

## Inflating Lorentzian wormholes

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It has been speculated that Lorentzian wormholes of the Morris-Thorne type might be allowed by the laws of physics at submicroscopic, e.g., Planck, scales and that a sufficiently advanced civilization might be able to enlarge them to classical size. The purpose of this paper is to explore the possibility that inflation might provide a natural mechanism for the enlargement of such wormholes to macroscopic size. A new classical metric is presented for a Lorentzian wormhole which is embedded in a flat de Sitter space. It is shown that the throat and the proper length of the wormhole inflate. The resulting properties and stress-energy tensor associated with this metric are discussed.

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### I. INTRODUCTION

There has been much interest recently in the Lorentzian signature, traversable wormholes conjectured by Morris and Thorne (MT) [1,2]. These wormholes have no horizons and thus allow two-way passage through them. As a result, violations of all known energy conditions, including the weak energy condition (WEC) [3] and averaged weak energy condition (AWEC) [4], must unavoidably occur at the throat of the wormhole. Another disturbing (or intriguing, depending on one's point of view) property of these wormholes is the possibility of transforming them into time machines for backward time travel [5,6] and thereby, perhaps, for causality violation. Whether such wormholes are actually allowed by the laws of physics is currently unknown. However, recent research by Hawking [7] and others [8] indicates that it is very likely that nature employs a "chronology protection agency" which prevents the formation of closed timelike curves. The method of enforcement appears to be the divergences in vacuum expectation values of the stress-energy tensor of test fields which accompany the advent of any self-intersecting null geodesics. It appears that this behavior is generic with the formation of closed timelike curves [7,8]. At this point it is not clear whether these results imply that traversable wormholes cannot exist at all or that nature just does not permit their conversion into time machines.

It has been known for some time that quantum field theory allows local violations of the WEC [9] in the form of locally negative energy densities and fluxes, the most notable example being the Casimir effect [10]. A major unresolved issue is whether quantum field theory permits the macroscopic effects of negative energy required to maintain traversable wormholes against collapse. Wald and Yurtsever [11] have recently shown that the AWEC holds for massless scalar fields in a wide range of spacetimes, but that it apparently does not hold in an arbitrary curved four-dimensional spacetime. It is possible that although violations of the WEC (or AWEC) might be allowed, the magnitude and duration of these violations may be limited by uncertainty-principle-type inequalities

which could render gross macroscopic effects of negative energy unobservable. This appears to be the case for negative energy fluxes due to quantum coherence effects in flat spacetime [12]. Such quantum inequalities also appear to prevent the unambiguous observation of violations of cosmic censorship in the attempt to produce a naked singularity from an extreme Reissner-Nordström black hole, in both two and four dimensions [13]. Quantum inequalities also constrain the magnitude and duration of the negative energy flux seen by an observer freely into an evaporating two-dimensional Schwarzschild black hole [14].

Several equally important, though much less explored, questions are the following. (a) Do the laws of physics permit the topology change required to create the wormhole in the first place? In classical general relativity, such topology change must be accompanied by the creation of closed timelike curves [7,15]. Also, at least some topology change issues may be related to energy conditions [16]. (b) Do the laws of physics permit submicroscopic Lorentzian wormholes (e.g., on the Planck scale [17])? It may be that wormhole formation, although possibly prohibited on the classical level, might be allowed quantum mechanically. If so, then (c) are there processes, either natural or artificial, which could lead to their enlargement to classical size? The present paper will attempt to address one aspect of the last question.

MT suggest that "[o]ne can imagine an advanced civilization pulling a wormhole out of the quantum foam and enlarging it to classical size." This would seem to be, at best, wishful thinking. However, consider the following scenario. Suppose that a submicroscopic MT-type wormhole could form in the very early Universe via, say, a quantum fluctuation (the nature of which we will leave conveniently vague). Is it possible that subsequent inflation of the universe, if it occurs, could enlarge the wormhole to classical size? Or perhaps it might be possible to artificially enlarge a tiny wormhole by embedding it in a false-vacuum bubble, as in the "creation of a universe in the laboratory" scenario [18]. The inflation of quantum fluctuations of a scalar field has previously been invoked as a mechanism for providing the seeds of galaxy

formation [19]. Basu, Guth, and Vilenkin [20] have examined the nucleation and evolution of topological defects during inflation. Mallett [21] has modeled the effects of inflation on the evaporation of a black hole using a Vaidya metric embedded in a de Sitter background. His results suggest that inflation depresses the rate of black hole evaporation. Sato *et al.* [22] have studied the formation of a Schwarzschild–de Sitter wormhole in an inflationary universe. More recently, Kim [23] has constructed a traversable wormhole solution by gluing together two Schwarzschild–de Sitter metrics across a  $\delta$ -function boundary layer, following the methods of Visser [24]. Hochberg [25] has used a similar technique to construct Lorentzian wormhole solutions in higher-derivative gravity theories. Hochberg and Kephart [26] have argued that gravitational squeezing of the vacuum might provide a natural mechanism for the production of the negative energy densities required for wormhole support. However, recent work of Kuo and Ford [27] indicates that many states of quantized fields which involve negative energy densities are accompanied by large fluctuations in the expectation value of the stress-energy tensor. For such states the semiclassical theory of gravity may not be a good approximation. The states they examined included squeezed states and the Casimir vacuum state.

The outline of the present paper is as follows. In Sec. II a new class of metrics is presented which represents a Lorentzian wormhole embedded in a de Sitter inflationary background. The embedding is quite “natural” in that it does not involve “thin shells” or  $\delta$ -function “transition layers.” The stress-energy tensor of the false vacuum for de Sitter space barely satisfies the weak energy condition, since the energy density is exactly equal to minus the pressure. So these models couple “exotic” (i.e., energy-condition-violating) to “near-exotic” matter, in the terminology of MT. In the limit of vanishing cosmological constant, the metric reduces to the static MT traversable wormhole. It is demonstrated that both the throat and proper length of the wormhole inflate. The resulting stress-energy tensor is constructed by plugging the metric into the Einstein equations. (Although it is possible that such a metric might represent a wormhole which was “caught” in an inflationary transition, to show this definitively one would need to solve the opposite problem. That is, one would have to come up with a physically plausible stress-energy tensor and solve the Einstein equations to find the metric, and then show that the resulting solution had the desired wormhole characteristics. This is a much more difficult problem than the one treated here.) The properties of our metrics are discussed in Sec. III. We use the same metric and curvature conventions as MT [1], and we work in units where  $G=c=1$ .

