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Isospin baryon mass differences in semibosonized SU(3) Nambu–Jona-Lasinio model

M. Praszałowicz,* A. Blotz, and K. Goeke
Institute of Theoretical Physics II, Ruhr-University Bochum, D-4630 Bochum, Germany
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Hadronic parts of the isospin mass differences for the octet and decuplet of baryons are calculated within a semibosonized SU(3) Nambu–Jona-Lasinio model. Striking agreement with experimental data is found.

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I. INTRODUCTION

The mass splittings of the baryons belonging to the isospin multiplets consist of two parts: hadronic and electromagnetic [1], namely,

$$\Delta M_B = (\Delta M_B)_h + (\Delta M_B)_e \ . \tag{1.1}$$

In this paper we will calculate $(\Delta M_B)_h$ in the framework of the semibosonized SU(3) Nambu–Jona-Lasinio (NJL) model. This model was recently proven to reproduce hyperon splittings both for the octet and decuplet of baryons [2,3]. Similarly to the hyperon splittings, the model makes specific predictions not only for the *n-p* mass difference but for all isospin splittings both in the octet and decuplet. We will show that the simple version of the model with only the octet of pseudoscalar mesons coupled to constituent quarks reproduces the existing data with high accuracy.

In spite of tremendous technical difficulties encountered in calculating the low-energy properties directly in quantum chromodynamics (QCD), one is prompted to use simpler *effective* models, which, in principle, should be derivable from the underlying QCD Lagrangian. Indeed, if one assumes that the QCD vacuum is dominated by instanton gluon field configurations, then one arrives at the model of the NJL type [4,5].

In chiral models, where baryons are described as solitons, the hadronic part of the isospin mass splittings is related to the current quark mass difference $m_d - m_u$. However, in the Skyrme model and also in the present model with only two quark flavors and three pseudoscalars (the pions) the hadronic part of *n-p* mass difference vanishes identically. One way to cure this disease is to enlarge the symmetry group. In Ref. [6] for example the U(2)⊗U(2) extension of the Skyrme model with pseudoscalar and vector fields was shown to predict a nonzero *n-p* mass difference; however, the result was still 35% too small. Some increase was obtained for the U(3)⊗U(3) case [6]. Similar results were obtained in the chiral bag model [7], where quark degrees of freedom are explicitly taken into account. Although in general the chiral model predictions are too small, in the recent paper of Clément and Stern [8] it is shown that in the linear SU(3) σ model the *n-p* mass difference is approximately two times too large. In contrast to the above examples our calculation shows surprisingly good accuracy as far as *all* isospin mass splittings are concerned—not only the *n-p* mass difference.

In Sec. II we will collect the known experimental results for the octet isospin splittings and present a useful parametrization, which we will use to estimate the hadronic part of the decuplet isospin splittings. In Sec. III we will formulate the model and fix its parameters from the meson sector. Then the mass splitting operator as well as its matrix elements will be evaluated in Sec. IV. It is perhaps worthwhile to stress already here that the pattern of the symmetry breaking due to the current quark mass matrix depends crucially on the SU(3) structure of the model. In the SU(2) version we would get a zero result in close analogy to the Skyrme model. Section V summarizes our results and in Sec. VI we present concluding remarks.

II. PHENOMENOLOGY OF THE ISOSPIN SPLITTINGS

The origin of the formula (1.1) is theoretically clear: the electromagnetic part is due to the baryon electromagnetic self-energy and the hadronic part is due to the mass difference of the quarks constituting different baryons. It is, however, a difficult task to disentangle the two contributions from the experimental data. Gasser and Leutwyler [1] quote the following values for experimental estimates of the electromagnetic parts (in MeV):

$$\begin{aligned} &(n-p)_e = -0.76 \pm 0.3, \quad (\Sigma^- - \Sigma^+)_e = 0.17 \pm 0.3 \ , \\ &(\Xi^- - \Xi^0)_e = 0.86 \pm 0.3 \ . \end{aligned} \tag{2.1}$$

These values have to be subtracted from the measured mass differences (also in MeV) [9]:

$$\begin{aligned} &n - p = 1.29, \quad \Sigma^- - \Sigma^+ = 8.07 \pm 0.09 \ , \\ &\Xi^- - \Xi^0 = 6.4 \pm 0.6 \ . \end{aligned} \tag{2.2}$$

By subtracting (2.1) from (2.2) we get the hadronic parts

*On leave of absence from the Institute of Physics, Jagellonian University, Reymonta 4, 30-059 Kraków, Poland.

of the isospin splittings:

$$(n-p)_h=2.05\pm0.30,\quad (\Sigma^- - \Sigma^+)_h=7.89\pm0.31\ ,$$
$$(\Xi^- - \Xi^0)_h=5.5\pm0.7\ . \tag{2.3}$$

Unfortunately, the similar estimates do not exist for the decuplet. Let us therefore propose a simple parametrization for both electromagnetic and hadronic splittings which we will subsequently use to estimate the hadronic parts of the isospin mass differences in the decuplet.

