PHYSICAL REVIEW D VOLUME 47, NUMBER 4 15 FEBRUARY 1993

Adiabatic regularization of the quantum stress-energy tensor in curved spacetimes: Path-integral quantization method

Wung-Hong Huang
Department of Physics, National Cheng Kung University, Tainan, Taiwan 70101, Republic of China
(Received 27 July 1992)

A possible way to obtain the adiabatic regularized quantum stress-energy tensor in a background curved spacetime is developed within the method of path-integral quantization. As an illustration, we study the mode of a massive scalar field with arbitrary curvature coupling to the inhomogeneous conformally flat spacetime. The results so obtained are useful to investigate the back-reaction effects of a quantized field in the early Universe with an inhomogeneity.

PACS number(s): 03.70.+k, 04.60.+n

I. INTRODUCTION

This is the third of a series of papers in which we study the adiabatic regularized quantum stress-energy tensor [1] in inhomogeneous spacetime. For a complete discussion of the motivation and related references, we refer the reader to Refs. [2] and [3]. In this section we only briefly mention the motivation for the present work.

In the theory of quantum fields in curved spacetime [4], one treats the gravitational field as a classical background field and the expectation values of some matter stress tensors, $\langle T_{\mu\nu}\rangle$, are regarded as the sources of the generalized Einstein equation. Although $\langle T_{\mu\nu}\rangle$ is formally divergent, it can be made finite by the renormalization procedure in the DeWitt-Schwinger formalism and point-splitting method [5]. However, in this approach the finite part of the stress tensor is rather difficult to evaluate. On the other hand, the adiabatic regularization method (ARM) [1–3], which can be used to find the finite parts of the quantum stress tensor, is a particularly efficient method in the numerical study of the dynamics of quantum fields in curved spacetime [6]. As noted by Parker and Fulling [1] in their original work, the ARM is applicable to any spacetime which has sufficient symmetry to allow a decomposition of the quantized field into modes. Therefore, when one wants to extend the ARM to the more general spacetime with an inhomogeneity, then one will immediately suffer the difficulty of mode-mixing behavior and the adiabatic approximation of the mode function cannot be straightforwardly obtained.

In a previous paper [2], we made a small step toward obtaining the adiabatic regularized quantum stress tensor (ARQST) in the spacetime with a small inhomogeneity which has a spatial reflection symmetry. In that paper we adopted the early-time approximation. In a recent paper [3], we used the stochastic quantization method [7] to obtain the ARQST in the general spacetime which is conformally flat. In that paper we invented some algorithms to simplify the tremendous work of integrating the stochastic time to obtain the desired results. However, the remaining task is still very lengthy. Therefore it appears worthwhile to find a more simple method to evaluate the ARQST for matter in curved spacetime, and in this paper we will report our work on this. This time we will use a more traditional method: the path-integral quantization method. We will show how the path-integral quantization method can be used to obtain the ARQST. Our results are useful to investigate the back-reaction effects of a quantized field in the early Universe with an inhomogeneity [8].

This paper is organized as follows. In Sec. II we define the mode and calculate the associated classical stress tensors. In Sec. III we present our method to evaluate the adiabatic expansion of quantum stress-energy tensors. The results are collected in Sec. IV. Finally, Sec. V is devoted to a discussion of the results of this paper.

II. MODE

We consider the action describing a massive scalar field (ϕ) coupled arbitrarily (ξ) to the curvature (R) of a gravitational background:

