Strange axial-vector mesons

Mahiko Suzuki
Department of Physics and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 5 October 1992)

The strange axial-vector mesons K_1 (1270) and K_1 (1400) are reanalyzed in the light of the updated experimental information and compared with the recent result on the $K\pi\pi$ production in τ decay. The mixing angle between the strange mesons of $^3P_1$ and $^1P_1$ is determined by the partial decay rates, and, independently, by the masses. They lead to $\theta_K \approx 33°$ or 57°. The observed K_1 (1400) production dominance in the τ decay favors $\theta_K \approx 33°$. Flavor-SU(3) breaking of 20% or so in the production amplitudes can explain quantitatively the observed production ratio.

PACS number(s): 14.40.Ev, 13.25.+m, 13.35.+s, 13.60.Le

I. INTRODUCTION

The TPC/Two-Gamma Collaboration [1] recently observed that $K\pi\pi$ production in τ decay is dominated by the $K_1(1400)$ meson with little evidence for $K_1(1270)$. It has been known [2,3] that, qualitatively speaking, they decay as

$$\begin{aligned} K_1(1270) &\to K\rho \\ &\not\to K^*\pi \\ K_1(1400) &\to K^*\pi \\ &\not\to K\rho\ . \end{aligned} \tag{1.1}$$

This decay pattern immediately suggests that $K_1(1270)$ and $K_1(1400)$ are approximately the 50–50 mixtures of the strange members of two axial-vector SU(3) octets $^3P_1$ and $^1P_1$. Carnegie *et al.* [4] actually obtained the mixing angle $\theta_K=(41\pm4)°$ as the optimum fit to the data as of 1977. Theoretically, if the $I=\frac{1}{2}$ members of the two octets are degenerate before mixing, SU(3)-symmetry breaking always leads to the maximal mixing $\theta_K=45°$ [5,6]. However, the recent $K\pi\pi$ data of the TPC/Two-Gamma Collaboration appears to contradict this simple picture: If the mixture were 50–50, production of $K_1(1270)$ and $K_1(1400)$ would be one to one up to the kinematical corrections since in the SU(3) limit only the linear combination $[K_1(1270)+K_1(1400)]/\sqrt{2}$ would have the right quantum number to be produced in the τ decay. After the phase-space correction, $K_1(1270)$ production would even be favored over $K_1(1400)$ production by nearly a factor of 2.

The purpose of this report is to redetermine the mixing angle θ_K from the latest experimental information on the masses and the partial decay rates and then to compare the result with the recently observed K_1 production in τ decay. The K_1 masses and decay branchings point to $\theta_K \approx 33°$ or 57°. However, the observed $K_1(1270)/K_1(1400)$ production ratio favors $\theta_K\approx 33°$, though some SU(3)-symmetry breaking effect is needed to obtain a good quantitative agreement between theory and experiment. In fact, contamination by SU(3) breaking as much as 20% in the K_1 production amplitudes can explain the observed $K_1(1400)$ dominance.

II. MIXING ANGLE

There are two ground-state axial-vector nonets $(\mathbf{1}+\mathbf{8})$ in the quark model. One is the $^3P_1$ state and the other is the $^1P_1$ state of the quark-antiquark pair. If the meson states are represented in the 3×3 matrix or tensor, they transform under charge conjugation as

$$\begin{aligned} &M_a^b(^3P_1)\to M_b^a(^3P_1)\ , \\ &M_a^b(^1P_1)\to -M_b^a(^1P_1)\quad (a,b=1,2,3)\ . \end{aligned} \tag{2.1}$$

Since the weak axial-vector current transforms as $(J_{5\mu})_a^b\to(J_{5\mu})_b^a$ under charge conjugation, only the $^3P_1$ states can be produced through the weak axial-vector current in the SU(3) symmetry limit.

