Becchi-Rouet-Stora-Tyutin symmetry in chiral electrodynamics

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We show how a Becchi-Rouet-Stora-Tyutin constant charge can be used to determine the asymptotic states of an anomalous theory when the quantization is defined through a functional integral over all the configurations. We also prove the equivalence of possible alternatives, which correspond to the decoupling of longitudinal modes, to cure nonrenormalization problems appearing in the Abelian (3+1)-dimensional case.

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I. INTRODUCTION

Becchi-Rouet-Stora-Tyutin (BRST) symmetry is a useful tool to determine the physical states in gauge theories [1]. In chiral electrodynamics one might think that the BRST treatment becomes useless because of the appearance of the anomaly. However, since the quantum nonconservation of the chiral fermionic current and the Lagrangian gauge invariance may be rendered compatible giving up the Dirac equation as an operatorial identity [2], it is conversely possible to have a BRST-noninvariant Lagrangian accompanied by a conserved BRST current [3]. Exploiting the validity of the Euler-Lagrange equations of motion as averages over the corresponding fields in the functional integral, the usefulness of the BRST approach for anomalous theories appears when the Faddeev-Popov gauge-fixing procedure is introduced and the consistent anomaly manifests itself through the one-cocycle emerging from the regularized transformation of the fermionic measure. In this way there arises a Lagrangian allowing the definition of canonical variables apparently without constraints, even though the chiral current is not conserved. The requirement of invariance of the physical states under BRST transformation leads in general to the restriction given by second-class constraints [4]. The choice of an additional condition determines asymptotic states which may correspond to massive gauge bosons also when no classical mass was initially present. However, in 3+1 dimensions the anomaly introduces a nonrenormalizable interaction if it is used perturbatively [5]. Modifications of the quantization procedure must be introduced to recover the renormalizability of the theory in the Abelian case. We prove that different approaches are equivalent to a restriction of the functional integration over configurations which satisfy the Lorentz condition, losing, however, the possibility of mass generation through the anomaly.

In Sec. II we show that the validity of bosonic equations as quantum identities allows the nonconservation of the fermionic current in a chosen gauge. Section III indicates, using the BRST symmetry with an additional condition, the cases in which unitarity may be expected. After a short survey of the renormalization difficulties in 3+1 dimensions with the usual quantization procedure in Sec. IV, alternative rules consistent with unitarity and renormalizability are included in Sec. V, where we prove their equivalence.

II. USE OF BOSONIC EQUATIONS AS QUANTUM IDENTITIES

If we define the quantum theory summing over all the configurations,

$$Z=\int \mathcal{D}A_\mu \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, \exp\left[i\int d^Dx\mathcal{L}\right] \qquad (1)$$

with the classical Lagrangian

$$\mathcal{L}=-\tfrac{1}{4}F^2+\frac{M^2}{2}A^2+i\bar{\psi}\partial\!\!\!/\psi+eA\cdot J_L \ , \qquad (2)$$

where only the left fermionic current is coupled to the massive vector field, no anomaly appears as average over electromagnetic configurations in the massless limit $M^2=0$.

In fact using the following identity for any particular field φ,

$$\int \mathcal{D}\varphi \frac{\delta}{\delta\varphi}e^{iS}=0 \ , \qquad (3)$$

one obtains the validity of the corresponding classical equation of motion $E_\varphi=0$ averaged over configurations of φ with the other fields kept fixed as background:

$$\langle E_\varphi\rangle=\int \mathcal{D}\varphi\frac{\delta S}{\delta\varphi}e^{iS}=0 \ . \qquad (4)$$

For the Lagrangian Eq. (2), the application to A of the identity Eq. (3) leads to

$$\langle M^2\partial\cdot A+e\partial\cdot J_L\rangle=0 \ . \tag{5}$$

In the limit $M^2\to 0$ Eq. (5) gives the conservation of the chiral fermionic current which is consistent with the cancellation of the anomaly when averaged over all electromagnetic configurations.

Adopting the usual view that the average over the configurations of a particular field of the corresponding equation of motion is equivalent to its operatorial validity, Eq. (5) agrees with the anomalous Poisson brackets treatment of chiral four-dimensional QED without fixing the gauge [6] where because of cancellations the normal Gauss and Ampere laws are valid as quantum identities. As a consequence, the vanishing of the anomaly is one of the four second-class constraints which do not allow the increase of the number of degrees of freedom as also occurs with an analogous analysis [7] of the chiral Schwinger model.