Let us assume, in analogy with the Gell-Mann–Okubo mass formula for hyperons [10,11], that the quark mass operator responsible for isospin breaking transforms like an octet isovector operator corresponding to $I_3=0$. This assumption will be further confirmed within the present model. If so, the matrix elements of the mass operator are given in terms of the SU(3) Clebsch-Gordan coefficients:

$$(\Delta M_B^{(8)})_h=\frac{1}{\sqrt{3}}f\begin{bmatrix} 8 & 8 & 8_- \\ 010 & B & B \end{bmatrix}+\sqrt{5/3}d\begin{bmatrix} 8 & 8 & 8_+ \\ 010 & B & B \end{bmatrix} \tag{2.4}$$

for the octet, and

$$(\Delta M_B^{(10)})_h=\sqrt{2/3}c\begin{bmatrix} 8 & 10 & 10 \\ 010 & B & B \end{bmatrix} \tag{2.5}$$

for the decuplet (normalization factors in front of the f, d, and c coefficients are chosen for future convenience). B stands in short for the baryon quantum numbers, namely, I, I_3, and Y. Reduced matrix elements f, d, and c are here treated as free parameters, however, in Sec. IV we will calculate them within the present model.

Evaluating the SU(3) Clebsch-Gordan coefficients gives [12,13]

$$(\Delta M_B^{(8)})_h=-\tfrac{1}{3}fI_3+dYI_3\ ,$$
$$(\Delta M_B^{(10)})_h=-\tfrac{1}{3}cI_3\ . \tag{2.6}$$

As far as the electromagnetic part is concerned we will assume, in analogy to the Dashen ansatz [14] for mesons, that $(\Delta M_B)_e$ is proportional to the baryon mass and charge squared:

$$(\Delta M_B)_e=\alpha Q_B^2 M_B \tag{2.7}$$

both for the octet and decuplet. Here Q_B is a baryon charge, M_B denotes an average isospin multiplet mass and α is a constant. We assume the proportionality to the baryon mass in order to make $(\Delta M_B)_e$ vanishing for massless baryons and also to account for the ratio of n-p and $\Xi^0-\Xi^-$ electromagnetic mass differences. This assumption results also in a prediction that electromagnetic part of $\Sigma^- - \Sigma^+$ and $\Sigma^{*-}-\Sigma^{*+}$ vanishes identically.

We fit α from the mass difference $\Sigma^-+\Sigma^+-2\Sigma^0=1.70\pm0.14$ MeV which, as can be seen from Eq. (2.6), is purely electromagnetic ($M_\Sigma=1192.55$):

$$\alpha=(7.13\pm0.59)\times10^{-4}\ . \tag{2.8}$$

Next we fix parameters f and d. f is entirely given by $\Sigma^- - \Sigma^+$ mass difference and d can be calculated from n-p mass difference. Taking $M_N=938.59$ MeV we get

$$f=12.11\pm0.14\ \text{MeV},\quad d=2.08\pm0.08\ \text{MeV}\ . \tag{2.9}$$

With these parameters fixed we get one prediction for the octet splitting of Ξ which is displayed in Table I (for $M_\Xi=1317.59$ MeV).

Having tested our parametrization for the octet we shall use it to estimate the hadronic part of the decuplet isospin splittings. Before that let us, however, check how well parametrization [Eqs. (2.6) and (2.7)] reproduces the absolute decuplet isospin splittings. We need to fix only one parameter, namely, c. Taking for $M_{\Sigma^*}=1383.7$ MeV we get from $\Sigma^{*-}-\Sigma^{*+}=4.40\pm0.64$ MeV (which in analogy to Σ we assume to be purely hadronic):

$$c=6.6\pm1.0\ \text{MeV}\ . \tag{2.10}$$

The remaining mass splittings come out as predictions. The results are presented in Table I where $M_{\Xi^*}=1532.85$ MeV and $M_\Delta=1231$ MeV were assumed. Although the agreement is striking the calculation of $(\Delta M_B)_e$ in the chiral quark model is certainly strongly needed (see however Refs. [15,16]).

