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Discrete quark-lepton symmetry need not pose a cosmological domain wall problem

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Quarks and leptons may be related to each other through a spontaneously broken discrete symmetry. Models with acceptable and interesting collider phenomenology have been constructed which incorporate this idea. However, the standard hot big bang model of cosmology is generally considered to eschew spontaneously broken discrete symmetries because they often lead to the formation of unacceptably massive domain walls. We point out that there are a number of plausible quark-lepton-symmetric models which do *not* produce cosmologically troublesome domain walls. We also raise what we think are some interesting questions concerning anomalous discrete symmetries.

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I. INTRODUCTION

In the early 1960s, a disconcerting imbalance in the spectrum of quarks and leptons was uncovered. With the discovery that the muon-neutrino was a distinct flavor it seemed that there were four fundamental leptons, but only three quarks. Largely on the basis of aesthetics, several people speculated that this asymmetry would eventually be rectified by the discovery of a fourth quark [1]. Their sense of aesthetics was vindicated in the mid 1970s with the experimental detection of charm. The idea that quarks and leptons are paired up in each fermion generation is now a familiar and pleasing fact of nature.

Although the charm quark was introduced on the basis of a desired "symmetry" between quarks and leptons, there really is no symmetry in the rigorous sense of the word between these fermions in the standard model (SM). Quarks are colored; leptons are not. Leptons have integral charge; quarks do not. Quark and lepton masses are quite different. Furthermore, there is no definitive evidence for the existence of the right-handed neutrino, which is the putative partner of the right-handed up quark. Does all of this mean that the successful aesthetic of the 1960s is in truth only partially adhered to?

The answer is actually "that we do not know" rather than a loud "no." Recently it has become clear that quarks and leptons *might* be more closely related to each other than is currently evident. Furthermore, evidence for such a relationship could be uncovered at energy scales as low as a few hundred GeV. This represents an attractive confluence between theoretical speculation and hard-core phenomenology.

This speculative relationship between quarks and leptons involves the ideas of "leptonic color" and "discrete quark-lepton (q-ℓ) symmetry" [2]. It is a gauge-theoretic fact that the leptons we observe might be just the lightest components of triplets under a spontaneously broken SU(3) gauge symmetry for leptons. This leptonic color group, if it exists, would nicely reflect the attributes of its quark cousin. Quarks and leptons would appear much more like each other than they do in the standard model.

But having gone to the trouble of introducing a spontaneously broken leptonic color group, it is very tempting to push the quark-lepton association still further by postulating that a rigorous, but spontaneously broken, discrete symmetry exist between the two sectors. This would be the logical culmination of the primordial aesthetic which lead to the experimentally vindicated hypothesis of charm. Nature may or may not make use of leptonic color or discrete quark-lepton symmetry. But, it is surely very interesting to find out.[1]

Models with q-ℓ symmetry yield a rich nonstandard phenomenology [3]: exotic charge $\pm\frac{1}{2}$ fermions (liptons) confined by a new unbroken asymptotically free SU(2) gauge interaction, light exotic SU(2) glueball states [4], new heavy gauge bosons and a number of new Higgs bosons. Since much of this new phenomenology is al-

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[1] It is also interesting to wonder about why the quark-lepton symmetry idea took so long to be proposed. A possible reason is that grand unified theories (GUT's) were proposed very soon after the wide-spread acceptance of the SM in the early 1970s. This alternative way of connecting the quarks and the leptons became, and still is, very influential, and so people may have felt that nothing qualitatively different from this was possible.

lowed to exist in the 100 GeV to 1 TeV energy regime, q-ℓ symmetric models should be of great interest to the phenomenologists and experimentalists of today.

Despite the appeal of discrete q-ℓ symmetry from a purely particle physics perspective, aficionados of the now standard hot big bang model (HBBM) of cosmology are likely to be less than enthusiastic about it, for reasons we will now explain.

In its simplest form, discrete q-ℓ symmetry is isomorphic to the group Z_2. When a Z_2 discrete symmetry spontaneously breaks, the vacuum manifold consists of two disconnected pieces which are related by a Z_2 transformation. If we ignore all of the other isometries of this manifold, then we can consider it to consist of only two (isolated) states which can be transformed into each other by the discrete symmetry. Since the actual vacuum state in a causally connected region of spacetime has to evolve into a unique state after a suitable relaxation time, one of these two candidate vacua is selected as the actual vacuum. A cosmological problem arises here, however, because spacetime immediately after the big bang consists of causally *disconnected* regions. This means that, at the time of the cosmological phase transition associated with the spontaneous breaking of the discrete symmetry, randomly different choices for the actual vacuum will in general be made in the various causally disconnected pieces of spacetime. But, as the Universe continues to expand after this phase transition, regions that previously had no influence over each other come into causal contact. Two such regions that happen to have different vacuum states therefore have to form a domain wall structure at their boundary, if there is insufficient energy to flip the vacuum state in one of the domains. (Although our discussion was restricted to the Z_2 case for simplicity, the existence of domain walls follows for all discrete symmetries.)

This reasoning is born out by examining the classical solutions of field theories which display spontaneously broken discrete symmetries, because they include solutions describing topologically stable domain walls. Using these classical solutions, one can calculate the energy per unit area of a domain wall and hence conclude that such structures should dominate the energy content of the observable Universe (unless the scale of discrete symmetry breaking is much less than the electroweak scale). Since this is contrary to observation, theories predicting stable domain walls are inconsistent with the HBBM of cosmology [5]. The purpose of this paper is to show that certain classes of q-ℓ symmetric theories evade this potential problem.

The conclusion that domain walls are a cosmological calamity relies on a number of assumptions: (i) that the HBBM is the correct model to use; (ii) that the domain walls are stable; (iii) that there is a cosmological phase transition associated with the spontaneous breaking of the discrete symmetry (in other words, that there exists a critical temperature T_c above which the discrete symmetry is restored); (iv) that an inflationary period did not occur after the discrete symmetry phase transition; and (v) that the two states in the vacuum manifold are really degenerate. There may also be other important assumptions that we have failed to notice.

In the remainder of this paper we will discuss each of these five assumptions with special emphasis on their role in determining the cosmological consequences of spontaneously broken q-ℓ symmetry. In Sec. II we discuss the status of the HBBM and its connection with particle physics [see assumption (i) above]. This puts into perspective the analysis that is to follow. Section III is devoted to a brief review of the minimal q-ℓ symmetric model. We then go on to show in Secs. IV–VI that assumptions (ii), (iii), and (iv), respectively, need not hold in plausible q-ℓ symmetric models, thereby demonstrating the existence of cosmologically benign gauge theories with discrete q-ℓ symmetry. In all of these cases, we will emphasize that the resulting theories can yield interesting new phenomenology in the 100 GeV to 1 TeV regime. We will pose some interesting questions concerning assumption (v) in Sec. VII, but we will not be able to answer them fully. Section VIII contains our conclusions.

II. STATUS OF THE HOT BIG BANG MODEL

In the last decade or so, the fields of cosmology and particle physics have become deeply intertwined. The classical evidence used to support the HBBM (redshifts and the blackbody microwave background) can now be augmented by precise calculations relating the abundances of the light nuclei H, ^{4}He, D, ^{3}He, and ^{7}Li to the expansion rate of the Universe about 1 second after the big bang. The expansion rate in turn can be connected with the number of relativistic degrees of freedom at the time of nucleosynthesis. If the SM of particle physics is used, then the only relativistic particles during this era are electrons, positrons, photons, neutrinos, and antineutrinos. Agreement between theory and observation is achieved only if the number of light neutrino flavors is three, which concurs with the number of light neutrino species that have been determined by measurements of the Z width at the CERN e^+e^- collider LEP. This establishes an interesting quantitative link between cosmology and particle physics.

Because of the success so far obtained through this linkage of microscopic with macroscopic physics, the derivation of "cosmological constraints" on nonstandard particle physics models has become commonplace. One such constraint says that domain walls formed from the spontaneous breaking of a discrete symmetry are cosmologically ruinous, unless the breaking scale is really very low. This sort of practice has now become so deeply ingrained that we feel a few paragraphs devoted to a critical assessment of its validity are warranted. While some may claim that many of the views to be expressed below are well understood and appreciated by the community, we believe that there is considerable value in them appearing explicitly stated in a contemporary paper on the cosmological ramifications of an extension of the SM of particle physics.

Why should the validity of cosmological constraints on particle physics be questioned? There are basically two reasons. First, although the HBBM of cosmology is an impressive scenario, it is in the nature of cosmology that detailed experimental and/or observational data are hard to obtain, and so all such models face a serious problem with *testability*. Second, the naive HBBM has some theoretical shortcomings despite its acknowledged success, and so it cannot be accepted completely without reservation.

As an example of the first difficulty, let us have a closer look at big bang nucleosynthesis (BBN). In order for these calculations to come out correctly, we need to postulate several times the number density of baryons that we can readily account for in luminous bodies. *Until this baryonic dark matter is found, there is a serious loose end in BBN.* We feel that this obvious point is not emphasized enough in the literature. There is some hope that these extra baryons may be located in galactic haloes. Astronomers are currently trying to test this idea through the observation of microlensing. This hope may or may not be realized. So, we should be aware that one of the three legs of the tripod upon which the HBBM stands is yet to be thoroughly checked out.