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\,\partial_\beta\phi-\frac{1}{2}m^2\phi^2-\frac{\xi}{2}R\phi^2\right], \tag{2.1}$$

where R is the Ricci scalar. The scalar wave equation, obtained by varying S with respect to ϕ, is

$$\Box\phi+m^2\phi+\xi R\phi=0, \tag{2.2}$$

where $\Box\phi=g^{\mu\nu}\nabla_\mu\nabla_\nu\phi=g^{\mu\nu}\phi_{;\mu\nu}$. The stress tensor is defined by

$$\begin{aligned}T_{\mu\nu}&=\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}\\&=(2\xi-1)\phi_{,\mu}\phi_{,\nu}+(\tfrac{1}{2}-2\xi)g_{\mu\nu}g^{ab}\phi_{,a}\phi_{,b}\\&\quad-2\xi g_{\mu\nu}\phi\Box\phi+2\xi\phi\phi_{;\mu\nu}-\tfrac{1}{2}g_{\mu\nu}m^2\phi^2+\xi G_{\mu\nu}\phi^2.\end{aligned} \tag{2.3}$$

Using the wave equation (2.2), the stress tensor can be written as

$$T_{\mu\nu}=(2\xi-1)\phi_{,\mu}\phi_{,\nu}+(\tfrac{1}{2}-2\xi)g_{\mu\nu}g^{ab}\phi_{,a}\phi_{,b}+2\xi\phi\phi_{;\mu\nu}-(\tfrac{1}{2}-2\xi)g_{\mu\nu}m^2\phi^2+\xi G_{\mu\nu}\phi^2+2\xi^2 g_{\mu\nu}R\phi^2 . \quad (2.4)$$

We consider the model spacetime with the inhomogeneous conformally flat metric form

$$ds^2=C^2(x_0,\mathbf{x})(dx_0^2-d\mathbf{x}^2) . \quad (2.5)$$

After introducing a modified wave function

$$\phi=C^{-1}\chi , \quad (2.6)$$

the wave equation of Eq. (2.2) becomes

$$\ddot{\chi}-\nabla^2\chi+C^2[(\xi-\tfrac{1}{6})R+m^2]\chi=0 , \quad (2.7)$$

where the Riemann-Christoffel curvature is

$$R=6C^{-3}\left[C_{,00}-\sum_i C_{,ii}\right] . \quad (2.8)$$

Note that Eq. (2.7) cannot be solved by splitting it into separated ordinary differential equations, as the spacetime is inhomogeneous.

The stress tensors in this metric are given by

$$T_{00}=\left[(6\xi-\tfrac{1}{2})C^{-4}(C_{,0})^2+(4\xi-\tfrac{1}{2})C^{-4}\sum_j(C_{,j})^2-(\tfrac{1}{2}-2\xi)m^2+\xi C^{-2}G_{00}+2\xi^2R\right]\chi^2 +(1-6\xi)C^{-3}C_{,0}\chi\chi_{,0}+(1-6\xi)C^{-3}\sum_j C_{,j}\chi\chi_{,j}-\tfrac{1}{2}C^{-2}(\chi_{,0})^2+2\xi C^{-2}\chi\chi_{,00}-(\tfrac{1}{2}-2\xi)C^{-2}\sum_j(\chi_{,j})^2 , \quad (2.9)$$

$$T_{ii}=\left[(4\xi-\tfrac{1}{2})C^{-4}(C_{,0})^2+(10\xi-1)C^{-4}(C_{,i})^2+(\tfrac{1}{2}-4\xi)C^{-4}\sum_j(C_{,j})^2+(\tfrac{1}{2}-2\xi)m^2+\xi C^{-2}G_{ii}-2\xi^2R\right]\chi^2 +(2\xi-\tfrac{1}{2})C^{-2}(\chi_{,0})^2+[(2-8\xi)C^{-3}C_{,i}+4\xi C^{-4}(C_{,i})^2]\chi\chi_{,i}-(6\xi-1)C^{-3}C{,}0\chi\chi_{,0} +(6\xi-1)C^{-3}\sum_j C_{,j}\chi\chi_{,j}+2\xi C^{-2}\chi\chi_{,ii}+(2\xi-1)C^{-2}(\chi_{,i})^2+(\tfrac{1}{2}-2\xi)C^{-2}\sum_j(\chi_{,j})^2 , \quad (2.10)$$

