Influence of the Landau-Pomeranchuk effect on lepton-pair production in a hadronic gas

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An estimate is made of the Landau-Pomeranchuk effect on the production of dileptons in a hadronic gas. For low-mass dilepton pairs this effect reduces the production rate for bremsstrahlung in a dense pion gas by an order of magnitude. For high invariant masses its influence is negligible. This behavior is of importance for the theoretical analysis of low-mass dilepton pairs produced in relativistic heavy ion collisions.

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I. INTRODUCTION

The prime objective of ultrarelativistic heavy ion collisions is to find physical observables sensitive to the phase transition from hadronic matter to a quark-gluon plasma. It has been suggested that dilepton pairs would be one of the best signals for this [1–13]. In order to describe them one needs to estimate quantitatively dilepton production in a hot hadronic medium and in a quark-gluon plasma.

Recently [8] it has been shown that low-invariant-mass dilepton pairs mainly come from Dalitz decays of mesons and from virtual bremsstrahlung. As the Dalitz background is of nonthermal origin, it will not be influenced by the medium. This is not the case for the bremsstrahlung type of emission. Here the medium effects could modify substantially the emission rate of dilepton pairs.

In this paper we investigate the influence of the Landau-Pomeranchuk effect on the rate of dilepton production in a dense hadronic gas. This effect was first investigated theoretically by Landau and Pomeranchuk [14]. A detailed investigation was done shortly afterwards by Migdal [15]. The importance of this effect has not been investigated previously (to the best of our knowledge) for dense hadronic matter or for a quark-gluon plasma. Our calculations show that the effect is important for invariant masses smaller than the ρ-meson mass when the temperature is above 150 MeV. It reduces the dilepton production rate from bremsstrahlung an order of magnitude. For masses heavier than 1 GeV it becomes negligible. The origin of this result is intuitively clear: radiation being of electromagnetic origin is relatively slow while the time between hadronic collisions becomes very short in a dense medium. It seems natural that the number of photons is proportional to the number of collisions, but if these happen too often the photons emitted at different points of the trajectory start to interfere with each other and the intensity of radiation is accordingly reduced.

For heavy masses the relevant time scale for the emission of a lepton pair is given by its inverse mass; therefore, heavy mass pairs are emitted in a very short time interval while light pairs are emitted over a much longer time. Thus a dense medium where many collisions occur will reduce the bremsstrahlung radiation of low-mass dilepton pairs but will not affect heavy pairs.

In this paper we estimate the importance of the Landau-Pomeranchuk effect for virtual photon emission. The scale of this effect is determined by the average time between two collisions. In our calculation we assume that the system is a thermalized static gas. We have also used a classical soft photon approximation. For low invariant masses and high temperatures the effect is substantial and will reduce the rate by a big factor.

The paper is organized as follows: in Sec. II we first investigate the average time between two collisions in a pion gas; we then continue to calculate the rate of lepton pair production via bremsstrahlung in a hot pion gas. In Sec. III we present numerical results and in Sec. IV we present final conclusions.

II. LANDAU-POMERANCHUK EFFECT

A. Time between two collisions

The most important parameter characterizing the Landau-Pomeranchuk effect is the average time between two collisions. We study here the inverse of this quantity and denote it by a. It can be calculated from

$$a \equiv n\sigma v \,, \tag{1}$$

where n is the density of the medium which depends on its composition and temperature T. For simplicity we will consider here only a pion gas. In Eq. (1) σ denotes the cross section of a test pion having velocity v interacting with the particles of the medium. Thus a is a function of T and of the momentum of the test particle. We do not include any dependence on chemical potentials as this is inessential for our considerations.