This is just an example of the type of testability problem the HBBM has. Another obvious prediction yet to be verified is the existence of a relic neutrino background. Of course this background is extremely difficult to detect. However technical difficulty does not absolve us of the requirement that scientific theories have to be well tested before they can be considered as established beyond reasonable doubt.

As a completely general statement, we should understand clearly that cosmological models can only be tested by looking at the present day structure of the Universe and interpreting various objects as "relics" from an earlier epoch. No direct test of the evolution of the Universe as such can ever be done, for obvious reasons. Therefore, cosmological models will never be as testable as, say, those particle physics models that are humble enough to pertain to terrestrially accessible energy scales.

Let us now discuss the theoretical shortcomings of the HBBM. The evident large-scale homogeneity and isotropy of the observed Universe (its "smoothness") is at odds with the existence of causally disconnected regions of spacetime in the early Universe. For instance, the HBBM asserts that the observable Universe of today evolved from about 10^6 causally disconnected spacetime volumes at the time of radiation decoupling. How then can the isotropy of the microwave background radiation be explained? On another tack, we know observationally that the Universe is very close to being spatially flat. However, the Einstein equations describing the expansion of the Universe require very special initial conditions in order to bring this about. In particular, the average mass density of the Universe must be equal to the critical density to one part in 10^{59} at the Planck time. Such a special value demands an explanation which is not forthcoming in the HBBM.

An interesting hypothesis advanced to rid the HBBM of the smoothness and flatness problems is that of inflation [6]. This phenomenon allows the present day universe to have evolved from within a causally connected region of spacetime, and thus the smoothness of our Universe is no longer a mystery. Also, the spatial metric after the inflationary epoch is flat to an extremely high precision. As a bonus, the inflationary scenario also provides a framework for the formation of large scale structure [7]. There are also other ideas concerning the smoothness problem [8].

Whatever scenario one adopts in response to this problem, its characteristic predictions will have to be observationally verified. For instance, a major prediction of inflationary models is that the energy density of the Universe is equal to the critical density at an extraordinarily high precision. Thus far observation has only limited the energy density to within an order of magnitude or so of this critical value, which is not nearly precise enough to be considered a good test of inflation. In addition, the nature of the "dark matter" is crucial to the formation of large scale structure. For inflation to be well tested, a detailed model of structure formation together with the appropriate experimentally verified dark matter is necessary. In addition, a well-motivated, theoretically consistent, and experimentally verified Higgs field to drive inflation is needed (for a recent suggestion about how inflation may be rendered more experimentally testable see Ref. [9]). We therefore conclude that although inflation is an *interesting* idea, it is not a *well-tested* idea (the same conclusion holds for the alternative suggestions of Ref. [8] as well). And since without inflation (or the other ideas) the HBBM has theoretical deficiencies, it is sensible to spend some effort in searching for alternatives to the HBBM framework itself [10].

By way of contrast, the testability of the SM of particle physics is manifest; extremely detailed and repeatable experiments can be performed in the laboratory. It is therefore somewhat ironic that great currency is given to constraints on new particle physics derived by demanding that the standard cosmological scenario not be disturbed. Furthermore, low-energy extensions of the SM are testable at terrestrial facilities, and so we do not need to use cosmology to evaluate these theories. Since cosmological models can only be tested with quite limited precision, it is not reasonable to view cosmological constraints completely without suspicion. Indeed, we may unwisely dismiss some interesting and potentially important ideas in particle physics if we take cosmological constraints as completely rigorous. On the other hand the HBBM (or its inflationary extension) is an impressive scenario that seems to be consistent with all available observational data. Therefore we conclude that although the compatibility of spontaneously broken discrete q-ℓ symmetry (and any other idea which appears to have cosmological problems) with standard cosmology is not necessary for it to be an important idea, it is nevertheless *interesting* to see under what circumstances it is in fact compatible. As we have said, the purpose of this paper is to study these circumstances.

III. THE MINIMAL QUARK-LEPTON SYMMETRIC MODEL

In the following we will give a brief summary of the essential features of the minimal quark-lepton symmetric model [2,3]. This will serve as a starting point from which discussions of other models involving q-ℓ symmetry are based while at the same time establishing the notation of the paper.

The minimal gauge model illustrating the basic idea of q-ℓ symmetry is obtained by enlarging the standard model gauge group to $G_{q\ell}$, where

$$G_{q\ell} = \mathrm{SU}(3)_\ell \otimes \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_X. \tag{1}$$

Here $\mathrm{SU}(3)_q$ is the usual color group and $\mathrm{SU}(3)_\ell$ is its leptonic partner. This enlargement requires a tripling in the number of leptons. Each standard lepton (the left-handed electroweak doublet f_L, the right-handed charged lepton e_R, and the right-handed neutrino ν_R) has two exotic partners, hereafter called "liptons." The expanded fermionic generation is defined by the transformation laws

$$\begin{aligned} &F_L \sim (3,1,2)(-1/3), \quad E_R \sim (3,1,1)(-4/3), \quad N_R \sim (3,1,1)(2/3), \\ &Q_L \sim (1,3,2)(1/3), \quad u_R \sim (1,3,1)(4/3), \quad d_R \sim (1,3,1)(-2/3). \end{aligned} \tag{2}$$

The standard lepton doublet f_L is embedded in F_L, e_R in E_R, and ν_R in N_R. The Z_2 discrete symmetry

$$\begin{aligned} &F_L \leftrightarrow Q_L,\ E_R \leftrightarrow u_R,\ N_R \leftrightarrow d_R, \\ &G_q^\mu \leftrightarrow G_\ell^\mu,\ C^\mu \leftrightarrow -C^\mu \end{aligned} \tag{3}$$

can now be defined [where $G^\mu_{q,\ell}$ are the gauge bosons of $\mathrm{SU}(3)_{q,\ell}$ and C^μ is the gauge boson of $\mathrm{U}(1)_X$]. Standard hypercharge is identified as the linear combination $X+\frac{1}{3}T$ where $T = \mathrm{diag}(-2,1,1)$ is a generator of $\mathrm{SU}(3)_\ell$. Standard leptons are identified with the $T=-2$ components of the leptonic color triplets, while the $T=1$ components are the charge $\pm 1/2$ liptons.

In order to spontaneously break $\mathrm{SU}(3)_\ell$ and the quark-lepton discrete symmetry, and also to give masses to the liptons, the Higgs bosons χ_1 and χ_2 are introduced. These scalars are defined through the Yukawa Lagrangian

$$\begin{aligned} \mathcal{L}^{(1)}_{\text{Yuk}} = {} & h_1[\overline{F_L}(F_L)^c\chi_1 + \overline{Q_L}(Q_L)^c\chi_2] \\ & + h_2[\overline{E_R}(N_R)^c\chi_1 + \overline{u_R}(d_R)^c\chi_2] + \text{H.c.}, \end{aligned} \tag{4}$$

where $h_{1,2}$ are the Yukawa couplings and family indices have been suppressed. The quantum numbers of the Higgs fields, and their behavior under the discrete symmetry, are

$$\chi_1 \sim (\overline{3},1,1)(-2/3), \quad \chi_2 \sim (1,\overline{3},1)(2/3), \qquad \chi_1 \leftrightarrow \chi_2. \tag{5}$$

The $T=2$ component of χ_1 develops a nonzero vacuum expectation value (VEV), while the VEV of χ_2 is completely zero.

Electroweak symmetry breaking is achieved through a SM Higgs doublet, which is defined through the analogue of the standard Yukawa Lagrangian:

$$\begin{aligned} \mathcal{L}^{(2)}_{\text{Yuk}} = {} & \Gamma_1(\bar{F}_L E_R\phi + \bar{Q}_L u_R \phi^c) \\ & + \Gamma_2(\bar{F}_L N_R \phi^c + \bar{Q}_L d_R \phi) + \text{H.c.} \end{aligned} \tag{6}$$

This Lagrangian has the same purpose as in the SM. The Higgs field ϕ has quantum numbers given by

$$\phi \sim (1,1,2)(1). \tag{7}$$

Under quark-lepton symmetry ϕ has to transform into its charge conjugate field (i.e., $\phi \leftrightarrow \phi^c$) since the $\mathrm{U}(1)_X$ gauge field changes sign (i.e., $C^\mu \to -C^\mu$) under the operation of the quark-lepton discrete symmetry.

The Yukawa Lagrangian yields the tree-level mass relations

$$m_u = m_e, \quad m_d = m_\nu^{\text{Dirac}}. \tag{8}$$

Here $m_{u,e,d,\nu}$ refer to the 3×3 mass matrices (u refers to charge $2/3$ uplike quarks, e refers to the charged leptons, etc.). These mass relations arise as a consequence of (i) the assumption that quark-lepton symmetry is a symmetry of the Yukawa Lagrangian and, (ii) using the minimal Higgs sector of only one doublet. It would be impressive if a q-ℓ symmetric model could be found which contained radiative corrections that transformed these tree-level mass relations into correct and predictive results. No such model has as yet been constructed, although a certain q-ℓ symmetric model with a nonminimal gauge group has been shown to contain radiative corrections which can yield correct but unpredictive fermion masses [11] (indeed a further extension of this nonminimal model will be used in the next section). If the minimal model is extended to contain two Higgs doublets, then the relations of Eq. (8) can be avoided at the tree level but at the expense of predictivity. Therefore, discrete q-ℓ symmetry is certainly not incompatible with the measured quark and lepton masses.

