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Chiral Lagrangians for radiative decays of heavy hadrons

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The radiative decays of heavy mesons and heavy baryons are studied in a formalism which incorporates both heavy-quark symmetry and chiral symmetry. The chiral Lagrangians for the electromagnetic interactions of heavy hadrons consist of two pieces: one from gauging electromagnetically the strong-interaction chiral Lagrangian, and the other from the anomalous magnetic moment interactions of the heavy baryons and mesons. Because of the heavy-quark spin symmetry, the latter contains only one independent coupling constant in the meson sector and two in the baryon sector. These coupling constants only depend on the light quarks and can be calculated in the nonrelativistic quark model. However, the charmed quark is not heavy enough and the contribution from its magnetic moment must be included. Applications to the radiative decays $D^*\rightarrow D\gamma$, $B^*\rightarrow B\gamma$, $\Xi_c'\rightarrow\Xi_c\gamma$, $\Sigma_c\rightarrow\Lambda_c\gamma$, and $\Sigma_c\rightarrow\Lambda_c\pi\gamma$ are given. Together with our previous results on the strong decay rates of $D^*\rightarrow D\pi$ and $\Sigma_c\rightarrow\Lambda_c\pi$, predictions are obtained for the total widths and branching ratios of D^* and Σ_c. The decays $\Sigma_c^+\rightarrow\Lambda_c^+\pi^0\gamma$ and $\Sigma_c^0\rightarrow\Lambda_c^+\pi^-\gamma$ are discussed to illustrate the important roles played by both heavy-quark symmetry and chiral symmetry.

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I. INTRODUCTION

Mass differences are generally small among the different spin multiplets of the ground-state heavy mesons and heavy baryons which contain a heavy quark. This is a consequence of the heavy-quark symmetry [1,2] of QCD. As a result of the small available phase space, the dominant decay modes for many of these heavy particles are strong decays with one soft pion emission and/or radiative decays. Prominent examples are D^*, B^*, and Σ_c among the heavy particles already observed. As none of the absolute widths for these decays has been measured experimentally, it is important to have a single framework for treating the strong and radiative decays of these particles. It will be then possible to test the predictions on branching ratios of various decay modes with available data. An ideal theoretical framework for studying these decays is provided by the formalism recently developed to combine the heavy-quark and chiral symmetries of light quarks [3–8]. When supplemented by the nonrelativistic quark model, the formalism determines completely the low-energy dynamics of heavy hadrons. Among other things, the strong decays are treated in detail in Ref. [3]. The radiative decays are the subject of the present work.

The formalism of Ref. [3] is easily extended to incorporate the electromagnetic field. The electromagnetic interactions of heavy hadrons consist of two distinct contributions: one from gauging electromagnetically the chirally invariant strong-interaction Lagrangians for heavy mesons and baryons given in Ref. [3] and the other from the anomalous magnetic-moment couplings of the heavy particles. Heavy-quark symmetry reduces the number of free parameters needed to describe the magnetic couplings to the photon. For the ground-state mesons, there is only one undetermined parameter, and there are two for the ground-state heavy baryons. All three parameters are related simply to the magnetic moments of the light quarks in the nonrelativistic quark model. However, the charmed quark is not particularly heavy ($m_c\simeq 1.6$ GeV), and it carries a charge of $\frac{2}{3}e$. Consequently, the contribution from its magnetic moment cannot be neglected.

In the nonrelativistic quark model, all the magnetic moments of hadrons are due to those of the constituent

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quarks. Thus two of the $1/m_Q$ corrections can easily be taken into account. The first is to remove the magnetic-moment terms of the heavy hadrons arising from the minimal couplings to the electromagnetic field. The second is to include the contributions from the magnetic moment of the heavy quark.

In Secs. II and III we present for heavy mesons and heavy baryons, respectively, the details of the formalism and related considerations including the SU(3)-flavor symmetry breaking due to light-quark mass differences.

In Sec. IV we consider applications to the radiative decays of charmed mesons and charmed baryons. Some examples are $D^*\to D\gamma$, $\Xi_c'\to\Xi_c\gamma$, $\Sigma_c\to\Lambda_c\gamma$ and $\Sigma_c\to\Lambda_c\pi\gamma$. Among these, perhaps the results for the $D^*\to D\gamma$ decays are the most interesting. Experimentally, the most recent CLEO II data [9] on the branching ratios for D^{*+} and D^{*0} differ significantly from those listed by the Particle Data Group (PDG) [10]. Theoretically, when combined with our predictions for the strong decays $D^*\to D\pi$ given in Ref. [3], we are able to obtain the branching ratios for D^* decays in the same theoretical framework. Agreement is excellent between theory and experiment. This is very encouraging. Although our predicted total width for D^{*+}, $\Gamma_{\rm tot}(D^{*+})=141$ keV, is consistent with the upper limit $\Gamma_{\rm tot}(D^{*+})<131$ keV published by the Amsterdam-Bristol-CERN-Cracow-Munich-Rutherford (ACCMOR) Collaboration [11], more precision measurements of the quantity are needed.