If we instead introduce in the partition function Eq. (1) the Faddeev-Popov identity as it is necessary for perturbative purposes in the massless case,

$$\Delta\int \mathcal{D}g\,\delta(\partial\cdot A^g-b)=1 \ , \tag{6}$$

where A^g is the gauge transformed field with group element $g=\exp(i\theta)$, b is a generic function, and $\Delta=\det\Box$, performing a subsequent average over b with a Gaussian weight, a new Lagrangian without apparent constraints is obtained. With a notation suitable for $3+1$ dimensions, it is

$$\mathcal{L}_B=-\tfrac{1}{4}F^2+\frac{M^2}{2}(A-\partial\theta)^2+i\bar{\psi}\partial\!\!\!/\psi+eA\cdot J_L+\partial\bar{c}\cdot\partial c +\frac{b^2}{2}+A\cdot\partial b+m^2K(A,\partial\theta)+\lambda\theta P(\partial_\nu A_\mu) \tag{7}$$

if only the left fermionic field is gauge transformed. Now in addition to the functional integrations indicated in Eq. (1) one must integrate over b, which has A_0 as conjugate canonical momentum, the Grassmannian ghosts c and $\bar{c}$, decoupled from the rest in the present Abelian case, and the group field θ. The last one is coupled to A due to the noninvariance of the fermionic measure. This produces as a consistent anomaly not only the Chern-Pontryagin density P, but also a possible kinetic term K depending on the chosen regularization that may determine the dimensional parameter m^2.

It is now clear that the equations of motion for A and θ, which are valid when averaged over the corresponding fields with a weight given by the action coming from Eq. (7), may be taken as quantum identities. These combined give

$$-e\partial\cdot J_L+\lambda P-m^2\partial_\nu\left[\frac{\partial K}{\partial A_\nu}+\frac{\partial K}{\partial\partial_\nu\theta}\right]=0 \tag{8}$$

and allow the nonconservation of the fermionic current even in the case of massless electrodynamics $M^2\to 0$.

We must note that taking bosonic equations as operatorial identities implies $b=\partial\cdot A$ and that to obtain Eq. (8) we have chosen $\Box b=0$. Whereas the latter is a consequence of the equations for A and θ in the pure massive vector boson case, it must be imposed as an additional condition when there is a coupling to chiral fermionic current. This fact will be important in what follows.

A nonvanishing K is necessary to have $\partial\cdot J_L\neq 0$ in the case $M^2=0$ since otherwise the equation for θ would give $P=0$. The choice of, e.g., a term that behaves as a one-cocycle and that in $1+1$ dimensions arises from regularization [8]

$$K=\partial_\mu\theta(\tfrac{1}{2}\partial^\mu\theta-A^\mu) \ , \tag{9}$$

gives a mass contribution to Eq. (8):

$$m^2\partial\cdot A-e\partial\cdot J_L+\lambda P=0 \ . \tag{10}$$

It should be remarked that without fixing the gauge an appropriate regularization of the charge depending on the electromagnetic potential [9] and which mimics the existence of a K term allows us to establish a system of two second-class constraints for the chiral Schwinger model given by the usual Gauss law and the momentum π_0 conjugate to A_0. In this way there remain two independent degrees of freedom and the use of normal Poisson brackets for canonical fields leads to the functional quantization defined by integration over all configurations.

Our approach takes Euler-Lagrange equations coming from Eq. (7) for bosonic fields as operatorial identities without using those for Dirac fields. This agrees with the alternative view [2] which relates the nonconservation of the chiral current to the nonvalidity of Dirac equation when multiplied by the change of the field under a global chiral transformation and averaged over fermionic configurations.

III. BRST CHARGE AND UNITARITY

We now turn to the analysis of BRST symmetry defined through the nilpotent transformations

$$\begin{aligned}&\delta A=\partial c \ , \quad \delta\bar{c}=-b \ , \quad \delta\theta=c \ ,\\ &\delta\psi_L=iec\psi \ , \quad \delta\bar{\psi}_L=-iec\bar{\psi} \ ,\end{aligned} \tag{11}$$

the rest of the changes being zero. The Lagrangian $\mathcal{L}_B$ will remain invariant except for its last two terms. With the choice Eq. (9) for K, as an example, we obtain

$$\delta\mathcal{L}_B=-m^2A\cdot\partial c+\lambda cP \ , \tag{12}$$

which corresponds to a Noether current whose divergence is

$$\partial\cdot J_B=m^2\partial_\mu(cA^\mu) \tag{13}$$

using classical equations of motion for the bosonic fields. From the quantum point of view we may start from the identity averaged over Dirac field configurations with the weight given by Eq. (7):

$$\langle\delta\mathcal{L}_B\rangle=\sum_\varphi\langle E_\varphi\delta\varphi\rangle-\langle\partial\cdot J_B\rangle \ . \tag{14}$$

If all the equations of motion $E_\varphi=0$ are taken as identities except for Dirac fields

$$\sum_{\varphi} \langle E_\varphi \delta\varphi \rangle = \langle e\partial \cdot J_L c \rangle \tag{15}$$

and consequently using Eq. (10), Eq. (13) is obtained in an operatorial sense.

