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Evolution of soliton stars in the Lee-Wick model

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We study the evolution of soliton stars in the Lee-Wick model at finite temperatures. Our discussion uses exact temperature dependence at T_c, the critical temperature of the phase transition, and carries on until the solitons attain their present cold configuration. The effects of gravity at T_c have been included by coupling the Newtonian gravitational field to the energy density. It turns out that the gravitational energy, though smaller by at least an order of magnitude compared to the surface term, has the important effect of removing the metastable equilibrium state for a rather small number of particles. This however, does not correspond to the Schwarzschild criterion of black-hole formation but precludes the existence of any stable soliton star configuration. Further, the critical number of particles at T_c depends on the parameters of the theory and, by carefully choosing these, it is indeed possible in some cases to have solitons that survive until the present time. We discuss the properties of such stars.

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I. INTRODUCTION

Recently in a series of papers Lee and collaborators [1] proposed that, by using the nontopological soliton solution in the Lee-Wick model [2], one can obtain cold stellar configurations having a very large number of particles ($>10^{60}$). They found that huge soliton stars with a mass of $\sim 10^{12}-10^{15}$ solar mass $M_\odot$ and a size of $\sim 0.1-100$ light years can be formed without becoming black holes. The existence of such large stars has not been verified experimentally and it is important to note that their properties depend critically on the choice of parameters in the Lee-Wick model.

Cottingham and Mau [3] have shown that by making a different choice of parameters in the Lee-Wick model one can obtain soliton stars with properties more like those of white dwarfs and neutron stars. In their choice of parameters the conditions for the existence of degenerate vacuum in the Lee-Wick model are relaxed. The interior of the soliton is now a false vacuum endowed with a constant energy density (much like bag pressure) and is separated by a shell of width $\sim m_\sigma^{-1}$ from the exterior which is the true vacuum. This results in a volume term contributing to the total energy of the soliton in addition to the usual surface term. They then showed that by taking a large number of fermions, N, at zero temperature and by taking the effect of gravity by coupling the Newtonian gravitational field to the energy density (treating this as a constant), one gets stars having critical parameters $N_c \sim 10^{58}$, $M_c \sim 3M_\odot$ and $R_c \sim 25$ km (for $B^{1/4}=100$ MeV) which are like those of neutron stars. In the limit of the vanishing volume term they obtained the same critical N_c, M_c, and R_c as obtained by Lee and Pang in Ref. [1] and showed their result to be consistent with the Schwarzschild criterion of stability (i.e., $R>2GM$) on the one hand and with detailed general relativistic calculations of Ref. [1] on the other.

The authors of Ref. [3] then introduced temperature dependence in the Lee-Wick model and explored the possibility of a phase transition from one type of soliton to another. They found that at a temperature greater than the critical temperature T_c [$T_c=(180B/7\pi^2)^{1/4} \sim 100$ MeV for $B^{1/4} \sim 100$ MeV] the soliton phase would fill the whole of space and, as the Universe cools, at $T=T_c$ a phase transition would occur and a soliton would be formed. The soliton would then have characteristics similar to those considered in Ref. [1] but at temperature T_c, and for large N would be stable against dispersion of fermions. Such a soliton could have been formed in the early Universe at a time $\sim 10^{-4}$ s after the big bang. They estimated the effect of gravity with the Schwarzschild limit and obtained a critical fermion number $N_c \sim 10^{52}$, much smaller than that found in Ref. [1] at zero temperature. This yields at the formation time a maximum soliton star of mass $20M_\odot$ and radius 60 km.

In these studies, since the early Universe was hot, the contribution of the internal Fermi pressure to the free energy density was approximated by taking the parameter $\beta^3 n$ (where $\beta=1/kT$ and n is the fermion number density) to be small, and taking only the first nonzero temperature-dependent contribution $3(\beta^3 n)^2/2$ in the free energy density. This is a perfectly valid approximation at $T=T_c$, but as the star cools, this approximation is eventually violated as has also been pointed out by the authors. In this paper we have carried out the above analysis with the exact free energy density correct at all temperatures for massless fermions. Also, the effects of gravity have been included at T_c by coupling the Newtonian gravitational field to the energy density. We find that stable solitons in such a case do not exist beyond $N \sim 10^{41}$, which is much smaller compared to the value obtained in Ref. [3], namely $N \sim 10^{52}$. The soliton star continues to be bound by the surface tension at $T=T_c$ and though the contribution of gravitational energy to the free energy remains more than an order of magnitude