For the radiative decays $\Sigma_c\to\Lambda_c\gamma$ and $\Xi_c'\to\Xi_c\gamma$, the two light quarks in the initial states have spin 1, while they have spin 0 in the final states. Consequently, the diquark system must undergo a spin-flip transition. The charmed quark is uninvolved in these transitions. Therefore our predictions for these decays are independent of the magnetic moment of the charmed quark.

Both chiral and heavy-quark symmetries play a critical role in radiative decays involving pions. Heavy-quark symmetry relates the strong coupling constants in the various pion emission vertices, while chiral symmetry dictates the structure of those vertices. The specific decays $\Sigma_c^0\to\Lambda_c^+\pi^-\gamma$ and $\Sigma_c^+\to\Lambda_c^+\pi^0\gamma$ are discussed in Sec. IV to expose the essential features of these processes.

II. CHIRAL LAGRANGIANS FOR ELECTROMAGNETIC INTERACTIONS OF HEAVY MESONS

To set up our notation, we denote the three light quarks by q,

$$q=\begin{pmatrix} u\\ d\\ s\end{pmatrix}, \tag{2.1}$$

and the associated charge matrix by $Q={\rm diag}(\frac{2}{3},-\frac{1}{3},-\frac{1}{3})$. The charge of the heavy quark Q is interchangeably denoted by e_Q or Q', depending on the circumstance of which one is more convenient to use. Under the electromagnetic gauge transformation for the vector potential A_μ

$$A_\mu\to A_\mu'=A_\mu-\frac{1}{e}\partial_\mu\lambda\ , \tag{2.2}$$

where λ is a U(1) gauge parameter, the quark fields transform as

$$q\to q'=e^{iQ\lambda}q,\quad Q\to Q'=e^{iQ'\lambda}Q\ . \tag{2.3}$$

Since the Goldstone-boson fields M given by

$$M=\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+\\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0\\ K^- & \bar{K}^0 & -\left(\frac{2}{3}\right)^{1/2}\eta \end{pmatrix} \tag{2.4}$$

are constructed from a light quark and an antiquark, they transform as

$$M\to M'=e^{iQ\lambda}Me^{-iQ\lambda}\ . \tag{2.5}$$

The meson field $\xi=\exp(iM/\sqrt{2}f_\pi)$, thus, has a simple gauge transformation property

$$\xi\to\xi'=e^{iQ\lambda}\xi e^{-iQ\lambda},\quad \xi^\dagger\to\xi'^\dagger=e^{iQ\lambda}\xi^\dagger e^{-iQ\lambda}\ . \tag{2.6}$$

A gauge-covariant derivative of the field ξ has the form

$$D_\mu\xi=\partial_\mu\xi+ieA_\mu[Q,\xi]\ , \tag{2.7}$$

with the gauge transformation

$$D_\mu\xi\to D_\mu'\xi'=e^{iQ\lambda}(D_\mu\xi)e^{-iQ\lambda}\ . \tag{2.8}$$

In the presence of electromagnetic interactions, the vector and axial-vector fields defined by

$$\mathcal{V}_\mu^{(0)}=\tfrac{1}{2}(\xi^\dagger\partial_\mu\xi+\xi\partial_\mu\xi^\dagger)\ , \tag{2.9a}$$

$$\mathcal{A}_\mu^{(0)}=\frac{i}{2}(\xi^\dagger\partial_\mu\xi-\xi\partial_\mu\xi^\dagger)\ , \tag{2.9b}$$

become

$$\mathcal{V}_\mu=\tfrac{1}{2}[\xi^\dagger D_\mu\xi+\xi(D_\mu\xi)^\dagger]\ , \tag{2.10a}$$

$$\mathcal{A}_\mu=\frac{i}{2}[\xi^\dagger D_\mu\xi-\xi(D_\mu\xi)^\dagger]\ , \tag{2.10b}$$

where we have used the script letters $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ to denote the chiral vector and axial-vector fields, respectively. More explicitly, $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are related to $\mathcal{V}_\mu^{(0)}$ and $\mathcal{A}_\mu^{(0)}$, respectively, by

$$\mathcal{V}_\mu=\mathcal{V}_\mu^{(0)}-ieQA_\mu+i\tfrac{1}{2}eA_\mu(\xi^\dagger Q\xi+\xi Q\xi^\dagger)\ , \tag{2.11a}$$

$$\mathcal{A}_\mu=\mathcal{A}_\mu^{(0)}-\tfrac{1}{2}eA_\mu(\xi^\dagger Q\xi-\xi Q\xi^\dagger)\ , \tag{2.11b}$$

$$\mathcal{V}_\mu^*=\mathcal{V}_\mu^{(0)*}+ieQA_\mu-i\tfrac{1}{2}eA_\mu(\xi^T Q\xi^*+\xi^* Q\xi^T)\ , \tag{2.11c}$$

$$\mathcal{A}_\mu^*=\mathcal{A}_\mu^{(0)*}-\tfrac{1}{2}eA_\mu(\xi^T Q\xi^*-\xi^* Q\xi^T)\ , \tag{2.11d}$$

where we have given $\mathcal{V}_\mu^*$ and $\mathcal{A}_\mu^*$ since they appear in the following discussion. The complex conjugate is related to operation of Hermitian conjugation and transposition, for example, $\mathcal{V}_\mu^*=(\mathcal{V}_\mu^\dagger)^T$.

