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London relation and fluxoid quantization for monopole currents in U(1) lattice gauge theory

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We explore the mechanism of quark confinement in four-dimensional U(1) lattice gauge theory by measuring the color-magnetic current distribution from Dirac magnetic monopoles in the presence of a static quark-antiquark pair. Our results give the first direct evidence that the quarks induce a solenoidal current distribution that screens the color-electric flux of the quarks in an electric analogue of the Meissner effect in a superconductor. We show that the vacuum state of U(1) lattice gauge theory obeys both a dual version of the London equation and an electric fluxoid quantization condition.

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Since free quarks have never been isolated, there must be a mechanism for permanently confining them within hadrons. This mechanism is expected to arise as a natural property of the vacuum state of the theory of strong interactions. It was suggested many years ago [1] that confinement would occur if the vacuum reacted to expel a color-electric field in a manner similar to the response of a superconductor to an external magnetic field, the Meissner effect. For example, if two magnetic monopoles of opposite magnetic charge are introduced into the interior of a superconductor, the Cooper pairs give rise to persistent currents to generate a countermagnetic field to expel the magnetic flux. As a result, the magnetic flux lines from one monopole to the other are confined to a narrow Abrikosov flux tube which is surrounded by a solenoidal distribution of persistent circulating currents. The energy of such a configuration is proportional to the separation of the two monopoles, thus permanently confining them. By analogy, therefore, if the vacuum naturally expels a color-electric flux, the field lines from a static quark-antiquark ($q\bar{q}$) pair would not spread out in a dipole field pattern but would instead form a narrow flux tube, leading to a quark potential proportional to their separation and confinement.

For this mechanism to work the vacuum must contain objects that react to a color-electric field in a fashion similar to the reaction of the Cooper pairs in a superconductor to an ordinary magnetic field. One possibility is to mimic [1] the Ginzburg-Landau theory of superconductivity by adding to the gauge theory an elementary charged Higgs field to act as the superconducting condensate, but there is no experimental evidence for any elementary scalar particles in particle physics. The dual superconductor mechanism [2] is an alternative that does not require the *ad hoc* introduction of a Higgs field but instead uses dynamically generated topological excitations to provide the screening supercurrents. For example, U(1) lattice gauge theory contains Dirac magnetic monopoles in addition to photons [3, 4]. The dual superconductor hypothesis postulates that these monopoles provide the circulating color-magnetic currents that constrain the color-electric flux lines into narrow flux tubes. 't Hooft has shown [5] that objects similar to the Dirac monopoles in U(1) gauge theory can also be found in non-Abelian SU(N) models.

We chose to study U(1) lattice gauge theory as a model for confinement for several reasons. U(1) lattice gauge theory in four dimensions has both a confined phase at large charge and a weak coupling deconfined phase corresponding to continuum electrodynamics with a Coulomb interaction between static quarks. Therefore confinement or its absence can be studied using U(1) lattice gauge theory as a prototype, before tackling the more complicated non-Abelian theories that actually describe quarks. Furthermore, there is evidence [5–7] that confinement in non-Abelian SU(N) gauge theory arises from monopoles associated with a residual $\mathrm{U}(1)^{N-1}$ gauge freedom that remains after gauge fixing.

Much evidence for the dual superconductor hypothesis has accumulated from studies [3, 4, 8–11] of U(1) lattice gauge theory. Polyakov [3] and Banks, Myerson, and Kogut [4] showed that U(1) lattice gauge theory in the presence of a quark-antiquark pair could be approximately transformed into a model describing magnetic current loops (the monopoles) interacting with the electric current generated by the $q\bar{q}$ pair. DeGrand and Toussaint [8] demonstrated via a numerical simulation that the vacuum of U(1) lattice gauge theory was populated by monopole currents, copious in the confined phase and rare in the deconfined phase. Similar behavior has also been seen in non-Abelian models with Dirac monopoles [6, 7, 12] or other topological excitations [13], although other studies [14] find no evidence for the dual superconductor hypothesis.