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smaller than the surface contribution, it has the most important effect of removing the metastable equilibrium for a rather small number of particles. It is worth noting that the Schwarzschild criterion for black-hole formation is not met at $N \sim 10^{41}$ but there cannot be any stable soliton star configuration. However, if our parameters, namely the volume energy density B and scalar mass m_σ, are adjusted, we can get stable solitons at $T = T_c$ for $N \sim 10^{41} - 10^{50}$, and some of these may survive to this date when the Universe has cooled to $T = 0$. We have studied their evolution with temperature.

The plan of the paper is as follows. In Sec. II we study the Lee-Wick model at finite temperatures and discuss the properties of stable soliton stars for a given number of particles in terms of the parameters of the model, and we study the evolution of such stars with temperature as the Universe cools. In Sec. III we incorporate the effects of gravity on the stability of such stars and find that stable solutions depend very much on the number of particles one starts with. By adjusting the parameters B and m_σ we can get solitons which would survive until the present age. Section IV is devoted to the study of soliton stars at $T = 0$, and we obtain the parameters of stable soliton stars for $N > 10^{22}$ (where the surface effects can be neglected) by solving the Tolman-Oppenheimer-Volkoff equation. We find that the maximum mass is $\sim 4 M_\odot$ with a radius of ~ 23 km for $N \sim 10^{58}$. We briefly discuss some of the properties of these stars. In Sec. V we discuss our results.

II. SOLITON STARS AT FINITE TEMPERATURE

The Lee-Wick model in the fermion case is defined by the Lagrangian [2]

$$\mathcal{L}(x) = \tfrac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}(1 - \sigma/\sigma_0)\psi - U(\sigma) \; . \tag{1}$$

The self-interaction of the σ field is taken to be of the form

$$U(\sigma) = \tfrac{1}{2} m_\sigma^2 \sigma^2 (1 - \sigma/\sigma_0)^2 + B\{4(\sigma/\sigma_0)^3 - 3(\sigma/\sigma_0)^4\} \; , \tag{2}$$

where $\sigma = 0$ is assigned to the normal absolute vacuum state and $\sigma = \sigma_0$ to the abnormal (local) minimum of U, which has an energy density B. A soliton of radius R has an interior where $\sigma = \sigma_0$, near the boundary a shell of width m_σ^{-1} over which σ changes from σ_0 to 0, and an exterior that is the true vacuum. The σ field energy in addition to a surface energy [4] $E_s = 4\pi R^2 s$, where the surface tension

$$s = \int_0^{\sigma_0} \sqrt{2U(\sigma)}\, d\sigma \simeq \tfrac{1}{6} m_\sigma \sigma_0^2 \; , \tag{3}$$

also contains a volume energy term given by

$$E_v = (4\pi/3) R^3 B \; . \tag{4}$$

When the temperature dependence is introduced in the Lee-Wick model, the effective potential energy (2) is modified and becomes temperature dependent [5]. However, for $T \ll m_\sigma$, the contribution from the scalar field is not significantly changed, whereas inside the soliton, fermions—being effectively massless due to their interaction with the σ field—would be copiously produced as fermion-antifermion pairs and would give a substantial contribution to the free energy.

The thermodynamic potential of fermions and antifermions valid at all temperatures for massless particles is given by

$$\Omega = \Omega_{f+\bar{f}} = -\left[\frac{\mu^4\beta^4}{12\pi^2} + \frac{\mu^2\beta^2}{6} + \frac{7\pi^2}{180}\right]\frac{1}{\beta^4} \; , \tag{5}$$

where μ is the chemical potential. The total free energy of a spherical soliton of radius R can be written as

$$\begin{aligned} F &= (\mu\tilde{n} + \Omega)\frac{4\pi}{3}R^3 + B\frac{4\pi}{3}R^3 + 4\pi R^2 s \\ &= \left[\frac{\mu^2\beta^2}{6} + \frac{\mu^4\beta^4}{4\pi^2} + B\beta^4 - \frac{7\pi^2}{180}\right]\frac{1}{\beta^4}\frac{4\pi}{3}R^3 + 4\pi R^2 s \; ; \end{aligned} \tag{6}$$