We next turn to the gauge transformation properties of heavy mesons. Following the notation of Ref. [3], the ground-state 1^- and 0^- heavy mesons are denoted by P^* and P, respectively. Since a heavy meson contains a heavy quark Q and a light antiquark $\bar{q}$, it obeys the gauge transformation law

$$P \to P' = e^{iQ'\lambda} P e^{-iQ\lambda} , \tag{2.12}$$

and a similar equation for the vector meson P^*. An electromagnetic gauge-covariant derivative can then be constructed to be

$$D_\mu P = \partial_\mu P + ieA_\mu (Q'P - PQ) , \tag{2.13}$$

which transforms as

$$D_\mu P \to D'_\mu P' = e^{iQ'\lambda}(D_\mu P) e^{-iQ\lambda} . \tag{2.14}$$

When the chiral field is included, the covariant derivative finally reads (see Ref. [3])

$$\begin{aligned} D_\mu P &= \partial_\mu P + \mathcal{V}^*_\mu P + ieA_\mu (Q'P - PQ) \\ &= \partial_\mu P + \mathcal{V}^{(0)*}_\mu P + ieQ' A_\mu P \\ &\quad - i\tfrac{1}{2} e A_\mu (\xi^T Q \xi^* + \xi^* Q \xi^T) P , \end{aligned} \tag{2.15a}$$

where use of Eq. (2.11c) has been made. Similarly,

$$\begin{aligned} D_\mu P^\dagger &= \partial_\mu P^\dagger + \mathcal{V}_\mu P^\dagger - ieA_\mu (P^\dagger Q' - QP^\dagger) \\ &= \partial_\mu P^\dagger + \mathcal{V}^{(0)}_\mu P^\dagger - ieQ' A_\mu P^\dagger \\ &\quad + \frac{i}{2} e A_\mu (\xi^\dagger Q \xi + \xi Q \xi^\dagger) P^\dagger . \end{aligned} \tag{2.15b}$$

Equation (2.15) shows that the electromagnetic interactions break the SU(3)-flavor symmetry. The charge operator Q has an equal mixture of $\mathbf{8}_L$ and $\mathbf{8}_R$ as it should be since the electromagnetic interactions conserve parity. The construction of the electromagnetic gauge-invariant chiral Lagrangian for heavy mesons simply follows from gauging the chiral-invariant meson Lagrangian presented in Ref. [3]. The relevant terms are

$$\begin{aligned} \mathcal{L}^{(1)}_{PP^*} &= D_\mu P D^\mu P^\dagger - M_P^2 P P^\dagger \\ &\quad + f\sqrt{M_P M_{P^*}} (P \mathcal{A}^\mu P^{*\dagger}_\mu + P^*_\mu \mathcal{A}^\mu P^\dagger) \\ &\quad - \tfrac{1}{2} P^{*\mu\nu} P^{*\dagger}_{\mu\nu} + M^2_{P^*} P^{*\mu} P^{*\dagger}_\mu \\ &\quad + \tfrac{1}{4} f \epsilon_{\mu\nu\lambda\kappa} (P^{*\mu\nu} \mathcal{A}^\lambda P^{*\kappa\dagger} + P^{*\kappa} \mathcal{A}^\lambda P^{*\mu\nu\dagger}) , \end{aligned} \tag{2.16}$$

where Eqs. (2.11) and (2.15) have been used,

$$P^{*\dagger}_{\mu\nu} = D_\mu P^{*\dagger}_\nu - D_\nu P^{*\dagger}_\mu , \tag{2.17}$$

and $D_\mu P^{*\dagger}_\nu$ is given by Eq. (2.15b) with $P^\dagger$ replaced by $P^{*\dagger}_\nu$. The universal coupling constant f is independent of heavy-quark masses and species. By expanding the meson-field matrix ξ into a power series,

$$\xi = 1 + i\frac{M}{\sqrt{2} f_\pi} - \frac{M^2}{4f_\pi^2} + \cdots , \tag{2.18}$$

it is evident that $\mathcal{V}_\mu$ ($\mathcal{A}_\mu$) contains only an even (odd) number of pions interacting electromagnetically. Consequently, the kinematic terms in (2.16) give rise to contact terms with one photon and even-number pion emissions, while the interacting terms yield electromagnetic contact terms with odd-number Goldstone-boson emission.