Equation (13) seems not to agree with the Lagrangian noninvariance Eq. (12). This may be understood in terms of the compensation provided by the noninvariance of the fermionic measure. In fact it is seen that the effective action [10], obtained by integrating over fermionic and group field configurations in the partition function, is BRST invariant.

From Eq. (13) we may define a constant charge operator

$$Q = \int d\mathbf{r} (\pi_{A^i} \partial_i c + b\partial_0 c - \pi_\theta c - ie\pi_{\psi_L} \psi_L c - m^2 A^0 c) \tag{16}$$

which generates, through canonical commutators, the transformations of quantum fields as in Eq. (11) except for b.

From the Lagrangian Eq. (7) in $1+1$ dimensions where $P = \varepsilon^{\mu\nu}\partial_\mu A_\nu$, and the choice Eq. (9) the Hamiltonian turns out to be

$$\begin{aligned} H_B = \int d\mathbf{r} \Bigg[& \frac{1}{2}\pi_{A^i}^2 + \frac{1}{2}\frac{\pi_\theta^2}{m^2+M^2} + \frac{M^2+m^2}{2}(A^i + \partial_i \theta)^2 \\ & - i\bar{\psi}\gamma^i \partial_i \psi + \pi_c \pi_{\bar{c}} + \partial_i \bar{c} \partial_i c + \frac{m^2}{2}(A^0)^2 \\ & + (\pi_\theta + \partial_i \pi_{A^i}) A^0 - eA \cdot J_L - \frac{b^2}{2} - A^i \partial_i b \\ & - \frac{m^2}{2}(A^i)^2 + \lambda\theta\pi_{A^i} + \frac{1}{2}\lambda^2\theta^2 \Bigg] . \end{aligned} \tag{17}$$

Its commutator with the constant charge Q gives

$$[iQ, H_B] = \int d\mathbf{r} (-\lambda F^{01} + m^2 b - e\partial \cdot J_L) c , \tag{18}$$

so that using Eq. (10), Q and H_B commute. A few comments are in order. Whereas in the simple massive vector-boson case (i.e., $\lambda = m^2 = e = 0$) the commutation is obvious, in the general case there are two sources of noncommutation: the term proportional to m^2 which comes from the term $b^2/2$ of the Hamiltonian used to avoid the strict gauge fixing $\partial \cdot A = 0$, and that depending on λ which appears as a consequence of the change $\lambda\theta$ produced by the Chern-Pontryagin density in the definition of momentum π_{A^1}, with Eq. (7). These two terms are compensated by the last one of the right-hand side of Eq. (18) which comes from writing $[\rho_L, H_B] = i\dot{\rho}_L$ for the chiral charge density which appears in the BRST charge Eq. (16).

If the explicit canonical commutators of ψ_L and $\bar{\psi}_L$ were used instead, which is equivalent to the validity of Dirac equation, the left charge would be conserved and the last term of Eq. (18) would not appear. But we are taking the left charge in Eq. (16) as the physical one constructed from positive energy states. It does not coincide with the formally conserved Dirac charge before consideration of the full infinite sea of negative energy states. This physical charge is not constant because of, on one hand, the generation of mass for vector bosons in a gauge different from $\partial \cdot A = 0$ and, on the other, the change in the number of left movers with positive energy when an electric field is applied [11].

If the physical states are defined to be invariant under transformations generated by Q, from Eqs. (16) and (17) this means the condition

$$\int d\mathbf{r} (-\dot{b}c + b\dot{c}) |\Phi_{\text{phys}}\rangle = 0 . \tag{19}$$