where the Einstein tensor $G_{\mu\nu}\equiv R_{\mu\nu}-\frac{1}{2}-g_{\mu\nu}R$ is

$$G_{00}=-2C^{-1}(C^{-1})_{,00}+\frac{C^{-2}}{2}\left[(C^2)_{,00}-\sum_j(C^2)_{,jj}\right]-3C^{-1}\left[C_{,00}-\sum_j C_{,jj}\right] , \quad (2.11)$$

$$G_{ii}=-2C^{-1}(C^{-1})_{,ii}-\frac{C^{-2}}{2}\left[(C^2)_{,00}-\sum_j(C^2)_{,jj}\right]+3C^{-1}\left[C_{,00}-\sum_i C_{,jj}\right] . \quad (2.12)$$

In the quantum theory, $T_{\mu\nu}$ is divergent and needs to be renormalized. In the next section, we will show how the path-integral quantization method can be used to obtain the adiabatic regularized quantum stress tensor. Using this, we can then begin to investigate the back-reaction effects of quantized fields in the early Universe with an inhomogeneity.

III. METHOD

The basic idea of our method is from the following argument. It is known that the infinity of a quantum expectation value comes from the large value of k, the wave vector of a quantum field. We also know that a large k corresponds to short-wavelength modes, and short-wavelength modes only probe the local behavior of the background spacetime. Therefore it is natural to conjecture that the ultraviolet divergence of a quantum observable may be made softer by taking into account more terms with a higher spacetime derivative. With this guide we have, in previous papers [2,3] and this paper, found the regularized quantum stress tensor.

It should be mentioned that the above ideal has been used by Bunch and Parker [9] to study the renormalization of an interacting field in curved spacetime. Although Bunch and Parker had obtained the so-called "adiabatic expansion" of the Feynman propagator, their result is different from ours. The reason is that they use the Riemann normal coordinate, while our coordinate system [Eq. (2.5)] is not normal. Of course, both coordinates will have a similar divergent behavior. Discussions of this point have been detailed in Appendix A of their paper [9].

Furthermore, it is well known that to obtain the finite part of $\langle T_{\mu\nu}\rangle$ one should subtract, mode by mode, the analytically divergent part (i.e., the adiabatic expansion term) from the numerically exact part. However, the exact part shall be examined (see the discussion in Sec. V) from the mode solution of Eq. (2.7), which is solved in a specified coordinate [Eq. (2.5)] and not in the Riemann normal coordinate frame. Therefore the adiabatic expansion of the quantum stress tensor evaluated in this paper is essential to obtain the finite part of $\langle T_{\mu\nu}\rangle$.

We also remark here that the results obtained in this paper can be, in principle, gotten by the method of Bunch and Parker [9] if the short-distance expansion is performed in a specified coordinate instead of the Riemann normal coordinate. However, our prescription for working within the path-integral quantization method, which need not solve the Green's function equation associated with the Feynman propagator, is of value, specifically when one wants to deal with the non-Abelian gauge theory and/or a more complex spacetime.

To proceed, let us adopt the path-integral quantization method to evaluate the ARQST at $x^\mu=0$. (We change the coordinate to get the interesting position at the original point.) Using the Taylor expansion

$$C^2(x)[(\xi-\tfrac{1}{6})R(x)+m^2] = B+B_\mu x^\mu+B_{\mu\nu}x^\mu x^\nu+B_{\mu\nu\lambda}x^\mu x^\nu x^\lambda + B_{\mu\nu\lambda\delta}x^\mu x^\nu x^\lambda x^\delta+\cdots \quad (3.1)$$

and Fourier transform

$$\chi(x_\mu)=\int d^4k\,\chi_k e^{ik\cdot x},\quad J(x_\mu)=\int d^4k\,J_k e^{-ik\cdot x}, \quad (3.2)$$