Note that the radiative transition $P^* \to P\gamma$ cannot arise from the electromagnetic Lagrangian (2.16). The lowest-order gauge- and chiral-invariant interaction that contributes to $P^* \to P\gamma$ is

$$\begin{aligned} \mathcal{L}^{(2)}_{PP^*} &= \sqrt{M_P M_{P^*}} \epsilon_{\mu\nu\alpha\beta} v^\alpha P^{*\beta} \\ &\quad \times [\tfrac{1}{2} d(\xi^\dagger Q \xi + \xi Q \xi^\dagger) + d' Q'] F^{\mu\nu} P^\dagger + \text{H.c.} \\ &\quad - ieF_{\mu\nu} P^{*\nu} [Q' - \tfrac{1}{2}(\xi^\dagger Q \xi + \xi Q \xi^\dagger)] P^{*\mu\dagger} \\ &\quad + id'' M_{P^*} F_{\mu\nu} P^{*\nu} [\gamma Q' - \tfrac{1}{2}(\xi^\dagger Q \xi + \xi Q \xi^\dagger)] P^{*\mu\dagger} . \end{aligned} \tag{2.19}$$

In (2.19), v^α is the four-velocity of the 1^- heavy meson and the second term is to remove the magnetic moment coupled to the electromagnetic field implied by the minimal couplings in (2.16), while the last term proportional to d'' is to account for the magnetic-moment couplings due to the constituent quarks, both light and heavy. The universal coupling constant d is independent of the heavy-quark masses and species. We have also included the d' and γ terms to account for the corrections due to the heavy-quark masses when $m_Q \neq \infty$.

The Lagrangian (2.19) describe the magnetic transitions $P^* \to P\gamma$ and $P^* \to P^*\gamma$. In the infinitely heavy-quark mass limit, only the two parameters d and d'' in Eq. (2.19) survive. The heavy-quark spin symmetry then relates them. To derive the relation, we will make use of the interpolating fields introducing in Ref. [3]:

$$\begin{aligned} P(v) &= \bar{q}_v \gamma_5 h_v \sqrt{M_P} , \\ P^*(v,\epsilon) &= \bar{q}_v \not{\epsilon} h_v \sqrt{M_{P^*}} , \end{aligned} \tag{2.20}$$

where $\bar{q}_v$ is a light antiquark which combines with a heavy quark h_v of velocity v to form the appropriate meson. Now let J_μ and j_μ be the electromagnetic currents of the heavy quark and light quarks, respectively. It is easy to show that J_μ does not contribute to the magnetic transitions of interest here. Consider

$$\begin{aligned} &\langle P(v') | J_\mu | P^*(v,\epsilon) \rangle \\ &= \sqrt{M_P M_{P^*}} \langle 0 | \bar{q}_{v'} \gamma_5 h_{v'} \bar{h}_{v'} \gamma_\mu h_v \bar{h}_v \not{\epsilon} q_v | 0 \rangle \\ &= -\sqrt{M_P M_{P^*}} \,\mathrm{tr} \left\{ \gamma_5 \frac{\not{v}' + 1}{2} \gamma_\mu \frac{\not{v} + 1}{2} \not{\epsilon} \langle 0 | q_v \bar{q}_{v'} | 0 \rangle \right\} . \end{aligned} \tag{2.21}$$

Sandwiched between the projection matrices, the matrix γ_μ can be replaced by

$$\gamma_\mu = \frac{1}{2}(v_\mu + v'_\mu) - \frac{i}{2m_Q} \sigma_{\mu\nu} k^\nu , \tag{2.22}$$

which shows that in the limit $m_Q \to \infty$, the heavy quark's electromagnetic current does not induce a magnetic coupling. We also note that the heavy-quark current is conserved by itself, and so the light-quark current must be separately conserved. We are now ready to examine the electromagnetic vertices associated with the light-quark current. We have

$$\langle P(v')|j_\mu|P^*(v,\epsilon)\rangle=-\sqrt{M_P M_{P^*}}\,\mathrm{tr}\left\{\gamma_5\frac{\not v+1}{2}\not\epsilon L_\mu\right\}, \quad (2.23)$$

where

$$L_\mu=\langle 0|q_v j_\mu \bar q_{v'}|0\rangle . \quad (2.24)$$

Lorentz covariance implies

$$L_\mu=c_1(v+v')_\mu+c_2\gamma_\mu+c_3\sigma_{\mu\nu}k^\nu . \quad (2.25)$$

Taking the trace, we find

$$\langle P(v')|j_\mu|P^*(v,\epsilon)\rangle=-2c_3\sqrt{M_P M_{P^*}}\epsilon_{\mu\nu\alpha\beta}k^\nu v^\alpha \epsilon^\beta . \quad (2.26)$$

Similarly, we have

$$\langle P^*(v',\epsilon_f)|j_\mu|P^*(v,\epsilon_i)\rangle=-M_{P^*}\mathrm{tr}\left\{\not\epsilon_f\frac{\not v+1}{2}\not\epsilon_i L_\mu\right\} . \quad (2.27)$$