In the case $m^2 = 0$ (but $M^2 \neq 0$) Eq. (19) implies the two separate conditions of annihilation of physical states by b and $\dot{b}$. Since b and $\dot{b}$ commute these conditions are therefore compatible as in the simple vector-boson case. In the general BRST treatment for second-class constraints [4] this corresponds to having enlarged the space to include the gauge group field so that the constraints turn into first-class constraints and Q becomes nilpotent. Therefore, it is possible to eliminate two degrees of freedom in the effective Hamiltonian. In the subspace of $|\Phi_{\text{phys}}\rangle$ this turns out to be

$$\begin{aligned} H_B^{\text{eff}} = \int d\mathbf{r} \Bigg[& \frac{1}{2}\pi_{A^i}^2 + \frac{M^2}{2}(A^i)^2 + \frac{1}{2M^2}(\partial_i \pi_{A^i} - e\rho_L)^2 \\ & + \frac{M^2}{2}(\partial_i\theta)^2 + \frac{\lambda^2}{2}\theta^2 + \lambda\theta\pi_{A^i} + \pi_c\pi_{\bar{c}} \\ & + \partial_i\bar{c}\partial_i c - i\bar{\psi}\gamma^i\partial_i\psi + eA^i J_L^i \Bigg] \end{aligned} \tag{20}$$

after having subtracted from H_B the terms $A^0\dot{b}$ and $\pi_\theta\dot{\theta}$, and used the classical definition of momenta for these modes followed by the minimization with respect to A^0 along the lines of Ref. [12]. In the same spirit H_B^{eff} must be understood with θ defined instantaneously in terms of π_{A^i} through

$$M^2\nabla^2\theta - \lambda^2\theta = \lambda\pi_{A^i} .$$

For the case $m^2 \neq 0$ (either with $M^2 = 0$ or $M^2 \neq 0$) Eq. (19) does not imply two separate conditions, in correspondence with the fact that Q is no longer nilpotent but satisfies $Q^3 = 0$ because it generates the transformation of b quoted above. In the language of the general treatment [4] this means that we may impose the validity of only one constraint on the physical states, e.g., the Gauss law $\dot{b}|\Phi_{\text{phys}}\rangle = 0$. The second part of Eq. (19) will be also satisfied requiring that the physical states do not contain either the antighosts $\bar{c}$ (whose conjugate momentum is $\dot{c}$) or the particles associated with b. The absence of the latter is consistent because, due to the additional condition $\Box b = 0$ we have adopted, they are free particles. In this way the number of physical degrees of freedom, other than the fermionic ones, may be equal to d in $d+1$ dimensions. This is in agreement with the exact solution of $(1+1)$-dimensional chiral QED which shows in total two degrees of freedom [13].

If both M^2 and m^2 are zero [14] there is no momentum conjugate to θ so that all terms of Eqs. (17) and (18) involving either the masses or π_θ vanish. Since θ is not a dynamical variable, the two first-class constraints implied by Eq. (19) leave $d-1$ gauge field degrees of freedom.

In the (3+1)-dimensional case there is the obvious replacement

$$P=\tfrac{1}{8}\varepsilon^{\mu\nu\rho\delta}F_{\mu\nu}F_{\rho\delta}\ .$$

If we keep the same K of Eq. (9) there are no changes in Eqs. (10)–(16). For the Hamiltonian treatment one must note the change in the definition of π_{A^i} induced by the Chern-Pontryagin density $\pi_{A^i}=F^{0i}-\lambda\theta F_{jk}$ (cyclic order). Therefore H_B, in addition to including the magnetic contribution $\frac{1}{2}F_{jk}^2$, will modify the λ-depending terms of Eq. (17) into $\lambda\theta\pi_{A^i}F_{jk}+\frac{1}{2}\lambda^2\theta^2F_{jk}^2$. As a consequence, since $[iQ,F_{jk}]=0$, Eq. (18) changes into

$$[iQ,H_B]=(-\lambda F^{0i}F_{jk}+m^2b-e\partial\cdot J_L)c\ , \tag{21}$$

with a sum in cyclic order, Q remains as a constant of motion, and Eq. (19) is still valid. Other alternatives for K will be considered in Sec. IV.

We assume that when $e\to 0$ the constants which couple the modes A^μ and θ because of the anomaly also vanish, i.e., $\lambda\to 0$ and $m^2\to 0$. Therefore the set of one-particle states for a definite four-momentum k_μ,

$$|A_\mu\rangle|\theta\rangle|c\rangle|\psi\rangle|\bar\psi\rangle\ ,$$

after excluding $|b\rangle$ and $|\bar c\rangle$ as explained above following the additional condition $\Box b=0$, is left invariant under transformations generated by Q. Note that with this restriction Q behaves as a nilpotent operator. We define the physical asymptotic states as those corresponding to $Q=0$. They will be the transverse vector bosons

$$|\Phi\rangle_{\rm as}=\varepsilon_\perp^\mu|A_\mu\rangle\ ,\quad Q\varepsilon_\perp^\mu|A_\mu\rangle=0\ ,\quad k\cdot\varepsilon_\perp=0 \tag{22}$$

which have a positive norm, with the possible addition of the zero-norm states [3]

$$|\Phi\rangle_0=\alpha|c\rangle+\beta(\varepsilon_\parallel^\mu|A_\mu\rangle-\varepsilon_\parallel^\mu|\partial_\mu\theta\rangle) \tag{23}$$

in addition to the fermionic states. For the massive case [15] $\varepsilon_\parallel^\mu=k^\mu/M$ with $k^2=M^2$ so that the longitudinal vector boson alone is not BRST invariant.