$\tilde{n}$ is the net fermion number density and is given by the usual relation

$$\tilde{n} = n_f - n_{\bar{f}} = -\frac{\partial\Omega}{\partial\mu} = \frac{\mu^3}{3\pi^2} + \frac{\mu}{3\beta^2} \; . \tag{7}$$

The total number of particles, N, is given by

$$N = \frac{4\pi R^3}{9\beta^3}\left[\mu\beta + \frac{(\mu\beta)^3}{\pi^2}\right] . \tag{8}$$

It is easy to see from Eqs. (6) and (8) that at temperatures above T_c free energy becomes negative and the whole of space is filled up with the soliton phase. At T_c a phase transition occurs and the phase $\sigma = \sigma_0$, with fermions in this false vacuum left as bubbles to form the soliton star. Below T_c the stable soliton-star configuration of a finite radius R is obtained by minimizing the free energy under the constraint that N and T are fixed. This is achieved by the Lagrange method of undetermined multipliers and amounts to setting the external pressure to zero, i.e.,

$$P = -\Omega - B - \frac{2s}{R} = 0 \; . \tag{9}$$

The minimization thus leads to

$$R = \frac{(24s\beta^3)\beta}{\pi^2 x^2 (2 + x^2 - 12\alpha/\pi^2 x^2)} \; , \tag{10}$$

$$N = \frac{4}{9\pi^4}(24s\beta^3)^3 \frac{1 + x^2}{x^5(2 + x^2 - 12\alpha/\pi^2 x^2)^3} \; , \tag{11}$$

$$\begin{aligned} F_{\min} &= \frac{4\pi}{3}R^2 s + \mu N \\ &= \frac{1}{3\pi^3\beta}(24s\beta^3)^3 \frac{\frac{5}{3} + \frac{3}{2}x^2 - 2\alpha/\pi^2 x^2}{x^4(2 + x^2 - 12\alpha/\pi^2 x^2)^3} \; , \end{aligned} \tag{12}$$

$$M=E=\frac{1}{9\pi^3\beta}(24s\beta^3)^3 \times\frac{9+\frac{9}{2}x^2-6B\beta^4/\pi^2x^2+21/10x^2}{x^4(2+x^2-12\alpha/\pi^2x^2)^3} =\frac{16\pi}{3}R^3B+12\pi R^2s \tag{13}$$

where $x=\beta\mu/\pi$ and $\alpha=B\beta^4-7\pi^2/180$.

The radius R, mass M, and $F_{\min}$ for the soliton star can now be obtained by solving Eq. (11) for x for fixed given values of N and β and then using Eqs. (10), (12), and (13). In Ref. [3] such a study was done under the approximation $x \ll 1$ which is quite, adequate for $N>10^{25}$ and at $T=T_c=(180B/7\pi^2)^{1/4}$, the critical temperature. Our expressions above reduce to their expressions under this approximation: namely,

$$R\simeq\left[\frac{1}{\pi}\right]^{2/5}\left[\frac{3}{4}\right]^{3/5}\left[\frac{\beta_c^2}{s}\right]^{1/5}N^{2/5} \tag{14}$$

and

$$M\simeq\frac{7}{270}\left[\frac{27\pi^3}{2s\beta_c^3}\right]^{3/5}\frac{N^{6/5}}{\beta_c} . \tag{15}$$

The soliton has properties similar to those considered by Lee and Pang and is stabilized by the surface term. The stabilization occurs at the mass and radius given in Eqs. (14) and (15). During this whole process the temperature remains fixed at T_c. As the Universe cools and the temperature falls below T_c, the volume energy term B takes over and plays the dominant role in compressing the soliton. Starting from the birth of a soliton at T_c with a fixed number of particles, one can then chart its evolution down to its cold configuration at the present time without being hampered by the constraint $x \ll 1$. The maximum number of particles in the soliton at the time of formation at T_c has been estimated by using the Schwarzschild limit $R>2GM$ and is found to be $N_c \sim 10^{52}$ for $B=(100\text{ MeV})^4$ and $s=(30\text{ GeV})^3/6$. In Fig. 1 we have plotted the radius of the soliton as a function of temperature T going from T_c to 0 for a fixed given number of particles (for fixed B and s). Changing B readjusts the critical temperature to $T_c=(180B/7\pi^2)^{1/4}$ and an increase in s and/or a decrease in B results in pushing the critical number to a higher value, beyond which the soliton becomes unstable. In this analysis we have neglected the evaporation from its surface [6] as well as the reabsorption of hadrons and nucleation of bubbles [7] inside the soliton. In Fig. 2 we have shown the variation in mass of the soliton as it cools from T_c to its cold configuration. As noted in Ref. [3], for particle number $N>10^{46}$ the star formed at T_c would survive to the present epoch. The critical number N_c, however, depends on the parameters and, as we show below, the effect of coupling is to remove the metastable minimum of the free energy, thereby pushing the soliton to an unstable configuration for a much smaller critical number N_c than is required by the gravitational singularity leading to black-hole formation.

