# Cross sections and multiplicities in hadron-nucleus collisions with interacting color strings

M. Braun and C. Pajares
*Department of Particle Physics, University of Santiago de Compostela, 15706 Santiago de Compostela, Spain*
(Received 8 July 1992)

The energy and $A$ dependence of the cross sections, number of wounded nucleons, and multiplicities in hadron-nucleus interactions at high energies are discussed in the framework of a probabilistic string model. All observables are calculated analytically. The results show that at present energies observable effects due to string fusion are small. However at higher energies, such as at the CERN Large Hadron Collider or the Superconducting Super Collider, these effects give rise to a spectacular reduction of the multiplicity, which becomes independent of $A$. The total cross section, on the contrary, remains proportional to $A^{2/3}$ and only its small reduction is predicted.

PACS number(s): 13.85.Ni, 12.40.Gg, 25.75.+r

## I. INTRODUCTION

Soft hadron interactions are currently described by interchanges of color strings, which play the role of Pomerons of the old Regge-Gribov theory. Models based on color strings [dual parton model (DPM) [1,2] or FRITIOF [3]] describe particle production at energies achieved so far reasonably well. Color strings exchanged during a $hh$ collision do not seem very numerous, but their number grows with energy and becomes large for $hA$ and especially $AB$ collisions for planned experiments at the BNL Relativistic Heavy Ion Collider (RHIC) and CERN Large Hadron Collider (LHC). This makes it necessary to take into account a possible interaction between the produced strings similar to the Pomeron-Pomeron interaction in the Regge-Gribov approach. Several attempts in that direction [4–6] point to the interaction between strings as a possible explanation for the rise in $\langle p\perp \rangle$ and the high baryon and strangeness content.

In recent papers [7,8] the authors have proposed a simple probabilistic model which gives a unified description of interacting color strings both for the elastic amplitude and particle production, thus incorporating unitarity. The model predicts, in particular, that the interaction between color strings mostly affects particle production in $hA$ and $AB$ collisions, its influence on $hh$ collisions and on the total $hA$ and $AB$ cross sections being relatively small, at least at present and near future energies. This conclusion was reached in [7,8] on the basis of crude asymptotic estimates made for very high energies. The model of string interactions introduced in [7,8] contains two independent physical assumptions. One is that strings interact as soon as their positions in the transverse space become closer than some critical distance, which determines the fusion area of an order of the proper string transverse area $s_0=\pi/\langle p_\perp^2\rangle$. As a result there can be no more than one string inside this area. A second assumption concerns the result of the interaction between strings. It was assumed in [7,8] that when $n$ ordinary strings get into the area $s_0$ they fuse to form a new string with a correspondingly higher color field flux and tension. The physical results obtained in [7,8] generally depend on both assumptions. Some of them, such as the rise in the $\langle p_\perp \rangle$ and central multiplicities, depend crucially on the assumed properties of the new strings, their decay law, and the cross sections related to them. However, there are some predictions following from the model which prove to be independent of the new string properties, and, in fact, of their existence, but reflect the mere fact that the strings do interact as soon as they come together close enough. These predictions refer to the total cross sections and the ratio of the multiplicities for $hA$ and $hN$ collisions. It seems important to us to study these results in some detail, since they give a possibility to investigate the interaction between strings irrespective of what becomes of them in the result and therefore they are free from inevitable uncertainties and additional parameters associated with possible strings of new types.

In this paper we accordingly study the $hA$ cross sections and particle production ratios for $hA$ and $hN$ collisions taking into account the interaction between color strings in the framework of the probabilistic model of [7,8]. The results of this study confirm the general predictions made in [8]: The total $hA$ cross sections stay practically unaffected by the string interaction up to energies of LHC (6.3 TeV center of mass), whereas the relative multiplicities reduce drastically at such high energies. This result is independent of the structure of the objects formed in the course of the interaction between strings and reflects only the fact of their fusion. The study of the distributions in the number of inelastic collisions $n$ in the nuclear target shows that this reduction is due to the substantial decrease in the probability of high $n$ events, in which the number of produced strings is inevitably large and the effect of their interaction is great. As a result the average number of inelastic collisions in $hA$ interactions at very high energies and therefore with a great number of strings produced becomes independent of $A$ and close to unity.

In Sec. II we review the main points of the model necessary for the following calculations. Section III is dedicated to the total $hA$ cross sections. In Sec. IV the multiplicities are studied. Numerical results and their discussion are presented in Sec. V.

 

## II. THE MODEL

The probabilistic model proposed in [7,8] is based on the interpretation of the eikonal formula for the scattering matrix $S(b)$ for fixed impact parameter $b$,

$$S(b)=\exp[i\chi(b)]=\sum_{N=0}[i\chi(b)]^N/N!\ , \tag{1}$$

as a certain average. Imagine that during the collision a variable number $N$ of elementary objects ("strings") are formed with a probability $p_N$ and that the contribution of such a configuration to the total $S$ matrix is $s^N(b)$. Then the $S$ matrix will be given by the expectation value

$$S(b)=\sum_N p_N s^N(b)\ . \tag{2}$$

With the Poisson distribution

$$p_N=\exp(-g)g^N/N! \tag{3}$$

we get $S(b)=\exp\{g[s(b)-1]\}=\exp[iga(b)]$, where $a(b)$ is the elementary amplitude. This is precisely the eikonal expression (1).

With this interpretation we hope to introduce the interaction between strings by changing the Poisson distribution (3). The concrete way to do it is suggested by the physical picture of string interaction as seen in the transverse plane. As noted in the Introduction, it is natural to assume that strings interact as soon as they are within a certain area $s_0$ in that plane, which has the meaning of the string proper transverse area. The result of such an interaction is fusion of several strings into a single string of a new type, which possesses an augmented color flux and so has a color quantum number $n$ equal to the number of fused ordinary strings, in the Abelian approximation. This physical picture is converted into the probability distribution $p(\nu_n)$ in the number $\nu_n$ of strings with color $n$ given by

$$p(\nu_n)=c\left[Q^N\Big/\prod_n \nu_n!(n!)^{\nu_n}\right]x^{N-M}\prod_{k=1}^{M-1}(1-kx)\ . \tag{4}$$

Here $x$ is the probability of string fusion determined by the ratio of the string interaction area $s_0$ to the total area where strings are formed of the order of the $hN$ total cross section $\sigma$. With the corresponding contribution to the $S$ matrix given by

$$s_{\nu_n}(b)=\prod_n s_n^{\nu_n}(b)\ , \tag{5}$$

where $s_n$ is the elementary $S$ matrix corresponding to the string of color $n$, the model is completely defined for $hN$ scattering.