## II. MORRIS-THORNE WORMHOLE IN AN INFLATING BACKGROUND

### A. Review of static Morris-Thorne wormholes

To make this paper relatively self-contained, we will review the results of MT [1]. The metric for a general

MT traversable wormhole is given by

$$ds^2 = -e^{2\Phi(r)} dt^2 + \frac{dr^2}{1-b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (2.1)$$

where the two adjustable functions  $b(r)$  and  $\Phi(r)$  are referred to as the “shape function” and “redshift function,” respectively. The shape function  $b(r)$  controls the shape of the wormhole as viewed, for example, in an embedding diagram. The metric Eq. (2.1) is spherically symmetric and static. The geometric significance of the radial coordinate  $r$  is that the circumference of a circle centered on the throat of the wormhole is given by  $2\pi r$ . The coordinate  $r$  is nonmonotonic in that it decreases from  $+\infty$  to a minimum value  $b_0$ , representing the location of the throat of the wormhole, and then it increases from  $b_0$  to  $+\infty$ . This behavior of the radial coordinate reflects the fact that the wormhole connects two separate external “universes” (or two regions of the same universe). At the throat, defined by  $r=b=b_0$ , there is a coordinate singularity where the metric coefficient  $g_{rr}$  becomes divergent, but the radial proper distance

$$l(r) = \pm \int_{b_0}^r \frac{dr}{[1-b(r)/r]^{1/2}} \quad (2.2)$$

must be required to be finite everywhere. At the throat,  $l=0$ , while  $l<0$  on the “left” side of the throat and  $l>0$  on the “right” side. For the wormhole to be traversable, it must have no horizons, which implies that  $g_{tt} = -e^{2\Phi(r)}$  must never be allowed to vanish. This condition in turn imposes the constraint that  $\Phi(r)$  must be finite everywhere.

To construct an embedding diagram [1,28] of the wormhole, one considers the geometry of a  $t=\text{const}$  slice. Using the spherical symmetry, we can set  $\theta=\pi/2$  (an “equatorial” slice). The metric on the resulting two-surface is

$$ds^2 = \frac{dr^2}{1-b(r)/r} + r^2 d\phi^2. \quad (2.3)$$

The three-dimensional Euclidean embedding space metric can be written as

$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2. \quad (2.4)$$

Since the embedded surface is axially symmetric, it can be described by  $z=z(r)$ , sometimes called the “lift function” (see [1,28]). The metric on the embedded surface can then be expressed as

$$ds^2 \left[ 1 + \left( \frac{dz}{dr} \right)^2 \right] dr^2 + r^2 d\phi^2. \quad (2.5)$$

Equation (2.5) will be the same as Eq. (2.4) if we identify the  $r, \phi$  coordinates of the embedding space with those of the wormhole spacetime and also require

$$\frac{dz}{dr} = \pm \left[ \frac{r}{b(r)} - 1 \right]^{-1/2}. \quad (2.6)$$

A graph of  $z(r)$  yields the characteristic wormhole pic-

tures found in [1,28]. For the space to be asymptotically flat far from the throat, MT require that  $dz/dr \rightarrow 0$  as  $l \rightarrow \pm\infty$ , i.e.,  $b/r \rightarrow 0$  as  $l \rightarrow \pm\infty$ . In order for this condition to be satisfied, the wormhole must flare outward near the throat, i.e.,

$$\frac{d^2r(z)}{dz^2} > 0, \quad (2.7)$$

at or near the throat. Therefore

$$\frac{d^2r(z)}{dz^2} = \frac{b-b'r}{2b^2} > 0, \quad (2.8)$$

at or near the throat,  $r=b=b_0$ , where the prime denotes differentiation with respect to  $r$ .

MT define an “exoticity function”

$$\xi \equiv \frac{\tau - \rho}{|\rho|} = \frac{b/r - b' - 2(r-b)\Phi'}{|b'|}, \quad (2.9)$$

where  $\rho$  and  $\tau$  are the energy density and radial tension, respectively, as measured by static observers in an orthonormal frame. MT show that Eq. (2.9) can be written as

$$\xi = \frac{2b^2}{r|b'|} \left[ \frac{d^2r(z)}{dz^2} \right] - \frac{2(r-b)\Phi'}{|b'|} \quad (2.10)$$

and argue (see Sec. III F2 of MT) that Eq. (2.10) reduces to

$$\xi_0 = \frac{\tau_0 - \rho_0}{|\rho_0|} > 0, \quad (2.11)$$

at or near  $r=b=b_0$ .

The general strategy is then to choose  $\Phi(r)$  and  $b(r)$  to get a “nice” wormhole and to compute the resulting stress-energy tensor components by plugging  $\Phi, b$  into the Einstein equations. One can show quite generally [1,5] that the resulting stress-energy tensor must violate all known energy conditions, including both the WEC and AWEC. It is known that quantum fields can violate the WEC [9]. Whether or not the laws of quantum field theory permit violations of AWEC large enough to support a macroscopic (or microscopic, for that matter) traversable wormhole is presently unknown [11].

One class of particularly simple solutions considered by MT is the class of so-called “zero-tidal-force” solutions, corresponding to the choice  $b=b(r)$ ,  $\Phi(r)=0$ . The choice of  $\Phi=0$  yields zero tidal force as seen by stationary observers. We write the metric for later reference as

$$ds^2 = -dt^2 + \frac{dr^2}{1-b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (2.12)$$

The energy density  $\rho(r)$ , radial tension per unit area,  $\tau(r)$ , and lateral pressure  $p(r)$  for this class of wormholes as seen by static observers in an orthonormal frame are given by

$$T_{\hat{t}\hat{t}} = \rho(r) = \frac{b'(r)}{8\pi r^2}, \quad (2.13)$$

$$-T_{\hat{r}\hat{r}} = \tau(r) = \frac{b(r)}{8\pi r^3}, \quad (2.14)$$

$$T_{\hat{\theta}\hat{\theta}} = T_{\hat{\phi}\hat{\phi}} = p(r) = \frac{b(r) - b'r}{16\pi r^3}. \quad (2.15)$$

Two examples of this class of wormholes are the following. The first is given by

$$b(r) = \frac{b_0^2}{r}, \quad \Phi(r) = 0. \quad (2.16)$$

This corresponds to

$$z(r) = b_0 \operatorname{arccosh} \left[ \frac{r}{b_0} \right], \quad (2.17)$$

which has the shape of a catenary, i.e.,

$$\frac{dz}{dr} = \frac{b_0}{(r^2 - b_0^2)^{1/2}}. \quad (2.18)$$

The wormhole material is everywhere exotic, i.e.,  $\xi > 0$  everywhere. It extends outward from the throat, with  $\rho$ ,  $\tau$ , and  $p$  asymptoting to zero as  $l = \pm\infty$ .