So far, studies of confinement have examined "bulk" properties such as the monopole density [8, 12, 15], the monopole susceptibility [9, 12], and the behavior of the static quark potential [6, 10]. However, the case for the dual superconductor hypothesis is incomplete without an explicit demonstration that a static $q\bar{q}$ pair actually induces the appropriate persistent current distribution. It is not the presence of the condensate, but the fact that it is magnetically charged and can generate the appropriate current distribution, that leads to flux expulsion. For example, ^{3}He exhibits a superfluid state with anomalous magnetic properties but no Meissner effect because the condensate is uncharged. In this Brief Report we present the first direct evidence for this behavior. We further show that there are exact U(1) lattice gauge theory analogues of two key relations [16] associated with the Meissner effect in a superconductor: the London equation and the fluxoid quantization condition.

Our simulations are done on a Euclidean spacetime grid of volume $L^3 \times L_t$, where L is the spatial size and L_t the temporal size of the lattice in units of the lattice spacing a. The U(1) gauge degrees of freedom are complex numbers of unit magnitude residing on the links of the lattice and are written $U_\mu(\mathbf{r}) = \exp[i\theta_\mu(\mathbf{r})]$, where $\mathbf{r}$ denotes a point on the lattice and μ the direction of the link from that point. The links form a directed lattice so that $U_{-\mu}(\mathbf{r}+\mu) = U_\mu^\dagger(\mathbf{r})$. We use a standard Wilson action S_0 supplemented with a Wilson loop W to represent a static $q\bar{q}$ pair with charges ± 1:

$$\begin{aligned} S &= S_0 - iW \\ &= \beta \sum_{\mathbf{r},\mu>\nu} [1 - \cos\theta_{\mu\nu}(\mathbf{r})] - i \sum_{\mathbf{r}\mu} J_\mu(\mathbf{r})\theta_\mu(\mathbf{r}). \end{aligned} \quad (1)$$

Here $\beta = \hbar c/e^2$ is a dimensionless measure of the strength of the charge and $\exp[i\theta_{\mu\nu}(\mathbf{r})] \equiv U_\mu(\mathbf{r})U_\nu(\mathbf{r}+\mu)U_\mu^\dagger(\mathbf{r}+\nu)U_\nu^\dagger(\mathbf{r})$ is an oriented product of gauge variables around an elementary plaquette of the lattice. The current $J_\mu(\mathbf{r})$ is ± 1 along the world line of the $q\bar{q}$ pair and 0 otherwise. In the naive continuum limit ($a \to 0$) S reduces to the action for a pure photon field in the presence of a current loop. Physical observables are given by expectation values

$$\langle A \rangle \equiv \frac{1}{Z} \mathrm{Tr}\left(e^{-S_0+iW} A\right) \quad (2)$$

where Tr denotes an integral over all angles $\theta_\mu(\mathbf{r})$ and $Z = \mathrm{Tr}\exp(-S_0 + iW)$ is the partition function.

The two observables we study here are the electric flux through a plaquette, which in lattice variables is $\mathcal{E}_\mu(\mathbf{r}) = \mathrm{Im}\exp[i\theta_{\mu 4}(\mathbf{r})]$, and the curl of the monopole current density $\nabla \times J_M$. The monopole current J_M is found by a prescription devised by DeGrand and Toussaint [8], which employs a lattice version of Gauss' law to locate the Dirac string attached to the monopole. The net flux into each plaquette is given by $\theta_{\mu\nu}(\mathbf{r}) \bmod 2\pi$. If the sum of the fluxes into the faces of a three-volume at fixed time is nonzero, a monopole is located in the box. The net flux into the box at fixed time thus yields the monopole "charge" density, or the time component of the monopole four-current $\mathbf{J}_M$, measured in units of the monopole charge $g_M = 2\pi\hbar c/e$. The spatial components are found in a similar manner. The monopole currents form closed loops due to the conservation of magnetic charge.

