

## ARTICLES

**Test of the gravitational inverse-square law at 0.4- to 1.4-m mass separations**

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Using a superconducting gravimeter, we have measured the force on a spherical shell of Nb due to the presence of a spherical mass  $M$  that is periodically moved up and down on an elevator underneath the gravimeter. Over a distance range from 0.4 to 1.4 m Newton's inverse square law is verified to a precision of  $\pm 1\%$ . At a 95% confidence level, our data restrict the coupling constant  $\alpha$  of a non-Newtonian Yukawa potential to be  $|\alpha| < 0.012G$  for Yukawa ranges from 0.2 to 2.0 m.

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## I. INTRODUCTION

The recent history of investigations of gravity includes such landmarks as the repeat of the Eötvös experiment [1,2], high-precision measurements of the gravitational constant  $G$  [3,4], precision tests of general relativity theory by radar-ranging studies of the solar system [5,6], and radio wave studies of binary pulsars that have yielded the first evidence of the existence of gravitational radiation [7,8]. There have also been several reports of unusual effects that have not been reproduced upon further investigation. Notable are a deviation from Newton's  $1/R^2$  law reported by Long [9] and laid to rest by the experiments of Spero *et al.* [10] and of Chen *et al.* [11], a possible anomaly in the acceleration of gravity in a mine shaft [12] that has since been accounted for [13], a possible "fifth force" affecting various Eötvös balance measurements [14–16] that has been ruled out by the work of Stubbs *et al.* [17] and many others [18–30], an anomalous acceleration due to gravity found on a 600-m

tower [31] that has been contradicted by later experiments [32–36], and an amazing gyroscopic effect on the weight of an object [37] that has also been confuted [38,39]. There is now apparently no possibility for a composition-dependent "fifth force" stronger than  $10^{-3}$  to  $10^{-4}$  of gravity with a range greater than 1 m, nor for a composition-independent nongravitational force stronger than  $10^{-4}$  of  $G$  at distances or order 10 cm. For the latter type of force there remains a window at ranges of order 1 m where there could be a new force as large as 1% of  $G$ .

Despite the negative result of all confirmed experiments to date, the fascinating possibility that a sufficiently sensitive experiment would be able to uncover a new force continues to be fueled by various theories predicting feeble Yukawa or van der Waals potentials arising from the exchange of light neutral particles such as neutrinos, axions, supersymmetry particles, and so forth [40–52]. We report here the development of a sensitive technique for searching for departures from the

$1/R^2$  law and our null results at the 1% level for mass separations in the range of 0.4 to 1.4 m.

## II. APPARATUS

Our apparatus, depicted in Figs. 1 and 2, consists of a superconducting gravimeter that measures the force on a small test mass  $m$  due to the presence of a large mass  $M$  that is moved periodically between two positions. The square-wave response of the gravimeter is separated out from the much larger tidal signal and compared to the signal calculated from the known distribution of moving mass and Newton's law. The use of a periodically moving large mass makes it possible to run the experiment in an ordinary laboratory, since the unwanted perturbations due to various local masses will average to zero for long times.

The gravimeter, used extensively for measurements of the gravitational field of the Earth [53–59] and to set limits on the effects of a universal preferred reference frame [60], is described in previous publications [59,61–63]. The principle of the gravimeter, shown in Fig. 1, is the persistent-current suspension of a hollow superconducting Nb sphere to obtain a balance that is, so far as we can determine, perfectly free of offset drift and changes in the

effective spring constant [61]. Displacements of the sphere caused by external forces are sensed by a capacitance bridge and nulled out by a feedback current  $I$  in a small coil. A voltage  $V$  proportional to  $I$  is analog filtered with a 20 sec time constant and digitized every 10 sec with a 17 bit converter. The 20 sec time constant was chosen to suppress the high frequency noise with a time constant intermediate between the digitization interval and the averaging time constant. Every minute, the average of the last six readings is recorded by a computer. It is suspended in a liquid-He Dewar that is mounted on a pier over a pit containing the moving mass, as shown in Fig. 2. The orientation of the gravimeter is maintained in the vertical direction by tilt-meter feedback to a pair of orthogonal leveling devices attached to the Dewar.