The second example corresponds to the confinement of the exotic matter to an arbitrarily small region around the throat. MT call this an “absurdly benign” wormhole. It is given by the choice

$$b(r) = \begin{cases} b_0 [1 - (r - b_0)/a_0]^2, \\ \Phi(r) = 0 \text{ for } b_0 \leq r \leq b_0 + a_0, \\ b = \Phi = 0 \text{ for } r \geq b_0 + a_0. \end{cases} \quad (2.19)$$

For  $b_0 < r < b_0 + a_0$ ,

$$\rho(r) = [(-b_0/a_0)/4\pi r^2][1 - (r - b_0)/a_0] < 0, \quad (2.20)$$

$$\tau(r) = b_0 [1 - (r - b_0)/a_0]^2 / 8\pi r^3, \quad (2.21)$$

$$p(r) = \frac{1}{2}(\tau - \rho). \quad (2.22)$$

For  $r \geq b_0 + a_0$ , the spacetime is Minkowski, and  $\rho = \tau = p = 0$ .

## B. $\Phi(r) \neq 0$ inflating wormholes

A simple generalization of the original MT wormhole metrics, characterized by Eq. (2.1), to a time-dependent inflationary background is

$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\chi t} \left[ \frac{dr^2}{1-b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]. \quad (2.23)$$

Here we have simply multiplied the spatial part of the metric Eq. (2.1) by a de Sitter scale factor  $e^{2\chi t}$ , where  $\chi = \sqrt{\Lambda/3}$  and  $\Lambda$  is the cosmological constant [29]. The coordinates  $r, \theta, \phi$  are chosen to have the same geometrical interpretation as before. In particular, circles of constant  $r$  are centered on the throat of the wormhole. Our coordinate system is chosen to be “comoving” with the wormhole geometry in the sense

that the throat of the wormhole is always located at  $r=b=b_0$  for all  $t$ . [Of course, this does not mean that two points at different (constant) values of  $r, \theta, \phi$  have constant proper distance separation.] For  $\Phi(r)=b(r)=0$ , our metric reduces to a flat de Sitter metric, while for  $\chi=0$ , it becomes the static wormhole metric Eq. (2.1). We may let  $\Phi(r)\rightarrow 0$ ,  $b/r\rightarrow 0$  as  $r\rightarrow\infty$ , so that the spacetime is asymptotically de Sitter or we may choose to let  $\Phi(r), b(r)$  go to zero at some finite value of  $r$ , outside of which the metric is de Sitter. The latter (together with a few other conditions) would correspond to a cutoff of the wormhole material at some fixed radius. Examples of each of these choices are given by Eqs. (2.16)–(2.18) and (2.19)–(2.22), respectively. However, our scheme should work for any of the original MT metrics. As before, we also demand that  $\Phi(r)$  be everywhere finite, so that the only horizons present are cosmological. The spacetime described by Eq. (2.23), unlike the usual flat de Sitter spacetime, is inhomogeneous because of the presence of the wormhole.

Our primary goal in this investigation is to use inflation to enlarge an initially small (possibly submicroscopic) wormhole. We choose  $\Phi(r)$  and  $b(r)$  to give a reasonable wormhole at  $t=0$ , which we assume to be the onset of inflation. To see that the wormhole expands in size, consider the proper circumference  $c$  of the wormhole throat,  $r=b=b_0$ , for  $\theta=\pi/2$ , at any time  $t=\text{const}$ :

$$c = \int_0^{2\pi} e^{\chi t} b_0 d\phi = e^{\chi t} (2\pi b_0). \quad (2.24)$$

This is simply  $e^{\chi t}$  times the initial circumference. The radial proper length through the wormhole between any two points  $A$  and  $B$  at any  $t=\text{const}$  is similarly given by

$$l(t) = \pm e^{\chi t} \int_{r_A}^{r_B} \frac{dr}{[1-b(r)/r]^{1/2}}, \quad (2.25)$$

which is just  $e^{\chi t}$  times the initial radial proper separation. Thus we see that both the size of the throat and the radial proper distance between the wormhole mouths increase exponentially with time.

To see that the “wormhole” form of the metric is preserved with time, let us embed a  $t=\text{const}$ ,  $\theta=\pi/2$  slice of the spacetime given by Eq. (2.23) in a flat three-dimensional Euclidean space with metric:

$$ds^2 = d\bar{z}^2 + d\bar{r}^2 + \bar{r}^2 d\phi^2. \quad (2.26)$$

The metric on our slice is

$$ds^2 = \frac{e^{2\chi t} dr^2}{1-b(r)/r} + e^{2\chi t} r^2 d\phi^2. \quad (2.27)$$

Comparing the coefficients of  $d\phi^2$ , we have

$$\bar{r} = e^{\chi t} r \Big|_{t=\text{const}}, \quad (2.28)$$

$$d\bar{r}^2 = e^{2\chi t} dr^2 \Big|_{t=\text{const}}. \quad (2.29)$$

With respect to the  $\bar{z}, \bar{r}, \phi$  coordinates, the “wormhole” form of the metric will be preserved if the metric on the embedded slice has the form

$$ds^2 = \frac{d\bar{r}^2}{1-\bar{b}(\bar{r})/\bar{r}} + \bar{r}^2 d\phi^2, \quad (2.30)$$

where  $\bar{b}(\bar{r})$  has a minimum at some  $\bar{b}(\bar{r}_0)=\bar{b}_0=\bar{r}_0$ . We can rewrite Eq. (2.27) in the form Eq. (2.30) by using Eqs. (2.28), (2.29), and

$$\bar{b}(\bar{r}) = e^{\chi t} b(r). \quad (2.31)$$

In particular, one can easily show that Eq. (2.31) is satisfied for the specific choices of  $b(r)$  given by Eqs. (2.16) and (2.19) by rewriting the right-hand sides of these equations in terms of  $\bar{r}$  and using Eq. (2.28). The inflated wormhole will have the same overall size and shape *relative to the  $\bar{z}, \bar{r}, \phi$  coordinate system*, as the initial wormhole had relative to the initial  $z, r, \phi$  embedding-space coordinate system. This is because the embedding scheme we have presented corresponds to an embedding space (or, more properly, a series of embedding spaces, each corresponding to a particular value of  $t=\text{const}$ ) whose  $z, r$  coordinates “scale” with time. To see this, we can follow the embedding procedure outlined in Eqs. (2.4)–(2.6), but using Eqs. (2.26) and (2.30). It is readily apparent that

$$\frac{d\bar{z}}{d\bar{r}} = \pm \left[ \frac{\bar{r}}{\bar{b}(\bar{r})} - 1 \right]^{-1/2} = \frac{dz}{dr}, \quad (2.32)$$

where we have used Eqs. (2.28), (2.29), and (2.31). Equation (2.32) implies

$$\begin{aligned} \bar{z}(\bar{r}) &= \pm \int \frac{d\bar{r}}{[\bar{r}/\bar{b}(\bar{r}) - 1]^{1/2}} \\ &= \pm e^{\chi t} \int \frac{dr}{[r/b(r) - 1]^{1/2}} = e^{\chi t} z(r). \end{aligned} \quad (2.33)$$