To calculate a we consider the expression

$$a(p,T)=g\int\frac{d^3q}{(2\pi)^3}e^{-E/T}\sigma_{\pi\pi}(s)v\ , \tag{2}$$

where the relative velocity v is given by

$$v\equiv|\mathbf{v}_{12}|=\sqrt{s(s-4m_\pi^2)}/2E_1E_2\ , \tag{3}$$

with $s\equiv(p+q)^2$ and g is the degeneracy factor (3 for the pion gas under consideration). For the cross section $\sigma_{\pi\pi}$ we consider different contributions [8]: (1) the low-energy part determined by chiral symmetry [16]:

$$\sigma_{\pi\pi}(s)=\frac{2}{3}\frac{1}{3F_\pi^4}\frac{s}{16\pi^2}\left[1-\frac{5m_\pi^2}{s}+\frac{7m_\pi^4}{s^2}\right]\ , \tag{4a}$$

with the pion decay constant $F_\pi=0.098$ GeV, (2) the ρ-pole contribution

$$\sigma_{\pi\pi}(s)=\frac{g_{\rho\pi\pi}^4}{48\pi s}\frac{(s-4m_\pi^2)^2}{(s-m_\rho^2)^2+\Gamma_\rho^2m_\rho^2}\ . \tag{4b}$$

The asymptotic behavior is taken to be constant. Its precise value is not very important for the range of temperatures we are considering. It was chosen to be $\sigma_{\pi\pi}(s)\simeq 15$ mb.

The resulting values for a^{-1} are shown in Fig. 1 as a function of momentum for different values of the temperature. As one can see the values for a^{-1} decrease rapidly for high temperatures. For a large range of values of incoming momenta a remains approximately constant. Below temperatures of about 150 MeV the values of a^{-1} are too large to be consistent with a hadronic gas interpretation, e.g., the mean free path for pions having $T\simeq 100$ MeV is of the order of 10 fm, this means that we cannot expect thermalization of pions anymore.

As we will show below the value of a is essential for the importance of the Landau-Pomeranchuk effect: if a is zero, the effect is absent.

In the subsequent calculations we will focus mainly on comparing results for $a=0$ and for a different from zero, i.e., respectively without and with the Landau-Pomeranchuk effect.

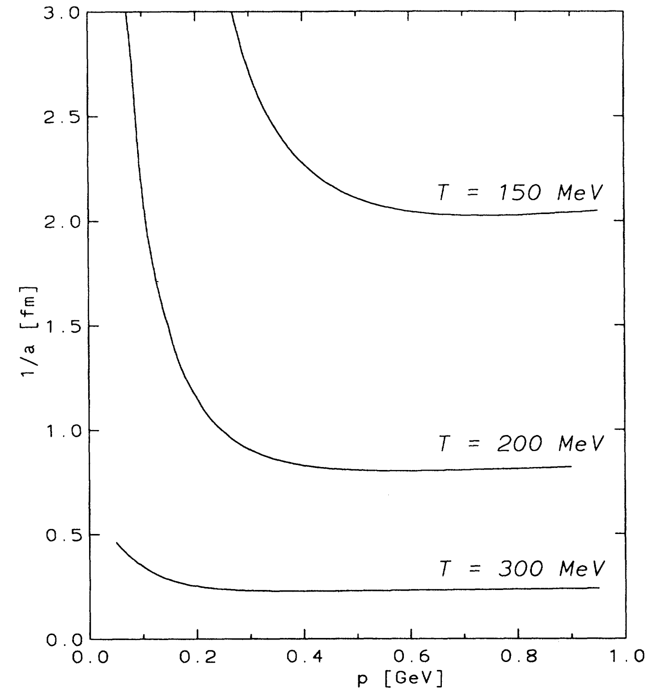

FIG. 1. Inverse value of a for different values of the temperature as a function of the momentum of the incoming particle.

