Causal dissipative Bianchi cosmology

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(Received 19 May 1992)

With the aid of causal thermodynamics, the Bianchi type-I cosmological model is analyzed under the assumption of plane symmetry. The initial anisotropy gets rapidly extinguished, and for an ample range of values of the parameters describing the cosmic fluid one has inflationary expansion.

PACS number(s): 98.80.Hw

I. INTRODUCTION

It has been argued for a long time that dissipative processes in the early stages of cosmic expansion may well account for the high degree of isotropy we observe today and the huge value of the ratio of the number of photons to baryons as well. However, most of the work done in that direction so far (see [1,2], and references therein) was based on the standard stationary Eckart theory of irreversible processes [3], which is now known to be unsatisfactory since it allows for the occurrence of some undesirable effects incompatible with relativistic causality. Among them there is the propagation of dissipative signals with boundless speed.

Recently, a more suitable theory free of such drawbacks, the so-called "extended irreversible thermodynamics" (EIT) [4], as formulated by Israel [5], Pavón, Jou, and Casas-Vázquez [6], and Hiscock and Lindblom [7], has been applied to the study of nonequilibrium processes involved in cosmic evolution. As a consequence, some novel results generalizing and/or correcting those arising from the standard theory have emerged (see papers by Calvão and Salim [8] and Pavón, Bafaluy, and Jou [9], and references therein). The key features of EIT lies in its considering the equilibrium and dissipative variables on the same footing, a hypothesis well supported by statistical fluctuation theory and kinetic theory. This makes EIT a suitable theory for dealing with nonstationary processes, such as those expected to arise in the early Universe, as relaxation terms appear naturally in the transport equations for dissipative fluxes (for a review, see the article by Jou, Casas-Vázquez, and Lebon [10]). The purpose of this paper is to study the damping of anisotropy and entropy production in a Bianchi type-I universe filled with a dissipative simple fluid characterized by transport coefficients of bulk and shear stresses as well as by the corresponding relaxation terms. The symmetries of spacetime prevent heat flows from arising. We shall make use of EIT to make sure that relativistic causality is fully respected. However, as explained below, we are forced to restrict ourselves to the "truncated" version of the transport equations of EIT. Nevertheless, this limitation is not a very serious one since the truncation does not affect in any way the causality of the theory. Owing to the fact that the system of differential equations governing the evolution of the Universe cannot be solved analytically, we turn to a qualitative analysis of the equations, in the case of plane symmetry, and we next proceed to solve them numerically for some values of certain parameters defined below. Some numerical solutions are shown graphically.

It turns out that the initial anisotropy disappears quickly and, in general, neither the Friedmann nor the de Sitter states happen to be attractors. However, for a wide range of values of the parameters occurring in the equations, there exists an invariant stable submanifold of phase space where the latter exhibits asymptotic stability. The entropy production diverges for some values of the parameters, while for some others it dies away quickly.

It is worth mentioning that there exist a variety of mechanisms able to generate non-negligible viscous stresses [11], notably the one connected with the matter-radiation interaction. In addition to this, the existence of dark matter has recently suggested a new one [12]. Nonbaryonic dark matter seems to exist abundantly, pervading the whole Universe, and it is thought to interact with normal matter only gravitationally. This interaction may conceivably give rise, on the average, to a "drag effect" of one kind of matter on the other; this, in its turn, may be viewed phenomenologically as viscosity. This is the usual outcome of mixing up two different fluids.

The outline of this paper is as follows. Section II is devoted to presenting the above-mentioned differential equations and analyzing them qualitatively for the case of plane symmetry. In Sec. III the numerical analysis is carried out and the entropy production studied. Finally, in Sec. IV the main conclusions are summarized.

II. EVOLUTION EQUATIONS

We consider the Bianchi type-I universe with the metric

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$$ds^2 = -dt^2 + \sum_{i=1}^{3} R_i^2(t) dx_i^2 , \qquad (1)$$

where the functions R_i denote the scale factors. From these, the Hubble parameters $H_i = d(\ln R_i)/dt$, the average Hubble parameter $H = \frac{1}{3}\sum_{i=1}^{3} H_i$, and the total expansion $W = \sum_{i=1}^{3} H_i$ are defined.

The source of the gravitational field is a simple viscous fluid with a stress-energy tensor given by

$$T^{\mu\nu} = \rho u^\mu u^\nu + P_{\text{eff}} h^{\mu\nu} + \sigma^{\mu\nu} , \qquad (2)$$

where ρ denotes the energy density and P_{eff} the effective pressure,

$$P_{\text{eff}} = P + \sigma , \qquad (3)$$

P and σ being the hydrostatic and scalar viscous pressure, respectively, and $\sigma^{\mu\nu}$ the shear viscous pressure. $h^{\mu\nu}$ stands for the spatial projector $g^{\mu\nu} + u^\mu u^\nu$.