For the $\mathcal{S}$-matrix elements $\langle\Phi_2|\mathcal{S}|\Phi_1\rangle$ it is clear that the time invariance of Q implies $[Q,\mathcal{S}]=0$ so that a physical state can only be connected to another physical state. Also for

$$|\Phi\rangle'_{\rm as}=|\Phi\rangle_{\rm as}+|\Phi\rangle_0$$

it would be important that

$${}_{\rm as}'\langle\Phi_2|\mathcal{S}|\Phi_1\rangle'_{\rm as}={}_{\rm as}\langle\Phi_2|\mathcal{S}|\Phi_1\rangle_{\rm as}\ , \tag{24}$$

allowing the definition of equivalence classes associated with a submatrix $\mathcal{S}$ among physical states. Equation (24) would be obviously satisfied if $|\Phi\rangle_0$ could be expressed as Q applied to a general state. This is true for $|c\rangle$ but not for the second part of the state $|\phi\rangle_0$ of Eq. (23). However, going back to Eq. (7), it is obvious that, for the normal massive vector-boson theory ($\lambda=0=m^2$),

$$|\partial\cdot A-\Box\theta\rangle=0 \tag{25}$$

so that this disturbing state vanishes.

For the chiral case with the choice of Eq. (9),

$$|\partial\cdot A-\Box\theta\rangle=-\frac{\lambda}{M^2+m^2}|P\rangle \tag{26}$$

showing that in the asymptotic limit it will again be possible to define the $\mathcal{S}$ matrix in the physical subspace. This will include the case $M=0$, provided λ tends to zero faster than m^2 when $e\to 0$, so that we may interpret that a quantum mass has appeared adding one degree of freedom to those of normal QED and consequently in the definition of $\varepsilon_\parallel^\mu$, M must be replaced by m.

IV. DIFFICULTIES WITH RENORMALIZATION IN 3+1 DIMENSIONS

In the (3+1)-dimensional case it is not obvious that the Wess-Zumino term of $\mathcal{L}_B$ must be iterated perturbatively, because of its topological nature and to the fact that it is not renormalized [16].

However, if a standard perturbative expansion of the term $\lambda\theta F\tilde F$ is performed, problems related to the renormalizability of the theory appear [5]. In fact, the one-loop correction to the inverse θ propagator of momentum p will contain divergent contributions of order λ^2 proportional to $(p^2)^0$, $(p^2)^1$, and $(p^2)^2$. The first must be compensated by a mass counterterm for $m\theta$ not present in $\mathcal{L}_B$, the second by a kinetic counterterm $(Z-1)m^2\,\partial_\mu\theta\partial^\mu\theta$, and the third by a term $\Box\theta\Box\theta$ which might be considered as part of a new kinetic term

$$K'=\kappa[(\partial\cdot A-\Box\theta)^2-(\partial\cdot A)^2] \tag{27}$$

that transforms as a one-cocycle.

It seems apparently impossible to reabsorb all the above infinities in a Lagrangian which keeps the form of Eq. (7) with the simple addition of kinetic terms transforming as one-cocycles. Moreover, a term such as that of Eq. (27) would make the Hamiltonian formalism obscure and it would be difficult to establish a criterion for avoiding the transition probability between transverse and longitudinal fields, spoiling therefore the unitarity requirement.

A renormalizable theory is attained suppressing the one-cocycle kinetic terms, and replacing m^2K of Eq. (7) by the more general gauge-invariant expression of dimension four [17]:

$$K''=\kappa_1(\partial\cdot A-\Box\theta)^2+\kappa_2[(A_\mu-\partial_\mu\theta)(A^\mu-\partial^\mu\theta)]^2 +\kappa_3(A_\mu-\partial_\mu\theta)J_L^\mu\ , \tag{28}$$

which must be thought as coming from additional terms to the original $\mathcal{L}$ of Eq. (2).

In this way we can absorb all the infinities generated by the Wess-Zumino term considering the dimensional field $\partial_\mu\theta$ instead of the dimensionless θ.

Since K'' is gauge invariant Eqs. (8), (10), (12), (13), and (16) will be valid with $m^2=0$. However, again because of the difficulty of handling the quantum states of a higher derivative theory, unitarity cannot be completely proved.