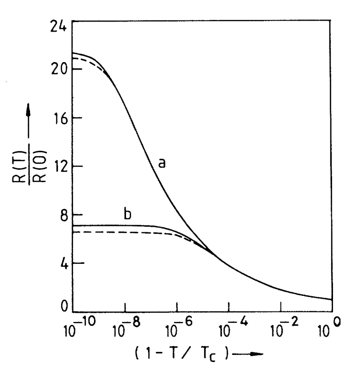

FIG. 1. Evolution of the soliton star radius with temperature. Graphs (a) and (b) correspond to the number of particles 10^{41} [$B^{1/4}=100$ MeV, $s=(30\text{ GeV})^3/6$] and 10^{49} [$B^{1/4}=40$ MeV, $s=(650\text{ GeV})^3/6$], respectively. The solid lines show the effects of gravity and the dashed lines give the evolution without gravity.

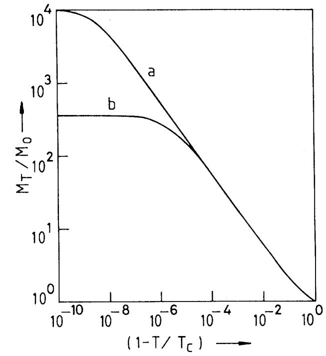

FIG. 2. Evolution of the soliton star mass with temperature. Graphs are labeled as in Fig. 1.

III. EFFECTS OF GRAVITY ON SOLITON STARS

For large N, one expects gravity to play an increasingly important role in determining the stability of the soliton star. As in Ref. [3], one can estimate it either by the Schwarzschild limit to get N_c beyond which a soliton star collapses into a black hole, or by coupling the Newtonian gravitational field to the free energy density inside the soliton, treating this as a constant throughout. Cottingham and Mau [9] showed that the two approaches give identical results at $T=0$ which are further in agreement with the calculations of the authors of Ref. [1] in a general relativistic framework. We find that the latter procedure for the existence of stable solutions gives a much smaller bound at T_c than that estimated by the Schwarzschild criterion. To see this, we write the total free energy of the soliton including the gravitational self-energy by treating the total energy density ρ as a constant throughout the star as

$$F=\left[\frac{\mu^2\beta^2}{6}+\frac{\mu^4\beta^4}{4\pi^2}\right]\frac{V}{\beta^4}+4\pi R^2s +\left[B-\frac{7\pi^2}{180\beta^4}\right]V-\frac{3G}{5R}(\rho V)^2 , \tag{16}$$

where the total energy density ρ is given by

$$\rho=\left[\frac{\mu^2\beta^2}{2}+\frac{\mu^4\beta^4}{4\pi^2}+\alpha+\frac{7\pi^2}{45}\right]\frac{1}{\beta^4}\ .$$

The mass of the star can be obtained from the expression

$$M=F-TV\left[\frac{\partial\Omega}{\partial T}\right]_{V,\mu}$$
$$=\left[\frac{\mu^2\beta^2}{2}+\frac{\mu^4\beta^4}{4\pi^2}+\alpha+\frac{7\pi^2}{45}\right]\frac{V}{\beta^4}$$
$$+4\pi R^2 s-\frac{3G}{5R}(\rho V)^2\ . \qquad (17)$$

At $T=T_c=(180B/7\pi^2)^{1/4}$. [$B^{1/4}=100$ MeV and $s=(30\ \text{GeV})^3/6$], the approximation $\mu\beta_c \ll 1$ is valid, and using $N=(4\pi R^3\mu)/(q\beta_c^2)$, F is given by

$$F\simeq\frac{3}{2}\frac{\beta_c^2N^2}{V}+4\pi R^2 s-\frac{3G}{5R}\left[\frac{9}{2}\frac{\beta_c^2N^2}{V}+\frac{7\pi^2V}{45\beta_c^4}\right]^2\ . \qquad (18)$$