To generalize it to $hA$ scattering one should take into account that in this case the volume where strings are formed corresponds to the transverse interaction areas for individual $hN$ collisions and not to the total nuclear transverse area. Accordingly we first separate geometrically the interaction area $\sigma$ for collisions of the projectile with each of $A$ nucleons of the target. As a result the total $S$ matrix becomes presented as a sum of contributions $S_l^{(A)}$ for a given number $l$ of interacting nucleons from the target

$$S_A(b)=\sum_{l=0}^{A}S_l(b)\ , \tag{6}$$

where

$$S_l(b)=C_A^l T_A^l(b)[1-\sigma T_A(b)]^{A-l}\int_\sigma \prod_1^l d^2b_i\, S_{l,b_1,\ldots,b_l} \tag{7}$$

and $S_{l,b_1,\ldots,b_l}$ is the scattering matrix with the positions of $l$ nucleons fixed in the interaction area $\sigma$. Each $S_l$ is then treated separately in the same probabilistic manner as before, taking into account that $l$ different nucleons from the target are present in the interaction area and form strings, which all fuse as soon as they come together close enough. For fixed transverse positions of these nucleons $b_1,\ldots,b_l$ in the interaction area this leads to an evident generalization of the probability (4):

$$p_l(\nu_n^{(1)},\nu_n^{(2)},\ldots)=c_l\left[Q^N\Big/\prod_{i,n}\nu_n^{(i)}!(n!)^{\nu_n^{(i)}}\right]x^{N-M}\prod_{k=1}^{M-1}(1-kx)\ . \tag{8}$$

Here $\nu_n^{(i)}$ is the number of strings of type $n$ produced in the interaction of the projectile with the $i$th target nucleon from $l$ active ones. Now $N=\sum_{i,n}n\nu_n^{(i)}$ and $M=\sum_{i,n}\nu_n^{(i)}$ are the total color and number of strings produced by all $l$ interaction nucleons. The $S$ matrix for a subprocess with fixed $l$ and transverse nucleon positions will be given by

$$S_{l,b_1,\ldots,b_l}=\prod_{i=1}^{l}\left[\sum_{\nu_n^{(i)}}s_n^{\nu_n^{(i)}}(b_i)\right]p_l(\nu_n^{(i)})\ ; \tag{9}$$

the sums here extend over all possible values of $\nu_n^{(i)}$. Equations (8) and (9) form the basis for calculations of the total and inclusive $hA$ cross sections with interacting strings.

Performing the summation over all configurations $\nu_n^{(i)}$ (see [8] for the details) we get for the $S$ matrix with nucleon transverse positions fixed

$$S_{l,b_1,\ldots,b_l}=\{1+s_l(b_1,\ldots,b_l)\}^{1/x}/\{1+lg\}^{1/x}\ . \tag{10}$$

Here the notation $\{1+x\}^a$ means the part of $(1+x)^a$ with positive binomial coefficients,

$$g=\exp(Qx)-1\ . \tag{11}$$

The effective elementary $S$ matrix corresponding to $l$ nucleons in the target, and $s_l(b_1,\ldots,b_l)$ is the sum over interacting nucleons

$$s_l(b_1,\ldots,b_l)=\sum_{i=1}^{l}s(b_i)\ , \tag{12}$$

where each individual scattering matrix is given by the sum over all types of strings

$$s(b)=\sum_{n=1}^{\infty}s_n(b)(Qx)^n/n!\ . \tag{13}$$

The $S$ matrix (10) goes over into the usual Glauber formula when there is no interaction between strings, which corresponds to $x \to 0$ and $s_l \sim x$.

Using the Abramovsky-Gribov-Kancheli (AGK) rules [9] one concludes from (10) that the cross section corresponding to $k$ cut strings of any type in the interaction with $l$ nucleons from the target with their impact parameters fixed is given by

$$\sigma_{lk,b_1,\ldots,b_l}=[\sigma_l(b_1,\ldots,b_l)]^k C_{1/x}^k \{1+lg-\sigma_l(b_1,\ldots,b_l)\}^{1/x-k}/\{1+lg\}^{1/x}\ , \tag{14}$$

where

$$\sigma_l(b_1,\ldots,b_l)=\sum_{i=1}^{l}\sigma(b_i) \tag{15}$$

and $\sigma(b)$ is the elementary cross section corresponding to $s(b)$:

$$\mathrm{Re}s(b)=1-\tfrac{1}{2}\sigma(b)\ . \tag{16}$$

The total inelastic cross section on the nucleus for $l$ interacting nucleons is the sum of (14) over all $k$

$$\sigma_{l,b_1,\ldots,b_l}=1-\{1+lg-\sigma_l(b_1,\ldots,b_l)\}^{1/x}/\{1+lg\}^{1/x}\ . \tag{17}$$

Of course the expressions (14) and (17) give the corresponding contributions only for $b$, $l$, and all fixed $b_i$. To obtain the physical cross sections one should integrate (14) or (17) over all $b_i$'s in the interaction area and sum over $l=1,2,\ldots,A$ with the weights

$$w_l=\int d^2b\, C_A^l T^l(b)[1-\sigma T_A(b)]^{A-l}\ . \tag{18}$$

The inclusive cross section for particle production from $k$ cut strings of any sort is obtained from (14) after multiplying it by the factor $kI(q)$ where $I(q)$ is the inclusive cross section for the production of a particle of momentum $q$ from one cut string averaged over its type (color). Summation over all $k$ gives the inclusive cross section for $l$ participant nucleons from the target with their impact parameters fixed

$$I_{l,b_1,\ldots,b_l}=I(q)\sigma_l(b_1,\ldots,b_l)\{1+lg\}^{1/x-1}/x\{1+lg\}^{1/x}\ . \tag{19}$$

It is linear in the elementary cross section as a consequence of the AGK cancellations [9].

### III. CROSS SECTIONS FOR A GIVEN NUMBER OF INELASTIC COLLISIONS INSIDE THE NUCLEUS

Beside the total cross sections (17) for a given number of interacting nucleons from the target with their impact parameters fixed, the cross sections for a given number $n$ of inelastic collisions inside the nucleus present a certain interest. The $n$ nucleons that interact inelastically (the "wounded" nucleons) get accelerated and subsequently give rise to the so-called "grey" nucleons, which can be detected experimentally and are in fact the wounded ones plus those which they produce during their cascading inside the nucleus. Thus the cross sections with a given number of wounded nucleons can in principle be measured experimentally.