the generating functional $Z[J]$ becomes

$$\begin{aligned} Z[J] &\equiv \int \mathcal{D}\phi \exp\left[i[\mathcal{S}+\int d^4x\,\chi(x)J(x)]\right] \\ &= \int \mathcal{D}\phi \exp\left[-iC^2\{\chi\ddot{\chi}-\chi\nabla^2\chi+C^2[(\xi-\tfrac{1}{6})R+m^2]\chi^2\}\right] \\ &= \left[\exp\left[\frac{-i}{2}\int d^4k\,d^4l\,\mathcal{H}(k,l)\frac{\delta}{\delta J_k}\frac{\delta}{\delta J_l}\right]\right]\exp\left[\frac{i}{2}\int d^4\tilde{k}\,d^4\tilde{l}\,J_{\tilde{k}}\delta^4(\tilde{k}+\tilde{l})(\tilde{k}^2+B)^{-1}J_{\tilde{l}}\right], \end{aligned} \quad (3.3)$$

where

$$\begin{aligned} \mathcal{H}(k,l) &= iB_\mu\frac{\partial\delta^4(k+l)}{\partial(k+l)_\mu}+B_{\mu\nu}\frac{\partial^2\delta(k+l)}{\partial(k+l)_\mu\partial(k+l)_\nu}-iB_{\mu\nu\lambda}\frac{\partial^3\delta(k+l)}{\partial(k+l)_\mu\partial(k+l)_\nu\partial(k+l)_\lambda} \\ &\quad -B_{\mu\nu\lambda\delta}\frac{\partial^4\delta(k+l)}{\partial(k+l)_\mu\partial(k+l)_\nu\partial(k+l)_\lambda\partial(k+l)_\delta}+\cdots, \end{aligned} \quad (3.4)$$

in which a summation shall always be taken on the same index in an equation. In Eq. (3.1) we have retained only terms with coefficients involving four derivatives or fewer. This will prove sufficient for dealing with all ultraviolet divergences.

From Eq. (3.3) we can straightforwardly evaluate the adiabatic expansions of $\langle\chi^2\rangle$, $\langle\chi\partial_\mu\chi\rangle$, and $\langle\chi\partial_\mu\partial_\nu\chi\rangle$ [see Eqs. (4.7)–(4.9)] by just the functional derivative with respective to $J(x)$ and thus from Eqs. (2.9) and (2.10) obtain the ARQST. During the calculation, we use integration by parts to get rid of the derivative to $\delta(k+l)$ for each term in $\mathcal{H}(k,l)$ and we only have to do some mathematical derivatives.

IV. RESULTS

As our result is mainly applied to the numerical investigation, we will present our results in a form which is more suitable to be written into a computer.

Defining

$$\begin{aligned} &H^\mu=iB_\mu,\quad H^{\mu\nu}=B_{\mu\nu}, \\ &H^{\mu\nu\lambda}=-iB_{\mu\nu\lambda},\quad H^{\mu\nu\lambda\delta}=-B_{\mu\nu\lambda\delta}, \end{aligned} \quad (4.1)$$

$$\begin{aligned} &\Omega=-(l^2-B)^{-1},\quad \Omega_\mu=l^\mu\Omega^2, \\ &\Omega_{\mu\nu}=2\eta^{\mu\nu}\Omega^2+8l^\mu l^\nu\Omega^3, \\ &\Omega_{\mu\nu\lambda}=8(\eta^{\mu\nu}l^\lambda+\eta^{\mu\lambda}l^\nu+\eta^{\nu\lambda}l^\mu)\Omega^3+48l^\mu l^\nu l^\lambda\Omega^4, \\ &\Omega_{\mu\nu\lambda\delta}=8(\eta^{\mu\nu}\eta^{\lambda\delta}+\eta^{\mu\lambda}\eta^{\nu\delta}+\eta^{\nu\lambda}\eta^{\mu\delta})\Omega^3 \\ &\qquad +48(\eta^{\mu\nu}l^\lambda l^\delta+\eta^{\mu\lambda}l^\nu l^\delta+\eta^{\mu\delta}l^\nu l^\lambda \\ &\qquad +\eta^{\nu\lambda}l^\mu l^\delta+\eta^{\nu\delta}l^\mu l^\nu+\eta^{\lambda\delta}l^\mu l^\nu)\Omega^4 \\ &\qquad +384l^\mu l^\nu l^\lambda l^\delta\Omega^5, \end{aligned} \quad (4.2)$$