The sensitivity of the gravimeter to external influences other than accelerations has been shown to be negligible. The test mass is shielded by a superconducting Pb shroud and by a high-permeability magnetic shield that surrounds the liquid He Dewar. Bringing a large permanent magnet near the Dewar, and thus changing the local magnetic field by more than an order of magnitude, produces no observable gravimeter signal. In another test, the signal from a nearby flux-gate magnetometer showed no correlation with the gravimeter signal. The outside temperature causes changes in the measured value of the local acceleration of gravity because of changes in atmospheric pressure and the concomitant changes in the distribution of the mass of the air. However, there is no no-

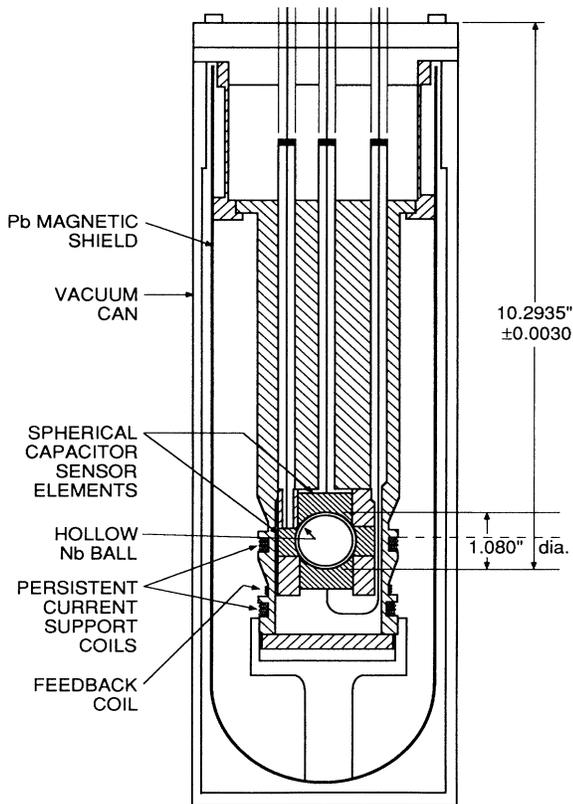


FIG. 1. Superconducting gravimeter, consisting of a hollow Nb ball that is suspended by persistent currents in vacuum.

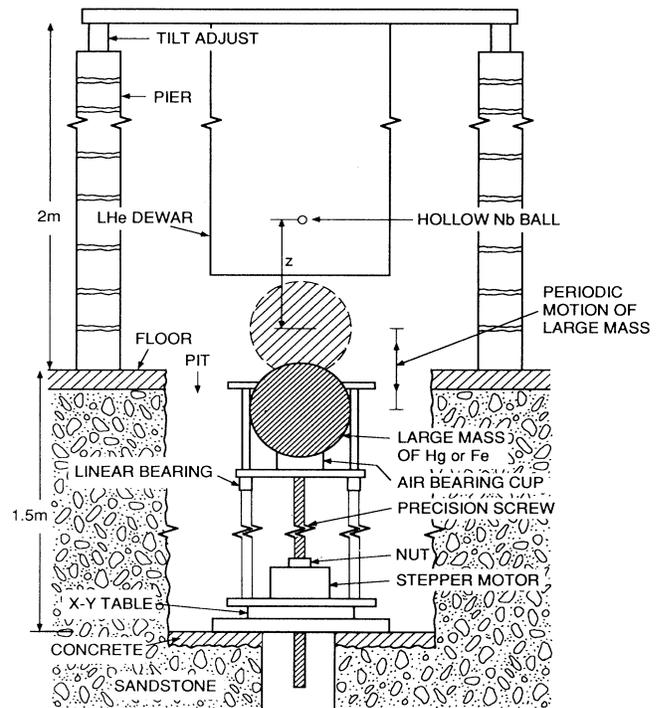


FIG. 2. Experiment for measuring the acceleration caused by a large mass that moves up and down on an elevator beneath the superconducting gravimeter.

ticeable correlation of the local room temperature and the gravimeter signal, principally because the temperature of the test mass and its environment is stabilized to  $\pm 5 \mu\text{K}$ .