An interesting feature of the axial-vector mesons is that their strange members of $^3P_1$ and $^1P_1$ can mix with each other through SU(3) breaking of $I=Y=0$. This mixing can be large if $^3P_1$ and $^1P_1$ are nearly degenerate in the symmetry limit. The magnitude of mixing can be determined from two sources; the partial decay rates of the K_1 mesons and the masses of the axial-vector nonets.

A. Branching ratios

The result from the most recent compilation of the data [7] is listed below:

$$\begin{aligned} K_1(1270) &\to K\rho \quad (42\pm6)0002 \quad (p_{\text{c.m.}}=71\ \text{MeV}) \\ &\to K^*\pi \quad (16\pm5)0002 \quad (p_{\text{c.m.}}=299\ \text{MeV}) \\ &\Gamma_{\text{tot}}=(90\pm2)\ \text{MeV}\ . \end{aligned} \tag{2.2}$$

$$\begin{aligned} K_1(1470) &\to K^*\pi \quad (94\pm6)0002 \quad (p_{\text{c.m.}}=401\ \text{MeV}) \\ &\to K\rho \quad (3.0\pm3.0)0002 \quad (p_{\text{c.m.}}=300\ \text{MeV}) \\ &\Gamma_{\text{tot}}=(174\pm13)\ \text{MeV}\ . \end{aligned}$$

Hereafter we will denote the K_1 of $^3P_1$ by K_a and the K_1

of ${}^1P_1$ by K_b, meaning that K_a and K_b are the strange partners of $a_1(1260)$ and $b_1(1235)$, respectively. By C invariance, the couplings of K_a and K_b to the 1^- octet and the 0^- octet are obtained by antisymmetrizing and symmetrizing the two octets, respectively:

$$H_{\text{int}}^{(a)}=(f_a/2)(K\rho-\pi K^*+K\phi_8-\eta_8K^*)K_a\ , \tag{2.3}$$

$$H_{\text{int}}^{(b)}=f_b\left[\left[\frac{3}{2\sqrt{5}}\right](K\rho+\pi K^*)-\left[\frac{1}{2\sqrt{5}}\right](K\phi_8+\eta_8K^*)\right]K_b\ . \tag{2.4}$$

The mixing between K_a and K_b is parametrized as

$$\begin{aligned}K_1(1400)&=K_a\cos\theta_K-K_b\sin\theta_K\ ,\\K_1(1270)&=K_a\sin\theta_K+K_b\cos\theta_K\ ,\end{aligned} \tag{2.5}$$

The d-to-s-wave decay ratio is 0.04 ± 0.01 for the decay $K_1(1400)\rightarrow K^*\pi$ and 1.0 ± 0.7 for the decay $K_1(1270)\rightarrow K^*\pi$. The d-to-s-wave ratio for $K_1(1270)\rightarrow K^*\pi$ appears to be large, but uncertainty is also large. We are sure that at least the decay $K_1(1270)\rightarrow K\rho$ must occur in the s wave because of its tiny phase space. For $K_1(1270)\rightarrow K^*\pi$, the branching ratio of Eq. (2.2) sets an upper bound on its s-wave decay contribution. By making the s-wave phase-space correction, we can impose constraints on the coupling ratio f_b/f_a and the mixing angle θ_K. From the ratio of the phase-space-corrected branching ratios $\bar{B}(K_1\rightarrow K^*\pi)$ and $\bar{B}(K_1\rightarrow K\rho)$, we obtain

$$\begin{aligned}&\left|\left[\frac{\sqrt{5}}{3}-x\tan\theta_K\right]\Big/\left[\frac{\sqrt{5}}{3}+x\tan\theta_K\right]\right|=0.21^{+0.08}_{-0.21}\ ,\\&\left|\left[\tan\theta_K-\frac{3x}{\sqrt{5}}\right]\Big/\left[\tan\theta_K+\frac{3x}{\sqrt{5}}\right]\right|=0.21^{+0.04}_{-0.03}\ .\end{aligned} \tag{2.6}$$