In taking the trace, the c_2 term does not contribute as a result of $\epsilon_i\cdot v=0$ and $\epsilon_f\cdot v=k/M_{P^*}\approx 0$, and while the c_1 term contributes, it is not of the magnetic type. Thus

$$\langle P^*(v',\epsilon_f)|j_\mu|P^*(v,\epsilon_i)\rangle_m =2ic_3M_{P^*}(\epsilon_f\cdot k\epsilon_{i\mu}-\epsilon_i\cdot k\epsilon_{f\mu}) , \quad (2.28)$$

where the subscript m is a remainder that we keep only the part dependent on the magnetic moment. By comparison with the matrix elements implied by Eq. (2.19) for $P^*\to P\gamma$ and $P^*\to P^*\gamma$, we find

$$d=-ic_3 ,\qquad d''=2ic_3 . \quad (2.29)$$

The relation we are looking for is

$$d''=-2d . \quad (2.30)$$

The SU(3)-breaking effects due to the light-quark mass differences can be incorporated in the Lagrangian (2.19) by replacing the charge matrix Q by (see also Sec. III)

$$Q\to\tilde Q=\begin{bmatrix}\frac{2}{3} & 0 & 0\\ 0 & -\frac{\alpha}{3} & 0\\ 0 & 0 & -\frac{\beta}{3}\end{bmatrix}, \quad (2.31)$$

where $\alpha=m_u/m_d$ and $\beta=m_u/m_s$.

We now show that the nonrelativistic quark model has a simple prediction for the couplings d, d', d'', and γ. The magnetic interaction of the quarks is

$$\mathcal{L}_{\mathrm{em}}=\sum_i e\frac{e_i}{2m_i}\bar\psi\sigma_i\psi\cdot\mathbf{H}=\bar\psi\left[\sum_i\mu_i\sigma_i\right]\psi\cdot\mathbf{H} , \quad (2.32)$$

where e_i is the charge of the ith quark in units of e. Instead of using the Lorentz-invariant normalization

$$\langle P|P'\rangle=2E(2\pi)^3\delta^3(P-P') , \quad (2.33)$$

it is more convenient to use a discrete normalization by enclosing the system in a large volume V, so that

$$\langle\langle P|P'\rangle\rangle=\delta_{\mathbf{PP'}} . \quad (2.34)$$

Then in the rest frame we get

$$\langle P|\mathcal{L}_{\mathrm{em}}|P^*\rangle =2\sqrt{M_PM_{P^*}}\left\langle\left\langle P\left|\sum_q\mu_q\sigma_q^z-\sum_{\bar q}\mu_q\sigma_{\bar q}^z\right|P^*\right\rangle\right\rangle H , \quad (2.35)$$

where we have chosen the magnetic field along the z direction and the minus sign for the antiquarks can be understood simply as having charges opposite to those of quarks. Next, we need the flavor-spin wave functions of the heavy mesons in the nonrelativistic quark model:

$$|P^*\rangle\rangle=\frac{1}{\sqrt2}|Q\uparrow\bar q\downarrow+Q\downarrow\bar q\uparrow\rangle ,\qquad |P\rangle\rangle=\frac{1}{\sqrt2}|Q\uparrow\bar q\downarrow-Q\downarrow\bar q\uparrow\rangle , \quad (2.36)$$

where $|P^*\rangle\rangle$ denotes the vector-meson state with the z component of its spin being zero. Let us denote the SU(3) P_i as

$$P_i=(Q\bar u,Q\bar d,Q\bar s)=(P^{-1/2},P^{1/2},P^0) , \quad (2.37)$$

where the superscript indicates the isospin quantum number I_3. We then have

$$\begin{aligned}\langle P^{1/2}|\mathcal{L}_{\mathrm{em}}|P^{*1/2}\rangle&=2\sqrt{M_PM_{P^*}}(\mu_d+\mu_Q) ,\\ \langle P^{-1/2}|\mathcal{L}_{\mathrm{em}}|P^{*-1/2}\rangle&=2\sqrt{M_PM_{P^*}}(\mu_u+\mu_Q) ,\\ \langle P^0|\mathcal{L}_{\mathrm{em}}|P^{*0}\rangle&=2\sqrt{M_PM_{P^*}}(\mu_s+\mu_Q) ,\end{aligned} \quad (2.38)$$

where we have dropped the magnetic field for convenience.

Note that in the rest frame of P^*, $v^\alpha=(1,\mathbf{0})$, so that

$$\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}v^\alpha\epsilon^{*\beta}=\epsilon_{ijk}F^{ij}\epsilon^{*k}=-2\boldsymbol{\epsilon}^*\cdot\mathbf{H} . \quad (2.39)$$

Choosing the $\mathbf{H}$ field along the z direction as before, we find from (2.19), (2.31), and (2.39) that

$$\begin{aligned}\langle P^{1/2}|\mathcal{L}^{(2)}_{PP^*}|P^{*1/2}\rangle&=-2\sqrt{M_PM_{P^*}}\left[-\frac{\alpha}{3}d+e_Qd'\right] ,\\ \langle P^{-1/2}|\mathcal{L}^{(2)}_{PP^*}|P^{*-1/2}\rangle&=-2\sqrt{M_PM_{P^*}}(\tfrac{2}{3}d+e_Qd') ,\\ \langle P^0|\mathcal{L}^{(2)}_{PP^*}|P^{*(0)}\rangle&=-2\sqrt{M_PM_{P^*}}\left[-\frac{\beta}{3}d+e_Qd'\right] .\end{aligned} \quad (2.40)$$

Comparing this with Eq. (2.38) gives the desired results

$$d=-\frac{e}{2m_u},\quad d'=-\frac{e}{2m_Q} . \quad (2.41)$$

A similar calculation gives

$$d''=\frac{e}{m_u}, \quad \gamma=\frac{m_u}{m_Q} . \qquad (2.42)$$

The quark-model results (2.41) and (2.42) satisfy the heavy-quark symmetry relation (2.30). This is not surprising, as SU(3) breakings preserve heavy-quark symmetry.