The small mass is a  $(2.5751 \pm 0.0025)$  cm diameter hollow sphere of Nb that was prepared by electrodepositing Nb on a  $(2.540 \pm 0.001)$  cm diameter Cu sphere. The Cu was etched out through a small hole in the top of the sphere. When floated on acetone, the oscillation period of the sphere was  $(0.60 \pm 0.02)$  sec/cycle, from which we conclude that the center of mass of the sphere is  $(0.122 \pm 0.008)$  cm below the center of the figure. When suspended in the gravimeter, the orientation of the spherical shell is such that the hole is at the top.

The large mass was either a solid sphere of type 304 stainless steel or a shell of type 304 stainless steel and  $(0.20 \pm 0.05)$  cm thick filled with mercury. The solid stainless steel sphere is  $(35.560 \pm 0.0025)$  cm in diameter and weighs  $(184\,441.1 \pm 1.0)$  g as determined by the National Institute of Standards and Technology. When the sphere was floated on an air bearing, a pressure instability caused it to rotate once in 3 min about an axis tilted at  $45^\circ$  from the vertical. Assuming that a displacement  $\delta r$  of the center of mass from the center of the figure would have been noticeable if it has been sufficient to cause a pendulum oscillation with a 30 sec period or less, we conclude that  $\delta r < 0.006$  cm.

The total mass of the mercury-filled shell is  $(323\,865 \pm 100)$  g [53], as determined by a beam balance calibrated using standard masses. The shell was filled with mercury in a cool room to ensure the absence of an air space at normal room temperature. The outer surface of the shell is an ellipsoid of revolution [63]. We measured the major axis to be 36.3525 cm and the minor axis 35.6235 cm. The major axis was within  $5^\circ$  of vertical during the experiment. For the purposes of calculating the force on the small mass, the departure from sphericity is approximated as a mass quadrupole [63] consisting of a 1125 g mass point located at each pole and a 2250 g missing mass (negative mass) at the center. (The poles are defined as the points where the major axis intersects the surface.)

The large mass was supported by a platform, shown in Fig. 2, that could be raised and lowered under the gravimeter, or moved independently in two horizontal directions by computer control. The platform was positioned by four linear ball bearings riding on 1.905 cm diameter rods that were supported by an  $x$ - $y$  motion table. A 3.81 cm diameter precision screw having a pitch of 1.5748 turns per cm was fixed to the platform upon which the ball rested and was supported at its lower end by a nut directly driven by a stepping motor. The vertical position was read to 0.0001 cm precision by an optical linear encoder. Displacements determined from the revolutions of the support screw agreed with the linear encoder distances to within  $\pm 0.006$  cm over the range of the large mass corresponding to all the upper positions. In the lower portion of the range, the platform tilted significantly as the screw rotated. Since the encoder was located on the outer edge of the platform and read in error by as much as  $\pm 0.02$  cm in this range, we determined

the vertical displacement from the rotations of the stepper motor.

The absolute distance from the large mass to the small mass was determined as follows. A Wild model KM273 cathetometer was set up about 200 cm both from the axis of the gravimeter and from a vertical Invar rod fixed to the floor. The cathetometer axis did not deviate from the vertical by more than  $3 \mu\text{rad}$  as the telescope was rotated about the axis. A fused silica rod 0.6 cm diameter and 122 cm long was inserted into a long thin-walled stainless steel tube capped by a  $(6.419 \pm 0.001)$  cm long drill bit. This combination was inserted into the liquid helium Dewar and twisted to drill through a several cm thick layer of solid air that had accumulated on top of the internal vacuum can holding the Nb ball. After electrical contact between the drill and the vacuum can was established, the cathetometer was used to measure the small vertical distance between the top of the fused silica rod and the bottom of a glass rod fixed to the Invar rod. The fused silica rod was then withdrawn from the Dewar and held against the Invar rod such that its top end was flush against the bottom of the glass rod. With the platform in a reference position for which the linear encoder reading was  $(0.000 \pm 0.001)$  cm, the cathetometer was used to measure the vertical distance between the bottom of the fused silica rod and the top of the large mass. The dimensions of the vacuum container, the radius of the large mass, the dimensions of the drill, and the known thermal contractions of various components are used to convert the cathetometer readings into a center-of-mass separation of the two masses corresponding to the reference position of the platform. Repeated measurements indicate that the uncertainty of the center-of-mass separation is  $\pm 0.020$  cm, including the small center-of-mass correction for the Nb ball.