There are eight sets of solutions for x and θ_K (see Fig. 1):

$$\begin{aligned}&\text{(a)}\ \theta_K=(33^{+6}_{-2})^\circ,\quad x=0.75^{+0.18}_{-0.07}\ ,\\&\text{(b)}\ \theta_K=(45^{+3}_{-6})^\circ,\quad x=1.14^{+0.13}_{-0.22}\ ,\\&\text{(c)}\ \theta_K=(57^{+2}_{-3})^\circ,\quad x=0.74^{+0.08}_{-0.14}\ ,\\&\text{(d)}\ \theta_K=(45^{+6}_{-3})^\circ,\quad x=0.48^{+0.08}_{-0.33}\ ,\end{aligned} \tag{2.7}$$

and four more sets of solutions (a')–(d') where the signs of both x and θ_K are flipped in (a)–(d). The solutions (a')–(d') are physically indistinguishable from the solutions (a)–(d). Therefore, hereafter we keep only the solutions (a)–(d) by choosing the convention $0^\circ\le\theta_K\le90^\circ$. Since the world averages of data from the Particle Data Group compilation [3] have been used here, in estimating uncertainties in Eq. (2.7) we have treated the errors quoted for the branching ratios as uncorrelated.

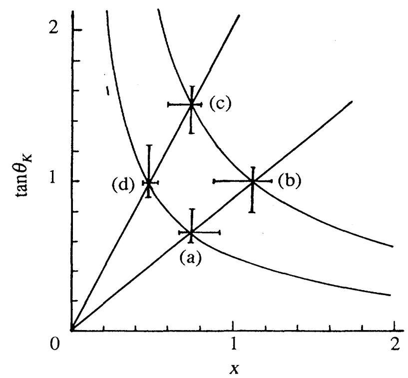

FIG. 1. Four sets of solutions for x and θ_K.

B. Mass formulas

Both the ${}^3P_1$ and ${}^1P_1$ states form an octet and a singlet. For the vector mesons (${}^3S_1$) and the tensor mesons (${}^3P_2$), the singlet and the $I=Y=0$ component of the octet mix with each other approximately by $\theta_{1-8}=\arctan(1/\sqrt{2})\approx35^\circ$ and lead to the *ideally mixed* mass eigenstates $(\bar{u}u+\bar{d}d)/\sqrt{2}$ and $-\bar{s}s$. Then one of the $I=Y=0$ meson decays into nonstrange mesons while the other decays into final states containing $K\bar{K}$. The *ideal mixing* seems to occur for the ${}^3P_1$ and ${}^1P_1$ nonets as well: The $h_1(1170)$ state with $J^{PC}=1^{+-}$ and $I=0$ decays predominantly into $\rho\pi$, while $h_1(1380)$ with same quantum numbers decays into $K\bar{K}^*$ and $\bar{K}K^*$. For ${}^3P_1$, $f_1(1282)$ decays predominantly into 4π and $\eta\pi\pi$, while there are two candidates for the $-\bar{s}s$ state, $f_1(1415)$ and $f_1(1510)$, both of which decay into resonant and/or nonresonant $K\bar{K}\pi$. When a singlet and an $I=Y=0$ member of octet mix *ideally*, they obey the mass formulas

$$\begin{aligned}&m^2(I=1)+m^2(I=0;-\bar{s}s)=2m^2(I=\tfrac{1}{2})\ ,\\&m^2(I=1)=m^2(I=0;(\bar{u}u+\bar{d}d)/\sqrt{2})\ .\end{aligned} \tag{2.8}$$