III. CHIRAL LAGRANGIANS FOR ELECTROMAGNETIC INTERACTIONS OF HEAVY BARYONS

We consider a heavy baryon containing a heavy quark and two light quarks. The two light quarks form either a symmetric sextet **6** or an antisymmetric antitriplet $\bar{\mathbf{3}}$ in flavor SU(3) space. We will denote these spin-$\frac{1}{2}$ baryons as B_6 and $B_{\bar{3}}$, respectively, and the spin-$\frac{3}{2}$ baryon by B_6^*. Explicitly, the baryon matrices read as in Ref. [3]:

$$B_6=\begin{bmatrix} \Sigma_Q^{+1} & \frac{1}{\sqrt{2}}\Sigma_Q^0 & \frac{1}{\sqrt{2}}\Xi_Q'^{+1/2} \\ \frac{1}{\sqrt{2}}\Sigma_Q^0 & \Sigma_Q^{-1} & \frac{1}{\sqrt{2}}\Xi_Q'^{-1/2} \\ \frac{1}{\sqrt{2}}\Xi_Q'^{+1/2} & \frac{1}{\sqrt{2}}\Xi_Q'^{-1/2} & \Omega_Q \end{bmatrix}, \qquad (3.1)$$

$$B_{\bar{3}}=\begin{bmatrix} 0 & \Lambda_Q & \Xi_Q^{+1/2} \\ -\Lambda_Q & 0 & \Xi_Q^{-1/2} \\ -\Xi_Q^{+1/2} & -\Xi_Q^{-1/2} & 0 \end{bmatrix}, \qquad (3.2)$$

and a matrix for B_6^* similar to B_6. The superscript in (3.1) and (3.2) refers to the value of the isospin quantum number I_3.

Under the electromagnetic gauge transformation Eq. (2.2), the heavy-baryon field transforms as

$$B \to B'=e^{iQ'\lambda}e^{iQ\lambda}Be^{iQ\lambda} . \qquad (3.3)$$

It is then easily shown that the electromagnetic gauge-covariant derivative has the form

$$D_\mu B=(\partial_\mu+ieQ'A_\mu)B+ieA_\mu\{Q,B\} , \qquad (3.4)$$

which transforms according to

$$D_\mu B \to D'_\mu B'=e^{iQ'\lambda}e^{iQ\lambda}(D_\mu B)e^{iQ\lambda} . \qquad (3.5)$$

With the chiral fields included, the covariant derivative is modified to (see Ref. [3])

$$D_\mu B=\partial_\mu B+\mathcal{V}_\mu B+B\mathcal{V}_\mu^T+ieQ'A_\mu B+ieA_\mu\{Q,B\} . \qquad (3.6)$$

It follows from Eq. (2.11a) that

$$D_\mu B=\partial_\mu B+\mathcal{V}_\mu^{(0)}B+B\mathcal{V}_\mu^{(0)T}+ieQ'A_\mu B +i\tfrac{1}{2}eA_\mu[(\xi^\dagger Q\xi+\xi Q\xi^\dagger)B+B(\xi^\dagger Q\xi+\xi Q\xi^\dagger)^T] . \qquad (3.7)$$

As in the meson case discussed in the previous section, a chiral and electromagnetic gauge-invariant Lagrangian for heavy baryons can be obtained by gauging the chiral Lagrangian (3.12) given in Ref. [3]. We write down the relevant terms

$$\begin{aligned}\mathcal{L}_B^{(1)}=&\tfrac{1}{2}\mathrm{tr}[\bar{B}_{\bar{3}}(i\not{D}-M_{\bar{3}})B_{\bar{3}}]+\mathrm{tr}[\bar{B}_6(i\not{D}-M_6)B_6]\\ &+\mathrm{tr}\{\bar{B}_6^{*\mu}[-g_{\mu\nu}(i\not{D}-M_{6^*})+i(\gamma_\mu D_\nu+\gamma_\nu D_\mu)-\gamma_\mu(i\not{D}+M_{6^*})\gamma_\nu]B_6^{*\nu}\}\\ &+g_1\mathrm{tr}(\bar{B}_6\gamma_\mu\gamma_5\mathcal{A}^\mu B_6)+g_2\mathrm{tr}(\bar{B}_6\gamma_\mu\gamma_5\mathcal{A}^\mu B_{\bar{3}})+\mathrm{H.c.}\\ &+\frac{\sqrt{3}}{2}g_1\mathrm{tr}(\bar{B}_{6\mu}^*\mathcal{A}^\mu B_6)+\mathrm{H.c.}-\sqrt{3}g_2\mathrm{tr}(\bar{B}_6^{*\mu}\mathcal{A}_\mu B_{\bar{3}})+\mathrm{H.c.}-\tfrac{3}{2}g_1\mathrm{tr}(\bar{B}_6^{*\nu}\gamma_\mu\gamma_5\mathcal{A}^\mu B_{6\nu}^*) ,\end{aligned} \qquad (3.8)$$