### III. CALIBRATION

The calibration factor of the superconducting gravimeter could be obtained by comparison of its tidal signal to that of an absolute free-fall gravimeter [64]. Unfortunately, such a comparison was not available at the time of this writing. An attempt to effect a comparison using a second gravimeter that was calibrated at La Jolla and moved to Miami for a comparison with a free-fall gravimeter [65] yielded a value for the calibration factor such that our measurements described below imply a value for the gravitational constant  $G$  within 2% of the expected value. We are unable to draw a more precise conclusion from the comparison because the effect of transportation on the calibration factor of the instrument has not been determined.

### IV. RESULTS

A typical one-day record of the gravimeter signal is presented in Fig. 3. Superimposed on the tidal signal is a small square wave due to the motion of the large mass up and down. From the expanded view of a portion of the record shown in Fig. 4 we can see that the noise level on the 40 mV signal is about 2 mV peak to peak for one-

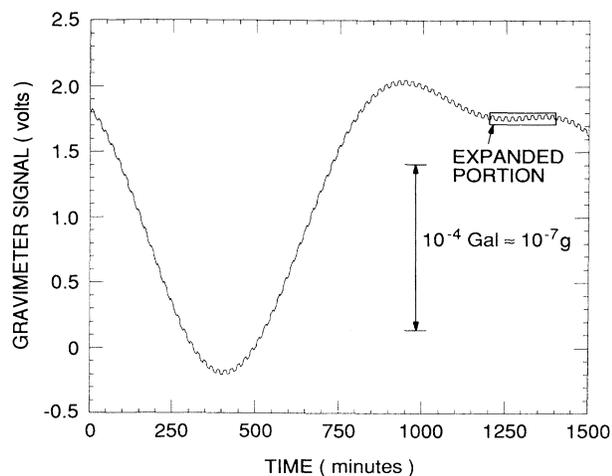


FIG. 3. A one-day segment of data from the gravimeter showing the large tidal signal and the small square wave due to the motion of the large mass on the elevator.

minute averaging. In order to extract the amplitude of the square-wave component, the data is first fitted with a theoretical tide expression computed from the ephemerides of the Moon and Sun after interpolating over any spikes caused by local disturbances. The fitted tide is subtracted and whole periods are deleted from the remaining square wave if the noise is greater than 2–3 times normal. A Hamming high-pass filter with a 60 min time constant is then applied. The amplitude of each transition (the rising and falling parts of the square wave) is computed by fitting a step function with a sloping additive background to the four data points on either side of the step that are not affected by the 1 min averaging time and the transit time of the large mass. The sloping background is included to remove the effects of slow offset drifts not eliminated by the high-pass filter. The average

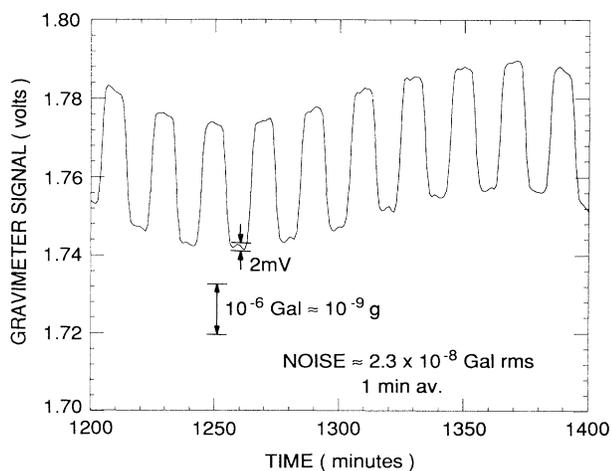


FIG. 4. Expanded portion of the gravimeter signal showing the large-mass signal in detail.

step height is determined from the average of all the steps in a run weighted by the inverse square of the error estimate determined by the fit to the step function. A minimum variance of  $(0.5 \text{ mV})^2$  is added in quadrature to the error estimate to avoid unduly weighting any datum that happens by chance to have an unusually small variance. Because of the restricted bandwidth of the Hamming filter, its use introduces a small error into the measured amplitude. We have determined the magnitude of this effect using an artificially generated square wave and have multiplied the data by the resulting correction factor of 1.001 24. The error estimate of the average is taken to be the inverse square root of the sum of the weights. This is in agreement in all cases with the error estimated from the weighted distribution of deviations from the mean. Because of the correlations introduced by the high-pass filter, the error so determined is underestimated by a factor  $1.282 \pm 0.066$  found from an examination of the statistics of the data when grouped into different size sets. The error estimate is multiplied by the factor 1.28 to account for this correlation effect.