B. Quantitative estimate

The starting point is the following textbook equation for the energy radiated per unit of momentum [17]:

$$\frac{dI}{d^3k}=\frac{\alpha}{(2\pi)^2}\left|\int_{-\infty}^{\infty}dte^{i[wt-\mathbf{k}\cdot\mathbf{r}(t)]}\mathbf{n}\times\mathbf{v}\right|^2\ , \tag{5}$$

where $\mathbf{r}(t)$ describes the trajectory of the charged particle, $\mathbf{v}(t)$ is its velocity, and

$$\mathbf{n}=\mathbf{k}/|\mathbf{k}|\ . \tag{6}$$

If only one collision occurs and if the velocity is a constant before (v_1) and after the collision (v_2) [see Fig. 2(a)] the energy radiated is given by

$$\begin{aligned}\frac{dI}{d^3k}&=\frac{\alpha}{(2\pi)^2}\left|\int_{-\infty}^{t_1}dte^{i(w-\mathbf{k}\cdot\mathbf{v}_1)t}\mathbf{n}\times\mathbf{v}_1+\int_{t_1}^{\infty}dte^{i(w-\mathbf{k}\cdot\mathbf{v}_2)t}\mathbf{n}\times\mathbf{v}_2\right|^2\\&=\frac{\alpha}{(2\pi)^2}\left|\frac{\mathbf{n}\times\mathbf{v}_1}{w-\mathbf{v}_1\cdot\mathbf{k}}-\frac{\mathbf{n}\times\mathbf{v}_2}{w-\mathbf{v}_2\cdot\mathbf{k}}\right|^2\end{aligned} \tag{7}$$

where one recognizes the two propagators familiar from quantum electrodynamics: $(p+k)^2-m^2=2p_0k_0-2\mathbf{p}\cdot\mathbf{k}=2p_0[w-\mathbf{v}\cdot\mathbf{k}]$.

If, however, a series of collisions occurs [see Fig. 2(b)], one has to change the above to a sum over all pieces of the trajectory

$$\frac{dI}{d^3k}=\frac{\alpha}{(2\pi)^2}\left|\int_{-\infty}^{t_1}dte^{i[wt-\mathbf{k}\cdot\mathbf{r}(t)]}\mathbf{n}\times\mathbf{v}_1+\int_{t_1}^{t_2}dte^{i[wt-\mathbf{k}\cdot\mathbf{r}(t)]}\mathbf{n}\times\mathbf{v}_2+\int_{t_2}^{t_3}dte^{i[wt-\mathbf{k}\cdot\mathbf{r}(t)]}\mathbf{n}\times\mathbf{v}_3 + \cdots + \int_{t_{n-1}}^{t_n}dte^{i[wt-\mathbf{k}\cdot\mathbf{r}(t)]}\mathbf{n}\times\mathbf{v}_n+\cdots\right|^2 . \tag{8}$$

Assuming the velocity is constant between two collisions, this leads to

$$\frac{dI}{d^3k}=\frac{\alpha}{(2\pi)^2}\left|\sum_{j=1}^{N}\frac{\mathbf{n}\times\mathbf{v}_j}{w-\mathbf{k}\cdot\mathbf{v}_j}e^{i(wt_{j-1}-\mathbf{k}\cdot\mathbf{r}_{j-1})}\left[1-e^{-i(w-\mathbf{k}\cdot\mathbf{v}_j)(t_j-t_{j-1})}\right]\right|^2 , \tag{9}$$

where N is the total number of collisions. The square leads to

$$\frac{dI}{d^3k}=\frac{\alpha}{(2\pi)^2}\sum_{j=1}^{N}\frac{v_j^2-(\mathbf{n}\cdot\mathbf{v}_j)^2}{(w-\mathbf{k}\cdot\mathbf{v}_j)^2}\left[2-e^{i\xi_j(w-\mathbf{k}\cdot\mathbf{v}_j)}-e^{-i\xi_j(w-\mathbf{k}\cdot\mathbf{v}_j)}\right] +2\,\mathrm{Re}\sum_{j>l}^{N}\frac{\mathbf{v}_j\cdot\mathbf{v}_l-(\mathbf{n}\cdot\mathbf{v}_j)(\mathbf{n}\cdot\mathbf{v}_l)}{(w-\mathbf{k}\cdot\mathbf{v}_j)(w-\mathbf{k}\cdot\mathbf{v}_l)}\exp\left[i\sum_{i=l}^{j-1}\xi_i(w-\mathbf{k}\cdot\mathbf{v}_i)\right](1-e^{-i(w-\mathbf{k}\cdot\mathbf{v}_j)\xi_j})(1-e^{i(w-\mathbf{k}\cdot\mathbf{v}_l)\xi_l}) , \tag{10}$$