A general analysis of the Faddeev-Popov gauge fixing of the sum over all configurations using lattice regularization indicates that the theory with a Wess-Zumino field cannot satisfy all the conditions of renormalizability, unitarity, and finiteness of the photon mass [18], showing the need for changes in the quantization prescription.

V. ALTERNATIVE QUANTIZATION RULES AND THEIR EQUIVALENCE

If we return to Eq. (7) and perform the separation

$$A_\mu = A_\mu^\perp + \partial_\mu\varphi \ , \quad \partial\cdot A^\perp = 0 \ , \tag{29}$$

we obtain

$$\begin{aligned}\mathcal{L}_B = & -\tfrac{1}{4}F^2[A^\perp] + eA^\perp\cdot J_L + \frac{b^2}{2} + A^\perp\cdot\partial b + \lambda\theta P[A^\perp] \\ & + i\bar\psi\partial\!\!\!/\psi + e\partial\varphi\cdot J_L + \partial\varphi\cdot\partial b + \partial\bar c\cdot\partial c \\ & + \frac{M^2+m^2}{2}(A^\perp + \partial\varphi - \partial\theta)^2 - \frac{m^2}{2}(A^\perp+\partial\varphi)^2 \ ,\end{aligned} \tag{30}$$

where the choice Eq. (9) for K has been made.

Using Eq. (10) together with $\Box b = 0$, and considering $\bar\theta = \theta - \varphi$ as a variable in the functional integral, Eq. (30) becomes

$$\begin{aligned}\mathcal{L}_B = & -\tfrac{1}{4}F^2[A^\perp] + eA^\perp\cdot J_L + \frac{b^2}{2} + i\bar\psi\partial\!\!\!/\psi - \frac{m^2}{2}(A^\perp)^2 \\ & + \frac{m^2}{2}(\partial\varphi)^2 + \lambda\bar\theta P[A^\perp] + \partial\bar c\cdot\partial c \\ & + \frac{M^2+m^2}{2}[(A^\perp)^2 + (\partial\bar\theta)^2]\end{aligned} \tag{31}$$

so that φ, in addition to b, is decoupled. This decoupling would remain true also if terms such as K' of Eq. (27) or K'' of Eq. (28) were added.

To avoid the perturbative difficulties originated by $\bar\theta P$ in the $(3+1)$-dimensional case one might add to Eq. (31) a regularizing term, a function of a new field [5], which cannot be derived from the original quantization Eq. (1):

$$\mathcal{L}' = \mathcal{L}_B + \lambda\eta P - \frac{M^2+m^2}{2}(\partial\eta)^2 \ . \tag{32}$$

$\mathcal{L}'$ gives the same Eqs. (8) and (10), and is left invariant under the nilpotent transformation

$$\tilde\delta\bar\theta = c \ , \quad \tilde\delta\eta = -c \ , \quad \tilde\delta\bar c = (-\bar\theta - \eta)(M^2+m^2) \ . \tag{33}$$

Changing variables to

$$\xi = \bar\theta + \eta \ , \quad \chi = \bar\theta - \eta \ , \tag{34}$$

the Lagrangian is rewritten as

$$\begin{aligned}\mathcal{L}' = & -\tfrac{1}{4}F^2 + eA^\perp\cdot J_L + \frac{b^2}{2} + i\bar\psi\partial\!\!\!/\psi + \frac{M^2}{2}(A^\perp)^2 \\ & + \frac{m^2}{2}(\partial\varphi)^2 + \partial\bar c\cdot\partial c + \lambda\xi P - \frac{M^2+m^2}{2}\chi\Box\xi \ .\end{aligned} \tag{35}$$

The functional integral over χ leads to $\delta(\Box\xi)$ showing that the field apparently coupled to $A^\perp$ is indeed free and consequently the anomaly becomes harmless. Defining the physical states as those invariant under the transformation Eq. (33), the possible state $|\bar\theta\rangle + |\eta\rangle = |\xi\rangle$ which satisfies this condition is irrelevant and the sole transverse states are important for the $\mathcal{S}$ matrix. These will be three independent modes provided $M^2 > 0$ since the effect of m^2 has disappeared from Eq. (35).

