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Tensor-meson dominance: Phenomenology of the f_2 meson

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The f_2 meson interaction is studied with the tensor-meson-dominance hypothesis of the energy-momentum tensor. A single parameter that determines the f_2 meson interactions is fixed by the decay $f_2 \to \pi\pi$. Then the hypothesis not only gives the right final photon helicities in the decay $f_2 \to \gamma\gamma$, but also reproduces its decay rate correctly. Further tests are suggested in other decay modes and two-photon production of the f_2 meson.

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I. INTRODUCTION

The vector-meson-dominance (VMD) hypothesis has been successful in low-energy phenomenology [1]. It appears that the ρ meson not only couples effectively to the isospin current, but also dominates in the isospin current at low energies. It often works with surprisingly good accuracy. One way to incorporate VMD in field theory was to postulate the field-current identity [2], which asserts that the currents be actually proportional to the vector fields themselves. In quantum chromodynamics, however, there is no doubt that the isospin current consists of quarks and that the ground-state vector mesons are the $^3S_1$ states of quark-antiquark. The other explanation was the kinematical argument which attributes the high accuracy of VDM to the *lightness* of the ρ meson. But the ρ meson is not so light in reality. Though both the field-current identity and the ρ meson pole saturation work as a computational tool, it is doubtful whether a satisfactory explanation has been really given as to why VMD works so well.

We can stretch our imagination and extend the VMD hypothesis to the tensor mesons of $J^P=2^+$, namely, the $^3P_2$ states of light quark-antiquark. We replace the currents in VMD by operators of right quantum numbers. The prime candidates of such operators are of course the energy-momentum tensor and its flavored partners. This hypothesis, called the tensor-meson-dominance (TMD) hypothesis, asserts that the f_2 meson dominates in the energy-momentum tensor $\Theta_{\mu\nu}$. Though the TMD hypothesis was studied in the past [3,4], its consequences were not fully explored. Here we will revisit the TMD hypothesis in the light of more recent theoretical and experimental progress. In Sec. II, the basic facts are discussed. Then the low-energy pion interaction of f_2 is viewed from the angle of the effective chiral Lagrangian in Sec. III. In Sec. IV, the experimental data on the decay $f_2 \to \gamma\gamma$ are analyzed by TMD. The decay rate and the photon helicities are in agreement with TMD if we apply it as a hypothesis on the effective Lagrangian. We make several suggestions on further tests of TMD in Sec. V.

II. TENSOR MESON DOMINANCE

In the language of the field-current identity, VMD asserts that the extrapolated field ϕ_μ of a vector meson is a vector current J_μ:

$$J_\mu = g_V \phi_\mu \,, \tag{2.1}$$

where g_V is the decay constant of the vector meson. In the language of dispersion theory, the absorptive part in q^2 of the matrix element

$$\langle a(p)|J_\mu(0)|b(p-q)\rangle \,, \tag{2.2}$$

is saturated by the contribution of a one-vector-meson state. A continuum contribution to the absorptive part lowers the precision of VMD. For instance, the high accuracy of the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation [5] implies that the $\pi\pi$ continuum contribution from outside the ρ mass region is only 6–7 % for the spectral function of the isospin-vector current.

We extend the VMD relation (2.1) to

$$\Theta'_{\mu\nu}(x) = g_f \phi_{\mu\nu}(x) \tag{2.3}$$

for the f_2 meson field $\phi_{\mu\nu}$. The left-hand side of Eq. (2.3) is the traceless part of the symmetrized, conserved energy-momentum tensor $\Theta_{\mu\nu}$ of quarks and gluons at the fundamental level or of *all* hadrons in phenomenological applications. The value of g_f can be determined from the decay rate for $f_2 \to \pi\pi$. For this purpose, define the matrix element of $\Theta_{\mu\nu}$ by

$$\begin{aligned}\langle \pi_a(p)\pi_b(q-p)|\Theta_{\mu\nu}(0)|0\rangle &= \delta_{ab}[(p_\mu - q_\mu/2)(p_\nu - q_\nu/2)T_1(q^2) \\ &\quad + (-g_{\mu\nu}q^2 + q_\mu q_\nu)T_2(q^2)] \,,\end{aligned} \tag{2.4}$$