We see that major contribution to mass comes essentially from the volume energy term B and is $\sim 4BV$. In order to study the existence of stable solitons we make a change of length scale:

$$R=\left[\frac{9N^2\beta_c^2}{32\pi^2 s}\right]^{1/5} y\ . \qquad (19)$$

We can write the free energy F as

$$F\simeq\left[\frac{9N^2\beta_c^2}{8\pi}\right]^{2/5}(4\pi s)^{3/5}\left[\frac{1}{y^3}+y^2-\epsilon y^5\left[1+\frac{\epsilon_1}{y^6}\right]^2\right] \qquad (20)$$

where

$$\epsilon=\frac{3G}{5}\left[\frac{9N^2\beta_c^2}{8\pi}\right]^{3/5}(4\pi s)^{-8/5}\left[\frac{16\pi}{3}B\right]^2$$

and

$$\epsilon_1=3\left[\frac{16\pi}{3}B\right]^{-1}\left[\frac{9N^2\beta_c^2}{8\pi}\right]^{-1/5}(4\pi s)^{6/5}\ . \qquad (21)$$

For large values of N it goes as

$$F\sim\text{const}\times\left[\frac{1}{y^3}+y^2-\epsilon y^5\right]\ . \qquad (22)$$

If $\epsilon=0$, (no gravity) then we have a stable minima at $y=(3/2)^{1/5}$. However for $\epsilon\neq 0$, there is a metastable minimum around $y=(3/2)^{1/5}$. The condition for the metastable minimum to exist is satisfied for $\epsilon<(4)^{-8/5}$. This will not be met for

$$N>\left[\frac{3G}{5}\right]^{-5/6}\left[\frac{16\pi}{3}B\right]^{-5/3}(\pi s)^{4/3}\left[\frac{8\pi}{9\beta_c^2}\right]^{1/2}\ . \qquad (23)$$

For the values of B and s considered in Ref. [3], we get $N_c>10^{41}$. This was also verified numerically without taking any approximations. The limits for mass and radius are $M_c\sim 5\times 10^{17}$ kg and $R_c\sim 1$ m. If the parameters are changed, N_c also changes and we can get a stable configuration having

$$N_c\sim 10^{46}\ \text{for}\ B=(80\ \text{MeV})^4\ \text{and}\ s=(300\ \text{GeV})^3/6$$
$$\text{and} \qquad (24)$$
$$N_c\sim 10^{49}\ \text{for}\ B=(40\ \text{MeV})^4\ \text{and}\ s=(650\ \text{GeV})^3/6\ .$$

It should be mentioned that, at T_c, though the surface term dominates over the gravitational term by roughly an order of magnitude, the gravity has the most important effect of removing the metastable minima for a smaller number of particles than is warranted by the Schwarzschild criterion for the formation of black holes. This can be clearly seen from Fig. 3(a) where we have plotted the total free energy at T_c as a function of chemical potential for different values of N. One finds that for $N\sim 10^{41}$ the contribution of gravity to the total free ener-

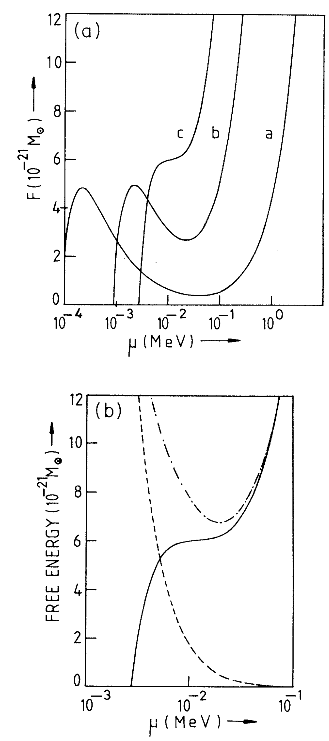

FIG. 3. (a) Free energy of the soliton as a function of chemical potential near the metastable minimum at T_c. Graphs a, b, and c correspond to $N=10^{40}$, 10^{41}, and 3.16×10^{41}, the critical number for $B^{1/4}=100$ MeV and $s=(30\ \text{GeV})^3/6$. Notice that graph c does not possess any metastable minimum and therefore does not lead to any stable soliton configuration. (b) Free energy as a function of chemical potential. The dashed-dotted line represents the free energy without gravity, the dashed line represents the magnitude of the contribution of the gravitational energy, and the solid line stands for the total free energy (including gravity) and shows the disappearance of the metastable minimum. All graphs are for the critical number of particles $N_c=3.16\times 10^{41}$ at T_c.