The aim of the above exercise was to show that the hadronic parts of the isospin splittings can be well parametrized by Eq. (2.6) rather than advocating the particular parametrization of the electromagnetic part. In fact, one may convince oneself that, in view of large un-

TABLE I. Mass differences in isospin multiplets calculated with parametrization [Eqs. (2.6) and (2.7)]. Experimental data from the Particle Data Group [9]. Data on Δ masses differ depending on experiment, no trustworthy result for Δ^- is quoted.

	Parametrization Eqs. (2.6) and (2.7) (MeV)	Experiment (MeV)
n-p	input	1.29
$\Sigma^- - \Sigma^+$	input	8.08±0.09
$\Sigma^- + \Sigma^+ - 2\Sigma^0$	input	1.70±0.14
$\Xi^- - \Xi^0$	7.06±0.12	6.4±0.6
$\Sigma^{*-}-\Sigma^{*+}$	input	4.4±0.7
$\Sigma^{*-}+\Sigma^{*+}-2\Sigma^{*0}$	1.97±0.16	2.6±1.2
$\Xi^{*-}-\Xi^{*0}$	3.29±0.33	3.20±0.68
Δ^{++}	1231.22±0.56	1230.9±0.3
		1230.6±0.2
		1231.1±0.1
Δ^+	1230.78±0.18	1234.9±1.4
		1231.6
		1231.2
		1231.8
Δ^0	1232.10±0.16	1233.6±0.5
		1232.5±0.3
		1233.8±0.2
Δ^-	1235.18±0.49	?

certainties of the experimental data for $(\Delta M_B)_e$, similar but M_B independent parametrization will work equally well.

Now we will extract the values of the reduced matrix elements independently of the particular form of $(\Delta M_B)_e$. We extract f and d directly from the experimental estimates of hadronic splittings for nucleon and Ξ [see Eq. (2.3)]. For c we will use the previous estimate which relies on the assumption that $\Sigma^{*-}-\Sigma^{*+}$ is purely hadronic. Altogether we get:

$$f=11.33\pm1.14 \text{ MeV}, \quad d=1.73\pm0.38 \text{ MeV} , \\ c=6.6\pm1.0 \text{ MeV} . \tag{2.11}$$

In Secs. IV and V we will calculate f, d, and c within the semibosonized SU(3) NJL model.

III. SEMIBOSONIZED SU(3) NAMBU–JONA-LASINIO MODEL

The generalization of the original SU(2) Nambu–Jona-Lasinio Lagrangian [17] to SU(3) is straightforward [2,3]. With the inclusion of the finite current quark mass matrix m

$$m=\begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{bmatrix}=\mu_0\lambda_0-\mu_3\lambda_3-\mu_8\lambda_8 , \tag{3.1}$$

where

$$\mu_0=(1/\sqrt{6})(m_u+m_d+m_s) , \\ \mu_8=(1/\sqrt{12})(2m_s-m_u-m_d) , \\ \mu_3=\tfrac{1}{2}(m_d-m_u) , \tag{3.2}$$

we consider the four-fermion interaction

$$\mathcal{L}_{\mathrm{NJL}}=\bar{q}(i\gamma_\mu\partial^\mu-m)q-(G/2)[(\bar{q}\lambda^a q)^2+(\bar{q}i\gamma_5\lambda^a q)^2] , \tag{3.3}$$

where the λ_a, $a=1,\ldots,8$ are the usual Gell-Mann matrices [10] and $\lambda_0=\sqrt{\frac{2}{3}}$. In the chiral limit $(m=0)$ $\mathcal{L}_{\mathrm{NJL}}$ is invariant under combined $\mathrm{SU}(3)_R\otimes\mathrm{SU}(3)_L$ transformations. There is also an additional $\mathrm{U}(1)_V\otimes\mathrm{U}(1)_A$ symmetry, so that the spontaneous symmetry breaking leads to appearance of nine Goldstone bosons [18]. In nature the redundant $\mathrm{U}_A(1)$ is assumed to be broken by instantons and this gives the η' a larger mass. Here we ignore this fact, because we are interested mainly in the baryon sector of the theory.