where $(p\cdot q)=q^2/2$ by the pion mass-shell condition. The $f_2 \to \pi\pi$ decay amplitude is given by

$$M(f_2 \to \pi_a \pi_b) = \lim_{q^2=m_f^2} (1/g_f)(m_f^2 - q^2) \times \epsilon^{\mu\nu} \langle \pi_a(p)\pi_b(q-p)|\Theta_{\mu\nu}(0)|0\rangle \,, \tag{2.5}$$

where $\epsilon^{\mu\nu}$ is the polarization tensor of f_2 which satisfies $\epsilon_{\mu\nu}=\epsilon_{\nu\mu}$, $\epsilon^{\mu}{}_{\mu}=0$, and $q^\mu \epsilon_{\mu\nu}=0$ for $q^2=m_f^2$. Only the amplitude T_1 contributes to the f_2 meson decay. TMD implies

$$T_1(q^2) = g_{f\pi\pi}/(m_f^2 - q^2) \tag{2.6}$$

and requires that this form be valid not only near the pole at $q^2=m_f^2$ but also away from $q^2=m_f^2$, in particular, even at $q^2=0$. The residue $g_{f\pi\pi}$ is equal to the $f_2\pi\pi$ coupling constant defined by

$$L_{\rm int} = -g_{f\pi\pi}(\partial^\mu \boldsymbol{\pi}\cdot\partial^\nu \boldsymbol{\pi})\phi_{\mu\nu} \,. \tag{2.7}$$

Thanks to TMD, we can evaluate the right-hand side of Eq. (2.5) at $q^2=0$ instead of $q^2=m_f^2$. Since $T_1(0)=2$ at $q^2=0$ according to the normalization condition of $\Theta_{\mu\nu}$, $g_{f\pi\pi}$ is related to g_f by

$$g_{f\pi\pi} = m_f^2/g_f \,. \tag{2.8}$$

Substituting the observed decay width $\Gamma(f_2\to\pi^+\pi^- + \pi^0\pi^0)=156^{+3.2}_{-1.3}$ MeV [6] in the decay width formula

$$\Gamma(f_2\to\pi^+\pi^-) = (m_f/15\pi)(m_f^3/g_f)^2(|\mathbf{p}_\pi|/m_f)^5 \,, \tag{2.9}$$

we obtain the constant g_f:

$$g_f = (0.084\pm0.001)\times m_f^3 \,. \tag{2.10}$$

III. EFFECTIVE CHIRAL LAGRANGIAN

The f_2 meson interaction affects low-energy pion dynamics. One way to describe this effect is to translate it into higher derivative terms of the effective chiral Lagrangian. Since we have some knowledge of the next-to-leading-order terms of the chiral Lagrangian, we look at the TMD hypothesis from this angle.

Before discussing the f_2 meson contribution, we would like to review the ρ meson in the chiral Lagrangian. The $\rho\pi$ interaction can be described by adding to the minimal Lagrangian the term [7]

$$\begin{aligned} L_\rho &= -\mathrm{tr}(G^{(\rho)}_{\mu\nu}G^{\mu\nu(\rho)})/2 \\ &\quad + m_\rho^2 \mathrm{tr}|\rho_\mu + i(g_{\rho\pi\pi}f_\pi^2/m_\rho^2)[\partial_\mu\xi^\dagger,\xi]|^2 \,, \\ &= -\mathrm{tr}(G^{(\rho)}_{\mu\nu}G^{\mu\nu(\rho)})/2 \\ &\quad + m_\rho^2 \mathrm{tr}|\rho_\mu - i(g_{\rho\pi\pi}/m_\rho^2)[\pi,\partial_\mu\pi] + \cdots |^2 \,, \end{aligned} \tag{3.1}$$