The Einstein field equations read

$$W^2 - \sum_{i=1}^{3} H_i^2 = 2\rho , \qquad (4)$$

$$\dot{H}_i + W H_i - \frac{1}{2}\left[2\dot{W} + W^2 + \sum_{i=1}^{3} H_i^2 \right] = P_{\text{eff}} + \frac{\sigma_{ii}}{R_i^2} \qquad (i = 1,2,3) . \qquad (5)$$

The trace condition takes the form

$$\dot{W} + W^2 = \tfrac{2}{3}(\rho - P_{\text{eff}}) . \qquad (6)$$

This set of equations has to be supplemented with the equation of state for the cosmic medium, which we take as a baratropic fluid, i.e.,

$$P = (\gamma - 1)\rho \qquad (7)$$

(the adiabatic index lies in the range $1 \le \gamma \le 2$, and it is assumed to be a constant or at least to vary much slower than any other variable in the theory) and with the transport equations for the dissipative stresses. The latter as given by EIT read

$$\sigma \left\{ 1 + \frac{1}{2} T\zeta \left[\frac{\tau_b}{T\zeta} u^\lambda \right]_{;\lambda} \right\} + \tau_b \dot{\sigma} = -\zeta W , \qquad (8)$$

$$\sigma^{\mu\nu} \left\{ 1 + \frac{1}{2} T\eta \left[\frac{\tau_s}{T\eta} u^\lambda \right]_{;\lambda} \right\} + \tau_s \dot{\sigma}^{\mu\nu} = -2\eta\left(h_\alpha^{(\mu} h_\beta^{\nu)} u^{(\alpha;\beta)} - \tfrac{1}{3} h^{\mu\nu} W \right) , \qquad (9)$$

where the second terms on the left-hand side of (8) and (9) stand for the relaxation terms alluded to above. These are absent from Eckart's transport equations since in that theory the unphysical assumption that they vanish is implicitly made. An overdot indicates derivation with respect to cosmic time, while ζ and η denote the bulk and shear viscosity coefficients, respectively. Also absent from Eckart's equations are the second terms within the curly brackets on the left-hand side of (8) and (9). These, which involve the temperature T of the cosmic fluid, do not vanish in general and may be significant, as proved by Hiscock and Salmonson [13], in some specific scenarios. However, because these terms involve the temperature of the fluid, they are very difficult to deal with. Effectively, in order to use them, a state equation relating the energy density ρ to T is needed. Unfortunately, for the rapidly expanding very early Universe, no such an equation, nor even an approximate one, is known. In these circumstances one may be tempted to resort to the relationship for thermal radiation, $\rho \propto T^4$. However, this would be preposterous since, on the one hand, the relationship is valid only for equilibrium and, on the other hand, the bulk viscosity coefficient would vanish, something that we do not wish as we are interested in studying the effects of bulk stresses on cosmic evolution. Any reliable expression linking ρ to T should depend very much on the different kinds of particle species making up the cosmic fluid and their proportions in it, something that no one knows. Confronted with this difficulty, the only sensible alternative is to drop the mentioned terms involving T from Eqs. (8) and (9) and to content ourselves with using the "truncated" transport equations instead:

$$\sigma + \tau_b \dot{\sigma} = -\zeta W , \qquad (8')$$

$$\sigma^{\mu\nu} + \tau_s \dot{\sigma}^{\mu\nu} = -2\eta\left(h_\alpha^{(\mu} h_\beta^{\nu)} u^{(\alpha;\beta)} - \tfrac{1}{3} h^{\mu\nu} W \right) . \qquad (9')$$

This obviously limits the range of our conclusions, which should be viewed as fully valid just for situations in which the divergence of the terms $(\tau_b / T\zeta) u^\lambda$ and $(\tau_s / T\eta) u^\lambda$ becomes vanishingly small. Of course, there is no reason *a priori* why it should be negligible, and what is more, Hiscock and Salmonson [13] have shown the importance of the first of these terms in the case of a Friedmann-Robertson-Walker universe with flat-space sections filled with a Boltzmann gas. However, from a cosmological point of view, the relevance of their findings is rather limited (as recognized by the authors) since a Boltzmann gas is not a realistic choice to illustrate cosmic evolution, for, among other things, the temperature, rather than decrease, tends asymptotically to a constant value as the Universe expands.

It is customary to express ζ and η in terms of the energy density according to

$$\zeta = \alpha \rho^m , \qquad (10)$$

$$\eta = \beta \rho^n , \qquad (11)$$

α and β being positive-definite parameters of first order satisfying the restrictions $\frac{1}{2} \le n - m$, $0 \le m \le \frac{1}{2}$. The relaxation times of bulk and shear stresses are given by

$$\tau_b = \zeta / \rho , \qquad (12)$$

$$\tau_s = \eta / \rho , \qquad (13)$$

respectively; these latter relationships follow from the requirement that the speeds of the dissipative signals do not exceed that of light in vacuum [14]. From (5) and (7), one obtains

$$P_{\text{eff}} = -\tfrac{1}{6} W^2 - \tfrac{1}{2} \sum_{i=1}^{3} H_i^2 - \tfrac{2}{3} \dot{W} , \qquad (14)$$

and, from (5) and (8)–(14),

$$\ddot{H}_i-\tfrac{1}{3}(\ddot{W}+2W\dot{W}-\dot{W}H_i)+2H_i(\dot{H}_i+WH_i-\tfrac{1}{3}W^2)+W\dot{H}_i+\beta^{-1}\left[\frac{D^2}{2}\right]^{1-n}\left[\dot{H}_i+WH_i-\tfrac{1}{3}(W^2+\dot{W})+2\beta\left[\frac{D^2}{2}\right]^{n}(H_i-\tfrac{1}{3}W)\right]=0\quad(i=1,2,3)\,, \tag{15}$$

whereas

$$\ddot{W}+2W\dot{W}-\frac{2}{3}\left\{(2-\gamma)\left[W\dot{W}-\sum_{i=1}^{3}H_i\dot{H}_i\right]+\alpha^{-1}\left[\frac{D^2}{2}\right]^{1-m}\left[\left[1-\frac{\gamma}{2}\right]D^2-\tfrac{2}{3}(\dot{W}+W^2)+\alpha\left[\frac{D^2}{2}\right]W\right]\right\}=0 \tag{16}$$

follows from (6) and (8)–(14). In order to simplify the notation, the short $D^2\equiv W^2-\sum_{i=1}^{3}H_i^2$ has been adopted.