In the bosonization procedure [19], we choose to retain the current masses within the quark Lagrangian. After inserting the following constant in the path integral

$$\int \mathcal{D}\sigma^a\mathcal{D}\pi^a \exp\left[i\mu^2/2\int d^4x[(\sigma^a-\bar{q}\lambda_a q)^2 + (\pi^a-\bar{q}i\gamma_5\lambda_a q)^2]\right] \tag{3.4}$$

we arrive at the generating functional:

$$W_{\mathrm{NJL}}=N\int \mathcal{D}\bar{q}\,\mathcal{D}q\,\mathcal{D}\sigma^a\mathcal{D}\pi^a \exp\left[i\int d^4x\,\mathcal{L}'_{\mathrm{NJL}}\right] , \tag{3.5}$$

where we have omitted the sources for simplicity. Going to Euclidean space according to Ref. [20] one obtains the semibosonized Lagrangian

$$\mathcal{L}'_{\mathrm{NJL}}=\bar{q}[-i\gamma_\mu\partial^\mu+m+g(\sigma_a\lambda_a+i\gamma_5\pi_a\lambda_a)]q \\ +(\mu^2/2)(\sigma_a^2+\pi_a^2) . \tag{3.6}$$

After that one makes a saddle point expansion and treats the mesonic fields as the classical ones. We can then immediately integrate over the quark fluctuations, which corresponds to a one-fermion and zero-boson-loop approximation. This yields—after making a well-known Legendre transformation [21] between the former sources and some new classical sources, which will play the role of the fields—the effective Euclidean action

$$S_{\mathrm{eff}}=-\mathrm{Sp}\ln[-i\gamma^\mu\partial_\mu+g(\sigma_a\lambda_a+i\gamma_5\pi_a\lambda_a)+m] \\ +\frac{\mu^2}{2}\int d^4x(\sigma_a^2+\pi_a^2) . \tag{3.7}$$

The remaining parameters of the theory are μ^2, g, the cutoff Λ, because the theory is nonrenormalizable, and the vacuum expectation values of the bosonic fields. We fix these parameters along the lines described in [20,3] for the SU(3) symmetric formulation and we mention here only the differences.

First we require a stationary vacuum state. From the effective potential

$$V_{\mathrm{eff}}=-\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}\ln[\not{k}+g(\sigma_a\lambda_a+i\gamma_5\pi_a\lambda_a)+m] \\ +(\mu^2/2)(\sigma_a^2+\pi_a^2) , \tag{3.8}$$

which is the local part in the expansion of the effective action, we obtain 3 nontrivial stationary equations $dV_{\mathrm{eff}}/d\sigma_a=0$; $a=0$, 3, and 8, which can be summarized in the so-called gap equations

$$\mu^2=8N_c g^2 I_1(M_i)+\frac{\mu^2 m_i}{M_i}, \quad a=u,\ d, \text{ and } s , \tag{3.9}$$

where the M_i are the constituent quark masses, which consist of current quark masses m_i and nonvanishing vacuum expectation values of σ_a, $a=0$, 3, and 8. Then we determine the pseudoscalar meson masses at zero momentum from the curvature of the effective potential via

$$m^2_{\pi_{ab}}=\frac{d^2V_{\mathrm{eff}}}{d\pi_a d\pi_b} \tag{3.10}$$

and obtain

$$\begin{aligned} m^2_{\pi_0} &= -4N_c g^2[I_1(M_u)+I_1(M_d)]+\mu^2 , \\ m^2_{\pi_\pm} &= -4N_c g^2[I_1(M_u)+I_1(M_d)] \\ &\quad -(M_u-M_d)^2 I_2(M_u,M_d)+\mu^2 , \\ m^2_{K_0} &= -4N_c g^2[I_1(M_d)+I_1(M_s)] \\ &\quad -(M_d-M_s)^2 I_2(M_d,M_s)+\mu^2 , \\ m^2_{K_\pm} &= -4N_c g^2[I_1(M_u)+I_1(M_s)] \\ &\quad -(M_u-M_s)^2 I_2(M_u,M_s)+\mu^2 , \end{aligned} \tag{3.11}$$

where $I_1(M_i)$ and $I_2(M_i,M_j)$ are certain divergent integrals, whose explicit form [3] is not needed here. In the first order of the perturbation theory for the current masses we get the following relations:

$$\begin{aligned} m^2_{\pi_{0,\pm}} &= (m_u+m_d)c_1 , \\ m^2_{K_\pm} &= (m_u+m_s)c_1 , \\ m^2_{K_0} &= (m_d+m_s)c_1 , \end{aligned} \tag{3.12}$$

where c_1 is some cutoff dependent constant constructed from the integrals $I_1(M_i)$ and $I_2(M_i,M_j)$. Making use of the Dashen ansatz [14] for the electromagnetic contribution to the pseudoscalar mesonic masses

$$m^2_{\rm ps} = m_h^2 + e^2 c_2 , \tag{3.13}$$

where e is the electric charge, m_h is the hadronic part given by Eqs. (3.12) and c_2 is some common factor, we can easily derive the cutoff independent relation

$$\frac{m_u-m_d}{m_u+m_d} = \frac{(m^2_{K^\pm}-m^2_{K^0})-(m^2_{\pi^\pm}-m^2_{\pi^0})}{m^2_{\pi^\pm}} = -0.28 , \tag{3.14}$$

with the experimental values for the mesonic masses. In this way for an average nonstrange current quark mass of $m=6.1$ MeV which is determined by our regularization [3] we obtain $m_d-m_u=3.4$ MeV, and therefore the absolute values $m_u=4.4$ MeV and $m_d=7.8$ MeV.