For future reference we mention that the mass relation involving the neutrinos can be avoided if Majorana masses are given to the right-handed neutrinos. This can be achieved through the Higgs multiplet Δ_1 as defined in

$$\mathcal{L}^{(3)}_{\text{Yuk}} = n\left[\overline{N_R}(N_R)^c\Delta_1 + \overline{d_R}(d_R)^c\Delta_2\right] + \text{H.c.}, \tag{9}$$

where

$$\begin{aligned} &\Delta_1 \sim (6,1,1)(4/3), \quad \Delta_2 \sim (1,6,1)(-4/3), \\ &\qquad\qquad \Delta_1 \leftrightarrow \Delta_2. \end{aligned} \tag{10}$$

It is assumed that the $T=-4$ component of Δ_1 develops a nonzero VEV while the VEV of Δ_2 remains zero.

The symmetry breaking pattern can be summarized as

$$\begin{array}{c}\mathrm{SU}(3)_\ell \otimes \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_X \\ \langle\Delta_1\rangle \downarrow \langle\chi_1\rangle \\ \mathrm{SU}(2)' \otimes \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \\ \downarrow \langle\phi\rangle \\ \mathrm{SU}(2)' \otimes \mathrm{SU}(3)_q \otimes \mathrm{U}(1)_Q . \end{array} \tag{11}$$

The $\mathrm{SU}(2)'$ is an unbroken gauge symmetry. This gauge force is expected to be asymptotically free. In analogy with QCD, we assume that it confines all $\mathrm{SU}(2)'$ colored states, so that at large distances only color-singlet states exist in the spectrum.

IV. UNSTABLE DOMAIN WALLS

We will now begin our investigation of assumptions (ii)–(v) as identified in the Introduction.

In this section, we will discuss one way in which assumption (ii), that domain walls are stable, can be evaded in q-ℓ symmetric models. The basic idea is not new: we find a way to embed the discrete symmetry inside a continuous symmetry [12]. We then envisage that the continuous symmetry spontaneously breaks to the discrete symmetry at a high scale, followed subsequently by the spontaneous breaking of the discrete symmetry at a lower scale. During the first cosmological phase transition, a network of cosmic strings forms. These cosmic strings then have to form the boundaries of the domain walls produced after the second phase transition. The dynamics of these hybrid string-wall structures is such that the domain walls are eventually ripped apart, thus rendering them unstable and cosmologically benign [12].

We will take the dynamics of the string-wall structures as given [12]. Our task is therefore to show how discrete q-ℓ symmetry can be embedded in a continuous symmetry and how the two stages of spontaneous symmetry breaking can be induced. We will also have to ensure that no other cosmological problems are introduced in the process.

The gauge group of the minimal q-ℓ symmetric model is given by $G_{q\ell}$ in Eq. (1). However, for the purposes of this section we have to begin with a slightly more complicated gauge group, given by $G'_{q\ell}$ where

$$G'_{q\ell} = \mathrm{SU}(3)_\ell \otimes \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_R \otimes \mathrm{U}(1)_V . \tag{12}$$

The group $\mathrm{U}(1)_R$ is just the Abelian subgroup of the (nonexistent) right-handed weak-isospin $\mathrm{SU}(2)_R$ symmetry, while $\mathrm{U}(1)_V$ is a new Abelian invariance intrinsic to q-ℓ symmetric models. Under this slightly extended gauge group, the fermion field representations are

$$\begin{gathered} F_L \sim (3,1,2)(0,-1), \quad E_R \sim (3,1,1)(-1,-1), \quad N_R \sim (3,1,1)(1,-1), \\ Q_L \sim (1,3,2)(0,1), \quad d_R \sim (1,3,1)(-1,1), \quad u_R \sim (1,3,1)(1,1), \end{gathered} \tag{13}$$

where F_L, E_R, and N_R are generalizations of the usual lepton fields $f_L \equiv (\nu_L,\ e_L)^T$, e_R, and ν_R respectively. The generator X of Eq. (1) is given by

$$X = R + V/3, \tag{14}$$

while, as for the minimal q-ℓ symmetric model, weak hypercharge is given by

$$Y = X + T/3, \tag{15}$$

where $T \equiv \mathrm{diag}(-2,1,1)$ in leptonic color space. As before, the formula for Y identifies the standard leptons as the $T=-2$ components of the $\mathrm{SU}(3)_\ell$ triplet fermions, while the $T=1$ components are the exotic charge $\pm 1/2$ liptons.

Many different types of discrete q-ℓ symmetries may be defined for the fermion spectrum of Eq. (13). We will consider the one which is defined by the transformations:

$$\begin{gathered} F_L \leftrightarrow Q_L, \quad E_R \leftrightarrow d_R, \quad N_R \leftrightarrow u_R, \\ G_q^\mu \leftrightarrow G_\ell^\mu, \quad W^\mu \leftrightarrow W^\mu, \quad R^\mu \leftrightarrow R^\mu, \quad V^\mu \leftrightarrow -V^\mu, \end{gathered} \tag{16}$$

where $G^\mu_{q,\ell}$ are quarklike and leptonic gluons respectively, W^μ are weak bosons, and R^μ and V^μ are the gauge boson fields of $\mathrm{U}(1)_R$ and $\mathrm{U}(1)_V$ respectively. Note that this discrete q-ℓ symmetry is different from the one in the minimal model [see Eq. (3)]. It is important to also realize that any q-ℓ symmetry may be modified by specifying a relative phase change for the quark and lepton fields when they interchange. The model specified by Eqs. (12)–(16) has not been explicitly analyzed before in the literature. However, a close cousin of it is discussed in Sec. III B of Ref. [3].

The gauge group $G'_{q\ell}$ and a phase-transformed version of the discrete symmetry given by Eq. (16) can be simultaneously embedded in a larger continuous symmetry. The new gauge group is given by G_6 where

$$G_6 = \mathrm{SU}(6)_{\mathrm{PS}} \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_R, \tag{17}$$

where the subscript "PS" refers to Pati-Salam [13]. The quarks and their corresponding generalized leptons are placed in the same multiplet under G_6. The fermion multiplet structure is, in fact,

$$\psi_L \sim (6,2)(0), \quad \psi_{1R} \sim (6,1)(1), \quad \psi_{2R} \sim (6,1)(-1), \tag{18}$$

where F_L and Q_L are inside ψ_L, N_R and u_R are inside ψ_{1R}, and E_R and d_R are inside ψ_{2R}. If we write the sextets as column matrices, then we will identify the quark colors with the upper three components while the lower three components will be the generalized leptons. The charge V is now the diagonal generator of $SU(6)_{PS}$ which is given by $\mathrm{diag}(\mathbb{1}, -\mathbb{1})$ where $\mathbb{1}$ is the 3×3 unit matrix.

How is the discrete quark-lepton symmetry embedded in $SU(6)_{PS}$? The most general matrix which is both an element of the sextet representation of SU(6) and a quark-lepton interchange operator is given by $\mathcal{C}$ where

$$\mathcal{C} = \begin{pmatrix} 0 & iD \\ iD^* & 0 \end{pmatrix}. \tag{19}$$

In this equation, $D = \mathrm{diag}\left(e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3}\right)$ and the phases $\theta_{1,2,3}$ correspond to the most generally allowed phase transformations of the various quark and lepton colors when they interchange. The matrix $\mathcal{C}$ represents the transformation in Eq. (16) but with a different phase structure. Since we are not particularly interested in most of these phase transformations, it is simplest to take $\theta_{1,2,3} = -\pi/2$. The simplified discrete symmetry matrix is then given by

$$\mathcal{C} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}. \tag{20}$$

Note that the minus sign in this simplified matrix is necessary to ensure that $\mathcal{C}$ has determinant equal to one. Therefore one cannot escape from complicating the phase structure of Eq. (16) a little. Actually, the discrete symmetry group left over after $SU(6)_{PS}$ breaking consists of the elements $\{\mathcal{I},\ -\mathcal{I} = \mathcal{C}^2,\ \mathcal{C},\ \mathcal{C}^{-1} = \mathcal{C}^\dagger\}$ and is isomorphic to Z_4, rather than the Z_2 of Eq. (16). The connection with Z_2 is provided by the homomorphism $\mathcal{I} \to \mathcal{I}'$, $-\mathcal{I} \to \mathcal{I}'$, $\mathcal{C} \to \mathcal{C}'$, $\mathcal{C}^{-1} \to \mathcal{C}'$ from Z_4 to Z_2 where $\mathcal{I}'$ is the Z_2 identity element and $\mathcal{C}'^2 = \mathcal{I}'$. This homomorphism identifies those elements of the Z_4 symmetry which are related to each other only by a phase transformation. Note also that the element $\mathcal{C}^2 = -\mathcal{I}$ of Z_4 is also the element $\exp(i\pi V)$ of $U(1)_V$. Therefore all of the purely phase transforming actions of the Z_4 symmetry can be undone by this $U(1)_V$ gauge transformation.