where $\rho_\mu=\boldsymbol{\rho}_\mu\cdot\boldsymbol{\tau}/2$, $\pi=\boldsymbol{\pi}\cdot\boldsymbol{\tau}/2$, and $\xi=\sqrt{U}=\exp(i\pi/f_\pi)$ with $f_\pi=93$ MeV. Here we have used the notation $|A_\mu|^2 \equiv A_0^2 - \mathbf{A}^2$. At $E \ll m_\rho$, we can drop the first term in the leading-order approximation. When we integrate out the ρ field, the entire L_ρ is gone and the minimal Lagrangian is recovered. This means that the ρ-meson pole diagrams counterbalance the $\pi\pi$ interaction coming from the ξ fields alone of Eq. (3.1). Namely, the elastic $\pi\pi$ scattering due to the ρ exchange is given by

$$L_{4\pi}^{(\rho)} = -m_\rho^2 \mathrm{tr}|-i(g_{\rho\pi\pi}/m_\rho^2)[\pi,\partial_\mu\pi]|^2 \,. \tag{3.2}$$

On the pion mass shell, this reduces with the KSRF relation, $g_{\rho\pi\pi}^2=m_\rho^2/2f_\pi^2$, to the form

$$L_{4\pi}^{(\rho)} = (1/4f_\pi^2)(|\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}|^2 - \boldsymbol{\pi}^2|\partial_\mu\boldsymbol{\pi}|^2) \,. \tag{3.3}$$

In comparison, the 4π contribution of the minimal term $(f_\pi^2/4)\mathrm{tr}(\partial_\mu U \partial^\mu U^\dagger)$ is

$$L_{4\pi} = (1/6f_\pi^2)(|\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}|^2 - \boldsymbol{\pi}^2|\partial_\mu\boldsymbol{\pi}|^2) \,. \tag{3.4}$$

That is, if we try to fit low-energy $\pi\pi$ scattering by extrapolating the ρ meson pole diagrams alone, we would overshoot by 50% [8].

Let us turn to the f_2 meson. The f_2 meson interaction can be introduced by adding to the minimal Lagrangian the term

$$\begin{aligned} L_f &= (\partial^\kappa\phi^{\mu\nu}\partial_\kappa\phi_{\mu\nu})/2 - (m_f^2/2)\phi_{\mu\nu}\phi^{\mu\nu} \\ &\quad - (m_f^2/g_f)\Theta'^{(\pi)}_{\mu\nu}\phi^{\mu\nu} \,, \end{aligned} \tag{3.5}$$

where

$$\begin{aligned} \Theta'^{(\pi)}_{\mu\nu} &= (f_\pi^2/2)\mathrm{tr}[\{\partial_\mu\xi^\dagger,\xi\}\{\partial_\nu\xi,\xi^\dagger\} \\ &\qquad - g_{\mu\nu}\{\partial^\kappa\xi^\dagger,\xi\}\{\partial_\kappa\xi^\dagger,\xi\}/4] \\ &= (f_\pi^2/2)\mathrm{tr}[\partial_\mu U\,\partial_\nu U^\dagger - g_{\mu\nu}(\partial^\kappa U\,\partial_\kappa U^\dagger)/4] \,. \end{aligned} \tag{3.6}$$

For the $\rho\pi\pi$ interaction, L_ρ must be of the squared form since otherwise integration of the ρ field would generate a wrong term of dimension 4. One may advocate such a form by a hidden local symmetry [7]. In contrast, there is no reason for putting L_f into a squared form for the f_2 meson. At low energies ($\ll m_f$), the first term in Eq. (3.5) can be ignored. In this limit, the Euler-Lagrange equation for $\phi_{\mu\nu}$ is the TMD relation Eq. (2.3) itself. If the f_2 meson field is integrated out, the chiral Lagrangian terms of dimension 6 appear:

$$\begin{aligned} \int L_f \mathcal{D}\phi_{\mu\nu} &= (m_f^2/2g_f^2)\Theta'^{\pi}_{\mu\nu}\Theta'^{\mu\nu(\pi)} \,, \\ &= (m_f^2 f_\pi^4/8g_f^2)\mathrm{tr}(\partial_\mu U\,\partial_\nu U^\dagger)\mathrm{tr}(\partial^\mu U\,\partial^\nu U^\dagger) \\ &\quad - (m_f^2 f_\pi^4/32g_f^2)\mathrm{tr}(\partial_\mu U\,\partial^\mu U^\dagger)^2 \,. \end{aligned} \tag{3.7}$$