$$\begin{aligned} &(\Omega H^\mu)_\mu=\Omega_\mu H^\mu,\quad (\Omega H^\mu)_\lambda=\Omega_\lambda H^\mu, \\ &(\Omega H^\mu)_{\mu\nu}=\Omega_{\mu\nu}H^\mu,\quad (\Omega H^\mu)_{\mu\nu\lambda}=\Omega_{\mu\nu\lambda}H^\mu, \\ &(\Omega H^\mu)_{\mu\nu\lambda\delta}=\Omega_{\mu\nu\lambda\delta}H^\mu, \end{aligned} \quad (4.3)$$

$$\begin{aligned} &(\Omega H^{\mu\nu})=\Omega_\mu H^{\mu\nu},\quad (\Omega H^{\mu\nu})_\lambda=\Omega H^{\mu\nu}, \\ &(\Omega H^{\mu\nu})_{\mu\nu}=\Omega_{\mu\nu}H^{\mu\nu},\quad (\Omega H^{\mu\nu})_{\mu\nu\lambda}=\Omega_{\mu\nu\lambda}H^{\mu\nu}, \\ &(\Omega H^{\mu\nu})_{\mu\nu\lambda\delta}=\Omega_{\mu\nu\lambda\delta}H^{\mu\nu}, \end{aligned} \quad (4.4)$$

$$\begin{aligned} &(\Omega H^{\mu\nu\lambda})_\mu=\Omega_\mu H^{\mu\nu\lambda},\quad (\Omega H^{\mu\nu\lambda})_{\mu\nu}=\Omega_{\mu\nu}H^{\mu\nu\lambda}, \\ &(\Omega H^{\mu\nu\lambda})_{\mu\nu\lambda}=\Omega_{\mu\nu\lambda}H^{\mu\nu\lambda},\quad (\Omega H^{\mu\nu\lambda})_{\mu\nu\delta}=\Omega_{\mu\nu\lambda\delta}H^{\mu\nu\lambda}, \\ &(\Omega H^{\mu\nu\lambda\delta})_{\mu\nu\lambda\delta}=\Omega_{\mu\nu\lambda\delta}H^{\mu\nu\lambda\delta}, \end{aligned} \quad (4.5)$$

we then have

$$\begin{aligned} &S_1=(\Omega H^\mu)_\mu, \\ &S_{11}=(\Omega H^\mu)_{\mu\nu}(\Omega H^\nu)+S_1^2, \\ &S_{11\lambda}=(\Omega H^\mu)_{\mu\nu\lambda}(\Omega H^\nu)+(\Omega H^\mu)_{\mu\nu}(\Omega H^\nu)_\lambda \\ &\qquad +2(\Omega H^\mu)_{\mu\lambda}(\Omega H^\nu)_\nu, \\ &S_{111}=s_{11\lambda}(\Omega H^\lambda)+S_{11}S_1, \\ &S_{11\lambda\delta}=(\Omega H^\mu)_{\mu\nu\lambda\delta}(\Omega H^\nu)+2(\Omega H^\mu)_{\mu\nu\lambda}(\Omega H^\nu)_\delta \\ &\qquad +(\Omega H^\mu)_{\mu\nu}(\Omega H^\nu)_{\lambda\delta}+2S_1(\Omega H^\mu)_{\mu\lambda\delta} \\ &\qquad +2(\Omega H^\mu)_{\mu\lambda}(\Omega H^\nu)_{\nu\delta}, \\ &S_{1111}=[S_{111\lambda}(\Omega H^\lambda)+S_{11\lambda}(\Omega H^\lambda)_\delta+S_{11\delta}S_1 \\ &\qquad +S_{11}(\Omega H^\lambda)_{\lambda\delta}](\Omega H^\delta)+S_{111}S_1, \\ &S_2=(\Omega H^{\mu\nu})_{\mu\nu}, \end{aligned} \quad (4.6)$$