Therefore we see that the relation between our embedding space at any time  $t$  and the initial embedding space at  $t=0$  is, from Eqs. (2.29) and (2.33),

$$\begin{aligned} ds^2 &= d\bar{z}^2 + d\bar{r}^2 + \bar{r}^2 d\phi^2 \\ &= e^{2\chi t} [dz^2 + dr^2 + r^2 d\phi^2]. \end{aligned} \quad (2.34)$$

It is quite important to keep in mind (especially when taking derivatives) that Eqs. (2.28) and (2.29) do not represent a “coordinate transformation,” but rather a “rescaling” of the  $r$  coordinate on each  $t=\text{const}$  slice. Relative to the  $\bar{z}, \bar{r}, \phi$  coordinate system, the wormhole will always remain the same size; the scaling of the embedding space compensates for the expansion of the wormhole. Of course, the wormhole will change size relative to the initial  $t=0$  embedding space.

If we write the analogue of the “flareout condition” [Eq. (2.7)] for the expanded wormhole, we have

$$\frac{d^2 \bar{r}(\bar{z})}{d\bar{z}^2} > 0, \quad (2.35)$$

at or near the throat. From Eqs. (2.28), (2.29), (2.31), and (2.32), it follows that

$$\frac{d^2 \bar{r}(\bar{z})}{d\bar{z}^2} = e^{-\chi t} \left[ \frac{b-b'r}{2b^2} \right] = e^{-\chi t} \left[ \frac{d^2 r(z)}{dz^2} \right] > 0, \quad (2.36)$$

at or near the throat. Rewriting the right-hand side of Eq. (2.36) relative to the barred coordinates, we obtain

$$\frac{d^2\bar{r}(\bar{z})}{d\bar{z}^2} = \left[ \frac{\bar{b} - \bar{b}'\bar{r}}{2\bar{b}^2} \right] > 0, \quad (2.37)$$

at or near the throat, where we have used Eqs. (2.28), (2.31), and

$$\bar{b}'(\bar{r}) = \frac{d\bar{b}}{d\bar{r}} = b'(r) = \frac{db}{dr}. \quad (2.38)$$

We observe that relative to the barred coordinates the “flareout condition” [Eq. (2.37)] has the same form as that for the static wormhole. With respect to the unbarred coordinates, the flareout condition [Eq. (2.36)] appears as though it might be harder to satisfy as time goes on because of the decaying exponential factor. However, this is due to the fact that as the wormhole inflates its throat size and proper length inflate along with the surrounding space. It therefore necessarily needs to “flare outward” less and less at its throat as the two external spaces connected by the wormhole move farther apart (again, relative to the initial “ $t=0$ ” embedding space). This behavior is confirmed in an animated “toy” model of an inflating wormhole produced with MATHEMATICA [30], where  $b(r)$  is given by Eq. (2.16) [31].

Let us now examine the stress-energy tensor that gives rise to the wormhole described by Eq. (2.23). First, switch to a set of orthonormal basis vectors defined by

$$T_{\hat{\theta}\hat{\theta}} = T_{\hat{\phi}\hat{\phi}} = p(r, t) = \frac{1}{8\pi} \left\{ -3\chi^2 e^{-2\Phi} + e^{-2\chi t} \left[ \frac{1}{2} \left( \frac{b}{r^3} - \frac{b'}{r^2} \right) + \frac{\Phi'}{r} \left( 1 - \frac{b}{2r} - \frac{b'}{2} \right) + \left( 1 - \frac{b}{r} \right) [\Phi'' + (\Phi')^2] \right] \right\}. \quad (2.44)$$

The quantities  $\rho$ ,  $\tau$ ,  $f$ , and  $p$  are respectively, the mass-energy density, radial tension per unit area, energy flux in the (outward) radial direction, and lateral pressures as measured by observers stationed at constant  $r, \theta, \phi$ . Note from Eq. (2.43) that the flux vanishes at the wormhole throat, as it must by symmetry. If we let  $\Phi(r) \rightarrow 0$ ,  $b/r \rightarrow 0$  as  $r \rightarrow \infty$ , the stress-energy tensor components asymptotically assume their de Sitter forms, i.e.,

$$T_{\hat{t}\hat{t}} = -T_{\hat{r}\hat{r}} = -T_{\hat{\theta}\hat{\theta}} = -T_{\hat{\phi}\hat{\phi}} = 3\chi^2.$$

Alternatively, we may wish to cut off the wormhole material at some fixed radius  $r=R$ . A sufficient condition for doing this would be to let  $\Phi(r) = \Phi' = \Phi'' = b = b' = 0$  for  $r \geq R$ . For completeness, the Riemann curvature tensor components are also included in an Appendix. Note that all the stress-energy and curvature components are finite for all  $t$  and  $r$ . For  $\chi=0$ , our expressions reduce to those of MT [1]. [Note the correction of a sign error in the  $(\Phi'b/2r^2)$  term of  $G_{\hat{\theta}\hat{\theta}}$  in their Eq. (12)].

### C. Simple examples: the $\Phi(r)=0$ cases

A particularly simple example of an inflating wormhole is obtained by setting  $\Phi(r)=0$  in Eq. (2.23):

$$\begin{aligned} e_{\hat{t}} &= e^{-\Phi} e_t, \\ e_{\hat{r}} &= e^{-\chi t} (1 - b/r)^{1/2} e_r, \\ e_{\hat{\theta}} &= e^{-\chi t} r^{-1} e_\theta, \\ e_{\hat{\phi}} &= e^{-\chi t} (r \sin\theta)^{-1} e_\phi. \end{aligned} \quad (2.39)$$