The transformation Eq. (33) leaves the fermion fields invariant. In the same spirit, if we keep the fermions invariant in the transformation Eq. (11) for the Lagrangian Eq. (7) with the choice of Eq. (9), instead of Eq. (12) we would obtain

$$\tilde\delta\mathcal{L}_B = \partial_\mu[(eJ_L^\mu - m^2A^\mu)c] \tag{36}$$

with a new BRST current $\tilde J_B^\mu = J_B^\mu - eJ_L^\mu c$ whose divergence would therefore be

$$\partial\cdot\tilde J_B = \partial_\mu[(m^2A^\mu - eJ_L^\mu)c] \ . \tag{37}$$

The nonconservation of $\tilde J_B^\mu$ is directly related to the change of the Lagrangian, a connection which is understood looking at Eq. (14) where now the Dirac equation does not contribute. From Eq. (37) it is clear that the charge Q defined previously in Eq. (16) is a quantum-mechanical constant of motion as verified using Eq. (18). Therefore both transformations δ and $\tilde\delta$ give rise to equivalent physical applications.

Another way of changing the quantization rule is to introduce a Lorentz restriction in the functional integration:

$$Z_L = \int \mathcal{D}A_\mu \mathcal{D}\psi\, \mathcal{D}\bar\psi\, \delta(\partial\cdot A)\exp\left[i\int d^Dx\mathcal{L}\right] \ . \tag{38}$$

This is consistent for massive vector bosons and avoids the sum with different weights of configurations which are gauge transformations of one another. Assuming no further regularization terms, we shall obtain a BRST Lagrangian of the type of Eq. (7), with a gauge-invariant kinetic term [3]

$$K = \partial f\cdot(\partial\theta - A) \ , \tag{39}$$

where the additional invariant scalar field f must be integrated over.

This functional integral over f gives a constraint for θ which indicates that the latter is not an independent degree of freedom and that Eq. (25) is satisfied leaving only the transverse states. In fact the restriction in Eq. (38) is equivalent to starting from a Lagrangian where A is replaced by the gauge-invariant [19] expression $A_\mu - \partial_\mu\Box^{-1}\partial\cdot A$ so that the fermion field must not be gauge transformed and the fermionic measure is therefore invariant giving no anomaly.

On the other hand, inserting the separation Eq. (29) into the Lagrangian Eq. (7) with the kinetic term Eq. (39) and using Eq. (10) and $\Box b = 0$, one obtains

$$\begin{aligned}\mathcal{L}'' = & -\tfrac{1}{4}F^2 + eA^\perp\cdot J_L + \frac{b^2}{2} + i\bar\psi\partial\!\!\!/\psi + \frac{M^2}{2}(A^\perp)^2 \\ & + \frac{M^2}{2}[\partial(\theta-\varphi)]^2 + \partial\bar c\cdot\partial c \\ & + \lambda(\theta-\varphi)P - m^2 f\Box(\theta-\varphi) \ .\end{aligned} \tag{40}$$

The functional integral over f shows that the field $\theta-\varphi$ is in fact free, proving that the treatments based on Lagrangians $\mathcal{L}'$ or $\mathcal{L}''$ are equivalent. The only difference comes from the choice of including the one-cocycle kinetic term Eq. (9) into $\mathcal{L}'$ but not in $\mathcal{L}''$. Adding it to the latter would not change the equivalence consideration.

A detailed analysis of the theory with the kinetic term Eq. (39) has been performed [20] considering the BRST symmetries for all the ghost fields involved, and this confirms the unitarity and renormalizability of this version of chiral QED in 3+1 dimensions. Even though the current J_L is still not conserved, its coupling to a transverse $A^\perp$ implies that A interacts with an automatically superconserved [21] current $J_L^\perp$.

It is interesting to see to what extent the restriction of the functional integral to configurations satisfying the Lorentz condition is determinant. If instead of Eq. (38) we introduce an average of gauges around the Lorentz one with a Gaussian weight of width $1/\alpha$ in the spirit of what was done for the chiral Schwinger model [22], we obtain a functional integral with a Lagrangian

$$\tilde{\mathcal{L}}=-\tfrac{1}{4}F^2+\frac{M^2}{2}(A-\partial\theta)^2+i\bar{\psi}\partial\!\!\!/\psi + eJ_L^\mu(A_\mu-\partial_\mu\theta)+\frac{\alpha}{2}(\partial\cdot A-\Box\theta)^2 \qquad (41)$$

and an additional sum over configurations of θ. Even though $\tilde{\mathcal{L}}$ is gauge invariant under the transformation of only A and θ so that no anomaly appears from the fermionic measure, for general values of α the field θ will not decouple from the physical degrees of freedom with the consequent difficulties related to its higher derivative term. Only for $\alpha\to\infty$, θ will be fixed as $\theta=\Box^{-1}\partial\cdot A$ recovering the model Eq. (38) in terms of transverse $A^\perp$, and Eq. (25) will be satisfied decoupling a dangerous state which could otherwise spoil unitarity and renormalizability [23]. For $\alpha=0$ one would instead return to the sum over all configurations Eq. (1), with the problems quoted above.