The experiment was operated for more than a year

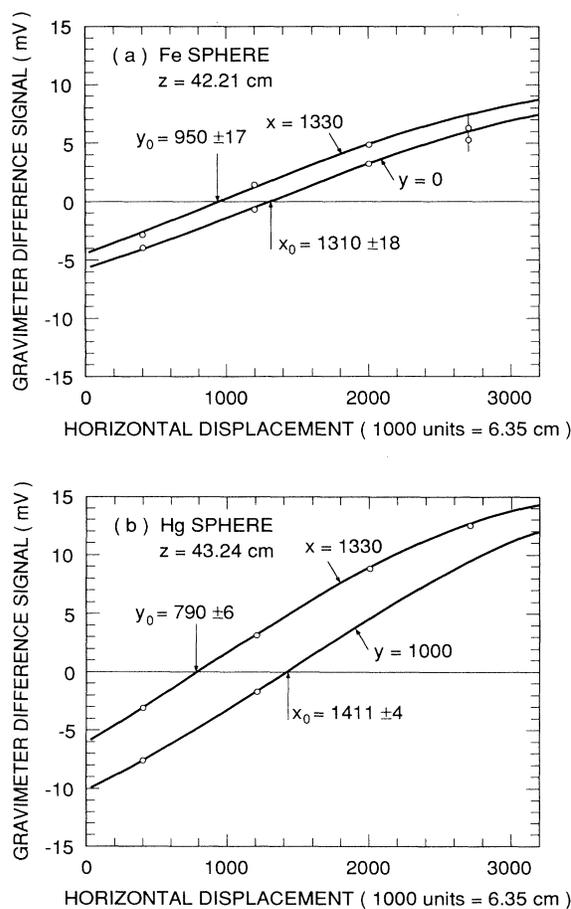


FIG. 5. Amplitude of the gravimeter signal square wave caused by horizontal motion of the elevator and large mass for determining the position  $(x_0, y_0)$  that is directly under the small mass: (a) steel mass, (b) Hg mass.

with each of the two masses, first the steel and then the mercury mass. As a preliminary measurement for choosing the proper horizontal positions of the elevator, the large masses were raised to near their uppermost positions, and data were obtained with the  $x$ - $y$  table moving periodically between two positions. Figures 5(a) and 5(b) show, for the steel and Hg spheres respectively, the square-wave amplitude of the gravimeter signal versus mean horizontal position as the table moves  $\pm 400$  units ( $\pm 2.54$  cm) in a square wave. The two curves in each figure are for motion first with  $x$  fixed and then with  $y$  fixed. Assuming cylindrical symmetry of the attracting mass, the two curves are sufficient to determine the position  $(x_0, y_0)$  for which the large mass is directly under the small mass. The gravimeter square-wave amplitude is proportional to  $V(x_1, y_1, z) - V(x_2, y_2, z)$ , where

$$V(x, y, z) = aMGz[(x - x_0)^2 + (y - y_0)^2 + z^2]^{-3/2}. \quad (1)$$

By fitting the above expression for the square-wave amplitude to the curves in Fig. 5 we determine the zero-crossing points  $(x_0, y_0)$  that are the desired horizontal positions for obtaining data with the elevator moving up and down. The statistical error bars are smaller than the size of the circles representing each datum; however, two of the measurements have been assigned an error of  $\pm 1$  mV and thus essentially ignored in the fit in Fig. 5 because of their unreasonably large deviation from the