This basis represents the proper reference frame of a set of observers who always remain at rest at constant  $r, \theta, \phi$ . The Einstein field equations will be written in the form

$$G_{\hat{\mu}\hat{\nu}} = R_{\hat{\mu}\hat{\nu}} - \frac{1}{2} g_{\hat{\mu}\hat{\nu}} R = 8\pi T_{\hat{\mu}\hat{\nu}}, \quad (2.40)$$

so that any “cosmological constant” terms will be incorporated as part of the stress-energy tensor  $T_{\hat{\mu}\hat{\nu}}$ . The components of  $T_{\hat{\mu}\hat{\nu}}$  are

$$T_{\hat{t}\hat{t}} = \rho(r, t) = \frac{1}{8\pi} \left[ 3\chi^2 e^{-2\Phi} + e^{-2\chi t} \frac{b'}{r^2} \right], \quad (2.41)$$

$$\begin{aligned} T_{\hat{r}\hat{r}} &= -\tau(r, t) \\ &= \frac{1}{8\pi} \left\{ -3\chi^2 e^{-2\Phi} - e^{-2\chi t} \left[ \frac{b}{r^3} - \frac{2\Phi'}{r} \left( 1 - \frac{b}{r} \right) \right] \right\}, \end{aligned} \quad (2.42)$$

$$\begin{aligned} T_{\hat{r}\hat{t}} &= -f(r, t) \\ &= \frac{1}{8\pi} \left[ 2e^{-\Phi - \chi t} \left( 1 - \frac{b}{r} \right)^{1/2} \chi \Phi' \right], \end{aligned} \quad (2.43)$$

$$ds^2 = -dt^2 + e^{2\chi t} \left[ \frac{dr^2}{1 - b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]. \quad (2.45)$$

The stress-energy tensor components in an orthonormal frame [Eq. (2.39) with  $\Phi=0$ ] become

$$T_{\hat{t}\hat{t}} = \rho(r, t) = \frac{1}{8\pi} \left[ 3\chi^2 + e^{-2\chi t} \frac{b'}{r^2} \right], \quad (2.46)$$

$$T_{\hat{r}\hat{r}} = -\tau(r, t) = \frac{1}{8\pi} \left[ -3\chi^2 - e^{-2\chi t} \left( \frac{b}{r^3} \right) \right], \quad (2.47)$$

$$T_{\hat{r}\hat{t}} = -f(r, t) = 0, \quad (2.48)$$

$$T_{\hat{\theta}\hat{\theta}} = T_{\hat{\phi}\hat{\phi}} = p(r, t) = \frac{1}{8\pi} \left[ -3\chi^2 + \frac{e^{-2\chi t}}{2} \left( \frac{b}{r^3} - \frac{b'}{r^2} \right) \right]. \quad (2.49)$$

The Riemann curvature tensor components for this metric are also included in the Appendix. Note that the stress-energy tensor and Riemann tensor components all approach their de Sitter space values for large  $t$ . [The

same is true for the expressions of these quantities associated with the metric Eq. (2.23), modulo some multiplicative factors of  $e^{-\Phi}$ , which would go to 1 outside the “wormhole” part of the spacetime, e.g., at large  $r$ .] When  $\chi=0$ , our metric reduces to that of a static “zero-tidal-force” wormhole [Eq. (2.12)].

### III. PROPERTIES OF THE SOLUTIONS AND DISCUSSION

A noticeable difference between the stress-energy tensors associated with the  $\Phi(r)\neq 0$  versus the  $\Phi=0$  wormholes is the presence of a flux term, given by Eq. (2.43). To understand this, we must clarify the difference between two “natural” coordinate systems associated with the wormhole. The first can be thought of as the rest frame of the wormhole geometry; i.e., an observer at rest in this frame is at constant  $r, \theta, \phi$ . The second can be thought of as the rest frame of the wormhole material. In the absence of a particulate model for the wormhole material, the best we can do is to define such a rest frame in terms of the properties of the stress-energy tensor. More specifically, we can define the rest frame of the wormhole material as the one in which an observer comoving with the material sees zero energy flux. From Eq. (2.43) we see that for  $\Phi(r)\neq 0$  the wormhole material is not at rest in the  $r, \theta, \phi$  coordinate system. For the  $\Phi(r)=0$  metrics given by Eq. (2.45), the two coordinate systems coincide.

Let  $U^\mu = dx^\mu/d\tau = (U^t, 0, 0, 0) = (e^{-\Phi(r)}, 0, 0, 0)$  be the four-velocity of an observer who is at rest with respect to the  $r, \theta, \phi$  coordinate system. The observer’s four-acceleration is

$$\begin{aligned} a^\mu &= \frac{DU^\mu}{D\tau} \\ &= U^\mu{}_{;v} U^v \\ &= (U^\mu{}_{,v} + \Gamma^\mu_{\beta\nu} U^\beta) U^\nu, \end{aligned} \quad (3.1)$$

which for the metric Eq. (2.23) gives the components

$$\begin{aligned} a^t &= 0, \\ a^r &= \Gamma^r_{tt} \left[ \frac{dt}{d\tau} \right]^2 \\ &= e^{-2\chi t} \Phi' (1 - b/r). \end{aligned} \quad (3.2)$$

From the geodesic equation, a radially moving test particle which is initially at rest has the equation of motion,

$$\frac{d^2 r}{d\tau^2} = -\Gamma^r_{tt} \left[ \frac{dt}{d\tau} \right]^2 = -a^r. \quad (3.3)$$

Therefore we see that  $a^r$  is the radial component of proper acceleration that an observer must maintain in order to remain at rest at constant  $r, \theta, \phi$ . From Eq. (3.3) it follows that for  $\Phi(r)\neq 0$  wormholes (whether static or inflating) such observers do not move geodesically (except

at the throat), whereas for  $\Phi(r)=0$  wormholes they do. In the  $\Phi(r)\neq 0$  case, for observers at fixed  $r, \theta, \phi$ ,

$$\frac{\partial}{\partial r} \left[ \frac{d\tau}{dt} \right] = \Phi' e^{\Phi(r)}. \quad (3.4)$$

Equation (3.4) can be thought of as the “radial gradient of the flow of proper time with respect to coordinate time.” Note that the flux component of the stress-energy tensor [Eq. (2.43)] goes like  $\chi\Phi'$ . It therefore depends both on the time dependence of the spatial part of the metric and on the “radial gradient of proper time flow.”

A wormhole will be called “attractive” if  $a^r > 0$  (observers must maintain an outward-directed radial acceleration to keep from being pulled into the wormhole) and “repulsive” if  $a^r < 0$  (observers must maintain an inward-directed radial acceleration to avoid being pushed away from the wormhole). For  $a^r = 0$ , the wormhole is neither attractive nor repulsive. The sign of the energy flux depends on the sign of  $\Phi'$  or, equivalently, on the sign of  $a^r$ . Since the flux  $f = -T_{\hat{r}\hat{r}}$ , then from Eq. (2.43) we see that if the wormhole is attractive, there is a negative energy flow out of it (or, equivalently, a positive energy flow into it); if it is repulsive, there is a negative energy flow into it (positive energy flow out of it). In the case where the wormhole material is cut off at a finite radius  $r=R$ , the energy flux vanishes at both  $r=R$  and  $r=b=b_0$ , though not necessarily inbetween. For this situation, we might think of the flux as being due to a redistribution of energy within the wormhole caused by its expansion.