IV. MASS SPLITTINGS—THEORY

In this section we will briefly recall how the mass splitting operator in the solitonic sector emerges. We will start from the effective action (3.7) on the chiral circle:

$$S_{\rm eff} = -\mathrm{Sp}\ln(-i\partial\!\!\!/ + m + MU^{\gamma_5}) . \tag{4.1}$$

U is an SU(3) matrix:

$$U = A(t)\begin{bmatrix} \overline{U}_0(\mathbf{x}) & 0 \\ 0 & 1 \end{bmatrix} A^\dagger(t) , \tag{4.2}$$

where $\overline{U}_0$ is the SU(2) *hedgehog* and $M=g\langle\sigma\rangle$ is the constituent quark mass, which is in fact the only free parameter of the model.

The energy of the soliton consists of two parts: the energy of the *continuum,* i.e., the energy corresponding to the effective action (4.1), and the energy of the *valence* level [22–27].

The effective action (4.1) can be rewritten in terms of the Euclidean spectral representation [2,3]:

$$S_{\rm eff} = -N_c T\int \frac{d\omega}{2\pi}\,\mathrm{Tr}\ln(i\omega+H) \times \left[1+\frac{1}{i\omega+H}(-i\gamma_4 A^\dagger m A + A^\dagger \dot{A})\right] , \tag{4.3}$$

where H is the Hermitian static Hamiltonian: $H=-i\gamma_4(i\gamma_i\partial_i+MU_0)$. The static soliton solutions for H reduce to the ones found in the SU(2) case and were extensively described in the literature [22–27]. Therefore we will proceed directly to the calculation of the mass splittings.

Formula (4.3) is already written in a form ready to be expanded in a power series in m and in generalized velocities $A^\dagger\dot{A}=\frac{1}{2}i\Omega_a\lambda_a$. Let us for the moment forget about the mass matrix m and expand (4.3) in powers of Ω. We get (back in the Minkowski metric):

$$L_{\rm rot} = \tfrac{1}{2}I_{ab}\Omega_a\Omega_b - (N_c/2\sqrt{3})\Omega_8 , \tag{4.4}$$

where the tensor of inertia I_{ab} is diagonal

$$I_{ab} = \frac{N_c}{4}\int\frac{d\omega}{2\pi}\,\mathrm{Tr}\left[\frac{1}{i\omega+H}\lambda_a\frac{1}{i\omega+H}\lambda_b\right] = \begin{cases} I_1\delta_{ab} & \text{for } a,b=1,\ldots,3 \\ I_2\delta_{ab} & \text{for } a,b=4,\ldots,7 \\ 0 & \text{for } a,b=8 . \end{cases} \tag{4.5}$$

For the baryon number $B=1$ sector I_{ab} splits into two parts, namely, a *valence* one,

$$I^{\rm val}_{ab} = \frac{N_c}{2}\sum_{n\neq \rm val}\frac{\langle n|\lambda_a|\mathrm{val}\rangle\langle \mathrm{val}|\lambda_b|n\rangle}{E_n-E_{\rm val}} , \tag{4.6}$$

and a sea part,

$$I^{\rm sea}_{ab} = \frac{N_c}{4}\sum_{m\neq n}\frac{\langle m|\lambda_a|n\rangle\langle n|\lambda_b|m\rangle}{E_m+E_n}\mathcal{R}_I(E_n,E_m) , \tag{4.7}$$

with the regularization function given by

$$\mathcal{R}_I(E_n,E_m) = \frac{-1}{2\sqrt{\pi}}\int_0^\infty \frac{dt}{\sqrt{t}}\phi(t)\left[E_n e^{-tE_n^2}+E_m e^{-tE_m^2} + \frac{e^{-tE_n^2}-e^{-tE_m^2}}{t(E_n-E_m)}\right] . \tag{4.8}$$

Here $|n\rangle$ and E_n are the eigenfunctions and eigenvalues of the Hamiltonian operator H.