Our simulations are performed on a $9^3 \times 10$ lattice using skew-periodic boundary conditions. Less extensive work on a $7^3 \times 8$ lattice yields similar results except for the expected increase in statistical fluctuations arising from the smaller lattice size. We used a 3×3 Wilson loop oriented in the zt plane and measured the electric flux and the monopole current in the transverse (xy) plane midway along the axis connecting the $q\bar{q}$ pair. A standard Metropolis algorithm [17] alternated with overrelaxation [18] is used to generate configurations distributed according to $\exp(-S_0)$. In the confined phase, we thermalize for 10 000 sweeps and sample the data every 10 sweeps for a total of 7000 measurements, which are then binned in groups of 5. In the deconfined phase only half as many measurements are taken since the fluctuations are much smaller. Because of the geometrical symmetry of the measurements only the z components of $\langle \boldsymbol{\mathcal{E}} \rangle$ and $\langle \nabla \times \mathbf{J}_M \rangle$ are nonzero. If the Wilson loop is removed, even the z components average to zero, so the response we observe is clearly induced by the presence of the $q\bar{q}$ pair.

Figure 1(a) shows the electric flux distribution for $\beta = 1.1$ where the vacuum is in the deconfined phase. The broad flux distribution seen is identical to the dipole field produced by placing two classical charges at the quark positions, except that the classical value of the flux on the $q\bar{q}$ axis is a factor of 2 smaller. We measure the total electric flux from one quark to the other, including

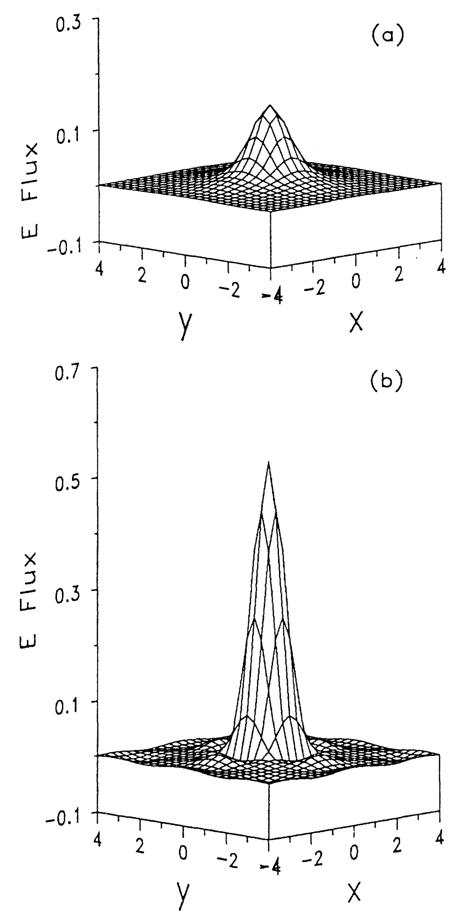

FIG. 1. Surface plot of the electric flux through the xy plane midway between the $q\bar{q}$ pair when the system is in (a) the deconfined phase ($\beta = 1.1$) and (b) the confined phase ($\beta = 0.95$). The line joining the pair is located at (0,0).

not only the flux through the plane between the charges (0.8504 ± 0.0045) but also the flux (0.0951 ± 0.0028) that flows through the lattice boundary because of the periodic boundary conditions. This yields a total flux of 0.9453 ± 0.0053, close to the theoretical value $\Phi_e = 1/\sqrt{\beta} = 0.9534$.

Figure 1(b) shows the electric flux in the confined phase ($\beta = 0.95$). In this case the flux is confined almost entirely within one lattice spacing of the axis and essentially no flux passes the long way around through the lattice boundary. The net flux is again equal to $1/\sqrt{\beta}$ within statistical error. This behavior is exactly what one would expect from the superconducting analogy, where the flux has been "squeezed" into a narrow tube. The data in Fig. 1 are consistent with flux profiles found in other studies of U(1) [19], SU(2) [20], and SU(3) [19] lattice gauge theory.