where

$$\xi_j=t_j-t_{j-1} . \tag{11}$$

As the hadronic gas is taken to be in thermal equilibrium, many possible velocities can result in between two collisions. For this reason the nondiagonal terms in the above equation will give zero contribution after one averages over all velocities, provided of course that no correlations exist between the velocity before and after the collision. Only the diagonal terms remain in this case.

The time between two successive collisions is given by ξ. Taking an average over the time between two collisions can be obtained using the following distribution

$$\frac{dW}{d\xi}=ae^{-\xi a} , \tag{12}$$

where a is the average time between two collisions and has been defined in Eq. (1).

The average energy radiated away is given by

$$\frac{dI}{d^3k}=N\frac{\alpha}{(2\pi)^2}a\int_0^\infty d\xi e^{-\xi a}\frac{v^2-(\mathbf{n}\cdot\mathbf{v})^2}{(w-\mathbf{k}\cdot\mathbf{v})^2}\times[2-e^{i\xi(w-\mathbf{k}\cdot\mathbf{v})}-e^{-i\xi(w-\mathbf{k}\cdot\mathbf{v})}] . \tag{13}$$

The averaging over the time between two collisions, ξ, is done using

$$a\int_0^\infty d\xi e^{-\xi a}[2-2\cos\xi b]=2\frac{b^2}{a^2+b^2} . \tag{14}$$

To relate the above expressions to cross sections, we note the following

$$\frac{dI}{d^3k}=w\frac{dN^{\gamma*}}{d^3k} =Nw\frac{1}{\sigma_{\pi\pi}}\frac{d\sigma^{\gamma*}}{d^3k} . \tag{15}$$

We are then left with

$$\left\langle\frac{d\sigma^{\gamma*}}{d^3k}\right\rangle=\sigma_{\pi\pi}\frac{1}{w}\frac{2\alpha}{(2\pi)^2}\left\langle v^2\frac{(1-\cos^2\theta)}{a^2+(w-kv\cos\theta)^2}\right\rangle , \tag{16}$$

the averaging is understood to be over the time between successive collisions and over all velocities after the collisions. In the remainder of this paper we will tacitly assume that such an averaging has been done and we will not write explicitly the brackets. We thus have

$$\sigma^{\gamma*}\equiv\int d^3k\left\langle\frac{d\sigma^{\gamma*}}{d^3k}\right\rangle . \tag{17}$$

For the angular integral we use

$$\int_{-1}^{+1}dY\frac{1-Y^2}{a^2+(w-kvY)^2}=\frac{1}{k^3v^3}\phi , \tag{18}$$

where the function ϕ is given by

$$\phi=\frac{a^2+k^2v^2-w^2}{a}\left[\arctan\frac{w+kv}{a}-\arctan\frac{w-kv}{a}\right] +w\ln\frac{a^2+(w+kv)^2}{a^2+(w-kv)^2}-2kv . \tag{19}$$

Equation (17) becomes

$$\sigma^{\gamma*}=\sigma_{\pi\pi}\frac{\alpha}{\pi}\frac{1}{v}\int_M^\Delta dw\frac{\phi}{k^2} . \tag{20}$$

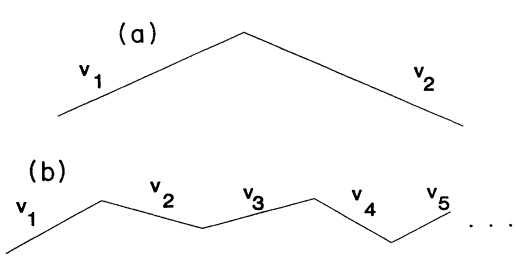

FIG. 2. A schematic representation of (a) the single collision process, and (b) the multiple collisions leading to the Landau-Pomeranchuk effect.