Numerically, the coefficient of the first term $L_2 \equiv f_\pi^4 m_f^2/8g_f^2$ is equal to

$$L_2 = 0.50\times10^{-3} \,, \tag{3.8}$$

which is about a third of the quark-loop contribution without gluon corrections [9], $L_2=N_c/192\pi^2$ with $N_c=3$. The estimate of L_2 from the low-energy d-wave $\pi\pi$ phase shifts, combined with the leading infrared singularity of the pion loops renormalized at m_η, was estimated as $L_2=(1.7\pm0.7)\times10^{-3}$ [10]. The positive sign of L_2 implies that the force is attractive in the $I=0$ d-wave channel, to be more precise, $\delta_{20}-\delta_{22}>0$. If we take this estimate of L_2, the low-energy limit of d-wave $\pi\pi$ scattering is not dominated by the s-channel f_2 meson pole. However, this does not necessarily contradict

TMD, since TMD demands f_2 dominance in the energy-momentum tensor, not in low-energy $\pi\pi$ scattering. It is not surprising at all that the f_2 meson poles fall short of accounting for the entire $|\mathrm{tr}(\partial_\mu U\,\partial_\nu U^\dagger)|^2$ term, because tails of other pion resonances in the crossed channels and iteration of the minimal Lagrangian term are certainly more important in d waves than in p waves. It is very desirable to find some principle that determines the $f_2\pi\pi$ coupling in analogy with the $\rho\pi\pi$ coupling.

IV. THE TWO-PHOTON DECAY

We proceed to analyze the decay $f_2\to\gamma\gamma$. This decay was studied through the two-photon annihilation process $\gamma^*\gamma^*\to f_2$ in e^+e^- collision. In applying the TMD hypothesis, we will relate the process $f_2\to\gamma\gamma$ to the process $f_2\to\rho\rho$ ($\omega\omega$) by VMD.

The effective Lagrangian of the f_2 interaction with a neutral vector meson ϕ_μ (ρ^0 or ω) under the TMD hypothesis is

$$L_{fVV}=-(m_f^2/g_f)\Theta'^{(V)}_{\mu\nu}\phi^{\mu\nu}\ , \tag{4.1}$$

where

$$\Theta'^{(V)}_{\mu\nu}=-G_\mu{}^{\kappa(V)}G^{(V)}_{\nu\kappa}+m_V^2\phi_\mu\phi_\nu +g_{\mu\nu}(G^{\kappa\lambda(V)}G^{(V)}_{\kappa\lambda}-m_V^2\phi^\kappa\phi_\kappa)/4\ . \tag{4.2}$$

It should be emphasized that, according to the TMD hypothesis, the constant g_f in Eq. (4.1) is the same universal constant that has also appeared in Eq. (3.5). Using L_{fVV} as an effective interaction and barring a gauge-noninvariant term, we obtain with VMD the interaction for $f_2\to\gamma\gamma$ of the form

$$L_{f\gamma\gamma}=-g_{f\gamma\gamma}\Theta^{(\gamma)}_{\mu\nu}\phi^{\mu\nu}\ , \tag{4.3}$$

where

$$\Theta^{(\gamma)}_{\mu\nu}=-F_\mu{}^\kappa F_{\nu\kappa}+g_{\mu\nu}F_{\kappa\lambda}F^{\kappa\lambda}/4\ . \tag{4.4}$$

As we can easily see by helicity counting with parity and gauge invariance, there are two independent amplitudes for $f_2\to\gamma\gamma$ in general, one for the final $\gamma\gamma$ helicity $h=\pm2$ and the other for $h=0$. The effective interaction Eq. (4.3) therefore means a specific relation between the two amplitudes. This form of the two-photon coupling of f_2 is a consequence of treating L_{fVV} as a tree-level effective interaction. We adopt this prescription of TMD and see how it fares with experiment.