The exoticity function [Eq. (2.9)] of MT can be written

$$\zeta = \frac{-T_{\hat{\rho}\hat{\nu}} W^{\hat{\rho}} W^{\hat{\nu}}}{|T_{\hat{r}\hat{r}}|}, \quad (3.5)$$

where  $W^{\hat{\rho}} = (W^{\hat{t}}, W^{\hat{r}}, 0, 0) = (1, \pm 1, 0, 0)$  is a radial outgoing (ingoing) null vector. This condition is, in some sense, a measure of the degree to which the wormhole material violates the WEC. In our case,

$$\zeta = \frac{\tau - \rho \mp f}{|\rho|}. \quad (3.6)$$

From Eqs. (2.41)–(2.43), it can be shown that

$$\begin{aligned} T_{\hat{\rho}\hat{\nu}} W^{\hat{\rho}} W^{\hat{\nu}} &= \frac{e^{-2\chi t}}{8\pi} \left[ \left[ \frac{b'}{r^2} - \frac{b}{r^3} \right] - \frac{2\Phi'}{r} \left[ 1 - \frac{b}{r} \right] \right] \\ &\quad \pm \frac{e^{-\chi t}}{4\pi} \left[ \left[ 1 - \frac{b}{r} \right]^{1/2} \chi \Phi' e^{-\Phi} \right]. \end{aligned} \quad (3.7)$$

For  $\Phi(r)=0$ , Eq. (3.6) reduces to

$$T_{\hat{\rho}\hat{\nu}} W^{\hat{\rho}} W^{\hat{\nu}} = \frac{e^{-2\chi t}}{8\pi} \left[ \frac{b'}{r^2} - \frac{b}{r^3} \right]. \quad (3.8)$$

Using Eq. (3.6), (3.7), and (2.8), the exoticity function at any radius and time can be written as

$$\zeta = \frac{e^{-2\chi t} \{ (2b^2/r^3) [d^2 r(z)/dz^2] + (2\Phi'/r)(1-b/r) \}}{|3\chi^2 e^{-2\Phi} + e^{-2\chi t} (b'/r^2)|} \mp \frac{2e^{-\chi t} [(1-b/r)^{1/2} \chi \Phi' e^{-\Phi}]}{|3\chi^2 e^{-2\Phi} + e^{-2\chi t} (b'/r^2)|}. \quad (3.9)$$

Comparing Eq. (2.10) with Eq. (3.9), we see that the relationship between the exotocity function and the flareout condition does not seem to be quite as simple as that for the static wormhole. The interpretation of Eq. (3.9) is complicated by the presence of the  $\chi^2$  term in the denominator, which could have the opposite sign from the  $b'$  term when the sign of the latter is negative, as well as by the addition of the flux term. If  $3\chi^2 e^{-2\Phi} \neq e^{-2\chi'(b'/r^2)}$  for all  $t$ , then, from Eq. (2.41),  $\rho$  is nonzero and finite. In this case the vanishing of terms such as  $\Phi'(1-b/r)$  at the throat and Eq. (2.8) allow us to write that

$$\zeta_0 > 0 \text{ at or near the throat, } r=b=b_0. \quad (3.10)$$

If  $\rho$  is nonzero and finite for all  $t$ , then it can be shown from Eq. (3.9) that the exotocity at the throat  $\zeta_0$  decays exponentially at large  $t$ . This is not terribly surprising in light of our earlier discussion regarding the “flareout” behavior of the wormhole throat during inflation.

Rather than examining the exotocity function, it is much simpler to just look at the WEC along the null vectors  $W^{\hat{\mu}}$  in the limit  $r \rightarrow b_0$ . At the throat this condition  $T_{\hat{\rho}\hat{\nu}} W^{\hat{\rho}} W^{\hat{\nu}} \geq 0$  simply reduces to the right-hand side of Eq. (3.8) evaluated at  $r=b=b_0$ , for both the  $\Phi \neq 0$  and  $\Phi=0$  cases. The term in parentheses is just the value of this expression at  $t=0$ , which is the same as that for the static wormhole and thus must be negative, from the original argument of MT. Therefore the violation of the WEC at the throat of the wormhole decreases exponentially with time.

To understand this behavior, one can give the following heuristic argument. Consider the simple static  $\Phi=0$  wormhole example given by Eqs. (2.13)–(2.16), for different throat sizes. For such a wormhole, the negative energy density, radial tension per unit area, and lateral pressure at the throat scale like  $1/b_0^2$ . They decrease in magnitude as the size of the throat increases. (Note, however, that for this wormhole the exotocity  $\zeta_0$  is independent of throat size.) This makes sense because the smaller the wormhole throat, the smaller its radius of curvature and hence the larger the curvature. The larger the curvature, the more “prone” is the wormhole to gravitational collapse, and therefore the larger the negative energy density required to hold it open. However, the total amount of negative energy near the throat scales like  $\rho V \sim (1/b_0^2 \times b_0^3) \sim b_0$  and therefore must increase as the throat size increases.

In general, because of the rapid expansion of the surrounding space, the two mouths of the wormhole will quickly lose causal contact with one another, i.e., they will move outside of each other’s cosmological horizon. Each mouth might reenter the other’s horizon after inflation [32]. If the mouths were to remain in causal contact throughout the duration of the inflationary period, then there would be a constraint on the initial size of the wormhole. To estimate this we will use the simple  $\Phi(r)=0$  wormhole metric [Eq. (2.45)]. Consider two observers stationed on opposite sides of the wormhole and separated by an initial radial proper distance at  $t=0$  of  $l_0$ . Let  $l(T)$  be their separation at the end of inflation,  $t=T$ . The proper distance  $l_H$  of each observer from

his/her horizon is  $l_H \sim 1/\chi$ . If we require that this distance be less than  $l(T)$ , then

$$l_0 < \frac{e^{-\chi T}}{\chi}. \quad (3.11)$$

For a typical inflationary scenario (see, for example, [33]),

$$\chi^{-1} \sim 10^{-34} \text{ sec} \sim 10^{-23} \text{ cm},$$

$\chi T \sim 100$ , which gives  $l_0 < 10^{-67} \text{ cm} \ll l_p \sim 10^{-33} \text{ cm}$ . Since the Planck length  $l_p$  is usually regarded as the smallest distance scale which makes physical sense, it seems that the condition Eq. (3.11) cannot be satisfied (at least in the usual inflationary scenarios). The same parameters yield an increase in size of the wormhole by a factor of  $\sim 10^{43}$ . An initially Planck-sized wormhole would be enlarged to a size of  $\sim 10^{10} \text{ cm} \sim 1R_{\odot}$  after inflation.

It is also possible that the wormhole will continue to be enlarged by the subsequent Friedmann-Robertson-Walker (FRW) phase of expansion. One could perform a similar analysis to ours by replacing the de Sitter scale factor in Eq. (2.23) by an FRW scale factor  $a(t)$ . A naive estimate yields a total enlargement of wormhole size which is larger than our present horizon size. However, since it is difficult to even say what effects the reheating at the end of inflation will have on the wormhole, we will not pursue this possibility further.