The upper limit on the integral over w is given by

$$\Delta = \frac{1}{s}(EA + P\sqrt{A^2 - M^2 s}) , \tag{21}$$

where

$$A = \frac{1}{2}(s - 4m_\pi^2 + M^2) , \tag{22}$$

and $E \equiv E_1 + E_2$, $P = |\mathbf{p}_1 + \mathbf{p}_2|$. Here Δ is the largest possible energy that can be radiated away in a collision; it is obtained from the kinematic constraints imposed by energy-momentum conservation.

C. Dilepton rate

We can now relate the expression derived previously to dilepton pair production using the standard expression [4]

$$\frac{d\sigma^{+-}}{dM^2} = \frac{\alpha}{3\pi}\frac{1}{M^2}\left[1 - \frac{4m_l^2}{M^2}\right]^{1/2}\left[1 + \frac{2m_l^2}{M^2}\right]\sigma^{\gamma *} . \tag{23}$$

For the order of magnitude estimates we ignore the various threshold factors in the following; this is certainly justified for dielectron production. Combining Eqs. (20) and (23) we obtain

$$\frac{d\sigma^{+-}}{dM^2} = \frac{\alpha^2}{3\pi^2}\frac{1}{M^2}\sigma_{\pi\pi}\frac{1}{v}\int_0^{\sqrt{\Delta^2 - M^2}}\frac{dk}{kw}\phi . \tag{24}$$

The expression for the rate is given by kinetic theory as being (we assume Boltzmann approximation) [3]

$$\frac{dN^{+-}}{dM^2 d^4x} = \int d^3p_1 \int d^3p_2 f(\mathbf{p}_1) f(\mathbf{p}_2)|\mathbf{v}_{12}|\frac{d\sigma^{+-}}{dM^2} . \tag{25}$$

Using Eq. (2) for the relative velocity this becomes

$$\frac{dN^{+-}}{dM^2 d^4x} = \frac{d}{(2\pi)^4}\frac{\alpha^2}{3\pi^2}\frac{1}{M^2}\int_{m_\pi}^{\infty} p_1 dE_1 \int_{m_\pi}^{\infty} p_2 dE_2 e^{-(E_1+E_2)/T}\int_{-1}^{+1} d(\cos\theta)\sqrt{s(s-4m_\pi^2)}\frac{1}{v_1}\sigma_{\pi\pi}(s)\int_M^{\Delta}\frac{dw}{k^2}\phi , \tag{26}$$

where d is a corresponding degeneracy factor.

The angular integration can be transformed into an integration over s using $s = 2(m_\pi^2 + E_1E_2 - p_1p_2\cos\theta)$. We obtain from Eq. (26)

$$\frac{dN^{+-}}{dM^2 d^4x} = \frac{d}{(2\pi)^4}\frac{\alpha^2}{3\pi^2}\frac{1}{M^2}\frac{1}{2}\int_{m_\pi}^{\infty} dE_1 \int_{m_\pi}^{\infty} dE_2 e^{-(E_1+E_2)/T}\int_{(2m_\pi+M)^2}^{\infty} ds\sqrt{s(s-4m_\pi^2)}\frac{1}{v_1}\sigma_{\pi\pi}(s)\int_M^{\Delta}\frac{dw}{k^2}\phi . \tag{27}$$