The quantization of the rotational Lagrangian (4.4) proceeds exactly as in the case of the Skyrme [28] model.

As a result one arrives at the Hamiltonian:

$$H_{\rm rot}=\frac{S(S+1)}{2I_1}+\frac{C_2[{\rm SU}(3)]-S(S+1)-N_c^2/12}{2I_2} \tag{4.9}$$

whose eigenfunctions are given in terms of Wigner $D^{(R)}$ matrices for SU(3) representation R:

$$\begin{aligned}\psi(A)&=\sqrt{\dim(R)}D_{ab}^{(R)}(A)\\ &\equiv\sqrt{\dim(R)}\langle Y,I,I_3|D^{(R)}(A)|Y_R,S,-S_3\rangle\ .\end{aligned} \tag{4.10}$$

Here S denotes baryon spin and the right hypercharge Y_R is a subject to a constraint:

$$Y_R=\frac{N_c}{3}\ . \tag{4.11}$$

The new part consists in the expansion in powers of the rotated matrix m:

$$\begin{aligned}L_m=&-\sigma[\sqrt{6}\mu_0-\sqrt{3}(\mu_8D_{88}^{(8)}+\mu_3D_{38}^{(8)})]\\ &-2(\mu_8D_{8a}^{(8)}+\mu_3D_{3a}^{(8)})K_{ab}\Omega_b\ ,\end{aligned} \tag{4.12}$$

where constant σ

$$i\frac{N_c}{4}\int\frac{d\omega}{2\pi}\,{\rm Tr}\left[\frac{1}{i\omega+H}\gamma_4\lambda_a\right]=\begin{cases}\sqrt{6}\sigma & \text{for } a=0\\ \sqrt{3}\sigma & \text{for } a=8\\ 0 & \text{for } a=1,\dots,7\end{cases} \tag{4.13}$$

is related to the pion-nucleon sigma term $\Sigma=3/2(m_u+m_d)\sigma$ and the *anomalous* tensor K_{ab} is defined as

$$\begin{aligned}K_{ab}&=i\frac{N_c}{4}\int\frac{d\omega}{2\pi}\,{\rm Tr}\left[\frac{1}{i\omega+H}\gamma_4\lambda_a\frac{1}{i\omega+H}\lambda_b\right]\\ &=\begin{cases}K_1\delta_{ab} & \text{for } a,b=1,\dots,3\\ K_2\delta_{ab} & \text{for } a,b=4,\dots,7\\ 0 & \text{for } a,b=8\ .\end{cases}\end{aligned} \tag{4.14}$$

We call K_{ab} *anomalous* since it comes from the imaginary part of the effective action, which is related to the anomalies, and as such does not require regularization [2,3,29]. In fact, K_{ab} gets contribution almost entirely from the *valence* level. We can rewrite K_{ab} as

$$K_{ab}=i\frac{N_c}{4}\sum_{m,n}\frac{\langle m|\lambda_a|n\rangle\langle n|\gamma_4\lambda_b|m\rangle}{|E_m|+|E_n|}\times\left[\frac{1-{\rm sgn}(E_n-\mu_F)\,{\rm sgn}(E_m-\mu_F)}{2}\right]. \tag{4.15}$$

Equation (4.15) represents the full (*sea* + *valence*) contribution, provided that the chemical potential μ_F is above the energy of the *valence* level.

The quantized Hamiltonian gets two new pieces corresponding to L_m: one which shifts all masses by a constant and another one which splits the spectrum:

$$\begin{aligned}H_{\rm br}=&\sqrt{3}\left[-\sigma+\frac{K_2}{I_2}\right][\mu_8D_{88}^{(8)}(A)+\mu_3D_{38}^{(8)}(A)]\\ &+2\left[\frac{K_1}{I_1}-\frac{K_2}{I_2}\right]\sum_{a=1}^{3}[\mu_8D_{8a}^{(8)}(A)+\mu_3D_{3a}^{(8)}(A)]\mathbf{S}_a\\ &+\frac{2K_2}{I_2}\left[\mu_8\frac{\sqrt{3}}{2}\mathbf{Y}+\mu_3\mathbf{I}_3\right],\end{aligned} \tag{4.16}$$

where index 8 corresponds to $Y=0$, $I=0$, and $I_3=0$ and index 3 to $Y=0$, $I=1$, and $I_3=0$.