Equations (15) and (16) are the evolution equations for the Hubble functions. In general, the latter must satisfy the energy conditions [15], namely, the weak energy condition (WEC)

$$\rho\geq 0\Longrightarrow D^2\geq 0\,, \tag{17}$$

the dominant energy condition (DEC)

$$\rho+P_{\rm eff}\geq 0\Longrightarrow \tfrac{1}{3}W^2-\sum_{i=1}^{3}H_i^2-\tfrac{2}{3}\dot{W}\geq 0\,, \tag{18}$$

and the strong energy condition (SEC)

$$\rho+3P_{\rm eff}>0\Longrightarrow \sum_{i=1}^{3}H_i^2+\dot{W}<0\Longrightarrow \dot{W}<0\,. \tag{19}$$

However, it is doubtful whether these conditions hold near the Planck era where strong quantum fields may dictate the cosmic evolution.

To facilitate the qualitative analysis of the system of equations (15) and (16), we restrict ourselves to the case of plane symmetry $H_2=H_3$. Admittedly, this is a limitation; nonetheless, we expect that no essential difference will arise with respect to the general case. Once the plane-symmetry assumption is taken up, it is easy to bring (15) and (16) into a first-order autonomous system:

$$\dot{H}_1=Y_1\,, \tag{20}$$

$$\dot{H}_2=Y_2\,, \tag{21}$$

$$\begin{aligned}\dot{Y}_1=&-\tfrac{2}{9}\dot{W}[4H_1+5H_2]-\tfrac{2}{3}W[2H_1^2-2H_2^2+Y_1-Y_2]\\&-\tfrac{4}{3}(H_1Y_1-H_2Y_2)+\tfrac{4}{9}W^2(H_1-H_2)-\frac{2}{3\beta}\left[\frac{D^2}{2}\right]^{1-n}\left\{Y_1-Y_2+(H_1-H_2)\left[W+2\beta\left[\frac{D^2}{2}\right]^{n}\right]\right\}\\&+\left\{(2-\gamma)[H_1Y_2+H_2Y_1+Y_2H_2]+\frac{1}{2\alpha}\left[\frac{D^2}{2}\right]^{1-m}\left[(2-\gamma)(2H_1H_2+H_2^2)-\tfrac{2}{3}(\dot{W}+W^2)+\alpha\left[\frac{D^2}{2}\right]^{m}W\right]\right\}\,,\end{aligned} \tag{22}$$

$$\begin{aligned}\dot{Y}_2=&-\frac{\dot{W}}{9}(5H_1+13H_2)+\frac{W}{3}(2H_1^2-2H_2^2+Y_1-Y_2)+\tfrac{2}{3}(H_1Y_1-H_2Y_2)-\tfrac{2}{9}W^2(H_1-H_2)\\&+(2-\gamma)(H_1Y_2+H_2Y_1+H_2Y_2)+\frac{1}{2\alpha}\left[\frac{D^2}{2}\right]^{1-m}\left\{(2-\gamma)(2H_1H_2+H_2^2)-\tfrac{2}{3}(\dot{W}+W^2)+\alpha\left[\frac{D^2}{2}\right]^{m}W\right\}\\&+\frac{1}{3\beta}\left[\frac{D^2}{2}\right]\left\{\dot{H}_1-\dot{H}_2+(H_1-H_2)\left[W+2\beta\left[\frac{D^2}{2}\right]^{n}\right]\right\}\,.\end{aligned} \tag{23}$$

Because of the algebraic difficulties inherent to this system, we have to give up finding all its singular points. Nevertheless, if we restrict ourselves to isotropic singular points, these can be found after letting $H_1=H_2=A$, A being a positive constant. From (22) it follows that

$$A^{2(1-m)}[A(\alpha 3^m A^{2m}-\gamma A)]=0\,. \tag{24}$$

The solutions of this equation determine the singular points. These are $Z_1\equiv(H_1=H_2=\dot{H}_1=\dot{H}_2=0)$, for $0<m<2$, a Friedmann-like state; and

$$Z_2\equiv\left[H_1=H_2=\left[\frac{3^m\alpha}{\gamma}\right]^{1/(1-2m)},\ \dot{H}_1=\dot{H}_2=0\right]\,,$$

for $m\neq\frac{1}{2}$, which corresponds to a de Sitter state. In the particular case $\gamma=3^m\alpha$, $m=\frac{1}{2}$, one has the critical point

$Z_3 \equiv (H_1 = H_2 = B, \dot{H}_1 = \dot{H}_2 = 0)$, where $B \in \mathbb{R}^+$.

Furthermore, (24) can be written as

$$\frac{\alpha 3^m - \gamma A^{1-2m}}{A^{-3}} = 0 , \tag{25}$$

and since

$$\lim_{A\to\infty} F(A) = \lim_{A\to\infty} \frac{\gamma(1-2m)}{3} A^{2(2-m)} , \tag{26}$$

where $F(A)$ denotes the left-hand side of (25), we have for $m = \frac{1}{2}$ and $m > 2$ the singular point $Z_4 \equiv (H_1 = H_2 = \infty, \dot{H}_1 = \dot{H}_2 = 0)$.

Likewise, the singular point $Z_5 \equiv (H_1 = H_2 = -\infty, \dot{H}_1 = \dot{H}_2 = 0)$, which follows too from (26), could be considered, but this point, along with Z_4, looks rather unphysical; therefore, we are not pursuing their study any further.