These formulas assume a little more than group theory of SU(3) alone; for instance, SU(6) of spin-flavor symmetry [8,9] or equivalently the static quark model. Both relations are satisfied with a good accuracy for the vector and tensor nonets. In the case of the axial-vector mesons, the proximity of $a_1(1260)$ and $f_1(1282)$, and, to a lesser degree, that of $b_1(1232)$ and $h_1(1170)$ are consistent with the second mass formulas of Eq. (2.8). By using the first mass formulas of Eq. (2.8), we are able to determine the masses of the $I=\frac{1}{2}$ members *in the absence of* ${}^3P_1$-${}^1P_1$ mixing, i.e., the masses of K_a and K_b:

$$\begin{aligned}m(K_a)&=[(1260^2+1415^2)/2]^{1/2}=1340\ \text{MeV with } f_1(1415)\\&=[(1260^2+1512^2)/2]^{1/2}=1392\ \text{MeV with } f_1(1510)\ ,\\m(K_b)&=[(1232^2+1380^2)/2]^{1/2}=1308\ \text{MeV}\ .\end{aligned} \tag{2.9}$$

After mixing occurs between K_a and K_b, the two mass eigenstates to be identified with $K_1(1270)$ and $K_1(1400)$ must obey the *center-of-gravity* relation

$$m^2(K_1(1270))+m^2(K_1(1400))=m^2(K_a)+m^2(K_b) , \tag{2.10}$$

which follows from diagonalization of the 2×2 mass matrix of K_a-K_b. By substituting $m(K_b)=1308$ MeV from Eq. (2.9), we find with Eq. (2.10) $m(K_a)=1365$ MeV, which does not agree with either the predicted K_a mass 1340 MeV from $f_1(1415)$ nor with 1392 MeV from $f_1(1510)$. It is halfway between them. This probably suggests that these two f_1 states are linear combinations of the quark-antiquark state $-\bar{s}s$ and some extra or exotic state such as a four-quark state or a glueball. We take this interpretation and proceed in our analysis, though this interpretation is not really needed for our numerical analysis that follows.

Diagonalization of the K_a-K_b mass matrix gives us the relation between the mixing angle θ_K and the mass differences:

$$\tan^2 2\theta_K=\{[m^2(K_1(1400))-m^2(K_1(1270))]/[m^2(K_a)-m^2(K_b)]\}^2-1 . \tag{2.11}$$

This gives $\theta_K\approx32°$ or 58°. It should be remarked that $\theta_K=45°$ would require $m(K_a)=m(K_b)$ by Eq. (2.11). Among the four sets of solutions from the partial decay rates, the two solutions are selected by the masses:

$$\begin{aligned}&(\mathrm{a})\ \theta_K=(32^{+6}_{-2})° , \quad x=0.75^{+0.18}_{-0.07} ,\\ &(\mathrm{c})\ \theta_K=(57^{+2}_{-3})° , \quad x=0.74^{+0.08}_{-0.14} .\end{aligned} \tag{2.12}$$

We can estimate the ratio of the rates for the main decay modes: $K_1(1400)\to K^*\pi$ and $K_1(1270)\to K\rho$. We find, for all four sets of solutions,

$$\Gamma(K_1(1400)\to K^*\pi)/\Gamma(K_1(1270)\to K\rho)\approx5.6 \tag{2.13}$$

with a typical error of about ±1.0, as compared with the experimental value 4.3 ± 1.2. Since the total decay widths vary fairly widely from one experiment to another, we have not included this information in fitting θ_K. Unfortunately we cannot resolve with this ratio the twofold ambiguity of the values for x and θ_K, We will see below, however, that the $K_1(1270)/K_1(1400)$ production ratio in the τ decay can resolve it.

III. PRODUCTION RATIO OF $K_1(1400)$ AND $K_1(1270)$

If the K_1 production amplitudes are perfectly SU(3) symmetric, only K_a is produced. Making the kinematical corrections by use of the decay-rate formula

$$\Gamma(\tau\to\nu K_1)=(G_F^2|V_{us}|^2f_{K1}^2/16\pi)(m_\tau^2+2m_{K1}^2)(m_\tau^2-m_{K1}^2)^2/m_\tau^3m_{K1}^2 , \tag{3.1}$$

we find

$$\frac{\Gamma(\tau\to\nu K_1(1270))}{\Gamma(\tau\to\nu K_1(1400))}=\tan^2\theta_K\times(\text{phase space and kinematical factors})$$
$$=\begin{cases}0.76^{+0.42}_{-0.11} & \text{for } \theta_K=(33^{+6}_{-2})° ,\\ 4.3^{+0.7}_{-0.9} & \text{for } \theta_K=(57^{+2}_{-3})° .\end{cases} \tag{3.2}$$