The electric field $\mathbf{E}$ produced by the monopole currents arises from the dual version of Ampere's law:

$$-c\nabla \times \mathbf{E} = \mathbf{J}_M. \tag{3}$$

As we will show below, we find that the Meissner effect appears because a dual version of the London relation holds between the field and the monopole current of the form

$$\mathbf{E} = \frac{\lambda^2}{c}\nabla \times \mathbf{J}_M. \tag{4}$$

Equations (3) and (4) result in the electric flux being confined to a region of size λ, which is the "London penetration depth" for the electric field.

We show in Fig. 2(a) $\langle \mathcal{E} \rangle$ and in Fig. 2(b) $-\langle \nabla \times \mathbf{J}_M \rangle$ in the confined phase as a function of the distance from the $q\bar{q}$ axis. The data show that the spatial variation of the flux and the curl of the current are very similar, except for the point on the axis which will be discussed below. Figure 2(c) shows the best fit found for $\langle \mathcal{E} \rangle - (\lambda^2/c)\langle \nabla \times \mathbf{J}_M \rangle$. We find a value of $\lambda/a = 0.482 \pm 0.008$, which is consistent with the range of penetration of the electric flux in Fig. 2(a) and the thickness of the current sheet in Fig. 2(b). The dashed curve in Fig. 2(a) is the result of using the continuum Eqs. (3) and (4) to yield a flux distribution of the form $\mathcal{E}(r) = (\Phi_e/2\pi\lambda^2)K_0(r/\lambda)$. Here $\Phi_e = e/\sqrt{\hbar c} = 1/\sqrt{\beta}$ is the quantum of electric flux. The agreement between the continuum version and the flux distribution from the lattice simulations is very good.

We also expect that, as in a superconductor, the transition to the deconfined phase will be signaled by a divergence of the London penetration depth. We have therefore measured λ further from the deconfinement transition at $\beta = 0.90$, and find a smaller penetration depth of $\lambda/a = 0.32 \pm 0.02$. In the deconfined phase we find an almost insignificant value of $\langle \nabla \times \mathbf{J}_M \rangle$ and fitted values of λ were larger than our lattice size.

The anomalous behavior of the point on the $q\bar{q}$ axis can be understood by recalling that a superconductor penetrated by an Abrikosov flux tube becomes multiply connected and the London relation is replaced by the more general quantization relation for the fluxoid. Since our U(1) vacuum is pierced by an electric flux tube, we expect a dual version of the fluxoid quantization relation to hold:

$$\int \mathbf{E} \cdot d\mathbf{S} - \frac{\lambda^2}{c} \oint \mathbf{J}_M \cdot d\boldsymbol{\ell} = n\Phi_e, \tag{5}$$