Except for the very particular case $n = m = 1$, the qualitative analysis of Z_1 is far too involved since the Jacobian vanishes and the singularity cannot be removed by a simple change of coordinates. Consequently, we have to content ourselves with the numerical analysis of next section. The qualitative analysis of the mentioned particular case ($n = m = 1$) is offered in the Appendix. It reveals that in this particular case Z_1 is not an attractor.

The qualitative analysis of Z_2 [with $A = (3^m \alpha / \gamma)^{1/(2m-1)}$, $m \neq \frac{1}{2}$] shows that the characteristic polynomial of the Jacobian matrix at Z_2 reads

$$P(\lambda) = (1-\lambda^2)[\lambda^2 - \lambda(Q_{11}+Q_{22}) + Q_{11}Q_{22} - Q_{12}Q_{21}] , \tag{27}$$

where

$$Q_{11} = -\tfrac{7}{2} A^2 - 2\beta^{-1} A (3A^2)^{1-n} + (m-2)\gamma A \alpha^{-1}(3A^2)^{1-m} ,$$

$$Q_{12} = 17 A^2 + 2\beta^{-1} A (3A^2)^{1-n} + 2(m-2)\gamma A \alpha^{-1}(3A^2)^{1-m} ,$$

$$Q_{21} = \tfrac{17}{2} A^2 + (m-2)\gamma A \alpha^{-1}(3A^2)^{1-m} + A\beta^{-1}(3A^2)^{1-n} ,$$

$$Q_{22} = 5A^2 + 2(m-2)\gamma A \alpha^{-1}(3A^2)^{1-m} - A\beta^{-1}(3A^2)^{1-n} .$$

From these expressions one has

$$Q_{11} + Q_{22} = 3A\{\tfrac{1}{2}A - 3\beta^{-1}(3A^2)^{1-n} + (m-2)\gamma\alpha^{-1}(3A^2)^{1-m}\} , \tag{28}$$

$$Q_{11}Q_{22} - Q_{12}Q_{21} = -A^2\{162A^2 + 36(m-2)\gamma A \alpha^{-1}(3A^2)^{1-m} + \tfrac{81}{2}\beta^{-1} A (3A^2)^{1-n} + 9(m-2)\gamma\alpha^{-1}\beta^{-1}(3A^2)^{2-m-n}\} . \tag{29}$$

For $m \geq 2$ one has $Q_{11}Q_{22} - Q_{12}Q_{21} < 0$, whence Z_2 is a saddle point. Accordingly, there exists a two-dimensional invariant manifold of solutions which tend to Z_2 as $t \to \infty$. For $m < 2$ the situation is a bit more involved. $Q_{11} + Q_{22}$ is negative if and only if

$$\beta^{-1}(3A^2)^{1-n} + (2-m)\gamma\alpha^{-1}(3A^2)^{1-m} > \tfrac{1}{2}A , \tag{30}$$

whereas $Q_{11}Q_{22} - Q_{12}Q_{21}$ is negative if and only if

$$162A^2 + \tfrac{81}{2}\beta^{-1} A (3A^2)^{1-n} > 9(2-m)\gamma\alpha^{-1}[4A(3A^2)^{1-m} + \beta^{-1}] . \tag{31}$$

If the last relationship is satisfied, the same result as that of the case $m \geq 2$, above, emerges. However, if the inequality signs in both (30) and (31) are reversed, the de Sitter state becomes unstable. Furthermore, if (30) holds and the inequality sign in (31) is reversed, then the de Sitter expansion turns out to be stable. It can be straightforwardly demonstrated that for $0 \leq m \leq 1/2$ and $0 < \alpha < 1$ the relationship (30) is satisfied quite independently of the values assumed by β and n. Likewise, it is a very easy matter to show that the relationship obtained from (31) by reversing the inequality sign there is satisfied for $0 \leq m \leq 1/2$ and $0 < \alpha < 1$ provided that $\gamma^2 > 3\alpha^2$. This result is independent of β and n as well. Accordingly, despite the de Sitter state not being an attractor in general, it is nonetheless stable for a wide range of values of the parameters describing the cosmic fluid.

It is worth noting that since A becomes very small for $m \gg 1$, Z_2 gets very close to Z_1. Hence we may loosely say that the qualitative analysis of Z_2 also applies to Z_1 when $m \gg 1$. However, it should be kept in mind that from a qualitative point of view the de Sitter and Friedmann states are altogether different.

III. NUMERICAL ANALYSIS

Owing to the fact that analytical solutions of the autonomous system of Eqs. (20)–(23) are very difficult to get, we have performed instead an extensive numerical analysis of the system in order to show those solutions that are not close to either of the singular points. As a matter of fact, the behavior of the solutions heavily depends on the parameter m and the difference between the initial values of the Hubble functions $H_1(0)$ and $H_2(0)$. The system is far more insensible to small changes in the values of the parameter n and the adiabatic index γ.

We can safely say that the initial anisotropy gets rapidly suppressed for any reasonable starting value of the Hubble parameters, whether they differ by two orders of magnitude or in sign or both. Furthermore, all the solutions fall into two categories: They end up either in a de Sitter or in a diverging state. No Friedmann-like solution was found. This, together with the qualitative analysis of Z_1 for $n = m = 1$ of the last section, reveals that Friedmann-like states are unstable. For illustrative purposes some solutions are shown graphically [Figs. 1(a), 2(a), and 3(a)] as well as the evolutionary behavior of $\rho + P_{\text{eff}}$ [Figs. 1(b), 2(b), and 3(b)] and $\rho + 3P_{\text{eff}}$ [Figs. 1(c), 2(c), and 3(c)]. We see that the de Sitter solution of Fig. 1(a) fails to comply with the SEC for late times. Figure 2(a) corresponds to an ever-diverging solution and is thereby incompatible with observation (aside from violating the SEC). However, Fig. 3(a), a de Sitter expansion again, satisfies the energy conditions; nevertheless, there is the problem of the exit from the inflation. In actual fact, what would be nice is a solution that after a period of isotropic inflation evolves toward a Friedmann-like expansion. We will have more to say about this in the next section.