For $\theta_K=45°$, this ratio would be 1.8. This clearly shows how important accurate determination of θ_K is for comparing experiment with theory. The preliminary data from TPC/Two-Gamma Collaboration show a signal of $K_1(1400)$ production but little evidence for $K_1(1270)$ production. In light of this experiment the solution $\theta_K\approx33°$ is definitely favored over the solution $\theta_K\approx57°$. To explain the $K_1(1400)$ production dominance, however, it is necessary to take account of SU(3) breaking in the production amplitudes. The first-order SU(3)-symmetry breaking can flip the SU(3) charge conjugation property of the strange weak current. The simplest way to parametrize such a breaking effect phenomenologically is to postulate that instead of just K_a, the linear combination

$$|K_a\rangle-\delta|K_b\rangle \quad (|\delta|\ll1) \tag{3.3}$$

is produced in the τ decay. The parameter δ is a complex number in general, but similarity of the ${}^3P_1$ and ${}^1P_1$ states in the static quark model makes it more likely that δ is a real number or close to it. Then the production ratio of Eq. (3.2) is modified into

$$\frac{\Gamma(\tau\to\nu K_1(1270))}{\Gamma(\tau\to\nu K_1(1400))}=1.8\left|\frac{\sin\theta_K-\delta\cos\theta_K}{\cos\theta_K+\delta\sin\theta_K}\right|^2 , \tag{3.4}$$

where the numerical factor 1.8 in front of the right-hand side is the phase space and other kinematical corrections based on Eq. (3.1). The production ratio is now very sensitive to the value of δ. Since there is no reason to expect that SU(3) breaking is abnormally enhanced in the axial-vector meson production, the magnitude of $|\delta|=O((m_s-m_u)/(m_s+m_u))\approx0.25$ is considered as normal. In the static limit of the quark model, the pa-

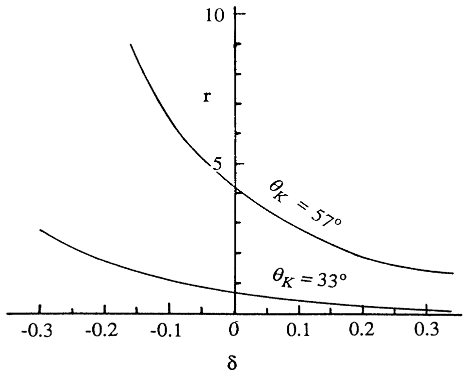

FIG. 2. The $K_1(1270)/K_1(1400)$ production ratio plotted against δ for $\theta_K = 33°$ and $57°$.

rameter δ is actually calculable and takes a very simple form

$$|\delta| = (m_s - m_u)/\sqrt{2}(m_s + m_u) \ . \quad (3.5)$$

With this value of $|\delta|$ and $\theta_K = 33°$,

$$\Gamma(\tau \to \nu K_1(1270))/\Gamma(\tau \to \nu K_1(1400)) \approx \begin{cases} 0.32 & \text{for } \delta = 0.18 \ , \\ 1.6 & \text{for } \delta = -0.18 \ . \end{cases} \quad (3.6)$$

Therefore, if the SU(3) symmetric and breaking amplitudes interfere destructively for $K_1(1270)$, the experimental data on the K_1 production [1] are fully consistent with the theoretical expectation. The ratio of Eq. (3.4) has been plotted against δ for $\theta_K = 33°$ and $57°$ in Fig. 2.