The idea of embedding quark and lepton color inside an extended Pati-Salam symmetry has already been considered in Ref. [14]. In this previous paper, the full $SU(2)_R$ right-handed weak-isospin group was postulated, together with an exact discrete left-right symmetry (parity) which swapped the two weak-isospin sectors. This made the model possess partial coupling constant unification, with some attendant constraints on symmetry breaking scales resulting from a renormalization-group analysis of the theory [15]. From the point of view of standard cosmology, however, neither the full $SU(2)_R$ symmetry nor the discrete left-right symmetry should be imposed. Imposition of the former would lead to a cosmological monopole problem, because the initial gauge group would not have a U(1) factor,[2] while imposition of the latter would result in its own domain wall problem [16]. Unlike its close relative in Ref. [14], the gauged G_6 model has no partial coupling constant unification, and so the symmetry-breaking scales are less constrained.

At the first stage of symmetry breaking we want to break $SU(6)_{PS}$ down to its $SU(3)_\ell \otimes SU(3)_q \otimes U(1)_V$ subgroup. We would also prefer to have the discrete q-ℓ symmetry, as given by $\mathcal{C}$ in Eq. (20), remain unbroken after this initial breakdown of the G_6 group. If the discrete symmetry were to break at the same scale as $SU(6)_{PS}$ then the domain-wall problem would be trivially "solved," because the discrete symmetry would have never existed as a free-standing invariance at any energy scale.[3] We prefer instead to ensure that the effective theory below the first symmetry-breaking scale is a model with an exact, unembedded q-ℓ symmetry.

This first stage of symmetry breaking can be accomplished in a number of ways. The simplest method is to introduce a real Higgs field Φ whose G_6 transformation law is given by

$$\Phi \sim (189, 1)(0). \tag{21}$$

Under the $SU(3)_\ell \otimes SU(3)_q \otimes U(1)_V$ subgroup of $SU(6)_{PS}$, the field

$$\begin{aligned} \Phi \to\ & (1,1)(0) \oplus (1,8)(0) \oplus (8,1)(0) \oplus (8,8)(0) \\ & \oplus (3,\bar{3})(-2) \oplus (\bar{3},3)(2) \oplus (3,\bar{3})(4) \oplus (\bar{3},3)(-4) \\ & \oplus (3,6)(-2) \oplus (6,3)(2) \oplus (\bar{3},\bar{6})(2) \oplus (\bar{6},\bar{3})(-2). \end{aligned} \tag{22}$$

A nonzero vacuum expectation value (VEV) for the singlet (1,1)(0) component of Φ performs the gauge symmetry breaking we require, this being

$$G_6 \to SU(3)_\ell \otimes SU(3)_q \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_V. \tag{23}$$

This breaking is of course just $G_6 \to G'_{q\ell}$. The discrete q-ℓ symmetry is also left unbroken, as we will now show.

The 189-plet is actually the lowest dimensional representation one can use to leave the discrete q-ℓ symmetry unbroken. The lower dimensional representations **35** and **175** also contain $SU(3)_\ell \otimes SU(3)_q \otimes U(1)_V$ singlets. However, VEV's for these components would break the discrete symmetry, because they are odd under the discrete

[2] Note that the generator R of the Abelian group factor in G_6 contributes to the formula for electric charge, as given by $Q = I_{3L} + R/2 + V/6 + T/6$. If the U(1) in G_6 had turned out not to contribute to Q, then topologically stable monopoles would have been produced at some stage in the symmetry-breaking process. We will make some more comments on monopoles later in this section.

[3] As a sidelight, we note that a Higgs boson transforming as a (20,1)(1) multiplet under G_6 can break G_6 to the gauge symmetry $G_{q\text{-}\ell}$ of the minimal model [see Eq. (1)] simultaneously with the discrete symmetry. Therefore, in this case there is definitely no domain-wall problem, but there is also never a free-standing discrete q-ℓ symmetry. Nevertheless, since the leptonic color group can remain exact to TeV scales even though the discrete q-ℓ symmetry might be broken at a high scale, this scenario is not completely devoid of phenomenological interest. Note also that a Higgs field in the (35,1)(0) representation can break the discrete symmetry at the same time as it induces the breaking of G_6 down to $G'_{q\ell}$ (see below).

symmetry.[4] To see this, consider the decomposition of the tensor product

$$\mathbf{6} \otimes \overline{\mathbf{6}} = \mathbf{1} \oplus \mathbf{35} \tag{24}$$

under the subgroup $SU(3)_\ell \otimes SU(3)_q \otimes U(1)_V$, where $\mathbf{6} \to (3,1)(-1) \oplus (1,3)(1)$. Denote the two singlets in the product by S_1 and S_2; they are given by

$$S_1 \subset (1,3)(1) \otimes (1,\overline{3})(-1)$$

and (25)

$$S_2 \subset (3,1)(-1) \otimes (\overline{3},1)(1).$$

Under the operation of the q-ℓ symmetry $S_1 \leftrightarrow S_2$. From these two singlets we can construct two independent combinations, $S_1 + S_2$ and $S_1 - S_2$, which transform as even and odd under the q-ℓ symmetry, respectively. By neccessity, the q-ℓ–even singlet corresponds to the SU(6) singlet in the right-hand side of Eq. (24). Therefore the singlet in the 35-plet must be q-ℓ odd. [One can check this explicitly by using the representation given in Eq. (20).] By using this result and the same method one can show that the singlet in the 189-plet is q-ℓ–even from the decomposition of

$$\mathbf{15} \otimes \overline{\mathbf{15}} = \mathbf{1} \oplus \mathbf{35} \oplus \mathbf{189}. \tag{26}$$

Similarly, the 175-plet can be shown to contain a q-ℓ–odd singlet by using the decomposition of $\mathbf{20} \otimes \mathbf{20} = \mathbf{1} \oplus \mathbf{35} \oplus \mathbf{175} \oplus \mathbf{189}$, while the 405-plet can be shown to contain a q-ℓ–even singlet by considering $\overline{\mathbf{21}} \otimes \mathbf{21} = \mathbf{1} \oplus \mathbf{35} \oplus \mathbf{405}$.

The second stage of symmetry breaking is induced through Higgs multiplets called χ and Δ (we require that $\langle\chi\rangle$, $\langle\Delta\rangle \ll \langle\Phi\rangle$ in order to create the possibility of interesting TeV-scale phenomenology). These fields possess Yukawa couplings to the fermions given by the Lagrangian $\mathcal{L}_{\rm Yuk}$ where

$$\begin{aligned}\mathcal{L}_{\rm Yuk} = {} & h_L \bar{\psi}_L \chi (\psi_L)^c + h_R \bar{\psi}_{1R} \chi (\psi_{2R})^c \\ & + n \bar{\psi}_{1R} \Delta (\psi_{1R})^c + \text{H.c.}\,,\end{aligned} \tag{27}$$

and their transformation properties under G_6 are

$$\chi \sim (15,1)(0) \quad \text{and} \quad \Delta \sim (21,1)(2). \tag{28}$$

Since $G_6 \to G'_{q\ell}$ at the first stage of symmetry breaking, we also need to know how χ and Δ transform under the unbroken subgroup. The branching rules to $G'_{q\ell}$ are

$$\begin{aligned}&\chi \to (\bar{3},1,1)(0,-2) \oplus (1,\bar{3},1)(0,2) \oplus (3,3,1)(0,0),\\ &\chi \to \chi_1 \oplus \chi_2 \oplus \chi',\end{aligned} \tag{29}$$

and

$$\begin{aligned}&\Delta \to (6,1,1)(2,-2) \oplus (1,6,1)(2,2) \oplus (3,3,1)(2,0),\\ &\Delta \to \Delta_1 \oplus \Delta_2 \oplus \Delta',\end{aligned} \tag{30}$$

where the equation below each branching rule establishes our notation for the multiplets which are irreducible under the unbroken gauge group. Clearly, under the discrete q-ℓ symmetry (ignoring the phases),

$$\chi_1 \leftrightarrow \chi_2 \qquad \text{and} \qquad \Delta_1 \leftrightarrow \Delta_2, \tag{31}$$

while the components of χ' and Δ' transform amongst themselves. The Higgs fields $\chi_{1,2}$ and $\Delta_{1,2}$ correspond to their namesakes in the minimal q-ℓ symmetric model reviewed in Sec. III.