Because of the presence of dissipative processes, there is a local entropy production given by [5–7]

$$\chi=\frac{1}{T}\left\{\frac{\sigma^2}{\zeta}+\frac{1}{2\eta}\sum_i \sigma^{ii}\sigma_{ii}\right\} \quad (i=1,2,3) , \tag{32}$$

with

$$\sigma=(2-\gamma)(2H_1H_2+H^2)-\tfrac{2}{3}(\dot W+W^2) , \tag{33}$$

$$\sigma_{ii}=R_i^2[\dot H_i+WH_i-\tfrac{1}{3}(\dot W+W^2)] . \tag{34}$$

We assume that the temperature of the cosmic fluid is a decreasing function of time, something very reasonable for expanding universes. By means of the numerical solutions of the autonomous system of Eqs. (20)–(23), we have numerically calculated the quantity $T\chi$ [Figs. 1(d), 2(d), and 3(d)]. Inspection of these figures show that, as expected, $T\chi$ remains constant for de Sitter evolutions, while it diverges for diverging Hubble functions.

IV. CONCLUDING REMARKS

We have studied the evolution of a Bianchi type-I universe with viscous dissipation. The relativistic causality of the approach is guaranteed by our use of EIT theory, even though we have used a "truncated" version

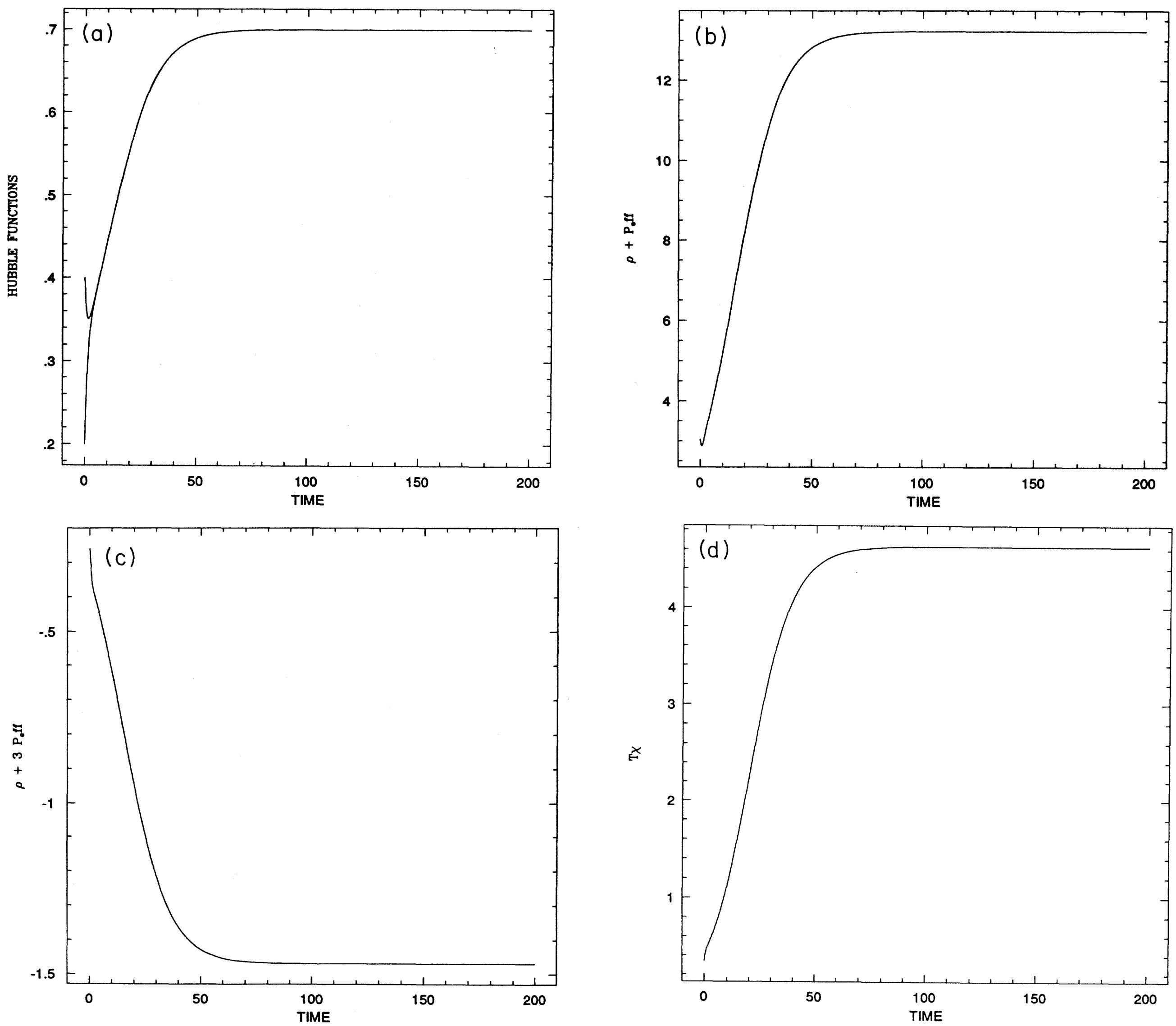

FIG. 1. Time evolution of (a) the Hubble functions and the quantities (b) $\rho+P_{\rm eff}$, (c) $\rho+3P_{\rm eff}$, and (d) $T\chi$ for the parameters values $m=0.4$, $n=3$, $\gamma=1.5$, $\alpha=0.5$, $\beta=0.9$, $H_1(0)=0.2$, $H_2(0)=0.4$, $\dot H_1(0)=0.10$, $\dot H_2(0)=-0.10$. The SEC is soon violated.