Since the two mouths of the wormhole lose causal contact during inflation, then presumably issues of traversability will arise only after inflation. In our discussion we have therefore avoided the enforcement of additional “usability criteria,” i.e., requirements proposed by MT which are designed to make wormhole traversal comfortable for human travelers. Also, a wormhole need not necessarily be traversable by human beings for it to be useful. Indeed, the more troubling characteristics of wormholes, such as their use for possible causality violation, should be realizable if it is possible to just send signals through them, in the form of light rays or particles. In passing, we again note that the Riemann curvature tensor components, given in the Appendix, are well behaved for all  $r$  and  $t$  (e.g., no “exponentially growing” tidal forces at the throat).

One might think that since two-way passage is practical only after inflation, the application of the present scenario to small ordinary Schwarzschild or Reissner-Nordström wormholes might yield large wormholes which could then later be made traversable. However, these wormholes have (noncosmological) horizons which tend to make them collapse very rapidly—an affliction which would probably be exacerbated by the positive energy released during the decay of the false vacuum. Assuming that one could circumvent the latter problem, then perhaps such a wormhole might be stabilized by the injection of a flux of negative energy. Unfortunately, the magnitude and duration of such fluxes would most likely be limited by “quantum-inequality-type” restrictions similar to those found to hold for negative fluxes injected into an extreme Reissner-Nordstrom black hole [13]. The same would likely be true for the pair-produced ex-

treme magnetically charged wormholes conjectured by Garfinkle and Strominger [34].

A nontrivial problem is the maintenance of the wormhole during and after the decay of the false vacuum. We saw earlier that although large (static) wormholes with  $\rho < 0$  required a smaller negative energy density for maintenance than small ones, the total amount of negative energy required should increase with increasing throat size. During inflation, the wormhole throat is greatly stretched in size because of the rapid cosmological expansion. However, note that in Eqs. (2.41)–(2.44) the “false-vacuum terms” remain constant with time while the “exotic wormhole material terms” decay exponentially with time. For example, in Eq. (2.41) the first term, which represents the energy density of the false vacuum, remains constant (at constant  $r$ ), while the second term, representing the “exotic” energy density of the wormhole, decreases with time. Consider the case where the latter is negative. Then the total amount of positive energy increases, since the positive energy density of the false vacuum remains constant as the volume increases. The total amount of negative energy decreases because the negative energy density exponentially decreases while the volume increases. When the false vacuum decays, the exponential stretching will cease and the positive energy in the false vacuum will be converted into more conventional forms, such radiation and/or particles. This potentially huge positive energy might flood the wormhole, triggering a gravitational collapse of the throat. Perhaps such a fate might be avoided if the two energy densities in Eq. (2.41) are roughly comparable in magnitude at the end of inflation.

As a simple example, let us first consider the inflating “absurdly benign” wormhole with  $\Phi$  and  $b$  given by Eq. (2.19). From Eq. (2.46), the energy density at the throat is

$$\rho_0 \sim 3\chi^2 - 2e^{-2\chi t} (b_0 a_0)^{-1}, \quad (3.12)$$

where  $a_0$  is the thickness (in  $r$ ) of the negative energy region near the throat. Let  $a_0 = \eta b_0$ , where  $\eta$  is some fraction, but require  $b_0 > l_p$ ,  $a_0 > l_p$ . For the two terms in Eq. (3.12) to be comparable at the end of inflation,  $t = T$ :

$$b_0 \sim \frac{e^{-\chi T}}{\eta\chi}. \quad (3.13)$$

which is almost identical to the condition Eq. (3.11). For the inflation parameters given earlier, we see that Eq. (3.13) also leads to a required initial wormhole size  $b_0 \ll l_p$ . One fares a little better with the  $\Phi(r) \neq 0$  wormhole. From Eq. (2.41), it appears that by making  $\Phi(b_0)$  large enough it might be possible to suppress the positive  $\chi^2$  “false-vacuum” term. The energy density at the throat goes like [35]

$$\rho_0 \sim 3\chi^2 e^{-2\Phi(b_0)} - (e^{-2\chi t}/b_0^2), \quad (3.14)$$

which leads to the condition that

$$b_0 \sim e^{\Phi(b_0)} \frac{e^{-\chi T}}{\chi}. \quad (3.15)$$

For  $b_0 \sim 10^{-33}$  cm,  $\Phi(b_0) \sim \ln(10^{34}) \sim 78$ . This corre-

sponds to a time dilation factor of  $d\tau/dt \sim 10^{34}$ ; i.e., clocks fixed at  $r = b_0$  must run  $\sim 10^{34}$  faster than clocks outside the wormhole.

These crude heuristic arguments suggest that in general it will be difficult for the negative-energy-density-type terms to overwhelm the false-vacuum-type terms. However, it should be mentioned that our simple argument does not take into account the effects of gravitational energy, and so it is not completely clear as to whether wormholes are unlikely to survive inflation. Also, the results in this paper represent only one possible generalization of MT wormholes to time-dependent situations. Even more general solutions might be obtained by allowing  $\Phi$  and  $b$  in our metrics to be functions of  $t$  as well as  $r$  [36].

On the other hand, if most of the wormholes in the quantum foam survived inflation, then the Universe might be far more inhomogeneous and topologically complicated than we observe [37] (unless they all inflated beyond our current horizon). Perhaps the wormholes were all destroyed by the flood of positive energy released during reheating. It is also possible that a given wormhole mouth might find itself in a slightly different gravitational potential from its counterpart. The quantum-field-theoretic instabilities associated with the tendency of such a wormhole to form closed timelike curves [6–8] might destroy it. Perhaps the probability for the existence of a wormhole in the quantum foam that has the right properties for inflation is extremely low, or perhaps none of the foam inflates (after all, galaxies in the FRW phase do not expand). Since we know very little about the quantum foam (or whether it even exists at all), these are difficult questions to answer. (The possibility of artificially enlarging a tiny wormhole by embedding it in a false-vacuum bubble is currently under investigation.)

Another part of the problem is that one does not know what constitutes a “generic” wormhole. In classical general relativity, the energy conditions determine the characteristics of “reasonable” sources. Quantum field theory allows *some* violation of the energy conditions, but with our present state of knowledge regarding the extent of these violations, we cannot yet say which types of wormholes, if any, are physically reasonable.