To find the expression for the cross section $\sigma^{(n)}_{l,b_1,\ldots,b_l}$ corresponding to $n$ wounded nucleons out of the total number $l$ taking part in the interaction we have to separate the contribution in (14) which comes from cutting $k$ strings in exactly $n$ different nucleons (evidently $k\geq n$) and then sum over all possible $k$'s. To do this denote $\sigma(b_i)\equiv x_i$ when it corresponds to an uncut string coming from the $i$th nucleon and $\sigma(b_i)\equiv y_i$ when it corresponds to a cut string. Then (14) may be rewritten as

$$\sigma_{lk}=c_l C_a^k\left[\sum_{i=1}^{l}y_i\right]^k\left\{1+lg-\sum_{i=1}^{l}x_i\right\}^{a-k},\quad x_i=y_i\ , \tag{20}$$

with $a=1/x$ and $c_l=\{1+lg\}^a$. Upon summing (20) over $k$ with $y_i\neq x_i$ we obtain

$$\sigma_l=c_l\left[\left\{1+lg+\sum_{i=1}^{l}(y_i-x_i)\right\}^a-\left\{1+lg-\sum_{i=1}^{l}x_i\right\}^a\right], \tag{21}$$

which goes over into (17) when $y_i=x_i$. Now consider all contributions to $\sigma_l$ which correspond to the events with only the $l$th nucleon wounded. These are those which contain factors $y_l,y_l^2,\ldots,y_l^k$ in (21) and no other $y_i$, $i\neq l$, that is, those which remain in (21) after we put $y_i=0$, $i\leq l-1$, $y_l=x_l$:

$$\sigma_{l(l\ \mathrm{wounded})}=c_l\left[\left\{1+lg-\sum_{i=1}^{l-1}x_i\right\}^a-\left\{1+lg-\sum_{i=1}^{l}x_i\right\}^a\right]. \tag{22}$$

Contributions with only two nucleons wounded, the $l$th and $(l-1)$th, are those which remain after we put $y_i=0$, $i\leq l-2$, $y_{l-1}=x_{l-1}$, and $y_l=x_l$, with $\sigma_{l(l-1\ \mathrm{wounded})}$ and $\sigma_{l(l\ \mathrm{wounded})}$ subtracted:

$$\sigma_{l(l,l-1\ \mathrm{wounded})}=c_l\left[\left\{1+lg-\sum_{i=1}^{l-2}x_i\right\}^a-\left\{1+lg-\sum_{i=1}^{l}x_i\right\}^a\right]-\sigma_{l(l-1\ \mathrm{wounded})}-\sigma_{l(l\ \mathrm{wounded})}\ . \tag{23}$$

Proceeding in this manner we can determine the cross sections with any number $n$ of wounded nucleons. As seen from (22) and (23) the corresponding explicit formulas with impact parameters fixed are rather cumbersome. Much simpler expressions follow after the integration over impact parameters.

We parametrize the $b$ dependence of the elementary cross sections as a Gaussian one:

$$\sigma(b)=\sigma_0\exp(-b^2/r^2)\ . \tag{24}$$

Here $r$ is the interaction radius for an exchange of a single string (one Pomeron contribution)

$$r^2=r_0^2+4\alpha' y \tag{25}$$

with $r_0$ twice the radius of the hadron, $y$ the laboratory rapidity, and $\alpha'$ the slope. According to the results of [7] the unitarity restricts $\sigma_0$ to be less than $g$; if $\sigma_0=\xi g$ then

$$0\le\xi\le 1\ . \tag{26}$$

The typical expression to be integrated over $b_i$ depends on

$$u=\sum_{i=1}^{n}\sigma(b_i)=\sigma_0\sum_{i=1}^{n}\exp(-b_i^2/r^2)\ . \tag{27}$$

The integral thus has the form

$$I=\int_\sigma\prod_1^l d^2b_i\,f(u)=\sigma^{l-n}\int_\sigma\prod_{i=1}^{n}d^2b_i\,f(u)\ , \tag{28}$$

where we have used that $f$ is independent of $b_i$ for $i>n$. Going over to variables $\xi_i=\exp(-b_i^2/r^2)$ and introducing a $\delta$ function to factorize the integrals over $\xi_i$ we present (28) in the form

$$I=\sigma^{l-n}\int du\,f(u)J_l(u) \tag{29}$$

with

$$J_l(u)=(\pi r^2)^l\int(dz/2\pi)\exp(-iuz)\times\left[\int_{\xi_0}^{1}(d\xi/\xi)\exp(i\sigma_0 z\xi)\right]^l\ . \tag{30}$$

Here $\xi_0=\exp(-\sigma/\pi r^2)$ and is supposed to be small. Performing the integration over $\xi$ we finally obtain

$$I=(\pi r^2)^l\rho^{l-n}f(\sigma_0\partial_z)\chi^l(z)|_{z=0}\ , \tag{31}$$

where the function $\chi(z)$ is defined according to

$$\chi(z)=\rho+\sum_{n=1}z^n/nn! \tag{32}$$

and

$$\rho=\sigma/\pi r^2 \tag{33}$$

is a known number.

With this technique the $b_i$ integrated total cross section results as

$$\sigma_l=c_l(\pi r^2)^l\{1+lg-\sigma_0\partial_z\}^{1/x}[\rho^l-\chi^l(z)]|_{z=0}\ . \tag{34}$$

This expression should be further multiplied by the nuclear factor $w_l$ (18) to give the final contribution to the total inelastic $hA$ cross section coming from $l$ interacting nucleons.

Applying (31) to the cross sections $\sigma_l^{(n)}$ with $n$ wounded nucleons we note that the operator which acts over $\rho^l-\chi^l$ in (34) is common to all terms. The only difference is in the function on which it operates. From (22) we conclude that with the $l$th nucleon wounded, the corresponding function is

$$\chi_{l(l\text{ wounded})}=\rho\chi^{l-1}-\chi^l\ . \tag{35}$$

For two wounded nucleons, $l$th and $(l-1)$th, according to (23) the function is

$$\chi_{l(l,l-1\text{ wounded})}=\rho^2\chi^{l-2}-\chi^l-2\chi_{l(l\text{ wounded})}\ , \tag{36}$$

where we have used the fact that after the integration over the $b_i$'s different wounded nucleons give the same contribution. Generally we then have

$$\chi_{l(l,l-1,\ldots,l-n+1\text{ wounded})}=\rho^n\chi^{l-n}-\chi^l-\sum_{j=1}^{n-1}C_n^j\chi_{l(l,l-1,\ldots,l-j+1\text{ wounded})}\ . \tag{37}$$