The multiplets χ_1 and Δ_2 can be represented by antisymmetric and symmetric 3×3 matrices, respectively. Under $SU(3)_\ell$ transformations

$$\chi_1 \to U_\ell \chi_1 U_\ell^T \qquad \text{and} \qquad \Delta_1 \to U_\ell \Delta_1 U_\ell^T\,, \tag{32}$$

where U_ℓ is a triplet representation matrix of an $SU(3)_\ell$ group element. We require the χ_1 and Δ_1 components of the full Higgs multiplets χ and Δ to develop nonzero VEV's in order to break both the discrete q-ℓ symmetry and the leptonic color group $SU(3)_\ell$. Of course, the other multiplets inside χ and Δ possess quark color and so we must demand that their VEV's be zero. The required pattern of VEV's is

$$\langle\chi_1\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & v \\ 0 & -v & 0 \end{pmatrix} \tag{33}$$

and

$$\langle\Delta_1\rangle = \begin{pmatrix} v' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{34}$$

with χ_2, χ', Δ_2, and Δ' all having zero VEV's.

It is important to note that the trilinear term $\chi^\dagger \chi \Phi$ appears in the Higgs potential for this model. This term ensures that the discrete transformation $\Phi \to -\Phi$ is not an accidental symmetry of the theory, and so there is no accidental domain-wall problem either. One can check that there are no other accidental discrete symmetries in the model that are not also elements of a continuous global symmetry. This term also serves to connect the Φ multiplet in a nontrivial way with the other Higgs fields of the theory.

After the second stage of symmetry breaking, the unbroken gauge group is $SU(2)' \otimes G_{\rm SM}$, where $G_{\rm SM}$ is just the standard model group $SU(3)_q \otimes SU(2)_L \otimes U(1)_Y$ [with Y given by Eq. (15)] and $SU(2)'$ is an unbroken remnant of leptonic color. The exotic partners of the leptons (the liptons) gain mass from the $h_{L,R}$ terms in Eq. (27), while the right-handed neutrinos develop Majorana masses from the n term in the Yukawa Lagrangian. All of these masses are thus expected to be heavy compared with the usual leptons and quarks (which are still

[4]The homomorphism from $Z_4 \to Z_2$ discussed above defines the representation of Z_4 under which $SU(3)_\ell \otimes SU(3)_q \otimes U(1)_V$ singlet component Higgs fields transform (with $\mathcal{C}' = -1$). Since this representation is isomorphic to Z_2, the terms "odd" and "even" are applicable.

massless). The heavy charge $\pm 1/2$ lipton fields are doublets under the unbroken shard $SU(2)'$. If the number of fermion generations is not too large (for instance, if there are three of them), then $SU(2)'$ is asymptotically free and thus is expected to be confining. All particles which have nontrivial quantum numbers under $SU(2)'$ (liptons, some Higgs bosons, and some heavy gauge bosons) are then confined into unstable, integrally charged bound states. This neatly evades a potential cosmological abundance problem, because the lightest half-integrally charged particle would be stable if it were free.[5] Finally, note that a large Majorana mass for the right-handed neutrinos sets the stage for the seesaw mechanism [17] once electroweak symmetry is broken.

The final stage of symmetry breaking just involves the usual spontaneous violation of the electroweak group. This is performed in the standard way through electroweak Higgs doublets, which also induce masses for quarks and the usual leptons (we of course require that $\langle\phi\rangle \ll \langle\chi_1\rangle, \langle\Delta_1\rangle$). If only one doublet is used, then there are quark-lepton mass relations at the tree level of the form $m_u = m_\nu^{\rm Dirac}$ and $m_d = m_e$ due to the discrete q-ℓ symmetry. Note that these mass relations are different from those obtained in the minimal q-ℓ symmetric model [see Eq. (8)]. Because of this, radiative corrections in the model that break the tree-level relations can yield correct but unpredictive quark and lepton masses [11]. Also, if more than one doublet is used, these mass relations no longer hold at the tree level (and predictivity is also unfortunately lost).

This essentially completes our demonstration that the domain wall problem for spontaneously broken discrete q-ℓ symmetry can be evaded by embedding the discrete transformation in a continuous gauge group. However, the attentive reader may have noticed a complication arising with regard to monopoles because of the way we have performed the spontaneous symmetry breaking. This issue requires some further discussion.

The point is that the first stage of symmetry breaking consists of $SU(6)_{PS} \to SU(3)_\ell \otimes SU(3)_q \otimes U(1)_V$, with no participation from $SU(2)_L \otimes U(1)_R$. The fact that $U(1)_V$ comes entirely from $SU(6)_{PS}$ means that monopoles exhibiting V-type magnetic charge will be created during the first phase transition.[6] Indeed, if no further symmetry breaking were to take place, these monopoles would be topologically stable. However, we know that after the final stage of symmetry breaking G_6 has broken to $SU(2)' \otimes SU(3)_q \otimes U(1)_Q$ where the generator R contributes to Q (see footnote 2). Since this breaking cannot support topologically stable monopoles, the monopolelike states produced at the first stage of symmetry breaking must disappear in some manner. Although a detailed analysis of how this occurs is well beyond the scope of this paper, we can fairly easily identify at least two important processes. First, since they are not topologically stable once all of the symmetry breaking is complete, it must be true that the monopoles can just decay into ordinary forms of energy. Second, at some point after the first phase transition we have to break a U(1) gauge group, which should lead to the formation of cosmic strings [which are different from the cosmic strings produced when the Pati-Salam SU(6) breaks to the discrete symmetry]. Since the generator V contributes to the generator of this broken U(1), we would expect these cosmic strings to end in monopoles and antimonopoles, so enhancing their annihilation rate [18]. We therefore conclude that although monopole-like states will exist during a certain epoch in the early Universe, they will ultimately disappear and thus in all probability not cause any cosmological problems.[7]

In order to round off the discussion, we will now briefly address some further issues. (i) There are many phenomenological constraints one could place on this model. We will not derive any bounds here, because we do not want to obscure our essential point about how the domain wall problem can be avoided. (ii) The scale at which $SU(2)'$ confines is approximately calculable, because the leptonic color coupling constant is equal to the strong coupling constant at the scale of q-ℓ symmetry-breaking. If the q-ℓ symmetry-breaking scale is not much higher than the lipton mass scale, then the confinement energy turns out to be about the same as for QCD. If there is a splitting between the discrete symmetry breaking scale and the lipton mass scale (i.e., if $\langle\Delta_1\rangle \gg \langle\chi_1\rangle$), then the $SU(2)'$ confinement energy is lower than its QCD counterpart. (iii) That the $SU(2)'$ confinement scale is of the order of hundreds of MeV or lower implies that the lowest mass survivors from the underlying q-ℓ symmetric model are the $SU(2)'$ glueballs. These objects may give rise to interesting phenomenology, and they are also of potential cosmological significance because they are long lived. If these glueballs are very light (~ 1 keV), then it has been shown that they do not interfere with standard big bang nucleosynthesis, and that they are a dark matter candidate [14]. If, on the other hand, the glueballs have masses in the 1 GeV range, then they have to decay in less than about 1 sec in order to be compatible with standard BBN. Although a detailed analysis of glueballs in this mass range has not as yet been carried

[5] Strictly speaking, this particle is stable even if it is confined. However, in this case its stability is not a problem for the same reason that there are no stable mesons.

[6] To be more precise, the global structure of the unbroken group is actually $SU(3)_\ell \otimes SU(3)_q \otimes U(1)_V / Z_3$ so the monopoles also carry some non-Abelian magnetic charge.

[7] If the $U(1)_V$ symmetry never exists as an exact symmetry in its unembedded form, then of course no monopoles, unstable or otherwise, ever form. This will be true, for instance, if $\langle\Phi\rangle \sim \langle\chi_1\rangle$ or $\langle\Phi\rangle \sim \langle\Delta_1\rangle$ (of course in this case the discrete q-ℓ symmetry would also not exist as a free-standing invariance). The requirement that the monopoles disappear quickly enough to be cosmologically acceptable therefore translates into an upper bound on $|\langle\Phi\rangle - \langle\chi_1\rangle|$ or $|\langle\Phi\rangle - \langle\Delta_1\rangle|$. A detailed dynamical calculation would be necessary to determine this bound, but we expect that reasonable values for the VEV's would be allowed.

out for q-ℓ symmetric models, a brief study was made in Ref. [4] which suggested that a range of parameters for the model allowing the glueballs to be cosmologically acceptable exists [19].

V. SPONTANEOUS DISCRETE SYMMETRY BREAKING AND THE COSMOLOGICAL PHASE TRANSITION

In this section we will examine whether or not there is *necessarily* a cosmological phase transition associated with the spontaneous breaking of the discrete symmetry [20]. If no such phase transition need exist (i.e., if no symmetry restoration need occur at some critical temperature T_c), then one can consistently attribute the broken symmetry as a special initial condition of the big bang. If this is the case then we can arrange for the vacua in casually disconnected regions to be the same. Hence the formation of domain walls is avoided. Of course, this very special initial condition would ultimately require a deep explanation. However, for our present purposes it is not necessary to push the analysis to this extreme, given our overwhelming ignorance of physics at the Planck scale [21].

Before proceeding, note that we are assuming that it is fundamental Higgs scalars which are responsible for the origin of spontaneous symmetry breaking. However, the Higgs sector of the SM is experimentally untested so the origin of spontaneous symmetry breaking remains unclear. It could well be that spontaneous symmetry breaking is dynamical in origin and has nothing to do with fundamental scalars [22]. If this is the case then it is still an open question as to whether symmetry restoration occurs at high temperatures [23].