It is straightforward to determine the coupling $g_{f\gamma\gamma}$ with the standard use of VMD. By use of Eq. (2.8), we find

$$g_{f\gamma\gamma}=e^2(m_f^2/g_f)[(g_\rho/m_\rho^2)^2+(g_\omega/3m_\omega^2)^2]\ , \tag{4.5}$$

where $g_{\rho,\omega}$ is defined by

$$\langle\rho(\omega)|J^\mu(0)|0\rangle=g_{\rho(\omega)}\epsilon^\mu\ , \tag{4.6}$$

where J^μ is the isospin current for ρ and the isoscalar current of the u and d quarks for ω. The value of $g_{\rho,\omega}$ can be found from the observed decay rate for $\rho(\omega)\to l^+l^-$ or from the KSRF relation, $g_\rho=2f_\pi^2g_{\rho\pi\pi}(\approx g_\omega)$.

We are able to make two tests with our TMD hypothesis, Eqs. (4.3)–(4.5). The first test is on the final photon helicities. Since only the traceless part is picked up by the f_2 polarization tensor in $L_{\rm int}$ of Eq. (4.3), the photon helicities are determined by the $F_\mu{}^\kappa F_{\nu\kappa}$ part of $\Theta^{(\gamma)}_{\mu\nu}$. In the rest frame of f_2, the terms proportional to δ_{ij} in Θ_{ij} also go away, and the $f_2\gamma\gamma$ interaction reduces to

$$L_{f\gamma\gamma}\to-g_{f\gamma\gamma}(E_iE_j+B_iB_j)\phi_{ij}\ , \tag{4.7}$$

where **E** and **B** are electric and magnetic fields of the two photons, respectively. This leads us to the ratio of the f_2 production cross sections by slightly off-shell photons,

$$\frac{\sigma[\gamma^*(\pm1)\gamma^*(\pm1)\to f_2(0)]}{\sigma[\gamma^*(\pm1)\gamma^*(\mp1)\to f_2(\pm2)]}=\frac{q_0{}^2-\mathbf{q}^2}{6(q_0{}^2+\mathbf{q}^2)}\ , \tag{4.8}$$

where the photon momenta are chosen to be $(q_0,\mathbf{q})$ and $(q_0,-\mathbf{q})$. The entries in the parentheses following γ^* denote helicities, and the entries in the parentheses following f_2 are the f_2 spins along the direction of $\mathbf{q}$. For the on-shell photons ($q_0=|\mathbf{q}|$), this ratio of cross sections is zero. Therefore,

$$\Gamma[f_2\to\gamma(\pm1)\gamma(\pm1)]/\Gamma[f_2\to\gamma(\pm1)\gamma(\mp1)]=0\ . \tag{4.9}$$

This is in agreement with what was observed in experiment. The most stringent experimental upper limit on this ratio is [11]

$$\Gamma[f_2\to\gamma(\pm1)\gamma(\pm1)]/\Gamma[f_2\to\gamma(\pm1)\gamma(\mp1)] <0.15\ (95\%\ \mathrm{CL})\ . \tag{4.10}$$

When the TMD hypothesis is extended to the other tensor mesons, the same prediction follows for photon helicities in $a_2(1320)\to\gamma\gamma$ and $f_2'(1525)\to\gamma\gamma$. Dominance of the $h=\pm2$ states for the final photons has been observed in $\gamma\gamma\to a_2\to\pi^0\eta$ [12] and $\gamma\gamma\to f_2'\to K_SK_S$ [13] as well. In the past, argument was made in favor of helicity-two-dominance with resort to the finite-energy sum rule [14] and to a superconvergence-type sum rule [15]. However, the origin of helicity-two-dominance in those arguments was mostly due to the kinematical factor of 6 that also appears in Eq. (4.8). TMD predicts helicity-two-dominance aside from this numerical factor. It is interesting to note that the naive quark model would also favor helicity-two-dominance *if* we took the *static limit* for the quarks inside f_2 [16]. They are actually highly relativistic. In our TMD argument, nowhere does the assumption of static quarks enter.