gy results in wiping out the metastable minima [Fig. 3(b)]. In Fig. 1 we have also shown the evolution of radius with temperature by including gravity.

IV. COLD SOLITON STARS

At $T=0$ we have a Lee-Pang model of solitons where the confinement is provided by surface tension, and we have another model discussed by Cottingham and Mau in which the bag pressure as well as the surface tension prevents the fermions from dispersing. It is found that, at the critical temperature, a phase transition from one type of soliton to another takes place, and that for $N>10^{22}$ the surface contribution is negligible and the confinement is mainly provided by the volume term. In this section we study the stellar parameters and other properties of cold soliton stars by ignoring the surface term.

It is well known that the mass-radius relation of a spherically symmetric cold steller configuration can be obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations of hydrostatic equilibrium in general relativity for a given equation of state. The equation of state $P=P(\rho)$ is easily obtained for fermions in the Lee-Wick model. The total energy density ρ and external pressure P at $T=0$ are given by

$$\rho=\mu^4/(4\pi^2)+B$$

and

$$P=-\Omega-B=\mu^4/(12\pi^2)-B \tag{25}$$

giving a very simple equation of state, namely,

$$P=(\rho-4B)/3 \; . \tag{26}$$

For a given fixed value of B, we now solve the TOV equations

$$\frac{dM(r)}{dr}=4\pi r^2\rho(r)$$

and

$$\frac{dP(r)}{dr}=-\frac{GM(r)}{r^2}[\rho(r)+P(r)]\frac{1+4\pi r^3P(r)/M(r)}{1-\frac{2GM(r)}{r}} \tag{27}$$

with the boundary condition that $M\to 0$ as $r\to 0$ and $P\to 0$ at the surface $r=R$. Once the equation of state is known, the properties of the star are determined by choosing a central density and integrating outwards till $P\to 0$. The total number of particles N in the star is given by

$$N=4\pi\int_0^R n(r)r^2\left[1-\frac{2GM(r)}{r}\right]^{-1/2}dr \; , \tag{28}$$

where the number density

$$n(r)=\mu^3(r)/(3\pi^2)=\tfrac{2}{3}\sqrt{2/\pi}(\rho-B)^{3/4} \; . \tag{29}$$

It has been shown that, for our simple equation of state, the TOV equation admits a simple scaling law [8], and one can obtain a new sequence of model stars with different values of B by taking each model from the original sequence and applying the scale transformations

$$\rho'=a\rho \; , \quad P'=aP \; , \quad M'=a^{-1/2}M \; , \quad \text{and} \quad R'=a^{-1/2}R \; , \tag{30}$$

where

$$a=(B'/B) \; . \tag{31}$$

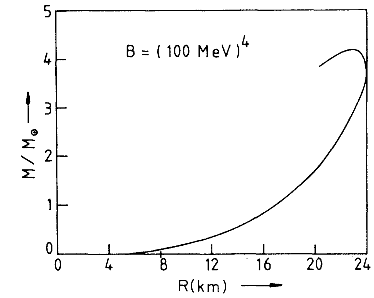

FIG. 4. Mass-radius relation of a cold soliton star for $B^{1/4}=100$ MeV and neglecting surface effects.