IV. CONCLUDING REMARKS

When the mixing angle θ_K and the coupling ratio $f_b/f_a(=x)$ are determined from the decay branching ratios in Eq. (2.7), generous errors have been attached. However, we have quoted no errors when θ_K is determined from the mass formulas. It is because the mass formulas of Eq. (2.8) involve, in addition to the relatively small experimental uncertainty in the axial-vector meson masses, theoretical uncertainties which are not easy to estimate. We have used the mass formulas for the purpose of selecting right solutions out of those which are allowed by the branching ratios alone.

The decay mode $K_1 \to K\omega$ has been observed for both $K_1(1400)$ and $K_1(1270)$. The ${}^1P_1$ octet has an SU(3)-allowed coupling to the vector singlet and the pseudoscalar octet. Most generally, this coupling constant is independent of f_b. However, in the *naive* quark model or with the nonet ansatz of Okubo [10], this coupling can be related to f_b. If we use this relation, we find

$$\Gamma(K_1(1400) \to K\omega)/\Gamma(K_1(1400) \to K\rho) = \left| x \tan\theta_K \Big/ \left[\frac{\sqrt{5}}{3} - x \tan\theta_K \right] \right|^2 \Big/ 3 \approx \begin{cases} 1.3 & \text{for } \theta_K \approx 33° \ , \\ 2.8 & \text{for } \theta_K \approx 57° \ . \end{cases} \quad (4.1)$$

The experimental data are $B(K_1(1400) \to K\omega) = (1.0 \pm 1.0)\%$ and $B(K_1(1400) \to K\rho) = (3.0 \pm 3.0)\%$. It might appear that Eq. (4.1) slightly favors $\theta = 33°$ over $\theta = 57°$, but the experimental uncertainties are too large to draw any conclusion. For $K_1(1270)$, the decay rates are very sensitive to the shape of the resonance because the Q value is virtually zero or even negative if one substitutes the peak values of resonances. In terms of the decay couplings, the prediction is

$$f(K_1(1270) \to K\omega)/f(K_1(1270) \to K\rho^0) = x \Big/ \left[x + \frac{\sqrt{5}}{3} \tan\theta_K \right] \approx \begin{cases} 0.61 & \text{for } \theta_K \approx 33° \ , \\ 0.40 & \text{for } \theta_K \approx 57° \ , \end{cases} \quad (4.2)$$

while $B(K_1(1270) \to K\omega) = (11 \pm 2)\%$ and $B(K_1(1270) \to K\rho) = (42 \pm 6)\%$ have been observed. For $K_1(1270) \to K\omega$, the coupling ratio of 0.40 for $\theta_K = 57°$ seems to give too small a branching ratio: $B(K_1(1270) \to K\omega)/B(K_1(1270) \to K\rho) < 0.05$. This may be regarded as another evidence in favor of the solution (a) over (c) and actually over all other solutions.

ACKNOWLEDGMENTS

I thank Michael T. Ronan for bringing the TPC/Two-Gamma Collaboration data to my attention. This work was supported in part by National Science Foundation under Grant No. PHY90-21139 and in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

[1] TPC/Two-Gamma Collaboration, LBL Report No. LBL-32377, 1992 (unpublished).
[2] G. W. Brandenburg *et al.*, Phys. Rev. Lett. **36**, 703 (1976).
[3] R. K. Carnegie *et al.*, Nucl. Phys. **B127**, 509 (1977).
[4] R. K. Carnegie *et al.*, Phys. Lett. **68B**, 289 (1977).
[5] E. W. Colglazier and J. L. Rosner, Nucl. Phys. **B27**, 349 (1971).
[6] H. J. Lipkin, Phys. Lett. **72B**, 249 (1977).
[7] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).
[8] F. Gürsey and L. A. Radicati, Phys. Rev. Lett. **13**, 173 (1964).
[9] B. Sakita, Phys. Rev. **136**, B1756 (1964).
[10] S. Okubo, Phys. Lett. **5**, 165 (1963).