The other test of TMD is for the decay rate of $f_2\to\gamma\gamma$. With $L_{f\gamma\gamma}$ of Eq. (4.3),

$$\Gamma(f_2\to\gamma\gamma)=(20\pi/81)(e^2/4\pi)^2(m_f^3/g_f)^2(g_\rho/m_\rho^2)^4m_f\ , \tag{4.11}$$

which includes both the $\rho\rho$ and $\omega\omega$ contributions in the approximation of $m_\omega=m_\rho$ and $g_\omega=g_\rho$. In terms of the ratio $\Gamma(f_2\to\gamma\gamma)/\Gamma(f_2\to\pi^+\pi^-)$, TMD predicts

$$\Gamma(f_2\to\gamma\gamma)/\Gamma(f_2\to\pi^+\pi^-)$$
$$=(100\pi^2/27)(e^2/4\pi)^2(g_\rho/m_\rho^2)^4(m_f/|\mathbf{p}_\pi|)^5\ ,$$
$$=5.1\times10^{-5}\ . \tag{4.12}$$

According to the latest high-statistics data [17],

$$\Gamma(f_2\to\gamma\gamma)=3.15\pm0.04\pm0.39\ \text{keV}\ ,$$

which translates into

$$[\Gamma(f_2\to\gamma\gamma)/\Gamma(f_2\to\pi^+\pi^-)]_{\text{exp}}=(3.0\pm0.4)\times10^{-5}\ . \tag{4.13}$$

It may appear that agreement of TMD with experiment is not impressive. However, our TMD prediction involves the assumption of perfect VMD *twice*. If VMD is short by 10%, for instance, the right-hand side of Eq. (4.12) would be lowered by a factor of $(0.9)^4$ down to 3.3×10^{-5}, which is in line with the experimental value. Unlike the helicity property, the decay rate is more sensitive to small deviation from the perfect VMD and TMD, so closeness of the perfect TMD prediction to experiment should be taken as a positive evidence for TMD [18].

Our helicity prediction for the on-shell photon final states can be extended to the off-shell photon final states. In the rest frame of f_2, TMD gives for the matrix elements of $f_2\to\gamma^*\gamma^*$,

$$M[f_2\to\gamma^*(\pm1)\gamma^*(\mp1)]$$
$$=C(q_0q_0'+\mathbf{q}^2)/(m_\rho^2-q^2)(m_\rho^2-q'^2)\ ,$$
$$M[f_2\to\gamma^*(\pm1)\gamma^*(0)]$$
$$=(C/\sqrt{2})q_0\sqrt{-q'^2}/(m_\rho^2-q^2)(m_\rho^2-q'^2)\ , \tag{4.14}$$
$$M[f_2\to\gamma^*(\pm1)\gamma^*(\pm1)]$$
$$=(C/\sqrt{6})(q_0q_0'-\mathbf{q}^2)/(m_\rho^2-q^2)(m_\rho^2-q'^2)\ ,$$
$$M[f_2\to\gamma^*(0)\gamma^*(0)]$$
$$=\sqrt{2/3}C\sqrt{-q^2}\sqrt{-q'^2}/(m_\rho^2-q^2)(m_\rho^2-q'^2)\ ,$$

where the photon momenta are $(q_0,\mathbf{q})$ and $(q_0',-\mathbf{q})$, and $C=2(e^2g_\rho^2m_f^2/g_f)$. These matrix elements are applicable when $|q^2|/m_\rho^2$, $|q'^2|/m_\rho^2\ll1$.

V. OTHER DECAY MODES

Among other decay modes of f_2, the decay $f_2\to\rho\gamma$ is interesting from the viewpoint of TMD since the longitudinally polarized state of ρ can probe further details of the TMD hypothesis. For the decay rate, TMD gives as a ratio to $\Gamma(f_2\to\gamma\gamma)$

$$\Gamma(f_2\to\rho\gamma)/\Gamma(f_2\to\gamma\gamma)=(81/200\pi)(4\pi/e^2)(m_\rho^2/g_\rho)^2\times F(m_\rho^2/m_f^2)\ , \tag{5.1}$$

where $F(x)=(1-x)^3(1+x/2+x^2/6)$. The right-hand side of Eq. (5.1) is equal to 2×10^2, which means the branching ratio $B(f_2\to\rho\gamma)=0.6\times10^{-2}$. No experimental value has been available to date.