Consider the zero-temperature Higgs potential of the minimal q-ℓ symmetric model given by

$$\begin{aligned} V_0 = {} & \lambda_1 \left[\chi_1^\dagger \chi_1 + \chi_2^\dagger \chi_2 - v^2\right]^2 \\ & + \lambda_2 \chi_1^\dagger \chi_1 \chi_2^\dagger \chi_2 + \lambda_3 \left(\phi^\dagger \phi - u^2\right)^2 \\ & + \lambda_4 \left[\phi^\dagger \phi - u^2 + \chi_1^\dagger \chi_1 + \chi_2^\dagger \chi_2 - v^2\right]^2 . \end{aligned} \tag{35}$$

(For illustrative purposes, we have kept the Higgs potential simple by not including the fields $\Delta_{1,2}$ or multiple copies of ϕ.) If the coefficients, λ_i, are all positive then the above potential is minimized where u and v are the nonzero VEV's of the ϕ and χ_1 fields respectively. This then leads to the symmetry breaking pattern given in Eq. (11). As a result of this symmetry breaking, there will be two residual neutral Higgs bosons (coming from ϕ and χ_1) whose mass (squared) matrix is given by

$$\begin{pmatrix} 4(\lambda_3 + \lambda_4) u^2 & 4\lambda_4 u v \\ 4\lambda_4 u v & 4(\lambda_1 + \lambda_4) v^2 \end{pmatrix} . \tag{36}$$

There will also be the charge 1/3 color triplet Higgs multiplet χ_2 with mass given by $M_{\chi_2}^2 = \lambda_2 v^2$.

For the purposes of this section we will rewrite Eq. (35) in a more convenient form as

$$\begin{aligned} V_0 = {} & -\mu_\phi^2 \phi^\dagger \phi + (\lambda_3 + \lambda_4)\left(\phi^\dagger \phi\right)^2 + 2\lambda_4 \left(\phi^\dagger \phi\right)\left[\chi_1^\dagger \chi_1 + \chi_2^\dagger \chi_2\right] + (2\lambda_1 + 2\lambda_4 + \lambda_2)\chi_1^\dagger \chi_1 \chi_2^\dagger \chi_2 \\ & -\mu_\chi^2 \left[\chi_1^\dagger \chi_1 + \chi_2^\dagger \chi_2\right] + (\lambda_1 + \lambda_4)\left[\left(\chi_1^\dagger \chi_1\right)^2 + \left(\chi_2^\dagger \chi_2\right)^2\right] . \end{aligned} \tag{37}$$

The minimization conditions then become

$$\begin{aligned} k u^2 &= (\lambda_1 + \lambda_4)\mu_\phi^2 - \lambda_4 \mu_\chi^2, \\ k v^2 &= (\lambda_3 + \lambda_4)\mu_\chi^2 - \lambda_4 \mu_\phi^2, \end{aligned} \tag{38}$$

where $k \equiv 2(\lambda_1 \lambda_3 + \lambda_1 \lambda_4 + \lambda_3 \lambda_4)$.

Now consider the finite temperature contributions to the effective Higgs potential. We will only consider the terms proportional to T^2 since it is sufficient for us to work within the high-temperature expansion approximation. By using the usual calculational techniques [24], the finite-temperature corrections[8] modify the minimization conditions of Eq. (38) to become

$$\begin{aligned} k u^2 &= (\lambda_1 + \lambda_4)\left(\mu_\phi^2 - \zeta_1 T^2\right) - \lambda_4 \left(\mu_\chi^2 - \zeta_2 T^2\right), \\ k v^2 &= (\lambda_3 + \lambda_4)\left(\mu_\chi^2 - \zeta_2 T^2\right) - \lambda_4 \left(\mu_\phi^2 - \zeta_1 T^2\right), \end{aligned} \tag{39}$$

where

$$\begin{aligned} \zeta_1 &= \frac{1}{2}\lambda_3 + \frac{3}{2}\lambda_4 + \frac{3}{16} g_2^2 + \frac{1}{16} g_X^2 + \frac{1}{2}\Gamma_t^2, \\ \zeta_2 &= \frac{7}{6}\lambda_1 + \frac{1}{4}\lambda_2 + \frac{3}{2}\lambda_4 + \frac{1}{3} g_3^2 + \frac{1}{36} g_X^2, \end{aligned} \tag{40}$$

and $g_{X,2,3}$ are the $U(1)_X$, $SU(2)_L$, $SU(3)_{q,\ell}$ gauge coupling constants respectively and Γ_t is the t-quark Yukawa coupling constant. (Note that our theory has two Yukawa coupling constants equal to Γ_t because of the discrete symmetry.) To simplify Eq. (39) let $\lambda_3 \ll \lambda_4$ and $\mu \equiv \mu_\phi \simeq \mu_\chi$. Then

$$\begin{aligned} k u^2 &\simeq \lambda_1 \mu^2 - \lambda_4 \zeta T^2, \\ k v^2 &\simeq \lambda_3 \mu^2 + \lambda_4 \zeta T^2, \end{aligned} \tag{41}$$

[8] In our high T approximation we have neglected the contributions that are proportional to T and the logarithmic corrections to the coefficients of the quartic terms in the potential.

where

$$\zeta = \frac{1}{2}\lambda_3 - \frac{7}{6}\lambda_1 - \frac{1}{4}\lambda_2 + \frac{1}{2}\Gamma_t^2 - \frac{1}{3}g_3^2 + \frac{3}{16}g_2^2 + \frac{5}{144}g_X^2. \tag{42}$$

Therefore v^2 can remain nonzero, and hence q-ℓ symmetry unbroken, provided $\zeta \geq 0$. Clearly, a range of parameters exists for which this is true.[9] More specifically we can choose, for example,

$$\begin{aligned} \lambda_3 &\geq \frac{7}{3}\lambda_1 + \frac{1}{2}\lambda_2, \\ \Gamma_t^2 &\geq \frac{2}{3}g_3^2 - \frac{3}{8}g_2^2 - \frac{5}{72}g_X^2, \end{aligned} \tag{43}$$

where the couplings are evaluated at high T. For such a range of parameters, the neutral Higgs-boson masses at zero temperature from Eq. (36) are given by

$$\begin{aligned} M_\phi^2 &\simeq 4(\lambda_1 + \lambda_3)\frac{u^2}{1 + \frac{u^2}{v^2}}, \\ M_{\chi_1}^2 &\simeq 4\lambda_4\left(u^2 + v^2\right). \end{aligned} \tag{44}$$

For a Higgs boson, mass greater than about 50 GeV (the current lower limit is 48 GeV [25]) gives $\lambda_1 + \lambda_3 \geq 0.02$ which is consistent with the chosen range of parameters in our example.

The Higgs sector we analyzed above is of course unrealistic from the point of view of fermion mass relations (see Sec III). We have, however, checked that a realistic theory containing two electroweak Higgs doublets yields the same qualitative conclusions as we reached in our simple illustrative model. For reasons of clarity we have therefore chosen to explicitly display only the simplified analysis.

So the minimal q-ℓ symmetric model has the necessary ingredients to prevent a restoration of q-ℓ discrete symmetry at high temperatures (and, for that matter, a restoration of leptonic color symmetry). Clearly, in extensions of the minimal model, where the Higgs sector will generally be more complicated, this will also be the case (the two electroweak doublet extension alluded to in the preceding paragraph is an example). Such a scenario provides one way of evading the domain-wall problem. It is interesting to also note that electroweak symmetry can be restored even though the q-ℓ symmetry remains broken. This may prove useful for baryogenesis at the electroweak scale.

VI. DOMAIN WALLS AND INFLATION

As we discussed in Sec. II, the unadorned hot big bang model cannot account for the smoothness and flatness of our Universe (although it is compatible with it). The interesting idea of "inflation" has been much studied as a way of remedying this deficiency [6]. At some very early stage in the evolution of the Universe, a finite period of exponential expansion is postulated, which renders spacetime almost perfectly flat after the exponential expansion ceases. Also, the present observable Universe arises from within a causally connected region of the very early Universe, thus explaining its palpable smoothness.

Since its inception, inflation has also served to rid the Universe of otherwise troubling topological defects, provided the period of inflation occurs after the cosmological phase transition that creates the topological defects. For instance, one of the original motivations for inflation was to cure grand unified theories (GUT's) of their monopole abundance problem. The cure is so efficacious, in fact, that from the pre-inflation prediction that GUT monopoles dominate the energy density of the Universe by many orders of magnitude, the Universe observable to us today after inflation is predicted to contain at most one monopole.

As for monopoles, an inflationary epoch after a cosmological phase transition associated with spontaneous discrete symmetry breaking also eliminates domain walls from the observable Universe. Clearly, therefore, domain walls generated by discrete q-ℓ symmetry can be rendered innocuous by this means.