of it [Eqs. (8′) and (9′)] since, at present, there is no sensible way of using the full theory. Because of the latter, our results should be viewed just as provisional until the terms we have ignored in the full EIT transport equations (8) and (9) can be taken into account, i.e., until a nonequilibrium state equation $\rho=\rho(T)$ for the very early Universe becomes available. This we feel still lies a long way ahead.

The initial anisotropy vanishes rapidly, and both the qualitative and numerical analyses show that neither the Friedmann nor the de Sitter states are attractors. This latter finding is at variance with the results of Huang [16], who concludes that the Bianchi type-I model asymptotically evolves toward Friedmann or de Sitter expansions. The root of this discrepancy may be traced to his relying on the traditional Eckart theory of irreversible processes. Furthermore, he restricts himself to the case of stiff matter ($P=\rho$). However, as mentioned before, there is still a wide range of values of the parameters α, β, m, and n for which the universe undergoes a period of inflationary expansion. As is well known [17], many inflationary scenarios are beset with the problem of the exit from the inflation. In our case the exit arises naturally provided the main contribution to the bulk stress comes from the interaction between unstable superheavy particles (e.g., fundamental strings, gauge bosons, etc.) and relativistic particles. Once the former have decayed or got sufficiently diluted, the bulk pressure vanishes and the universe enters a radiation-dominated phase characterized by a stable Friedmann-like expansion [18].

Accordingly, our results hint that the Bianchi type-I cosmological model may well account for the main obser-