This recurrency relation is solved with

$$\chi_{l(l,l-1,\ldots,l-n+1\text{ wounded})}=(\rho-\chi)^n\chi^{l-n}\ . \tag{38}$$

The cross section $\sigma_l^{(n)}$ with $n$ wounded nucleons is thus determined as

$$\sigma_l^{(n)}=c_l(\pi r^2)^l\{1+lg-\sigma_0\partial_z\}^{1/x}\chi_l^{(n)}(z)|_{z=0}\ , \tag{39}$$

where the function $\chi_l^{(n)}$ is given by (38) multiplied by a symmetry factor

$$\chi_l^{(n)}=C_l^n(\rho-\chi)^n\chi^{l-n}\ . \tag{40}$$

The final cross section for $n$ wounded nucleons is obtained as a sum over $l$ of (39) with appropriate nuclear factors

$$\sigma^{(n)}=\sum_{l=1}^{A}\sigma_l^{(n)}w_l\ . \tag{41}$$

The sum over $\chi_l^{(n)}$ over all $n>0$ gives $\rho^l-\chi^l$ in accordance with (34).

Equations (34) and (41) form the basis for numerical calculations of the $hA$ cross sections. We postpone the discussion of their results until Sec. V to study the multiplicities.

## IV. MULTIPLICITIES

The differential multiplicities $\mu_A(q)$ are determined as the inclusive cross sections $I_A(q)$ divided by the total inelastic cross section $\sigma_A$. According to (19) they involve an unknown factor $I(q)$ which describes particle production from one string, averaged over its types. However, in the ratio of the multiplicities on the nucleus and on the single nucleon,

$$\alpha=\mu_A(q)/\mu_1(q)\ , \tag{42}$$

the factor $I(q)$ cancels out and the ratio becomes independent of the properties of individual strings. In the following we shall study precisely these relative multiplicities $\alpha$, which in our approximation (all strings taken at the same energy) do not depend on the observed particle momentum $q$.

According to (19) the partial relative multiplicity $\alpha_l$ corresponding to $l$ interacting nucleons with their impact parameters fixed is given by

$$\alpha_{l,b_1,\ldots,b_l}=\kappa_l\sigma_l(b_1,\ldots,b_l)\sigma/\sigma_P\sigma_A \quad (43)$$

with $\sigma_P$ being the cross section for one string

$$\sigma_P=\int d^2b\,\sigma(b)=\sigma_0\pi r^2$$

and the coefficients $\kappa_l$ given by

$$\kappa_l=\{1+lg\}^{1/x-1}\{1+g\}^{1/x}/\{1+lg\}^{1/x}\{1+g\}^{1/x-1} . \quad (44)$$

The integration of (43) over $b_i$ gives

$$\alpha_l=\kappa_l l\sigma^l/\sigma_A . \quad (45)$$

The physical $\alpha$'s are obtained from (45) upon multiplying it with the nuclear factor $w_l$ (18). The effect of string interactions is contained in the factors $\kappa_l$, smaller than unity. They reduce the multiplicity for an interacting nucleon when surrounded by $l-1$ others as compared to an isolated one. The total relative multiplicity $\alpha$ is the sum

$$\alpha=\sum_{l=1}^{A}\alpha_l w_l . \quad (46)$$

As with the cross sections, we shall be also interested in the multiplicities $\alpha^{(n)}$ for $n$ nucleons interacting inelastically (wounded). They are determined as ratios of the corresponding inclusive cross sections $I^{(n)}$ to the cross sections $\sigma^{(n)}$ for $n$ wounded nucleons calculated above, normalized to the case $A=1$. To calculate the inclusive cross sections $I^{(n)}$ we apply the same technique as for the cross sections $\sigma^{(n)}$.

The multiplicity corresponding to $k$ cut strings of any sort for $l$ interacting nucleons, up to a factor which cancels out in $\alpha$, is given by (14) multiplied by $k$. We rewrite it as

$$\mu_{lk}=kc_lC_a^k\left[\sum_{i=1}^{l}y_i\right]^k\left\{1+lg-\sum_{i=1}^{l}x_i\right\}^{a-k},\quad x_i=y_i , \quad (47)$$

where we have used the same notation as in Sec. II: $a=1/x$, $x_i=\sigma(b_i)$ for uncut strings and $y_i=\sigma(b_i)$ for cut strings. The sum of (47) over all $k$ with $x_i\neq y_i$ gives

$$\mu_l=c_la\left[\sum_{i=1}^{l}y_i\right]\left\{1+lg+\sum_{i=1}^{l}(y_i-x_i)\right\}^{a-1} . \quad (48)$$

With $x_i=y_i$ this brings us back to (19) for the total multiplicity for $l$ interacting nucleons.

Consider again all events with only the $l$th nucleon wounded. The corresponding inclusive cross section is given by what is left after we put in (48) $y_i=0$, $i\le l-1$, $x_l=y_l$:

$$\mu_{l(l\text{ wounded})}=c_lax_l\left\{1+lg-\sum_{i=1}^{l-1}x_i\right\}^{a-1} . \quad (49)$$

Likewise the inclusive cross section for only two nucleons wounded, $l$th and $(l-1)$th, is given by what remains in (48) after we put $y_i=0$, $i\le l-2$, $x_{l-1}=y_{l-1}$, and $x_l=y_l$ minus the contributions with only the $l$th and only the $(l-1)$th nucleon wounded:

$$\mu_{l(l,l-1\text{ wounded})}=c_la(x_{l-1}+x_l)\left\{1+lg-\sum_{i=1}^{l-2}x_i\right\}^{a-1}-\mu_{l(l-1\text{ wounded})}-\mu_{l(l\text{ wounded})} . \quad (50)$$

Proceeding in the same manner we can calculate the inclusive cross sections for an arbitrary number $n$ of wounded nucleons out of their total number $l$.