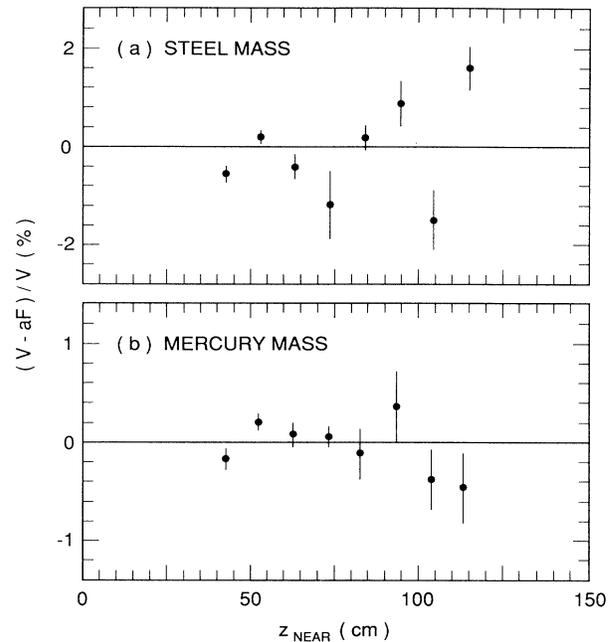


FIG. 6. Average residuals from fitting the measurements of Tables I and II assuming the validity of Newton's law of gravitation. The error flags are computed from the weighted standard deviations of the data averaged from the Tables.

TABLE I. Data obtained with the steel mass.  $z$ , distance between the large mass and the small mass in the upper position;  $dz$ , displacement between the upper and lower positions of the large mass;  $V$ , amplitude of the square-wave component of the gravimeter signal;  $F$ , computed difference in the Newtonian acceleration on the small mass due to the displacement of the large mass between the upper and lower positions;  $a$ , fitted calibration coefficient;  $V - aF$ , residual.  $1/a = 77.66 \pm 0.18 \mu\text{Gal}/\text{V}$ ,  $\chi^2 = 71.58$ , 24 degrees of freedom,  $a = (12\,877 \pm 30) \text{V}/\text{Gal}$ .

No.	Date d m y	$z$ [cm]	$dz$ [cm]	$V$ [mV]	$F$ [ $\mu\text{Gal}$ ]	$V - aF$ [mV]	$(V - aF)/V$ [%]
1	05 08 1989	104.372	41.340	7.328 $\pm$ 0.044	0.578	-0.112	-1.527
2	10 08 1989	73.370	31.046	15.278 $\pm$ 0.165	1.202	-0.201	-1.315
3	12 08 1989	63.038	31.005	22.634 $\pm$ 0.053	1.764	-0.079	-0.349
4	20 08 1989	52.702	20.670	28.295 $\pm$ 0.064	2.205	-0.093	-0.329
5	22 08 1989	42.368	20.670	49.202 $\pm$ 0.077	3.843	-0.285	-0.578
6	24 08 1989	114.726	31.005	4.847 $\pm$ 0.033	0.374	0.030	0.613
7	31 08 1989	83.724	31.005	11.277 $\pm$ 0.247	0.856	0.259	2.301
8	05 09 1989	94.166	31.005	8.217 $\pm$ 0.095	0.630	0.102	1.244
9	09 09 1989	94.160	31.005	8.192 $\pm$ 0.067	0.630	0.076	0.927
10	10 09 1989	63.154	31.005	22.805 $\pm$ 0.300	1.756	0.197	0.862
11	16 09 1989	114.962	31.005	4.941 $\pm$ 0.048	0.372	0.150	3.036
12	20 09 1989	114.966	31.005	4.890 $\pm$ 0.045	0.372	0.099	2.034
13	26 09 1989	94.322	31.005	8.139 $\pm$ 0.049	0.627	0.059	0.728
14	01 10 1989	83.978	31.005	10.919 $\pm$ 0.037	0.849	-0.013	-0.115
15	07 10 1989	83.978	31.005	10.981 $\pm$ 0.049	0.849	0.049	0.451
16	10 11 1989	52.975	31.046	35.010 $\pm$ 0.055	2.720	-0.015	-0.044
17	04 11 1989	114.862	31.005	4.859 $\pm$ 0.150	0.373	0.057	1.170
18	08 11 1989	73.510	31.005	15.219 $\pm$ 0.130	1.195	-0.173	-1.136
19	08 12 1989	114.791	31.005	4.893 $\pm$ 0.058	0.374	0.083	1.696
20	12 12 1989	83.791	31.005	11.031 $\pm$ 0.058	0.854	0.036	0.328
21	27 12 1989	52.789	31.005	35.430 $\pm$ 0.053	2.742	0.124	0.349
22	02 01 1990	52.797	31.005	35.443 $\pm$ 0.058	2.741	0.150	0.423
23	07 01 1990	52.806	10.335	17.507 $\pm$ 0.081	1.361	-0.019	-0.109
24	11 01 1990	63.756	10.335	10.263 $\pm$ 0.074	0.810	-0.171	-1.670
25	04 02 1990	52.793	31.005	35.574 $\pm$ 0.214	2.741	0.274	0.771