In the next section the expectation values of the Hamiltonian $H_{\rm br}$ are calculated. We adopt the following numerical procedure: first we find the solitonic solution for a range of constituent masses M, then we find the optimal value of M which reproduces the octet-decuplet splitting due to the rotational Hamiltonian $H_{\rm rot}$ (4.9). It turns out [2,3] that $M=390$ MeV and the corresponding moments of inertia take the following values: $I_1=1.31$ fm, $I_2=0.62$ fm, $K_1=0.45$ fm, $K_2=0.30$ fm, and $\sigma\simeq3$.

Next we calculate the mass splittings as functions of the current quark masses and compare with experiment.

V. MASS SPLITTINGS—NUMERICAL RESULTS

The expectation values of $H_{\rm br}$ between the baryonic wave functions (4.10) are easily expressed as products of two SU(3) Clebsch-Gordon (CG) coefficients [12]:

$$\int dA\, D_{\nu\nu'}^{(n)*}(A)D_{\nu_1\nu_1'}^{(n_1)}(A)D_{\nu_2\nu_2'}^{(n_2)}(A)=\frac{1}{\dim(n)}\sum_{\mu}\begin{bmatrix}n_1 & n_2 & n_\mu\\ \nu_1 & \nu_2 & \nu\end{bmatrix}\begin{bmatrix}n_1 & n_2 & n_\mu\\ \nu_1' & \nu_2' & \nu'\end{bmatrix}. \tag{5.1}$$

Matrix elements of the operator $D_{a8}^{(8)}$ are, of course, straightforward to calculate by means of formula (5.1). It turns out that diagonal matrix elements of the operator $D_{ab}^{(8)}\mathbf{S}_b$ are proportional to

$$D_{ab}^{(8)}\mathbf{S}_b=\delta\, D_{a3}^{(8)}\ , \tag{5.2}$$

where $\delta=S(S+1)/S_3$ for the octet and decuplet representations of SU(3) flavor. One can convince oneself by a direct computation that these matrix elements do not depend on S_3.

It is now easy to see that one of the CG coefficients in matrix elements in question, namely, the one corresponding to *left* indices of the wave functions coincides with the one present in Gell-Mann–Okubo mass formulas [Eqs. (2.4) and (2.5)]. In Eq. (4.16) there are two additional terms proportional to $\mathbf{Y}$ and $\mathbf{I}_3$; however, the same $\mathbf{I}_3$ term is present in the CG coefficients of Eqs. (2.4) and (2.5) and as a result the mass splittings of Eq. (2.6) are naturally reproduced. The *right* CG coefficients correspond to the reduced matrix elements which we discussed in Sec. II.

In order to calculate the reduced matrix elements let us define the following quantities:

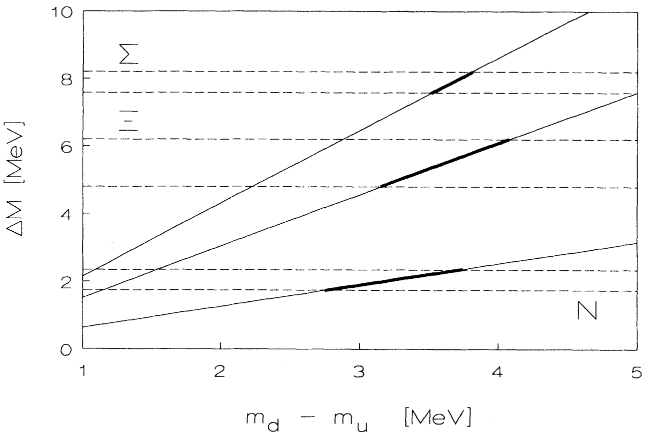

FIG. 1. Hadronic part of the octet isospin splittings: n-p (N), $\Xi^- - \Xi^0$ (Ξ), $\Sigma^- - \Sigma^+$ (Σ). Solid lines represent model predictions as functions of $m_d - m_u$ mass difference. Dashed lines correspond to the error bars of Eq. (2.3).

$$\varphi = \sigma + 2\frac{I_2}{K_2} + \frac{I_1}{K_1}, \quad \gamma = \sigma + 2\frac{I_2}{K_2} + 5\frac{I_1}{K_1} ,$$
$$\delta = \sigma + 2\frac{I_2}{K_2} - 3\frac{I_1}{K_1} . \tag{5.3}$$

Then we get

$$f = \tfrac{3}{4}\varphi(m_d - m_u), \quad d = \tfrac{3}{20}\delta(m_d - m_u) ,$$
$$c = \tfrac{3}{8}\gamma(m_d - m_u) . \tag{5.4}$$

Let us first confront the model with experimental data on hadronic part of isospin splitting in the octet. In Fig. 1 we plot the dependence of our predictions for the isospin splittings as functions of the mass difference $m_d - m_u$ for the set of parameters given at the end of Sec. IV. The dashed lines correspond to the error bars of Eq. (2.3). One can see that for $m_d - m_u = 3.43 - 3.62$ MeV all three mass differences are reproduced within the experimental errors.