As in the case of ordinary cross sections, simpler formulas are obtained after the integration over nucleon impact parameters $b_i$. The typical integral that appears here has a structure

$$I=\int_\sigma\prod_1^l d^2b_i\left[\sum_{i=l-n+1}^{l}x_i\right]\left\{1+lg-\sum_{i=1}^{l-n}x_i\right\}^{a-1} . \quad (51)$$

The integration over $b_i$ with $l-n+1\le i\le l$ is done directly resulting in the factor $n\pi r^2\sigma^{n-1}$. The rest of the integration is performed according to Eq. (31), so that the integral (51) is finally given by

$$I=n(\pi r^2)^l\rho^{n-1}\{1+lg-\sigma_0\partial_z\}^{1/x-1}\chi^{l-n}(z)|_{z=0} , \quad (52)$$

where $\rho$ is defined in (33). The part of the operator applied to the function $\chi^{l-n}$ which is independent of $n$ is common to all expressions for multiplicities with a different number of wounded nucleons. So we present each multiplicity as

$$\mu_l=c_l(\pi r^2)^l\{1+lg-\sigma_0\partial_z\}^{1/x-1}\lambda_l(z)|_{z=0} . \quad (53)$$

Then we find from (49) and (50)

$$\lambda_{l(l\text{ wounded})}=\chi^{l-1} , \quad (54)$$

$$\lambda_{l(l,l-1\text{ wounded})}=2\rho\chi^{l-2}-2\lambda_{l(l\text{ wounded})} , \quad (55)$$

and generally

$$\lambda_{l(l,l-1,\ldots,l-n+1\text{ wounded})}=n\rho^{n-1}\chi^{l-n}-\sum_{j=1}^{n-1}C_n^j\lambda_{l(l,l-1,\ldots,l-j+1\text{ wounded})} . \quad (56)$$

Equation (56) gives a recurrency relation with the solution

$$\lambda_{l(l,l-1,\ldots,l-n+1\text{ wounded})}=n(\rho-\chi)^{n-1}\chi^{l-n} . \quad (57)$$

The multiplicity $\mu_l^{(n)}$ for $n$ wounded nucleons is obtained from (53) with a function $\lambda_l^{(n)}$ given by (57) multiplied by a symmetry factor

$$\lambda_l^{(n)} = nC_l^n(\rho-\chi)^{n-1}\chi^{l-n} . \tag{58}$$

The sum of $\lambda_l^{(n)}$ over all $n$ gives a constant $l\rho^{l-1}$ which upon substitution in (53) and proper normalization returns us to (45). The final multiplicities $\mu^{(n)}$ for a given number of wounded nucleons are obtained from (53) after summation over $l$ with nuclear factors attached and adequate normalization:

$$\mu^{(n)} = \sum_{l=1}^{A} \mu_l^{(n)} w_l / \sigma^{(n)} . \tag{59}$$

## V. NUMERICAL RESULTS AND DISCUSSION

Calculations of the cross sections and multiplicities in the model require knowledge of the parameters $x$, $g$, and $\sigma_0$ which determine the probability of string fusion, their number, and the cross section for one string averaged over its types, respectively. In accordance with the physical picture explained before, the probability $x$ was chosen proportional to the ratio of the proper string transverse area to the total interaction area

$$x = 0.199\pi/\sigma\langle p_\perp^2\rangle . \tag{60}$$

The numerical coefficient 0.199 was taken to ensure the unitarity of the model at the energy as low as corresponding to $p_{\text{lab}} = 200$ GeV/$c$ ($y = 5.93$). With a higher value of the fusion probability the resulting $pp$ cross sections at $y = 5.93$ become lower than the experimental ones. The cross-section parameter $\sigma_0$ was taken in the same form as in the theory of a supercritical Pomeron,

$$\sigma_0 = C(r_0/r)^2 \exp(\Delta y) , \tag{61}$$

with $r$ and $r_0$ related according to (25). Two values for the intercept $\Delta$ were considered: $\Delta = 0.12$ and $\Delta = 0.5$. This latter high value is favored if one wishes to describe the rise in $\langle p_\perp \rangle$ and multiplicity with energy by string fusion [7]. The parameter $g$ was determined from the condition that the values of the cross section $\sigma$ for $pp$ (or $p\bar{p}$) interaction given by the model coincide with the experimental ones, extrapolated for energies unaccessible so far as $a + by^2$. It turns out that this procedure essentially eliminates the dependence of the results on the value of the numerical coefficient $C$ in (61): A change of $C$ by an order of magnitude changes the results by a few percent. We have chosen $C = 0.1$ in the calculation.

For the nuclear profile function $T_A(b)$ a simple form was taken that corresponds to a constant nuclear density inside a sphere of a radius $R_A$. To take into account the growth of the interaction range at very high energies the radius $R_A$ was taken as a sum of a constant proper nucleus radius plus the hadron radius, which grows with energy in accordance with the values of $\sigma$. The value of $R_A$ was determined to be $A^{1/3}1.26$ fm at $p_{\text{lab}} = 800$ GeV/$c$ from the known experimental data on $pA$ inelastic cross sections [10]. The results depend very weakly on a particular form of $T_A(b)$ and are practically the same even with a constant $T_A(b)$.

The results of the calculation of $hA$ cross sections and relative multiplicities are presented in Figs. 1–5.

As expected, the total cross sections (Fig. 1) are only slightly affected by the interaction between strings. Even at the energies as high as $\sqrt{s} = 6.3$ TeV and with the choice $\Delta = 0.5$ (that is, with a very strong string interaction) they are reduced by 15% at most. This can also be seen in Fig. 2 where we compare the energy dependence of the $p$-air inelastic cross sections without ($x = 0$) and with string interactions and of the experimental ones taken from cosmic ray data [11]. The interaction of strings reduces the cross sections by 10% at the highest energy. The experimental data suffer from various uncertainties and depend on the way they are extracted, making the comparison inconclusive. It is remarkable that the string interaction does not change the $A$ dependence of $\sigma_A$, which stays proportional to $A^{2/3}$, although with a slightly lower coefficient.

The relative multiplicities $\alpha$ (Fig. 3), on the contrary, change drastically both in their absolute value and $A$