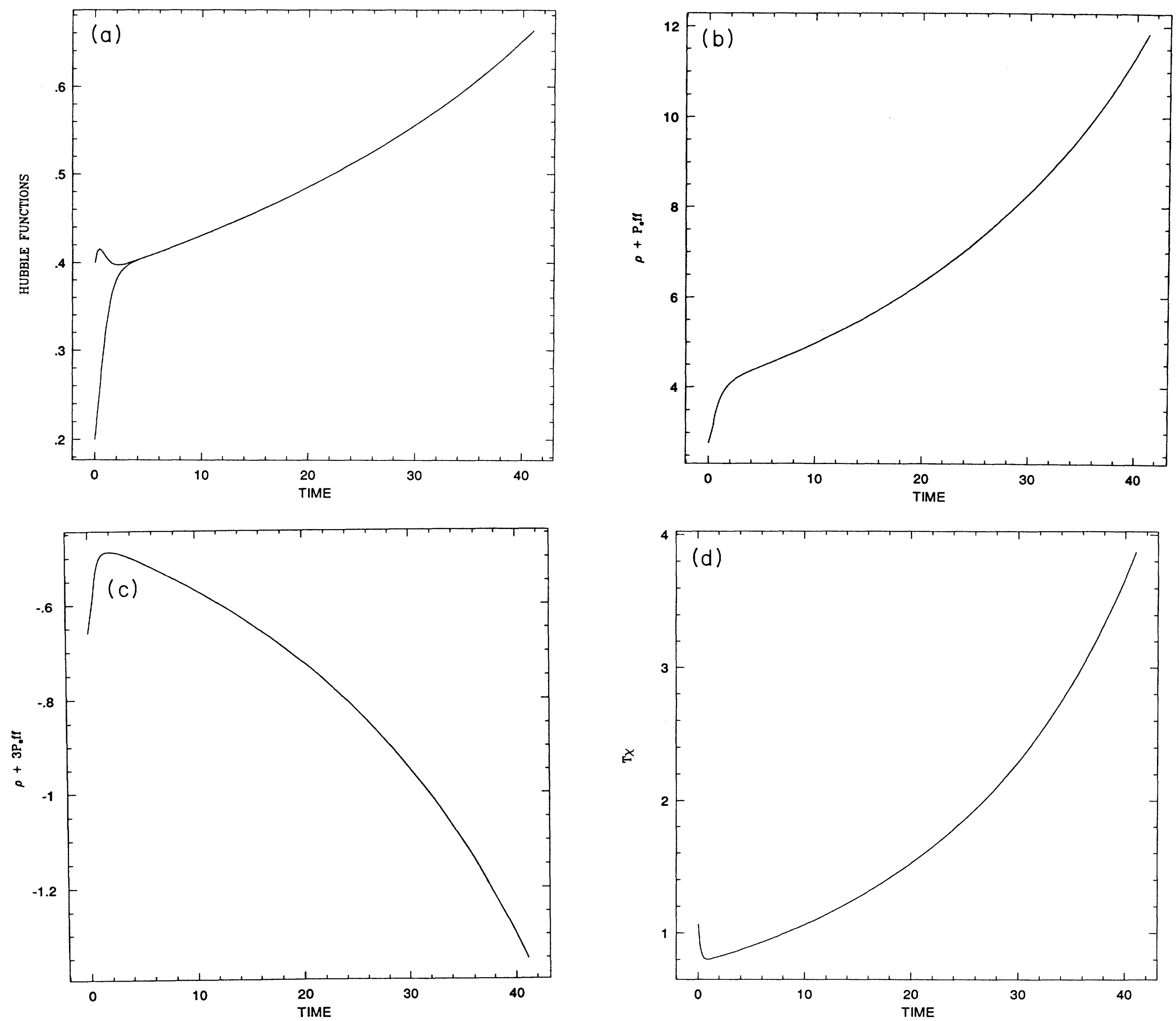

FIG. 2. Time evolution of (a) the Huble functions and the quantities (b) $\rho+P_{\rm eff}$, (c) $\rho+3P_{\rm eff}$, and (d) $T\chi$ for the parameters values $m=0.5$, $n=1$, $\gamma=1.5$, $\alpha=0.5$, $\beta=0.9$, $H_1(0)=0.2$, $H_2(0)=0.4$, $\dot{H}_1(0)=0.10$, $\dot{H}_2(0)=0.10$. Both Hubble functions diverge as well as $\rho+P_{\rm eff}$ and $T\chi$. The SEC is not satisfied at late times.

vational features of the Universe. It remains to be seen if for more general Bianchi models the range of fluid parameters allowing for inflationary expansion is widened or, on the contrary, shortened while still retaining the attractive feature of a quick decay of the initial anisotropy.

ACKNOWLEDGMENTS

One of the authors (V.R.) wishes to thank Professor A. M. Anile for useful suggestions and encouragement. This work has been partially supported by the Spanish Ministry of Education under Grant No. PB89-0290.

APPENDIX

In this appendix we detail the qualitative analysis of the singular point Z_1 for $n=m=1$. Linearization around Z_1 results in the dynamical system of equations,

$$\begin{aligned} 3\ddot{H}_1 &= -(2\beta^{-1}+\alpha^{-1})\dot{H}_1 + 2(\beta^{-1}-\alpha^{-1})\dot{H}_2 , \\ 3\ddot{H}_2 &= (\beta^{-1}-\alpha^{-1})\dot{H}_1 - (\beta^{-1}+2\alpha^{-1})\dot{H}_2 , \end{aligned} \tag{A1}$$

whose characteristic polynomial is found to be

$$P(\lambda) = \lambda^2 + (\beta^{-1}+\alpha^{-1})\lambda + \tfrac{5}{9}\alpha^{-1}\beta^{-1} + \tfrac{4}{9}\alpha^{-2} . \tag{A2}$$

The following cases arise;

(i) $9\alpha^2 - 7\beta^2 - 2\alpha\beta = 0$. In this case

$$\dot{H}_i = A_i \exp(\lambda t) + B_i t \exp(\lambda t) \quad (i=1,2) , \tag{A3}$$

with $\lambda < 0$. It can be trivially integrated to give

$$\begin{aligned} H_i = {} & A_i \lambda^{-1} \exp(\lambda t) \\ & + B_i \lambda^{-1} \exp(\lambda t)[t - \lambda^{-1}] + C_i , \end{aligned} \tag{A4}$$