Next we compare our predictions with experimental data for the reduced matrix elements both for the octet and decuplet case. In Fig. 2 we plot the splitting constants of (5.4) as functions of $m_d - m_u$ together with the error bars corresponding to (2.11). The constants f, d and c are simultaneously reproduced within experimental errors for $m_d - m_u \approx 3.5$ MeV.

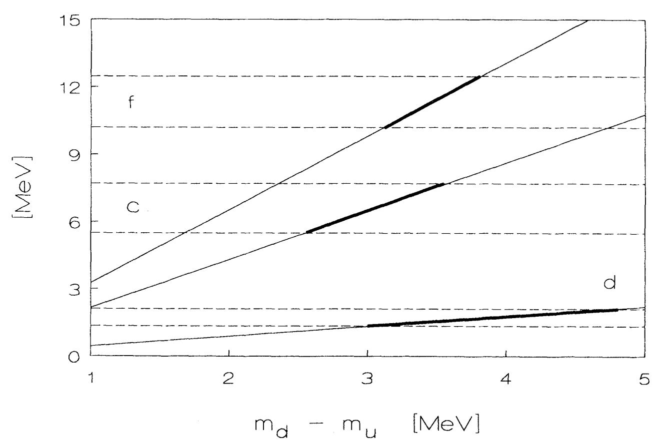

FIG. 2. Constants f, d, and c. Solid lines represent model predictions as functions of $m_d - m_u$ mass difference. Dashed lines correspond to the error bars of Eq. (2.11).

VI. CONCLUSIONS

In this paper we have calculated the isospin mass splittings within the semibosonized SU(3) Nambu–Jona-Lasinio model taking into account the linear term in the current quark mass matrix m. We have found a very good agreement with the experimental estimates for these splittings both in the octet case, where the explicit data exist, and in the decuplet case, where we have used phenomenological parametrization worked out in Sec. II. Theoretical predictions are functions of the mass difference between up and down quarks, and we have found that the required mass difference is about 3.5 MeV in striking agreement with the value needed to reproduce kaon mass splittings in the meson sector. Altogether the model offers a satisfactory and consistent description of the isospin hadronic mass differences both in the mesonic and baryonic sectors.

A short inspection of Eq. (4.16) reveals that the isospin and hyperon splittings are related. Indeed, if one parametrizes the hyperon splittings in terms of Gell-Mann–Okubo mass formula [10,11]:

$$\Delta M_B^{(8)} = -\frac{F}{2}\mathbf{Y} - \frac{D}{\sqrt{5}}\left[1 - \mathbf{I}^2 + \frac{1}{4}\mathbf{Y}^2\right] ,$$
$$\Delta M_B^{(10)} = -\frac{C}{2\sqrt{2}}\mathbf{Y} , \tag{6.1}$$

one immediately discovers that the following relation follows from Eq. (4.16):

$$\frac{f}{F}[(2.99\pm0.30)\times10^{-2}] = \sqrt{5}\frac{d}{D}[(4.90\pm2.15)\times10^{-2}]$$
$$= \sqrt{2}\frac{c}{C}[(2.25\pm0.42)\times10^{-2}] , \tag{6.2}$$

where the numbers in brackets correspond to the experimental values of Eq. (2.11) and $F=379$, $D=79\pm17$, and $C=415\pm15$ MeV. Certainly the central values are fairly scattered. We would like to offer the following explanation of this discrepancy. The isospin splittings are proportional to a tiny parameter, namely, $m_d - m_u$, and therefore the first order of the perturbation theory is legitimate. On the contrary, for the hyperon splittings which are proportional to the much larger parameter, namely, strange quark mass, one may expect some corrections from the higher orders of the perturbative expansion in m_s. Indeed, already the second order brings the splittings to their experimental values with an accuracy of a few MeV [2,3]. A fully consistent incorporation of higher-order effects would, however, require to abandon the *hedgehog* ansatz. Therefore, throughout this paper we have used the SU(3) symmetric wave functions and we have confined our calculations to the first order in m.

It is well known that the chiral models give too large values as far as the absolute masses are concerned. Let us only comment that there exist several mechanisms which may bring them down, namely, gluon corrections [30], rotational and translational band subtraction [3], and Casimir energies of quantum fluctuations [31].

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