theoretical curve. The fitted values of  $a$  ( $14\,471 \pm 258$ ) V/Gal, and ( $13\,160 \pm 53$ ) V/Gal, respectively for Fe and Hg, are not to be compared with the calibration constant in Tables I and II below because we have not included the mass of the platform in Eq. (1). It should be pointed out that the zero-cross points determined in Fig. 5 by Eq. (1) are not significantly sensitive to the  $\approx 10\%$  mass contribution of the elevator. During the vertical displacement experiments, the elevator was actually slightly displaced from the  $(x_0, y_0)$  position by distances  $dr_{\text{Fe}} = (0.342 \pm 0.090)$  cm and  $dr_{\text{Hg}} = (0.577 \pm 0.028)$  cm.

Tables I and II contain a list of all the measurements using the steel and Hg masses moving vertically. After the date of the beginning of each run is listed the minimum distance between the large and small masses,  $z = z_{\text{near}}$ , the difference between the maximum and minimum distances,  $dz$ , and the measured average step height  $V$ . Using Newton's law and the known masses and

positions of all the moving components of the elevator, and assuming that the gravitational constant is  $G = 6.6726 \times 10^{-8}$  cgs units, we calculate the difference in acceleration,  $F$ , corresponding to the two positions of the elevator. The residual  $V - aF$  is found after a weighted least-squares fit that yielded the values  $a_{\text{Fe}} = (12\,877 \pm 30)$  V/Gal and  $a_{\text{Hg}} = (12\,849 \pm 13)$  V/Gal. These error bars have been increased over the error bars obtained from the sum of the weights by a factor of the square root of  $\chi^2$  divided by the number of degrees of freedom. The first line in Table II is a measurement taken with no large mass on the elevator, and so tests our ability to account for the elevator mass. The measurement is in agreement with the computed acceleration. Figure 6 displays the percentage residuals, shown in the last column of the Tables, averaged into 10 cm wide bins of values of  $z_{\text{near}}$ . The measurement with only the platform moving is not included in Fig. 6.

TABLE II. Data obtained with the mercury mass.  $z$ , distance between the large mass and the small mass in the upper position;  $dz$ , displacement between the upper and lower positions of the large mass;  $V$ , amplitude of the square-wave component of the gravimeter signal;  $F$ , computed difference in the Newtonian acceleration on the small mass due to the displacement of the large mass between the upper and lower positions;  $a$ , fitted calibration coefficient;  $V - aF$ , residual.  $1/a = (77.83 \pm 0.08) \mu\text{Gal}/V$ ,  $\chi^2 = 54.86$ , 32 degrees of freedom,  $a = (12\,849 \pm 13)$  V/Gal.