One of the integrations over energies can be done analytically; for this purpose one interchanges the w and E_2 integrals. The final result for dilepton production including the Landau-Pomeranchuk effect then is

$$\frac{dN^{+-}}{dM^2 d^4x} = \frac{d}{(2\pi)^4}\frac{\alpha^2}{3\pi^2}\frac{T}{2M^2}\int_{(2m_\pi+M)^2}^{\infty} ds\sqrt{s(s-4m_\pi^2)}\sigma_{\pi\pi}(s)\int_0^{\infty} dp_1 \int_0^{k_+}\frac{dk}{kw}\phi$$
$$\times\begin{cases}(e^{-E_-/T} - e^{-E_+/T}) & \text{for } 0 < k < k_- , \\ (e^{-E_*/T} - e^{-E_+/T}) & \text{for } k_- < k < k_+ ,\end{cases} \tag{28}$$

where

$$E_\pm = \frac{1}{2m_\pi^2}\left[E_1 s \pm p_1\sqrt{s(s-4m_\pi^2)}\right] , \tag{29}$$

$$k_\pm = \frac{1}{2m_\pi^2 s}\left\{\left[E_1 s \pm p_1\sqrt{s(s-4m_\pi^2)}\right]\sqrt{A^2 - M^2 s} + A\left|E_1\sqrt{s(s-4m_\pi^2)} \pm p_1 s\right|\right\} , \tag{30}$$

and

$$E* = \frac{1}{M^2}(wA - k\sqrt{A^2 - M^2 s}) . \tag{31}$$

The value of d was chosen to be 6 corresponding to the number of possible processes contributing to bremsstrahlung. In the following section we present numerical evaluations of the above expression for different values of the parameters.

III. NUMERICAL RESULTS

We will now discuss, quantitatively, the influence of the Landau-Pomeranchuk effect on the bremsstrahlung of dilepton pairs in a hot hadronic gas. We shall assume for simplicity that the system is composed of only pions. The temperature and other thermodynamic quantities are taken as time independent. In Figs. 3 and 4 we show the dilepton production rate as a function of the invariant mass of the pair at temperatures of 200 and 300 MeV, respectively. The results without the Landau-Pomeranchuk effect are indicated by the dashed line ($a = 0$). The solid line takes into account the full effect. It is seen from the figures that, in the low-mass region, the Landau-Pomeranchuk effect decreases the bremsstrahlung rate by up to an order of magnitude for the temperature of 300 MeV. It is also seen, comparing Figs. 3 and 4, that decreasing T will also decrease the effect.

The change of slope observed in Figs. 3 and 4 at $M \simeq 0.4$ GeV is related to the peak structure of the pion-

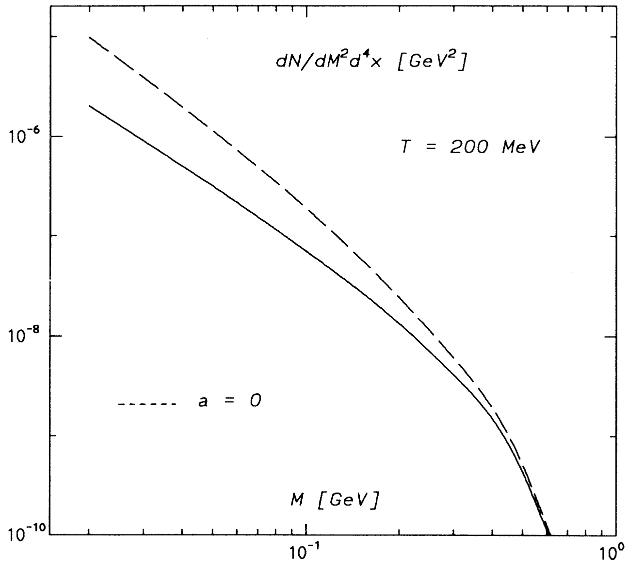

FIG. 3. Rate as a function of the invariant mass of the dilepton pair at a fixed temperature ($T = 200$ MeV) with and without the Landau-Pomeranchuk effect.