The decay mode $f_2\to\pi\pi\pi\pi$ has not been actively studied for years. As was pointed out by Ascoli *et al.* [19], if $f_2\to(\pi\pi)_{I=1}(\pi\pi)_{I=1}$ dominates, the decay branching ratios should be

$$\Gamma(f_2\to\pi^+\pi^-\pi^0\pi^0):\Gamma(f_2\to\pi^+\pi^-\pi^+\pi^-):\Gamma(f\to\pi^0\pi^0\pi^0\pi^0)=2:1:0\ . \tag{5.2}$$

The world averages of the currently available data for these ratios are [6]

$$2^{+0.43}_{-0.78}\ :\ 0.81\pm0.16\ :\ 0.09\pm0.03\ , \tag{5.3}$$

which are consistent with $(\pi\pi)_{I=1}$ dominance. Encouraged with this observation, we adopt a ρ-dominance model, $f_2\to\rho\rho\to\pi\pi\pi\pi$, for the 4π decay. Since at least one of the two ρ's must be off shell, we compute the decay rate with the assumption,

$$\Gamma(f_2\to\pi^+\pi^-\pi^0\pi^0)=\Gamma(f_2\to\rho^+\rho^{*-}\to\rho^+\pi^-\pi^0)+\Gamma(f\to\rho^{*+}\rho^-\to\pi^+\pi^0\rho^-)\ , \tag{5.4}$$

where ρ^* denotes an off-shell ρ. The result is

$$\Gamma(f_2\to\pi^+\pi^-\pi^0\pi^0)=(g_{\rho\pi\pi}^2/480\pi^3)(m_f^3/g_f)^2\times(m_\rho/m_f)^4m_\rho\int_a^b f(y)dy\ ,$$
$$f(y)=(y-4m_\pi^2/m_\rho^2)^{3/2}\sqrt{P(y)/y}\,[(y-1)^2+\Gamma_\rho^2/m_\rho^2]^{-1}$$
$$\times\{4(2+1/y)P(y)^2/3+10[m_f^2/m_r^2+(1-y)^2/y]\times P(y)/3+10m_f^2/m_\rho^2\}\ , \tag{5.5}$$
$$P(y)=(m_\rho^2/m_f^2)(m_f^2/m_\rho^2-y+1)^2/4-1\ ,$$

where the integral variable y is the invariant mass square of the nonresonant $\pi\pi$ pair in units of m_ρ^2 with the lower and upper limits of the integral being $a=4m_\pi^2/m_\rho^2$ and $b=(m_f/m_\rho-1)^2$. Numerically, the rate computed with Eq. (5.5) is

$$\Gamma(f_2\to\pi^+\pi^-\pi^0\pi^0)=13.8\ \text{MeV}\ , \tag{5.6}$$

as compared with

$$\Gamma(f_2 \to \pi^+\pi^-\pi^0\pi^0)_{\rm exp} = (12.7^{+2.9}_{-5.1})\ \text{MeV [6]} .$$

Though the contribution for both ρ's off shell is a little more cumbersome, a crude estimate indicates that it is much smaller than the $\rho\pi\pi$ contribution. At present, statistics is not high enough to determine from the $\pi\pi$ invariant-mass plot whether the decay actually occurs through $\rho\pi\pi$ or not. For the time being, agreement of our estimate of $\Gamma(f_2 \to \pi^+\pi^-\pi^0\pi^0)$ with experiment may be counted as another support for TMD.

VI. CONCLUDING REMARKS

f_2 meson dominance in the energy-momentum tensor seems to be a viable hypothesis in light of the current experimental data on the f_2 meson decay. At present, we are unable to clarify the role of the f_2 meson in low-energy $\pi\pi$ scattering in the context of the chiral Lagrangian. It is tempting to constrain the f_2 meson interaction by some fundamental principle, e.g., conformal invariance. When the hypothesis is extended in isospin and SU(3) flavors, we will encounter new problems. For instance, how can one explain the large decay width for $a_2 \to \rho\pi$? We will try to look at these problems in the future.

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