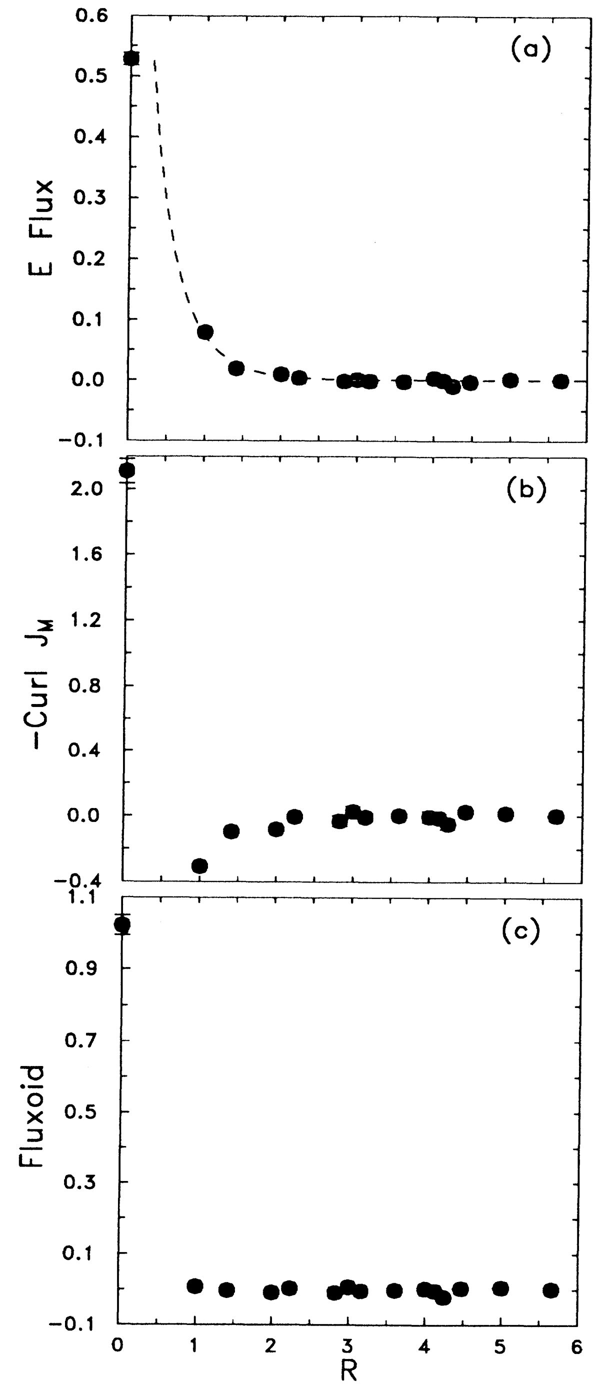

FIG. 2. Behavior of (a) the electric flux, (b) the curl of the monopole current, and (c) the fluxoid in the confined phase ($\beta = 0.95$) as a function of the perpendicular distance R from the $q\bar{q}$ axis. The dashed line in (a) shows the flux expected using the continuum relations (3) and (4).

where n is an integer. In fact, the data in Fig. 2(c) represent a lattice version of a δ function whose strength (1.016 ± 0.014) is very close to $\Phi_e = 1/\sqrt{\beta} = 1.026$. Thus, if the surface integral in Eq. (5) includes the axis of the $q\bar{q}$ pair, we obtain $n = 1$, while if the axis is excluded from the integral we obtain $n = 0$ and Eq. (4) holds. We examined the response to doubly charged quarks to find $n = 2$ electric flux quantization but did not get data of sufficient quality to draw any conclusions.

Equations (4) and (5) show that, except for the interchange of electric and magnetic quantities under duality, the confined phase of U(1) lattice gauge theory behaves exactly like a superconductor in an external magnetic field. It is perhaps surprising that a nonlinear, strongly interacting, model such as U(1) lattice gauge theory could be described by such a simple model as the linear London equations, but our results indicate that the operators $\boldsymbol{\mathcal{E}}$ and $\nabla \times \mathbf{J}_M$, when measured in the presence of a source of external flux such as a Wilson loop, give an unambiguous indication of the confinement of electric flux by a monopole current distribution. The simulation yields a large signal even with modest amounts of computer time on a Sun workstation. Although the Meissner effect itself requires only that Eq. (4) hold, our data also support the more restrictive fluxoid quantization relation (5). This additional relation reflects the single-valued nature of the order parameter in a Ginzburg-Landau description of the monopole condensate. Because the monopoles appear pointlike in our simulations, lattice gauge theory looks like an extreme type-II superconductor and it is tempting to argue that the phase transition in lattice gauge theory is a Bose condensation of magnetically charged particles, similar to the Bose condensation of charged local pairs [21]. Analogous studies of SU(2) lattice gauge theory along these lines are currently in progress.

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