\hat{F}_I^*}^2-m_K^2}\right]+\left[\frac{m_F^2-m_\pi^2}{m_{\hat{D}^{s*}}^2-m_\pi^2}\right]\left[\frac{f_\pi}{f_K}\right]\right]f_a^*\\&\left.+\left[\left[\frac{m_F^2-m_K^2}{m_F^2-m_{E^*_{\pi K}}^2}\right]+\left[\frac{m_F^2-m_\pi^2}{m_F^2-m_{E^*_{\pi K}}^2}\right]\left[\frac{f_\pi}{f_K}\right]-\left[\frac{m_F^2-m_\pi^2}{m_{C^{s*}_D}^2-m_\pi^2}\right]\left[\frac{f_\pi}{f_K}\right]\right]f_s^*\right\},\end{aligned} \tag{B12}$$

where the asymptotic $SU_f(4)$ parametrizations of the matrix elements of V_α, A_α, and H_w have been used. h is the asymptotic invariant matrix element of the axial charge A_α and is defined by $h=\sqrt{2}\langle K^{*+}|A_{\pi^+}|K^0\rangle$. The last terms of Eqs. (B5) and (B6) denote the glueball contributions through $f_0(1770)$.

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