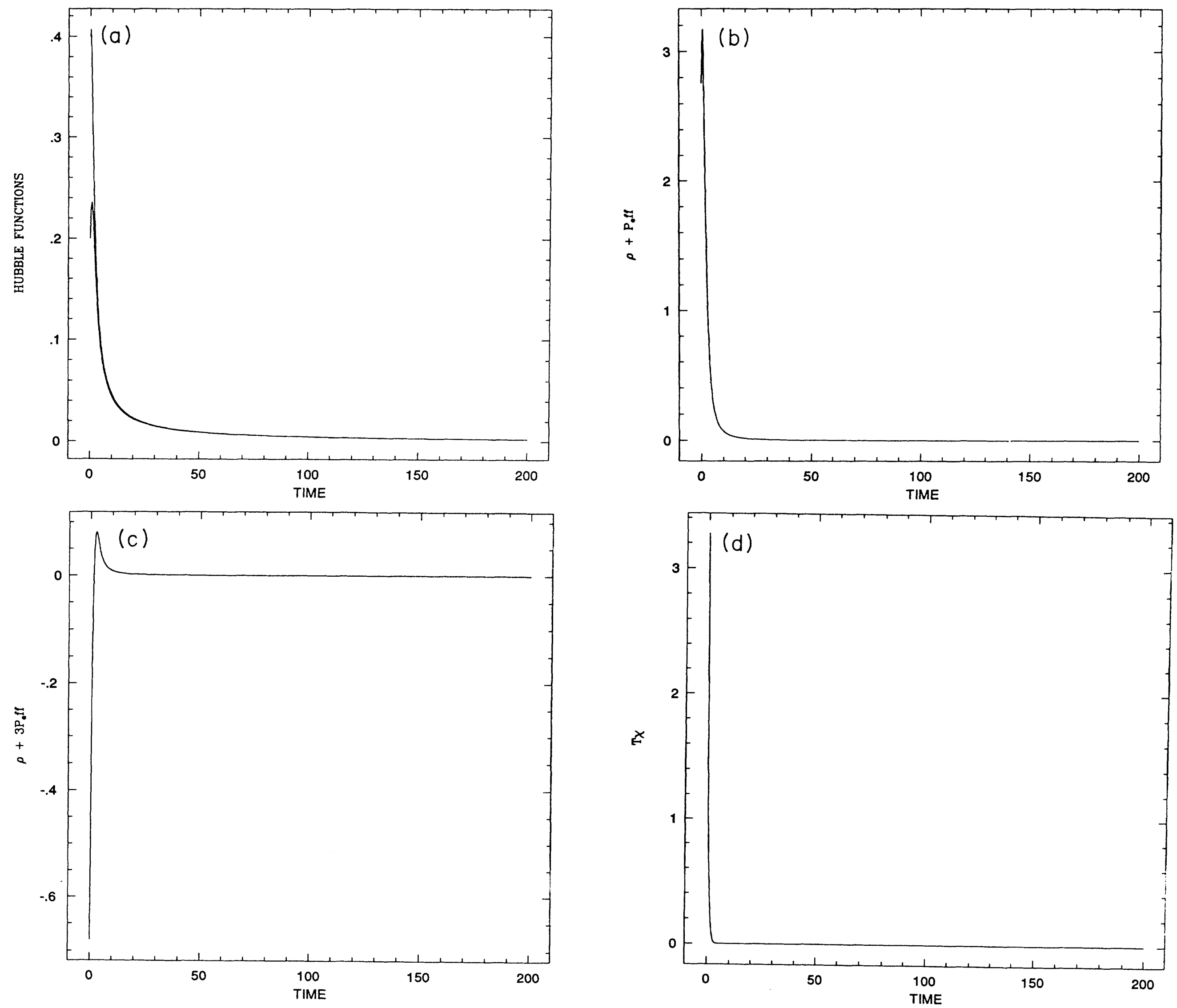

FIG. 3. Time evolution of (a) the Hubble functions and the quantities (b) $\rho + P_{\text{eff}}$, (c) $\rho + 3P_{\text{eff}}$, and (d) $T\chi$ for the parameters values $m=1.5$, $n=1$, $\gamma=1.5$, $\alpha=0.5$, $\beta=0.9$, $H_1(0)=0.2$, $H_2(0)=0.4$, $\dot{H}_1(0)=0.10$, $H_2(0)=0.11$. Both Hubble functions soon reach a common constant value (de Sitter expansion) and $T\chi$ vanishes almost at once. The SEC is not satisfied at late times.

where the six constants A_i, B_i, and C_i depend on the initial values of H_i and $\dot{H}_i$. Asymptotically, one has

$$\lim_{t\to\infty} H_i = C_i \ , \tag{A5}$$

whence the Friedmann state will be an attractor if and only if $C_1 = C_2 = 0$. However, from (A4) one has

$$C_i = H_i(0) - A_i\lambda^{-1} + B_i\lambda^{-2} \ , \tag{A6}$$

which leads to the conditions

$$H_i(0) = A_i\lambda^{-1} - B_i\lambda^{-2} \ , \tag{A7}$$

for the Friedmann state being an attractor. Obviously, these will not be met in general.

(ii) $9\alpha^2 - 7\beta^2 - 2\alpha\beta < 0$. In this case $P(\lambda)$ happens to have two complex roots: namely,

$$\lambda_{1,2} = \tfrac{1}{2}\{-(\alpha^{-1}+\beta^{-1}) \pm i\sqrt{-\Delta}\} \ ,$$

$$\Delta \equiv \frac{9\alpha^2 - 7\beta^2 - 2\alpha\beta}{9\alpha^2\beta^2} \ .$$

The Hubble functions take the form

$$H_i = A_i^* \exp(-\Gamma t)\sin\Theta t + B_i^* \exp(-\Gamma_t)\cos\Theta t + C_i^* \quad (i=1,2) \ , \tag{A8}$$

with $\Gamma \equiv (\alpha+\beta)\alpha^{-1}\beta^{-1}$ and $\Theta \equiv \sqrt{-\Delta}$. The constants A_i^*, B_i^*, and C_i^* depend on the initial conditions. A parallel reasoning to that of the previous case shows that the Friedmann state will not be an attractor in general.

(iii) $9\alpha^2 - 7\beta^2 - 2\alpha\beta > 0$. In this case the roots of $P(\lambda)$ are real and negative. The Hubble functions read

$$\begin{aligned} H_1 &= K_1 M \exp(\lambda_1 t) + K_2 N \exp(\lambda_2 t) + C_1' \ , \\ H_2 &= K_1 \exp(\lambda_1 t) + K_2 \exp(\lambda_2 t) + C_2' \ , \end{aligned} \tag{A9}$$

with

$$M = \frac{\alpha\beta[6\theta^* + \beta - \alpha]}{4(\alpha-\beta)} \ ,$$

$$N = \frac{\alpha\beta[-6\theta^* + \beta - \alpha]}{4(\alpha-\beta)} \ ,$$

and $\theta^* = \sqrt{\Delta}$. We parenthetically note that in this case α and β are necessarily different. The constants K_i and C_i' depend on the initial conditions and are given by

$$\begin{aligned} K_1 &= \dot{H}_2(0) - (N-M)[\dot{H}_1(0) - \dot{H}_2(0)] \ , \\ K_2 &= \dot{H}_2(0) - K_1 \ , \\ C_1' &= H_1(0) - K_1 M\lambda_1^{-1} - K_2 N\lambda_2^{-2} \ , \\ C_2' &= H_2(0) - K_1\lambda_1^{-1} - K_2\lambda_2^{-2} \ . \end{aligned}$$

Again, the Friedmann state will be realized if and only if $C_i' = 0$ $(i=1,2)$, that is to say, if the equations

$$\begin{aligned} H_1(0) &= K_1 M\lambda_1^{-1} + K_2 N\lambda_2^{-1} \ , \\ H_2(0) &= K_1\lambda_1^{-1} + K_2\lambda_2^{-1} \end{aligned} \tag{A10}$$

are satisfied. However, this can only happen for a very restricted set of initial conditions. Accordingly, we conclude by saying that Z_1 is not an attractor in general.

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