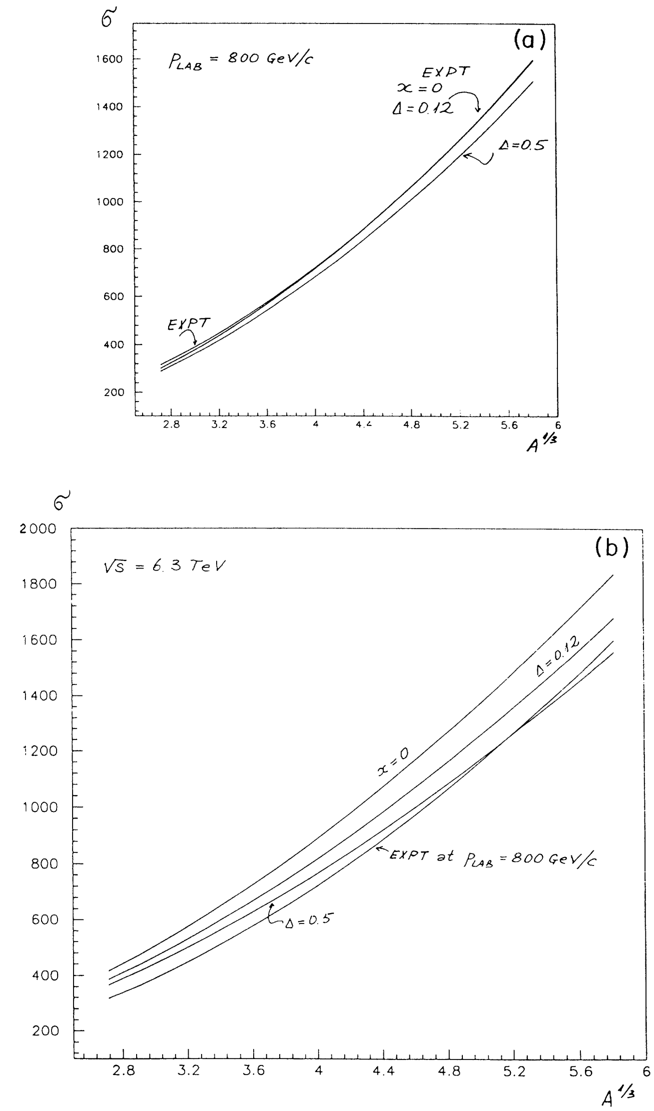


FIG. 1. The $hA$ inelastic cross sections as a function of $A^{1/3}$ at (a) $p_{\text{lab}} = 800$ GeV/$c$ and (b) $\sqrt{s} = 6.3$ TeV. The curve $x = 0$ corresponds to noninteracting strings. The experimental data are at $p_{\text{lab}} = 800$ GeV/$c$ [10].

dependence. As observed from Fig. 3(b), for high energies and therefore with a strong interaction between strings they become practically independent of $A$ and close to unity.

The physical picture underlying this change becomes clear from the dependence of the cross sections and multiplicities on the number of wounded nucleons presented in Figs. 4 and 5 for a Au target ( $A = 197$). The striking feature of these results is a very weak influence of string interaction on the multiplicity for a given number $n$ of wounded nucleons. It practically remains proportional to $n$ at least up to values $n = 8$, for which the cross sections are not too small (Fig. 5). There does not seem to appear any sign of a saturation. A possible explanation of this result is that the string interaction equally reduces the $hN$ interaction itself and absorption corrections to it, so that the net result is negligible (as happens normally with the AGK cancellations [9]). The cross sections $\sigma^{(n)}$ having $n$ wounded nucleons, on the other hand, change radically (Fig. 4). They fall with $n$ much more rapidly than without string interaction, so that at high energies very few wounded nucleons appear with appreciable probability. As a result, the mean number of inelastic collisions is substantially reduced, approaching unity for a very strong string interaction. This result is all the more interesting if we recall that the total cross section stays practically the same.

We tried to compare these results to the existing experimental data on the dependence of the cross sections and multiplicities on the number of inelastic collisions inside the nucleus [12]. Unfortunately these data belong to comparatively small energies ($p_{\text{lab}} = 100, 200$ GeV/$c$), where the effect of string interaction is not so pronounced. Besides, what one sees in experiment is not the number of wounded nucleons $n$ but rather the number of "grey" protons $p_n$, which is greater than $n$ because of cascade effects. To perform the comparison we have made use of the standard procedure to convert data on grey protons to wounded nucleons $n = n(p_n)$ [13]. The resulting experimental points for the cross sections and multiplicities on Au are shown in Figs. 4(a) and 5(a) by circles and triangles for two different choices of parameters in the conversion formula. In both cases the experimental data do not agree too well with theoretical predictions both with and without string interaction. The difference

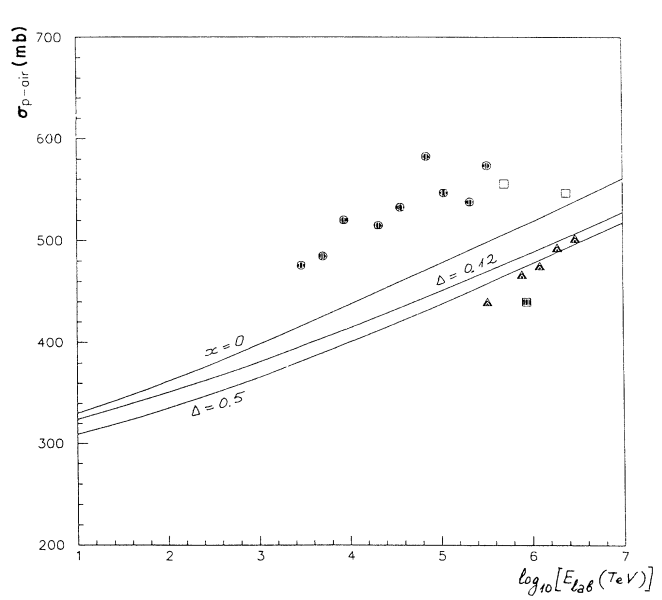


FIG. 2. The $p$-air inelastic cross sections as functions of the laboratory energy. The curve $x = 0$ corresponds to noninteracting strings. The experimental points are from [11].