The only issue we have to really discuss in this regard is the relative positioning of the scales of symmetry breaking in q-ℓ symmetric models and the scale Λ_{inf} at which inflation occurs.[10] In the past, the inflationary phase transition was generally arranged to occur at about 10^{14} GeV. Recently, it has been pointed out that inflation could occur at energies as low as the electroweak scale [9]. If inflation is taken to occur at the electroweak scale then clearly any extension of the SM which pro-

[9]Note, however, that the electroweak phase transition is still expected to take place. This is because for temperatures lower than the mass of the lightest exotic particle (be it a lipton or an exotic Higgs boson or whatever), the effect of all these nonstandard states is Boltzmann suppressed, and so the effective finite-temperature field theory is essentially that of the SM. The nature of the electroweak phase transition may be altered because the lightest Higgs-boson mass eigenstate may have different properties from the standard Higgs boson and because of the possibility that one or more of the exotic particles may fortuitously have masses as low as, say, 100 GeV. However, interesting though they may be, these details are unimportant for our present purpose.

[10]For inflation to solve the smoothness and flatness problems, the period of exponential expansion must be sufficiently long. The scale Λ_{inf} is then to be interpreted as corresponding to the temperature at which inflation begins. During the inflationary phase, the Universe supercools so that Λ_{inf} no longer even approximately corresponds to the temperature of the Universe at that time. When inflation ceases the Universe is reheated by the conversion of false vacuum energy into thermal energy for the particle soup. The reheating temperature turns out to be less than Λ_{inf} so inflation does not restart and the discrete q-ℓ symmetry is not restored.

duces naively troublesome topological defects at a higher scale will see these defects inflated away. Therefore, low-scale inflation can cure the q-ℓ symmetric models of their domain wall problem. However, if inflation is taken to occur at a much higher scale than the electroweak, then we must arrange spontaneous q-ℓ symmetry breaking to occur at an even higher scale yet. In the rest of this section we will assume that inflation occurs for $\Lambda_{\text{inf}} \gg M_W$, and see what needs to be done to ensure that the domain walls are still inflated away.

Since we would like a lot of new phenomenology to occur in the TeV energy regime, we may like to consider divorcing the discrete symmetry-breaking scale $\Lambda_{q\ell}$ from the leptonic color breaking scale Λ_3 and/or the lipton mass scale Λ_L. It is sensible, in fact, to consider three hierarchical patterns:

$$\Lambda_{q\ell} > \Lambda_{\text{Inf}} \gg \Lambda_3 \sim \Lambda_L \tag{45}$$

or

$$\Lambda_{q\ell} > \Lambda_{\text{Inf}} \gg \Lambda_3 \gg \Lambda_L \tag{46}$$

or

$$\Lambda_{q\ell} \sim \Lambda_3 > \Lambda_{\text{Inf}} \gg \Lambda_L. \tag{47}$$

Let us begin thinking about these patterns in terms of the minimal q-ℓ symmetric model introduced in Sec. III.[11] (Do not worry, for the near future, about how the scale of inflation Λ_{inf} is to be generated. We will just assume in an *ad hoc* way that an inflaton field can be added to the model to bring about inflation at any desired scale. Fitting the inflaton field into the rest of particle physics in an elegant way is a very deep and unsolved problem which we are not going to address.) It is immediately apparent that we have to extend the Higgs sector of the minimal q-ℓ symmetric model in order to generate the hierarchies of Eqs. (45) and (46) (call them hierarchy 1 and hierarchy 2 respectively). This is because the Higgs fields χ_1 and Δ_1 introduced in Sec. III both simultaneously break the discrete symmetry and leptonic color. Thus we need to introduce another Higgs field σ which is a gauge singlet but which is odd under q-ℓ symmetry. Note, however, that χ_1 and Δ_1 are sufficient in order to generate hierarchy 3 [see Eq. (47)].

In terms of VEV's for Higgs fields,

$$\begin{aligned}&\Lambda_{q\ell} \sim \max(\langle\sigma\rangle, \langle\chi_1\rangle, \langle\Delta_1\rangle),\\ &\Lambda_L \sim \langle\chi_1\rangle,\\ &\Lambda_3 \sim \max(\langle\chi_1\rangle, \langle\Delta_1\rangle).\end{aligned} \tag{48}$$

Hierarchies 1, 2, and 3 are generated, respectively, if

$$\langle\sigma\rangle > \Lambda_{\text{inf}} \gg \langle\Delta_1\rangle \sim \langle\chi_1\rangle, \tag{49}$$

or

$$\langle\sigma\rangle > \Lambda_{\text{inf}} \gg \langle\Delta_1\rangle \gg \langle\chi_1\rangle, \tag{50}$$

or

$$\langle\Delta_1\rangle > \Lambda_{\text{inf}} \gg \langle\chi_1\rangle. \tag{51}$$

We should be aware that some fine-tuning of parameters will be necessary in order to generate these hierarchies. Since the purpose of the present paper is to show how domain walls can be made cosmologically safe, we do not want to cloud the issue by including complicated speculations about how the gauge hierarchy problem might eventually be alleviated. It is sufficient for us that such hierarchies *can* be induced.

We do not need to discuss hierarchy 3 much further. We simply fine-tune the Higgs potential parameters to create this hierarchy, and we throw in an inflaton field. Note also that a fine-tuning is necessary to keep the lipton masses light after leptonic color is broken, because the gauge group $SU(2)' \otimes G_{\text{SM}}$ cannot by itself prevent the radiative generation of nonzero lipton masses. If we extend the minimal q-ℓ symmetric model gauge group, then it is possible to have a symmetry left over after the first stage of symmetry breaking in hierarchy 3 which does prevent the liptons from gaining mass [3].

Hierarchies 1 and 2 require the additional Higgs field σ. This is a real Higgs field which is odd under discrete q-ℓ symmetry. It couples to the other fields in the model through the Higgs potential terms

$$\begin{aligned}V_\sigma = &-\mu_\sigma^2\sigma^2 + \lambda_\sigma\sigma^4 + a_\chi(\chi_1^\dagger\chi_1 - \chi_2^\dagger\chi_2)\sigma\\ &+a_\Delta(\Delta_1^\dagger\Delta_1 - \Delta_2^\dagger\Delta_2)\sigma + \lambda_{\sigma\chi}(\chi_1^\dagger\chi_1 + \chi_2^\dagger\chi_2)\sigma^2\\ &+\lambda_{\sigma\Delta}(\Delta_1^\dagger\Delta_1 + \Delta_2^\dagger\Delta_2)\sigma^2 + V(\phi,\sigma),\end{aligned} \tag{52}$$

where $V(\phi,\sigma)$ describes the coupling of σ to however many electroweak doublets ϕ we have in our theory. The two trilinear terms in this equation establish σ's q-ℓ–odd credentials. Again, we fine-tune the parameters in the full Higgs potential in order to generate either hierarchy 1 or hierarchy 2. Note that after the discrete symmetry is spontaneously broken, the coupling constants of the two color forces will evolve a little differently under the renormalization group, due to the fact that the leptonically colored Higgs fields will now have different masses from those with quark color.

So, we conclude this section by saying that it is possible to inflate away the domain walls from q-ℓ symmetry breaking, while also preserving the feature of having new low-energy phenomenology. This new phenomenology can be either the existence of light liptons (say 100 GeV and above), or the existence of both light liptons and all the new physics associated with the breaking of leptonic color (Higgs fields and heavy gauge bosons).

VII. LIFTING THE VACUUM DEGENERACY

When a Z_2 discrete symmetry spontaneously breaks, the standard perturbative analysis of the Higgs potential reveals a vacuum manifold consisting of two degenerate states that can be transformed into each other. An exact degeneracy is necessary for the resulting domain walls to be completely stable.

If a perturbation is added to the theory that explic-

[11] Generically, we would expect the scales $\Lambda_{3,L}$ to roughly correspond to the temperatures at which the associated cosmological phase transitions occur. Note, however, that this need not be true for reasons outlined in the preceding section. Note also that if the reheating temperature after inflation is lower than either Λ_3 or Λ_L or both, then the associated phase transition(s) will not occur in the post-inflationary Universe.

itly breaks the discrete symmetry, then these two states are no longer exactly degenerate. Provided the explicit breaking is small enough, domain-wall structures can still form, but they will no longer be stable. Since it is energetically favored for the true vacuum state to be established throughout the Universe, these domain walls have to break up eventually. One can view this process as being caused by a pressure differential across the domain wall, due to the slightly different energy densities on each side.

A "cheap and nasty" way out of the domain-wall problem for q-ℓ symmetric models is therefore to include a small amount of explicit breaking. One can even be so sophisticated as to include only soft breaking terms. One would also have to be careful to make the explicit breaking strong enough so that the domain walls break up quickly enough. However, we view such models as unpalatable since they render the term "quark-lepton symmetry" a misnomer.

A somewhat more attractive possibility exists, however, for it could turn out that *nonperturbative effects* lift the degeneracy. A class of discrete symmetry models for which this is supposed to occur has recently been discussed in the literature [26]. They are known as theories with "anomalous discrete symmetries" [26, 27].