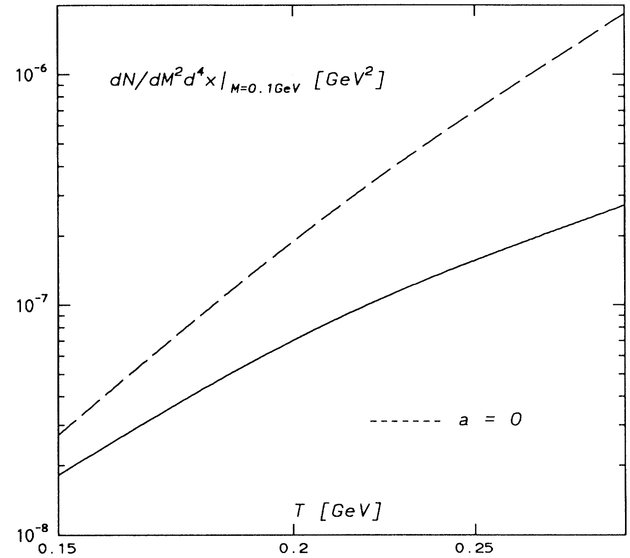

FIG. 5. Rate as a function of temperature for a fixed value of the invariant mass ($M = 100$ MeV).

pion elastic cross section coming from the ρ meson.

In Fig. 5 we show the rate as a function of temperature for a fixed value of the invariant mass ($M = 100$ MeV). The value of M was chosen so as to illustrate that the Landau-Pomeranchuk effect is of importance in the low-invariant-mass region. A very clear decrease of the rate with increasing temperature can be seen.

From the above properties one can conclude that for low-mass dilepton production in relativistic heavy-ion collisions one cannot neglect the Landau-Pomeranchuk effect as it leads to an important reduction of the bremsstrahlung contribution.

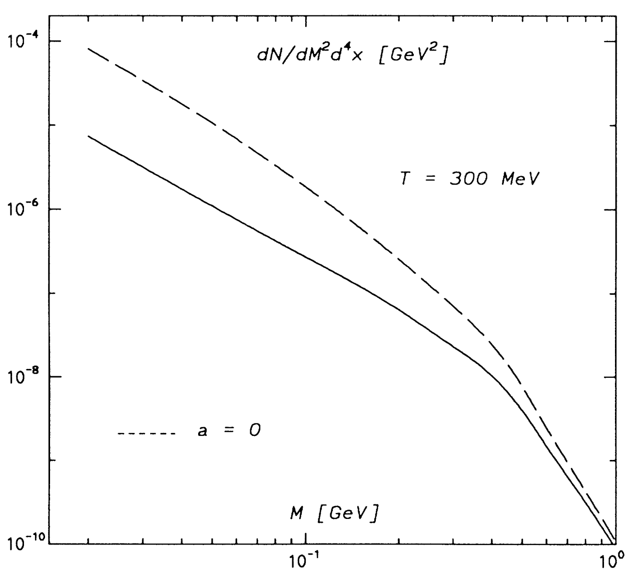

FIG. 4. Rate as a function of the invariant mass of the dilepton pair at a fixed temperature ($T = 300$ MeV) with and without the Landau-Pomeranchuk effect.

IV. CONCLUSIONS

In this paper we have estimated the Landau-Pomeranchuk effect on the dilepton production rate from virtual bremsstrahlung in hot hadronic matter. Our conclusion is that the effect is mainly important for low values of the invariant dilepton pair mass and for large temperatures. In a realistic situation, corresponding to a temperature of the pion gas of about 200 MeV, the effect reduces the dilepton production rate below the region dominated by the ρ meson by an order of magnitude. It is not relevant for larger values of the invariant dilepton pair mass. It becomes more pronounced if the temperature is increased.

We have shown that the Landau-Pomeranchuk effect is of importance for the analysis of the bremsstrahlung production of low-invariant-mass virtual photons in a dense hadronic system.

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