$$S_{21}=(\Omega H^{\mu\nu})_{\mu\nu\lambda}(\Omega H^{\lambda})+S_2S_1\ ,$$

$$S_{22}=(\Omega H^{\mu\nu})_{\mu\nu\lambda\delta}(\Omega H^{\lambda\delta})+2(\Omega H^{\mu\nu})_{\mu\nu\lambda}(\Omega H^{\lambda\delta})_{\delta}+S_2^2\ ,$$

$$S_{211}=[(\Omega H^{\mu\nu})_{\mu\nu\lambda\delta}(\Omega H^{\lambda})+(\Omega H^{\mu\nu})_{\mu\nu\lambda}(\Omega H^{\lambda})_{\delta}+(\Omega H^{\mu\nu})_{\mu\nu\delta}S_1+S_2(\Omega H^{\lambda})_{\lambda\delta}](\Omega H^{\delta})+S_{21}S_1\ ,$$

$$S_3=(\Omega H^{\mu\nu\lambda})_{\mu\nu\lambda}\ ,$$

$$S_{31}=(\Omega H^{\mu\nu\lambda})_{\mu\nu\lambda\delta}(\Omega H^{\delta})+S_3S_1\ ,$$

$$S_4=(\Omega H^{\mu\nu\lambda\delta})_{\mu\nu\lambda\delta}\ ,$$

$$S_{12}=(\Omega H^{\mu})_{\mu\nu\lambda}(\Omega H^{\nu\lambda})+2(\Omega H^{\mu})_{\nu\lambda}(\Omega H^{\nu\lambda})+S_1S_2\ ,$$

$$S_{13}=(\Omega H^{\mu})_{\mu\nu\lambda\delta}(\Omega H^{\nu\lambda\delta})+3(\Omega H^{\mu})_{\mu\nu\lambda}(\Omega H^{\nu\lambda\delta})_{\delta}+3(\Omega H^{\mu})_{\mu\nu}(\Omega H^{\nu\lambda\delta})_{\lambda\delta}+S_1S_3\ ,$$

$$S_{112}=[S_{11\lambda\delta}(\Omega H^{\lambda\delta})+2S_{11\lambda}(\Omega H^{\lambda\delta})_{\delta}+S_{11}S_2\ ,$$

$$S_{121}=[(\Omega H^{\mu})_{\mu\nu\lambda\delta}(\Omega H^{\nu\lambda})+(\Omega H^{\mu})_{\mu\nu\lambda}(\Omega H^{\nu\lambda})_{\delta}+2(\Omega H^{\mu})_{\mu\nu\delta}(\Omega H^{\nu\lambda})_{\lambda}+(\Omega H^{\nu\lambda})_{\nu\lambda\delta}S_1+2(\Omega H^{\mu})_{\mu\nu}(\Omega H^{\nu\lambda})_{\lambda\delta}+(\Omega H^{\mu})_{\mu\delta}(\Omega H^{\nu\lambda})_{\nu\lambda}](\Omega H^{\delta})+S_{12}S_1\ .$$

Using the above definition, we have the adiabatic expansions

$$\langle\chi^2\rangle_A=-i\int d^4l(S_2+S_4+S_{11}+S_{13}+S_{22}+S_{31}+S_{112}+S_{121}+S_{211}+S_{1111}-1)\Omega\ , \tag{4.7}$$

$$\langle\chi\partial_\mu\chi\rangle_A=\int d^4l(S_1+S_3+S_{12}+S_{21}+S_{111})l_\mu\Omega\ , \tag{4.8}$$

$$\langle\chi\partial_\mu\partial_\nu\chi\rangle_A=i\int d^4(S_2+S_4+S_{11}+S_{13}+S_{22}+S_{31}+S_{112}+S_{121}+S_{211}+S_{1111}-1)l_\mu l_\nu\Omega\ , \tag{4.9}$$

and we can then from Eqs. (2.9) and (2.10) obtain the ARQST.