It is therefore clear that only the Lorentz gauge restriction of Eq. (38) provides a consistent anomaly cancellation in 3+1 dimensions, whereas for the chiral Schwinger model the general restriction Eq. (41) allows an exact solution with a single massive mode when $M=0$ in much the same way as mass appears in the normal Schwinger model. A similar thing occurs with the temporal gauge $A_0=0$, an unnatural choice when aiming at massive bosons, which forces a modification of the Gauss law [24] and a single massive mode for the chiral Schwinger model [25].

In conclusion, we have shown that it is possible to envisage a unitary chiral QED in 3+1 dimensions using the sum over all the configurations through the consideration of invariance under a constant BRST charge which would allow the generation of mass originated by the anomaly. However, the renormalizability is only assured restricting the integration to configurations which satisfy the Lorentz condition in such a way that the Wess-Zumino field is effectively decoupled from the transverse electromagnetic modes without the possibility of mass generation.

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[1] C. Becchi, A. Rouet, and R. Stora, Phys. Lett. **52B**, 344 (1974); I. V. Tyutin, Lebedev Report No. FIAN 39, 1975 (unpublished).
[2] I. Tsutsui, Phys. Rev. D **40**, 3543 (1989).
[3] A. Della Selva, L. Masperi, and G. Thompson, Phys. Rev. D **37**, 2347 (1988).
[4] R. Marnelius, Nucl. Phys. **B294**, 685 (1987); **B370**, 165 (1992); **B372**, 218 (1992).
[5] A. Andrianov, A. Bassetto, and R. Soldati, Phys. Rev. Lett. **63**, 1554 (1989).
[6] H. Rothe and K. Rothe, Phys. Rev. D **40**, 545 (1989); F. Otto, *ibid.* **43**, 548 (1991).
[7] P. Mitra and R. Rajaraman, Phys. Lett. B **225**, 267 (1989).
[8] O. Babelon, F. Schaposnik, and C. Viallet, Phys. Lett. B **177**, 385 (1986); M. Manías, M. von Reichenbach, F. Schaposnik, and M. Trobo, J. Math. Phys. **28**, 1632 (1987); K. Harada and I. Tsutsui, Phys. Lett. B **183**, 311 (1987).
[9] K. Harada, Tokyo Institute of Technology Report No. TIT/HEP-118, 1987 (unpublished); R. Trinchero, Mod. Phys. Lett. A **4**, 187 (1989).
[10] F. Schaposnik and J. Solomin, Ann. Phys. (N.Y.) **189**, 363 (1989).
[11] H. B. Nielsen and M. Ninomiya, Int. J. Mod. Phys. A **6**, 2913 (1991).
[12] L. Masperi and R. Montemayor, Mod. Phys. Lett. A **4**, 953 (1989).
[13] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. **54**, 1219 (1985); R. Rajaraman, Phys. Lett. **154B**, 305 (1985); D. Boyanovsky, Nucl. Phys. **B294**, 223 (1987); N. Falck and G. Kramer, Phys. Lett. B **193**, 257 (1987).
[14] L. D. Faddeev and S. L. Shatashvili, Phys. Lett. **167B**, 225 (1986).
[15] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980), p. 136.
[16] J. Magnin and L. Masperi, An. Asoc. Fis. Arg. **2**, 63 (1990).
[17] T. D. Kieu, Phys. Lett. B **242**, 234 (1990).
[18] S. Aoki, Phys. Rev. D **42**, 2806 (1990).
[19] G. Thompson and R. Zhang, Mod. Phys. Lett. A **2**, 869 (1987).
[20] G. Demarco, C. Fosco, and R. Trinchero, Mod. Phys. Lett. A **6**, 1299 (1991); Int. J. Mod. Phys. A **7**, 5499 (1992).

[21] T. D. Kieu, "Gauge invariant renormalizable and unitary chiral theory," Oxford report, 1991 (unpublished).
[22] N. V. Krasnikov, Nuovo Cimento **89A**, 308 (1985).
[23] A. Andrianov, A. Bassetto, and R. Soldati, Phys. Rev. D **44**, 2602 (1991); C. Fosco and R. Montemayor, *ibid.* (to be published).
[24] K. Harada and I. Tsutsui, Prog. Theor. Phys. **78**, 675 (1987).
[25] F. M. Saradzhev, Baku Report No. 415, 1991 (unpublished) and No. 447, 1992 (unpublished).