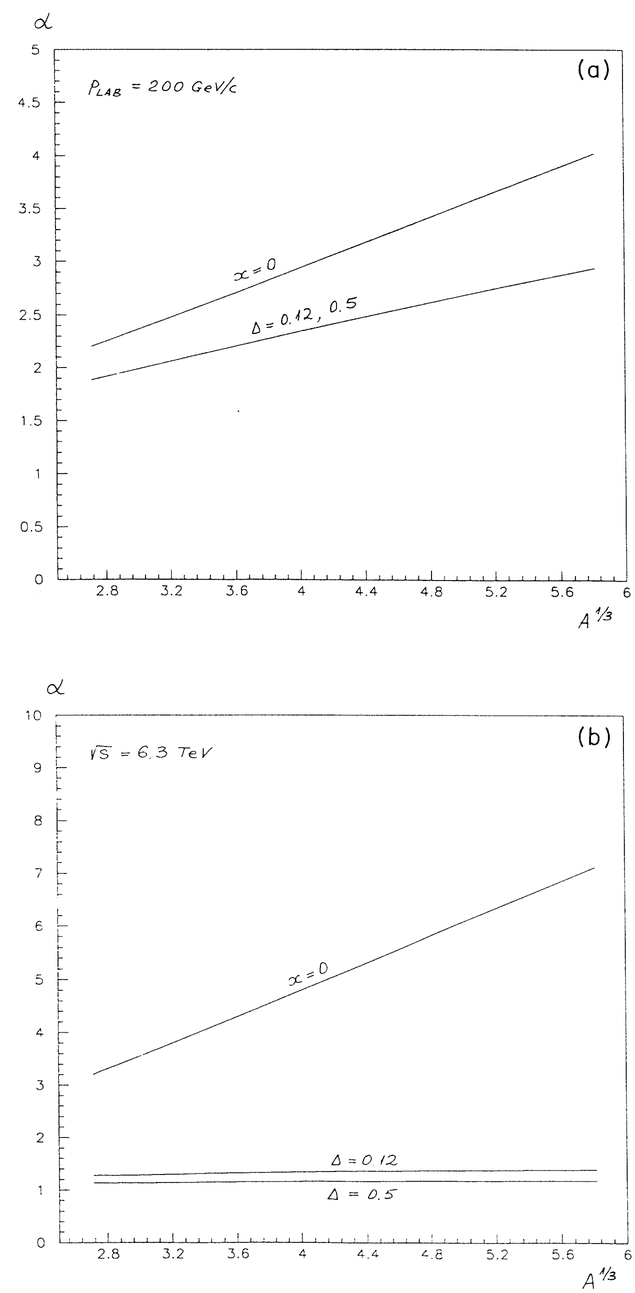


FIG. 3. The ratios of $hA$ to $hN$ multiplicities $\alpha$ as a function of $A^{1/3}$ at (a) $p_{\text{lab}} = 200$ GeV/$c$ and (b) $\sqrt{s} = 6.3$ TeV. The curve $x = 0$ corresponds to noninteracting strings.

between these latter is of the same order of magnitude as the one which results from different types of nuclear profile functions. Therefore more precise experimental data, preferably for higher energies, together with a detailed knowledge of the relation between $n$ and $p_n$, are needed to draw definite conclusions on the subject.

We want to stress that the present calculation was performed in the simplified version of the string interaction model where all strings are formed at the same energy and no attempt has been made to incorporate energy conservation. This corresponds to energies high enough, when all strings formed have a considerable "length" (that is, carry enough energy). It is known, however, that at present energies strings vary substantially in their energies and the majority of them are rather short. The generalization of our model to this realistic case requires considering strings with different locations in the rapidity space. Together with the energy conservation requirement, this considerably complicates the picture, so that

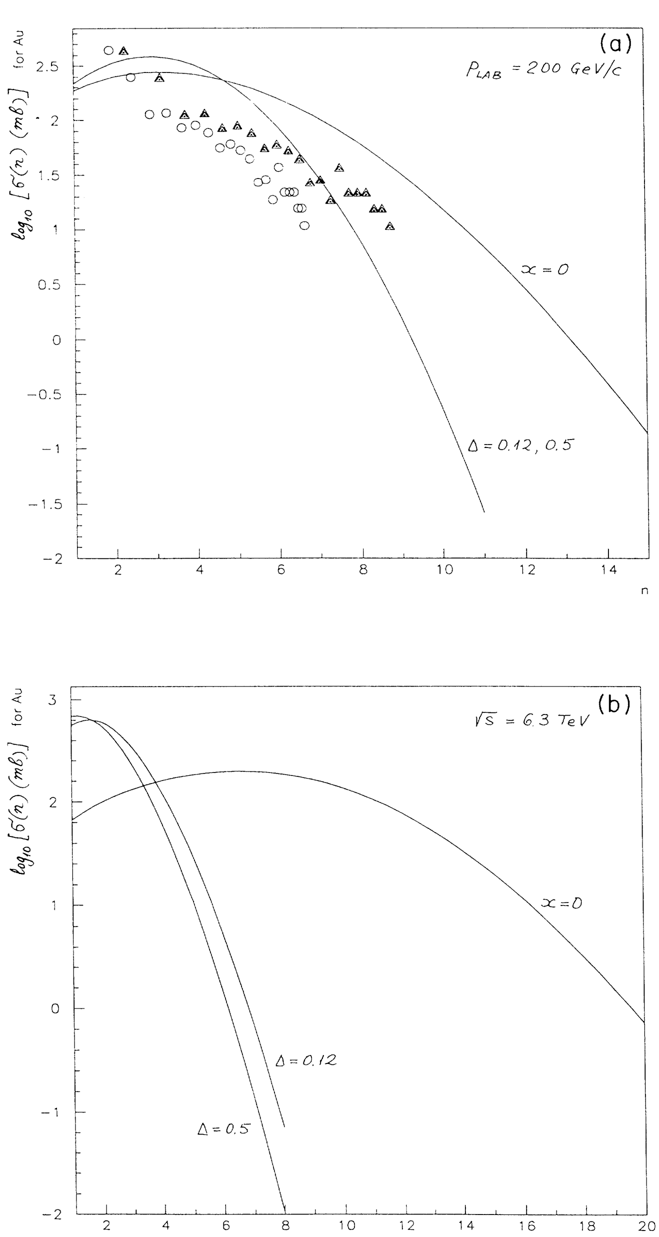


FIG. 4. The $h$Au inelastic cross sections for different numbers $n$ of wounded nucleons at (a) $p_{\mathrm{lab}}=200$ GeV/$c$ and (b) $\sqrt{s}=6.3$ TeV. The curve $x=0$ corresponds to noninteracting strings. The experimental data are at $p_{\mathrm{lab}}=200$ GeV/$c$ [12].