In Figs. 4 and 5 we show the mass and total number of particles as a function of radius. We find, just as for strange stars [9], $M\sim R^3$ for soliton stars of $M\sim 0.5M_\odot$. In this region gravity plays essentially no role and the confinement is provided by the bag pressure B. As mass increases gravity becomes important and the mass reaches a maximum, and then becomes double valued:

$$M_{\rm max}=4\left[\frac{B^{1/4}}{100\text{ MeV}}\right]^{-2}M_\odot \; ,$$
$$R_{\rm max}=23\left[\frac{B^{1/4}}{100\text{ MeV}}\right]^{-2}\text{km} \; , \tag{32}$$
$$N_{\rm max}=10^{58}\left[\frac{B^{1/4}}{100\text{ MeV}}\right]^{-3} .$$

In contrast, conventional (neutron) star radii decrease with increasing mass and there is a minimum mass be-

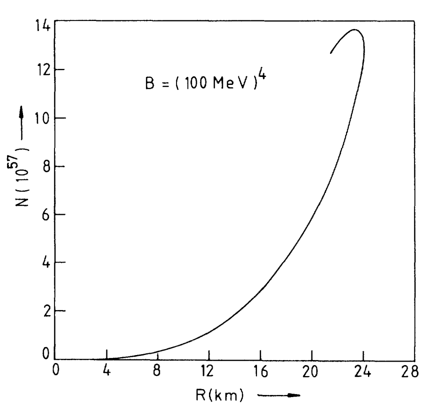

FIG. 5. Number of particles versus radius for the cold soliton star as in Fig. 4.

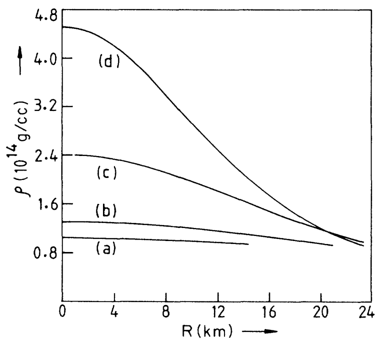

FIG. 6. Density profile for some soliton stars of Fig. 4. Profiles (a), (b), (c), and (d) correspond to $0.59M_\odot$, $2.01M_\odot$, $3.87M_\odot$, and maximum mass ($4.23M_\odot$) models respectively.

cause for neutron stars $\rho \to 0$ and $P \to 0$ on the surface, whereas for soliton stars density drops abruptly from $\rho = 4B = 0.933\times 10^{14}$ gm/cm^3 (for $B^{1/4} = 100$ MeV) to zero, and there is no minimum mass. We also find (Fig. 6) that as we move away from center to the surface, there is only a modest variation of density in contrast with neutron stars. The soliton stars, being very much like the strange stars, share some interesting properties with the latter, namely, (a) soliton stars being self-bound like strange stars have higher mean density and are therefore capable of sustaining faster rotation [10], and (b) the surface of a soliton star, being principally bound by bag pressure and not by gravity, is very different from a conventional star. An immediate consequence of this is that the luminosity of a soliton star can be much higher than that of a conventional star and is not bound by the Eddington limit.

V. CONCLUSIONS

We have studied the evolution of soliton stars formed during phase transition at some critical temperature $T_c = (180B/7\pi^2)^{1/4}$ in the early Universe and discussed under what parameters would they survive until the present time in their cold configuration. We used exact temperature dependence starting at T_c and carried on until the solitons cooled to their present state. The effects of gravity were included by coupling the Newtonian gravitational field to the energy density. It was found that at the time of formation at T_c the soliton star, though bound by surface tension, is stable for a rather small number of particles. The effect of gravity is to wipe out the metastable minima in the total free energy of the soliton thereby making the configuration unstable for a smaller number of particles than would be required for the formation of a black hole by the Schwarszchild criterion. We find that Lee-Pang-type solitons are indeed possible at $T = T_c$ but the choice $B^{1/4} = 100$ MeV and $s = (30 \text{ GeV})^3/6$ gives us stable solitons with $N_c \sim 10^{41}$, which probably would evaporate and would not survive. But for another choice of parameters [$B^{1/4} = 80$ MeV, $s = (300 \text{ GeV})^3/6$] it is possible to have stable solitons for $N \sim 10^{46}$ that would survive. The mass and radius of such a star at the time of formation and after evolution to the present epoch would be

$$M_{T_c} \sim 3.4\times 10^{-9} M_\odot \ , \quad R_{T_c} \sim 3.5\times 10^{-2} \text{ km} \ ,$$

$$M_0 \sim 2.4\times 10^{-12} M_\odot \ , \quad R_0 \sim 3.1\times 10^{-3} \text{ km} \ .$$

Finally, at $T = 0$ we studied the properties of soliton stars bound principally by the bag pressure by solving the TOV equation of hydrostatic equilibrium. We found the properties of such soliton stars to be similar to those of strange stars.

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