with $D_\mu B$ and $\mathcal{A}_\mu$ given by (3.7) and (2.11), respectively, where $B_{6\mu}^*$ is a Rarita-Schwinger vector-spinor field for a spin-$\frac{3}{2}$ particle, and use of heavy-quark symmetry has been applied to relate various coupling constants. As in the case of heavy mesons, electromagnetic contact terms with an even (odd) number of pions come from kinematic (interacting) terms in (3.8).

Since baryons do not behave much like Dirac point particles, they can have large anomalous magnetic moments. Apart from the nonanomalous electromagnetic interaction described by $\mathcal{L}_B^{(1)}$, the most general gauge invariant Lagrangian for anomalous magnetic transitions of heavy baryons is given by (we use the abbreviation $\sigma\cdot F\equiv\sigma_{\mu\nu}F^{\mu\nu}$)

$$\begin{aligned}\mathcal{L}_B^{(2)}=&a_1\mathrm{tr}(\bar{B}_6Q\sigma\cdot FB_6)+a_1'\mathrm{tr}(\bar{B}_6Q'\sigma\cdot FB_6)+a_2\mathrm{tr}(\bar{B}_6Q\sigma\cdot FB_{\bar{3}})+\mathrm{H.c.}+a_2'\mathrm{tr}(\bar{B}_6Q'\sigma\cdot FB_{\bar{3}})+\mathrm{H.c.}\\ &+a_3\mathrm{tr}(\epsilon_{\mu\nu\lambda\kappa}\bar{B}_6^{*\mu}Q\gamma^\nu F^{\lambda\kappa}B_6)+\mathrm{H.c.}+a_3'\mathrm{tr}(\epsilon_{\mu\nu\lambda\kappa}\bar{B}_6^{*\mu}Q'\gamma^\nu F^{\lambda\kappa}B_6)+\mathrm{H.c.}\\ &+a_4\mathrm{tr}(\epsilon_{\mu\nu\lambda\kappa}\bar{B}_6^{*\mu}Q\gamma^\nu F^{\lambda\kappa}B_{\bar{3}})+\mathrm{H.c.}+a_4'\mathrm{tr}(\epsilon_{\mu\nu\lambda\kappa}\bar{B}_6^{*\mu}Q'\gamma^\nu F^{\lambda\kappa}B_{\bar{3}})+\mathrm{H.c.}\\ &+a_5\mathrm{tr}(\bar{B}_6^{*\mu}Q\sigma\cdot FB_{6\mu}^*)+a_5'\mathrm{tr}(\bar{B}_6^{*\mu}Q'\sigma\cdot FB_{6\mu}^*)+a_6\mathrm{tr}(\bar{B}_{\bar{3}}Q\sigma\cdot FB_{\bar{3}})+a_6'\mathrm{tr}(\bar{B}_{\bar{3}}Q'\sigma\cdot FB_{\bar{3}})\\ &+\tfrac{1}{4}\mu_{B_{\bar{3}}}\mathrm{tr}(\bar{B}_{\bar{3}}Q_{\mathrm{tot}}\sigma\cdot FB_{\bar{3}})+\tfrac{1}{2}\mu_{B_6}\mathrm{tr}(\bar{B}_6Q_{\mathrm{tot}}\sigma\cdot FB_6)-\tfrac{1}{2}\mu_{B_6^*}\mathrm{tr}(\bar{B}_6^{*\mu}Q_{\mathrm{tot}}\sigma\cdot FB_{6\mu}^*) ,\end{aligned} \qquad (3.9)$$

where $Q_{\rm tot}=2Q+Q'$ and $\mu_B=e/2M_B$, recalling that Q is the charge matrix of light quarks, whereas Q' (or e_Q) is the charge of the heavy quark. The Lagrangian $\mathcal{L}_B^{(2)}$ is also the most general chiral-invariant one provided that one makes the replacement

$$Q\to\tfrac{1}{2}(\xi^\dagger Q\xi+\xi Q\xi^\dagger),\quad Q'\to Q' . \tag{3.10}$$

Note that we have subtracted out the Dirac magnetic moments of heavy baryons μ_B, so that in the quark model the coefficients a_i are simply related to the Dirac magnetic moments of the light quarks, while a_i' are connected to those of the heavy quarks. In the heavy-quark limit, both Dirac magnetic moments μ_B and the heavy-quark magnetic moments a_i' vanish as they are suppressed by the heavy-quark mass.