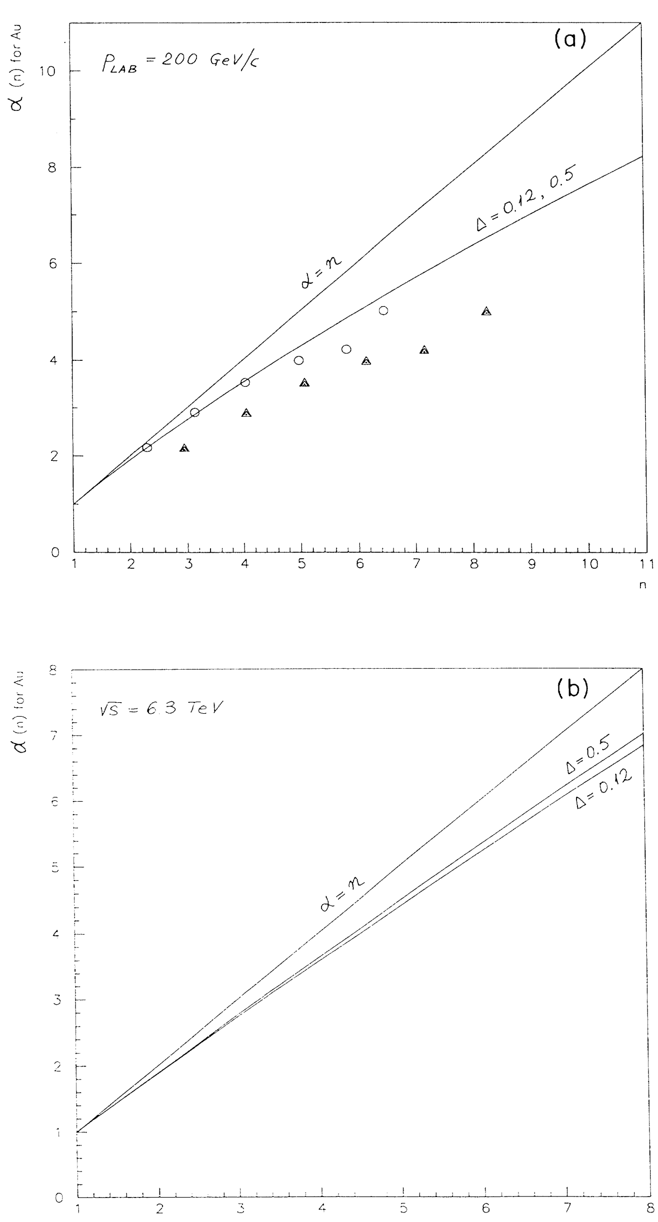


FIG. 5. The $h$Au multiplicities for different numbers $n$ of wounded nucleons relative to $n=1$ at (a) $p_{\mathrm{lab}}=200$ GeV/$c$ and (b) $\sqrt{s}=6.3$ TeV. The curve $\alpha=n$ corresponds to noninteracting strings ($x=0$). The experimental data are at $p_{\mathrm{lab}}=200$ GeV/$c$ [12].

calculations become possible only within a Monte Carlo approach. Such a program is now in progress. However, our strongest prediction, namely, the practical independence of the multiplicity on $A$ at very high energies, will remain valid independently of the above considerations and may have important consequences for heavy-ion collisions at RHIC and LHC energies. Several theoretical predictions on multiplicities at the LHC energy have been made using different models. The values of the central charged rapidity density for central Au-Au collisions range from 2500 in VENUS 3.11 [14] to 3200 or 7000 in the DPM depending on such details as the *pp* cross section used, the shape of the nuclear profile function, the definition of central collisions, and approximations made in the course of calculations [15,16]. The resulting $A$ dependence is $A^{1.16}$ in VENUS 3.11 and $A^{4/3}$ in DPM, the latter being the one which corresponds to the pure AGK rules and the Glauber model. The extension of our approach to nucleus-nucleus collisions is straightforward but does not allow to obtain analytic formulas like the ones obtained here for the $hA$ case. The exact behavior of cross sections and multiplicities in $AA$ collisions with $A$ and energy can thus be studied only by Monte Carlo simulations. Nevertheless, based on the analogy with Pomeron models for $AB$ collisions, one may conjecture that in the limiting case of strong string fusion the multiplicity in $AA$ collisions would become proportional to $A^{2/3}$. At LHC energies we would therefore expect it to behave between $A^{2/3}$ and $A$, with a reduction factor relative to the standard AGK prediction of the order $A^{1/3}$-$A^{2/3}$. For Au-Au collisions it will reduce the value of the central charged multiplicity density by a factor of 6/33, which means that this multiplicity density should be less than 1200. Needless to say, this spectacular reduction may have important consequences in many aspects, ranging from the LHC detectors to the extensive air shower data in cosmic rays.

## ACKNOWLEDGMENTS

The authors are grateful to the General Direction of the Scientific and Technical Investigation (DGICYT) of Spain for the financial support.

[1] A. Capella, U. Sukhatme, C. I Tan, and J. Tran Thanh Van, Phys. Lett. **81B**, 68 (1979).

[2] A. Kaidalov, Phys. Lett. **116B**, 459 (1982).

[3] B. Andersson, G. Gustafson, and B. Nilsson-Almquist, Nucl. Phys. **B281**, 289 (1987).

[4] C. Pajares, Phys. Lett. B **258**, 461 (1991); C. Merino, C. Pajares, and J. Ranft, Phys. Lett. B **276**, 168 (1992); C. Pajares, in *Quark Gluon Plasma Signatures,* Proceedings of the International Workshop, Strasbourg, France, 1991, edited by V. Bernard *et al.* (Editions Frontières, Gif-sur-Yvette, 1991).

[5] B. Andersson and P. A. Henning, Nucl. Phys. **B355**, 82 (1991).

[6] V. A. Abramovski, E. V. Gedalin, E. G. Gurvich, and O. V. Kancheli, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 281 (1988) [JETP Lett. **47**, 337 (1988)].

[7] M. Braun and C. Pajares, Nucl. Phys. B (to be published).

[8] M. Braun and C. Pajares, Phys. Lett. B **287**, 154 (1992).

[9] V. A. Abramovsky, V. N. Gribov, and O. V. Kancheli, Yad. Fiz. **18**, 595 (1973) [Sov. J. Nucl. Phys. **18**, 308 (1973)].

[10] A. S. Sambamurti *et al.*, Phys. Rev. D **41**, 1371 (1990).

[11] T. Yuda, in *Proceedings of the Twenty-Second International Cosmic Ray Conference,* Dublin, Ireland, 1991, edited by M. Cowley *et al.* (Dublin Institute for Advanced Research, Dublin, 1992), Vol. 5, p. 313.

[12] W. D. Walker and P. C. Bhat, in *Hadronic Multiparticle Production,* edited by P. Carruthers (World Scientific, Singapore, 1988), pp. 153–205.

[13] E. Stenlund and I. Otterlund, Nucl. Phys. **B198**, 407 (1982).

[14] P. Cerello, P. Giubellino, and K. Werner, in *Proceedings of the ECFA Large Hadron Collider Workshop,* Aachen, Germany, 1990, edited by G. Jarlskog and D. Rein (CERN Report No. 90-10, Geneva, Switzerland, 1990), Vol. 2.

[15] H. M. Moehring and J. Ranft, Z. Phys. C **52**, 643 (1992).

[16] A. Capella, C. Merino, and J. Trahn Thanh Van, Phys. Lett. B **265**, 415 (1991).