No.	Date d m y	$z$ [cm]	$dz$ [cm]	$V$ [mV]	$F$ [ $\mu\text{Gal}$ ]	$V - aF$ [mV]	$(V - aF)/V$ [%]
1	12 04 1990	52.624	31.005	$1.004 \pm 0.064$	0.078	0.001	0.060
2	27 04 1990	114.194	31.005	$8.417 \pm 0.087$	0.652	0.046	0.543
3	03 05 1990	93.558	31.005	$14.215 \pm 0.071$	1.106	0.006	0.043
4	08 05 1990	83.535	31.005	$19.183 \pm 0.096$	1.488	0.070	0.366
5	12 05 1990	62.866	31.005	$39.819 \pm 0.082$	3.085	0.182	0.456
6	18 05 1990	73.204	31.005	$27.055 \pm 0.069$	2.093	0.166	0.614
7	24 05 1990	73.206	31.005	$26.838 \pm 0.085$	2.093	-0.049	-0.183
8	01 06 1990	62.865	31.005	$39.774 \pm 0.071$	3.085	0.135	0.339
9	10 06 1990	52.533	31.005	$62.263 \pm 0.068$	4.838	0.106	0.171
10	14 06 1990	104.189	31.005	$10.694 \pm 0.073$	0.832	0.003	0.031
11	19 06 1990	104.185	31.005	$10.513 \pm 0.082$	0.832	-0.179	-1.698
12	23 06 1990	113.461	31.005	$8.532 \pm 0.054$	0.663	0.015	0.176
13	30 06 1990	113.454	31.005	$8.341 \pm 0.085$	0.663	-0.177	-2.126
14	03 07 1990	82.462	31.005	$19.704 \pm 0.059$	1.538	-0.063	-0.318
15	07 07 1990	93.840	31.005	$14.196 \pm 0.076$	1.097	0.099	0.699
16	13 07 1990	104.182	31.005	$10.631 \pm 0.061$	0.832	-0.061	-0.576
17	22 07 1990	73.242	31.005	$26.852 \pm 0.052$	2.090	-0.001	-0.004
18	30 07 1990	73.242	31.005	$26.842 \pm 0.059$	2.090	-0.011	-0.041
19	07 08 1990	73.242	31.005	$26.758 \pm 0.056$	2.090	-0.095	-0.355
20	14 09 1990	74.317	31.005	$25.930 \pm 0.057$	2.013	0.064	0.248
21	20 09 1990	74.317	31.005	$25.892 \pm 0.063$	2.013	0.026	0.102
22	14 10 1990	103.058	31.005	$11.013 \pm 0.096$	0.856	0.008	0.075
23	19 10 1990	103.058	31.005	$11.020 \pm 0.076$	0.856	0.015	0.139
24	30 10 1990	112.864	31.005	$8.537 \pm 0.067$	0.672	-0.101	-1.183
25	07 11 1990	42.601	31.005	$103.854 \pm 0.086$	8.093	-0.130	-0.126
26	11 11 1990	52.534	31.005	$62.291 \pm 0.090$	4.837	0.139	0.223
27	21 11 1990	42.614	31.005	$103.351 \pm 0.206$	8.087	-0.559	-0.541
28	23 11 1990	62.854	31.005	$39.601 \pm 0.103$	3.087	-0.056	-0.142
29	26 11 1990	62.854	31.005	$39.729 \pm 0.083$	3.087	0.072	0.181
30	01 12 1990	62.854	31.005	$39.595 \pm 0.073$	3.087	-0.062	-0.157
31	06 12 1990	62.854	31.005	$39.528 \pm 0.074$	3.087	-0.129	-0.327
32	06 03 1991	113.569	31.005	$8.454 \pm 0.085$	0.661	-0.041	-0.490
33	12 03 1991	113.569	31.005	$8.463 \pm 0.085$	0.661	-0.032	-0.383

## V. DISCUSSION AND CONCLUSIONS

The calibration constant of the gravimeter deduced from the measurement using the moving masses is  $a_{av} = (12\,853 \pm 12)$  V/Gal. The  $\pm 0.1\%$  error of this value has negligible contributions from electrical noise, from dimensional measurement errors, and from mass and tidal uncertainties. Rather, the error is attributed to short-term seismic noise and to long-term fluctuations of unknown origin. There is some source of random disturbances that causes a larger than statistical variation among measurements taken at the same value of  $z_{near}$  at different times of year. Given that the large and small masses were supported at points separated by nearly 4 m, it is possible that the variable expansion of the various components due to temperature, wind, barometric pressure, and humidity could account for the effect.

The difference between the calibration constants determined using the Hg and steel masses implies that the ra-

tio of active-to-passive gravitational mass of the two materials differs by less than 0.7% at a 95% confidence level.

Given an independent calibration of the gravimeter using the tidal comparison discussed above we would be able to determine the gravitational constant with a  $\pm 0.1\%$  precision. The residuals in Fig. 6 do not show any convincing trend that would require invoking a nongravitational force for explanation. Nevertheless, if we include a Yukawa potential  $\phi = \alpha \exp\{-\lambda R\}$  in our computation of the effect of the elevator, we find that  $|\alpha|$  must be less than 0.012 at a 95% confidence level ( $2\sigma$ ) for ranges  $\lambda^{-1}$  in the range from 0.2 to 2.0 m. Our result is comparable to others in the 1–2 m range [66]. If the full sensitivity of the superconducting instrument could be utilized by improved dimensional stability and a lower ground-noise contribution, it would be possible to improve our measurement by up to two orders of magnitude.

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