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## APPENDIX

The following curvature tensor components, as well as some of the stress-energy tensor components found in the



text, were computed using MATHTENSOR [38]. For the metric Eq. (2.23), the Riemann tensor components are

$$R_{\hat{t}\hat{t}\hat{t}\hat{t}} = -R_{\hat{t}\hat{t}\hat{t}\hat{t}} = -R_{\hat{t}\hat{t}\hat{t}\hat{t}} = R_{\hat{t}\hat{t}\hat{t}\hat{t}} \\ = -\chi^2 e^{-2\Phi} + e^{-2\chi t} (\Phi'/r^2)(r-b), \quad (\text{A1})$$

$$R_{\hat{t}\hat{t}\hat{r}\hat{r}} = R_{\hat{r}\hat{r}\hat{t}\hat{t}} = -R_{\hat{t}\hat{r}\hat{t}\hat{r}} = -R_{\hat{r}\hat{t}\hat{r}\hat{t}} = R_{\hat{r}\hat{t}\hat{r}\hat{t}} \\ = -\chi e^{-\chi t} (1-b/r)^{1/2} e^{-\Phi} \Phi', \quad (\text{A2})$$

$$R_{\hat{t}\hat{t}\hat{\theta}\hat{\theta}} = -R_{\hat{\theta}\hat{\theta}\hat{t}\hat{t}} = -R_{\hat{t}\hat{\theta}\hat{t}\hat{\theta}} = R_{\hat{\theta}\hat{t}\hat{\theta}\hat{t}} \\ = -\chi^2 e^{-2\Phi} + e^{-2\chi t} (\Phi'/r^2)(r-b), \quad (\text{A3})$$

$$R_{\hat{t}\hat{\theta}\hat{t}\hat{\theta}} = R_{\hat{\theta}\hat{t}\hat{\theta}\hat{t}} = -R_{\hat{t}\hat{\theta}\hat{t}\hat{\theta}} = -R_{\hat{\theta}\hat{t}\hat{\theta}\hat{t}} = R_{\hat{\theta}\hat{t}\hat{\theta}\hat{t}} \\ = -\chi e^{-\chi t} (1-b/r)^{1/2} e^{-\Phi} \Phi', \quad (\text{A4})$$

$$R_{\hat{r}\hat{r}\hat{r}\hat{r}} = -R_{\hat{r}\hat{r}\hat{r}\hat{r}} = -R_{\hat{r}\hat{r}\hat{r}\hat{r}} = R_{\hat{r}\hat{r}\hat{r}\hat{r}} \\ = -\chi^2 e^{-2\Phi} + e^{-2\chi t} (1-b/r) [\Phi'' + (\Phi')^2] \\ + (e^{-2\chi t}/2) \Phi' (b/r^2 - b'/r), \quad (\text{A5})$$

$$R_{\hat{r}\hat{\theta}\hat{r}\hat{\theta}} = -R_{\hat{\theta}\hat{\theta}\hat{r}\hat{r}} = -R_{\hat{r}\hat{\theta}\hat{r}\hat{\theta}} = R_{\hat{\theta}\hat{r}\hat{\theta}\hat{r}} \\ = \chi^2 e^{-2\Phi} + (e^{-2\chi t}/2) (b'/r^2 - b/r^3), \quad (\text{A6})$$

$$R_{\hat{\phi}\hat{\theta}\hat{\theta}\hat{\phi}} = -R_{\hat{\theta}\hat{\theta}\hat{\phi}\hat{\phi}} = -R_{\hat{\phi}\hat{\theta}\hat{\theta}\hat{\phi}} = R_{\hat{\theta}\hat{\phi}\hat{\theta}\hat{\phi}} \\ = \chi^2 e^{-2\Phi} + e^{-2\chi t} (b/r^3), \quad (\text{A7})$$

$$R_{\hat{\phi}\hat{r}\hat{r}\hat{\phi}} = -R_{\hat{r}\hat{r}\hat{\phi}\hat{\phi}} = -R_{\hat{\phi}\hat{r}\hat{r}\hat{\phi}} = R_{\hat{r}\hat{\phi}\hat{r}\hat{\phi}} \\ = \chi^2 e^{-2\Phi} + (e^{-2\chi t}/2) (b'/r^2 - b/r^3). \quad (\text{A8})$$

For the metric Eq. (2.45), the above components reduce to

$$R_{\hat{t}\hat{t}\hat{t}\hat{t}} = -\chi^2, \quad (\text{A9})$$

$$R_{\hat{t}\hat{t}\hat{\theta}\hat{\theta}} = -\chi^2, \quad (\text{A10})$$

$$R_{\hat{r}\hat{r}\hat{r}\hat{r}} = -\chi^2, \quad (\text{A11})$$

$$R_{\hat{\theta}\hat{r}\hat{\theta}\hat{r}} = \chi^2 + (e^{-2\chi t}/2) (b'/r^2 - b/r^3), \quad (\text{A12})$$

$$R_{\hat{\phi}\hat{\theta}\hat{\theta}\hat{\phi}} = \chi^2 + e^{-2\chi t} (b/r^3), \quad (\text{A13})$$

$$R_{\hat{\phi}\hat{r}\hat{r}\hat{\phi}} = \chi^2 + (e^{-2\chi t}/2) (b'/r^2 - b/r^3). \quad (\text{A14})$$

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- [33] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, New York, 1990), pp. 270–275.
- [34] D. Garfinkle and A. Strominger, *Phys. Lett. B* **256**, 146 (1991).
- [35] On dimensional grounds,  $b'/r^2$  must  $\sim 1/b_0^2$  at the throat. Here we have taken the case where  $b'$  is negative. However, it *need* not be. Recall that the violation of the WEC at the throat involves the combination  $\tau - \rho$  ( $f$  vanishes at the throat). Violation of the WEC implies that *some* observers will see negative energy densities, but those ob-
- servers need not be the “comoving” ones.
- [36] It is interesting that naively allowing  $\Phi$  and  $b$  to be functions of  $t$  as well as  $r$  in the original MT metric does not give a well-behaved, time-dependent wormhole solution. The flux term  $T_{\hat{r}\hat{r}} = (1/8\pi)(b_{,t}/r^2)(\exp[-2\Phi(r,t)])(1-b/r)^{-1/2}$  diverges at  $r=b=b_0$  [T. Roman (unpublished)]. The problem seems to come from the fact that if we demand that the geometric significance of  $r$  be that  $2\pi r$  is the circumference of a circle centered at the wormhole’s throat, then the *only* such circle which has any time dependence is the one at the throat  $r=b_0(t)$ .
- [37] R. Penrose, in *Proceedings of the Fourteenth Texas Symposium on Relativistic Astrophysics*, edited by E. J. Fenyves, [Ann. N.Y. Acad. Sci. **571**, 262 (1989)].
- [38] L. Parker and S. M. Christensen, MATHTENSOR (MathSolutions, Inc., Chapel Hill, NC, 1992).