The examples of this phenomenon studied in the literature to date refer to discrete Z_n symmetries that can be embedded inside the continuous group U(1) (in a number of different ways, in general) [27, 28]. Such a Z_n discrete symmetry is termed anomalous if all of the associated U(1) parents are anomalous with respect to the gauge symmetries of the model. In an interesting analysis, Preskill *et al.* [26] have argued that the discrete symmetry imposed to prevent Higgs-induced tree-level flavor-changing process in the two-Higgs-doublet model is anomalous, and thus the putative vacuum degeneracy is lifted by instanton effects.[12] They go on to argue that domain walls caused by this discrete symmetry decay in time to prevent cosmological difficulties.

Can such a phenomenon also occur for discrete quark-lepton symmetry? Before addressing this question, we have to generalize the notion of an anomalous discrete Z_n symmetry to include embeddings inside SU(N) rather than just U(1). This is because discrete symmetry subgroups of U(1) act by changing the relative phases of fields, while q-ℓ symmetry is an example of a discrete symmetry that *interchanges flavors.* We will call such a group a "flavor interchanging discrete (FID) symmetry."

The discussion pertaining to Eq. (20) illustrates how flavor interchanging embeddings of Z_4 inside SU($2N$) are established. To generalize the discussion slightly, consider the SU(N)$_1\otimes$ SU(N)$_2\otimes$ U(1) subgroup of SU($2N$). The element of the fundamental representation of SU($2N$) that interchanges the two SU(N) sectors is given by

$$\mathcal{C}_N = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}, \tag{53}$$

where $\mathbb{1}$ is the $N \times N$ unit matrix (and we have fixed the phases). We will call the resulting FID symmetry anomaly-free (anomalous) if the parent SU($2N$) gauge theory is anomaly-free (anomalous). This is, of course, a fairly obvious generalization of the U(1) example studied in the literature.

In Sec. IV we showed that the discrete symmetry of Eq. (16) could be embedded within an anomaly-free representation of G_6 = SU(6)$_{\rm PS}\otimes$SU(2)$_L$ $\otimes$U(1)$_R$. Therefore, this particular version of q-ℓ symmetry is anomaly-free according to our definition. Thus effects related to anomalies cannot lift the vacuum degeneracy, if the analysis of Ref. [26] can be validly extended to FID symmetries (and we see no obvious reason why it cannot be).

We now consider how the alternative version of q-ℓ symmetry given by Eq. (3), as used in the minimal model, may be embedded into the G_6 model. Since the minimal-model form of q-ℓ symmetry has the interchanges $E_R \leftrightarrow u_R$ and $N_R \leftrightarrow d_R$, it clearly is not an element of G_6. However, if the discrete symmetry $\mathcal{R}$ given by

$$\psi_{1R} \leftrightarrow \psi_{2R}, \quad R^\mu \leftrightarrow -R^\mu \tag{54}$$

is also imposed, then the discrete symmetry of the minimal model is given by the diagonal subgroup of $\mathcal{R}\otimes\mathcal{C}$ [where $\mathcal{C}$ is defined in Eq. (20)].

Is this new discrete symmetry $\mathcal{R}$ anomaly-free or not? It is trivial to embed (a phase-transformed version of) this discrete symmetry into an anomaly-free gauge theory. The symmetry $\mathcal{R}$ is just a remnant of the right-handed weak-isospin group SU(2)$_R$. Under the gauge group SU(6)$_{\rm PS}\otimes$ SU(2)$_L\otimes$ SU(2)$_R$, the fermion transformation laws are

$$\psi_L \sim (6,2,1), \qquad \psi_R \sim (6,1,2). \tag{55}$$

Since this fermion spectrum is anomaly-free, so is $\mathcal{R}$. Therefore it follows that the version of quark-lepton symmetry employed in the minimal model (up to phases) is anomaly-free. [Of course, if the symmetry SU(6)$_{\rm PS}\otimes$

[12] An as yet unresolved controversy exists as to whether anomalous symmetries are "explicitly" or "spontaneously" broken [29]. In the past, this contentious issue has of course revolved around anomalous continuous symmetries [and in particular the axial U(1) transformations that are an approximate symmetry of QCD]. It seems reasonable that a similar uncertainty should also exist about the status of anomalous discrete symmetries. If it turns out that anomalous symmetries are to be properly regarded as explicitly broken, then such transformations are not really symmetries in the first place. Anomalous q-ℓ symmetries, should they exist, would therefore be as misnamed as their "cheap and nasty" cousins in the case mentioned above. However, if the alternative view prevails that anomalous symmetries are in truth spontaneously broken, then this method of avoiding the domain-wall problem would be rather more attractive. Note that this point of view requires one to view the physical consequences of spontaneous breaking differently for anomalous symmetries compared with those broken in the more conventional manner. For the axial U(1) of QCD, for instance, Goldstone's theorem no longer holds, while for anomalous discrete symmetries the vacuum degeneracy does not occur. We have nothing new to contribute to this old debate, but merely wish to alert the uninitiated reader to its existence.

$SU(2)_L \otimes SU(2)_R$ were actually gauged, then the model would have a monopole problem, provided the monopoles were not inflated away, and provided that the full gauge symmetry were restored at high temperature.]

There are other versions of q-ℓ symmetry that we should also consider. Take for instance the q-ℓ symmetric model that also features left-right symmetry [3, 30]. The gauge group is $G_{q\ell LR}$ where

$$G_{q\ell LR} = SU(3)_\ell \otimes SU(3)_q \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_V, \tag{56}$$

under which the fermion classifications are

$$F_L \sim (3,1,2,1)(-1), \qquad F_R \sim (3,1,1,2)(-1),$$
$$Q_L \sim (1,3,2,1)(1), \qquad Q_R \sim (1,3,1,2)(1). \tag{57}$$

The two q-ℓ symmetries we have considered hitherto have involved interchanging left-handed leptons with left-handed quarks, and right-handed leptons with right-handed quarks (in two different ways). But the discrete q-ℓ symmetry

$$F_L \leftrightarrow (Q_R)^c, \qquad F_R \leftrightarrow (Q_L)^c \tag{58}$$

is also worth looking at. Note that we have not written down the obvious interchanges of gauge fields necessary to define this symmetry. In order for this discrete symmetry to be classified either as anomalous or anomaly-free, we have to find a way of placing $[F_L, (Q_R)^c]$ and $[F_R, (Q_L)^c]$ into non-Abelian gauge group representations. Such a group would have to be large enough to contain the whole of $G_{q\ell LR}$ as a subgroup (it would be a simple GUT group, in fact). It is clear that there is no such group with the necessary representations. (Note that we want to do this embedding without introducing any other fermions into the same GUT multiplets with the pre-existing leptons and quarks.) We therefore conclude that the symmetry of Eq. (58) is *neither* an anomalous *nor* an anomaly-free discrete symmetry.

So, we cannot use the argument pertaining to anomalous discrete symmetries to conclude that domain walls are unstable in this model, even though the discrete symmetry is not anomaly-free. The authors do not know if there are nonperturbative effects different from those associated with anomalies that might lift the vacuum degeneracy in a case such as this.

There is yet one more class of discrete q-ℓ symmetry we should discuss: those also involving the discrete spacetime symmetries of parity and time reversal [31, 2]. An example will suffice. Consider the minimal q-ℓ symmetric model gauge group $G_{q\ell}$ and its fermion spectrum Eq. (2). The q-ℓ symmetry,

$$F_L \leftrightarrow (Q_L)^c, \quad E_R \leftrightarrow (u_R)^c, \quad N_R \leftrightarrow (d_R)^c, \tag{59}$$

is also a parity symmetry, and requires the spacetime parity transformation for its consistent definition. (Gauge boson interchanges are not displayed, and the Lorentz structure is suppressed.) There are other examples of q-ℓ symmetries that are also spacetime symmetries [31]. Clearly these transformations cannot be embedded into any gauge group representation. Therefore, they are also neither anomalous nor anomaly-free discrete symmetries. Can any nonperturbative effects lift the vacuum degeneracies naively implied by spacetime discrete symmetries? Again, the authors do not know the answer to this question. (Note that the answer to this question would also be relevant to the usual discrete parity symmetry of left-right symmetric models, and to CP transformations, and so on.)

So, we conclude that all q-ℓ symmetries are either manifestly anomaly-free or not embeddable into a gauge group. Models using the former varieties are expected to have an exact vacuum degeneracy, while the latter varieties could perhaps do with some further analysis.

VIII. CONCLUSION

We have demonstrated that spontaneously broken discrete quark-lepton symmetry can be consistent with the standard hot big bang model of cosmology. The domain-wall problem can be avoided by rendering inoperative one or more of the usual assumptions made in the standard argument that domain walls are a cosmological disaster. We found (i) that domain walls can be made unstable by embedding the discrete symmetry into a continuous symmetry; (ii) that the necessary cosmological phase transition need not occur; and (iii) that stable domain walls can be inflated-away even if they form. In each of these scenarios, much interesting new phenomenology can occur at the TeV scale. The cosmological domain wall problem therefore does not in any way rule out the possibility of finding evidence at the TeV scale for an underlying quark-lepton symmetry in nature. We also discussed the idea of anomalous discrete symmetries, but did not find any clear-cut version of quark-lepton symmetry that could have its vacuum degeneracy lifted by nonperturbative effects.

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