It can be easily checked that in the case of homogeneous spacetime the leading term in Eq. (4.7) will produce that in Ref. [1].

V. DISCUSSION

We shall make some remarks about our method.

(1) From the above results, we see that the ultraviolet divergence coming from the terms containing more spacetime derivatives is softer. Thus, to regularize the divergence appearing in the quantum stress tensor, we only need to subtract the terms with a small power of the spacetime derivative and this will be sufficient to find the exact form of the ARQST.

(2) We have presented the calculations containing the terms up to the fourth spacetime derivative, although in evaluating the ARQST not all the terms in Eqs. (4.7)–(4.9) will appear as ultraviolet divergence. The reason will be like that discussed in the original paper of Fulling, Parker, and Hu [1], in which it has been shown that the original mode sum can be renormalized by subtracting from it the quantity calculated to fourth adiabatic order. We leave this to the future a more detailed discussion and an explicit proof.

(3) To study the back-reaction problem, it is necessary to solve numerically the Einstein and Klein-Gordon equations. Although, taking the Fourier transform, defined in Eq. (3.2), of the wave equation [Eq. (2.7)] the mode functions will mix with each other, this will, however, not cause much difficulty. The reason is that for the large-k model the mode-mixing behavior will disappear asymptotically [10] and a conventional method [1] (neglect the mixing term) can be used to find the ARQST. Such a value of k is determined at the point in which the ARQST (the mode value before mode integration) is equal in both methods. Thus the numerical work of solving the mode-mixing wave equation and mode integration is only performed to this determined value of k.

(4) Finally, it can be seen that our method can be applied to the model with other matter fields including the gauge field and in more general spacetimes. This is a more realistic model to be studied.

In conclusion, we have succeeded in using the path-integral method to find the adiabatic regularized quantum stress-energy tensor in a general spacetime. Using this, one can then begin to analyze the problems of the back reaction of a quantum field in general spacetime, which will be studied in a future paper.

[1] L. Parker and S. A. Fulling, Phys. Rev. D **9**, 341 (1974); S. A. Fulling, L. Parker, and B. L. Hu, *ibid.* **10**, 3905 (1974).
[2] W.-H. Huang, Phys. Rev. D **43**, 1262 (1991); Class. Quantum Grav. **8**, 83 (1991).
[3] W.-H. Huang, Phys. Rev. D **46**, 2560 (1992).
[4] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
[5] J. Schwinger, Phys. Rev. **82**, 914 (1951); B. S. DeWitt, Phys. Rep. **19**, 295 (1975); P. C. W. Davies, S. A. Fulling, and W. G. Unruh, Phys. Rev. D **10**, 2720 (1976); S. M. Christensen, *ibid.* **17**, 946 (1978).
[6] B. L. Hu and L. Parker, Phys. Rev. D **17**, 933 (1978); T. S. Bunch, J. Phys. A **11**, 603 (1978); **13**, 1297 (1980); W. M. Suen and P. R. Anderson, Phys. Rev. D **35**, 2940 (1987); P. R. Anderson and L. Parker, *ibid.* **36**, 2963 (1987).
[7] G. Parisi and Y. Wu, Sci. Sin. **24**, 483 (1981); P. H. Damgaard and H. Hüffel, Phys. Rep. **152**, 227 (1987).
[8] R. Brandenberger, Rev. Mod. Phys. **57**, 1 (1985).
[9] T. S. Bunch and L. Parker, Phys. Rev. D **20**, 2499 (1979).
[10] W.-H. Huang, Phys. Lett. B **244**, 378 (1990).