At first sight, it appears that other gauge invariants, e.g., $\bar B_6^{*\mu}F_{\mu\nu}\gamma^\nu\gamma_5B_6$ and $\bar B_6^{*\mu}F_{\mu\nu}B_6^{*\nu}$, can be added to $\mathcal{L}_B^{(2)}$. However, by applying the identity

$$i\epsilon^{\mu\nu\lambda\kappa}\gamma_\kappa=-\gamma^\mu\gamma^\nu\gamma^\lambda\gamma_5+g^{\mu\nu}\gamma^\lambda\gamma_5-g^{\mu\lambda}\gamma^\nu\gamma_5+g^{\nu\lambda}\gamma^\mu\gamma_5 , \tag{3.11}$$

we see that

$${\rm tr}(\bar B_6^{*\mu}F_{\mu\nu}\gamma^\nu\gamma_5B)=\frac{i}{2}{\rm tr}(\epsilon_{\mu\nu\lambda\kappa}\bar B_6^{*\mu}\gamma^\nu F^{\lambda\kappa}B) , \tag{3.12}$$

for $B=B_{\bar 3}$ or B_6. Next, using the fact that the Rarita-Schwinger vector spinor u_λ obeys the relations

$$u_\mu=i\sigma_{\mu\nu}u^\nu,\quad \bar u_\mu=i\bar u^{\,\nu}\sigma_{\nu\mu} , \tag{3.13}$$

it is straightforward to show that

$$\begin{aligned}\bar u^{\,\mu}F_{\mu\nu}u^\nu&=i^2\bar u^{\,\lambda}\sigma_\lambda{}^\mu F_{\mu\nu}\sigma^\nu{}_\kappa u^\kappa\\&=\bar u^{\,\mu}F_{\mu\nu}u^\nu+2\bar u^{\,\nu}F_{\mu\nu}u^\mu+i\bar u^{\,\lambda}\epsilon_{\lambda\mu\nu\kappa}\gamma_5F^{\mu\nu}u^\kappa\\&\quad-i\bar u^{\,\lambda}\sigma_{\mu\nu}F^{\mu\nu}g_{\lambda\kappa}u^\kappa\end{aligned} \tag{3.14}$$

and, hence,

$$\begin{aligned}\bar u^{\,\lambda}(v')\sigma\cdot Fu_\lambda(v)&=2i\bar u^{\,\mu}(v')F_{\mu\nu}u^\nu(v)\\&\quad+\bar u^{\,\lambda}(v')\epsilon_{\lambda\mu\nu\kappa}\gamma_5F^{\mu\nu}u^\kappa(v) .\end{aligned} \tag{3.15}$$

In the heavy-quark limit and $v'\sim v$, $\bar u^{\,\lambda}(v)\gamma_5u^\kappa(v)=0$. Therefore there are only six independent couplings in the heavy-quark limit for anomalous magnetic moment radiative baryonic transitions.

We shall see that the heavy-quark spin symmetry reduces the six couplings a_i to two independent ones. To embark on this task, we will apply the interpolating fields for the heavy baryons in terms of the diquark fields of the light quarks (see Ref. [3]):

$$B_{\bar 3}(v,s)=\bar u(v,s)\phi_vh_v , \tag{3.16}$$

$$B_6(v,s,\kappa)=\bar B_\mu(v,s,\kappa)\phi_v^\mu h_v , \tag{3.17}$$

where ϕ_v and ϕ_v^μ are the 0^+ and 1^+ diquarks, respectively, which combine with the heavy quark h_v of velocity v to form the appropriate heavy baryon. The argument κ indicates the spin of the baryon: $\kappa=1$ for spin-$\frac{1}{2}$ baryons (B_6) and $\kappa=2$ for spin-$\frac{3}{2}$ baryons (B_6^*). The wave function $\bar B_\mu$ is given by

$$\bar B_\mu(v,s,\kappa=1)=\frac{1}{\sqrt3}\bar u(v,s)\gamma_5(v_\mu+\gamma_\mu) , \tag{3.18a}$$

$$\bar B_\mu(v,s,\kappa=2)=\bar u_\mu(v,s) . \tag{3.18b}$$

We shall now apply heavy-quark symmetry to the magnetic-moment coupling constants a_i. As in the meson case, let us denote the electromagnetic current of light and heavy quarks by j_μ and J_μ, respectively. For the couplings $a_1,\dots,a_6$, we do not have to consider the heavy-quark current. For example, it is easily shown that

$$\langle B_{\bar 3}(v',s')|J_\mu|B_{\bar 3}(v,s)\rangle=e_Q\zeta(v\cdot v')\bar u(v',s')\gamma_\mu u(v,s) , \tag{3.19}$$

where $\zeta(v\cdot v')=\langle 0|\phi_{v'}\phi_v^\dagger|0\rangle$ is a universal Isgur-Wise function. Equation (2.22) is applicable here, and it shows that the heavy-quark electromagnetic current does not induce a magnetic-type coupling in the heavy-quark limit. As in the meson case, the heavy-quark current is conserved by itself, and so the light-quark current must be separately conserved. We next note that $a_6=0$ because the spin of the heavy quark cannot be flipped by a photon emission and because the radiative transition $0^+\to0^++\gamma$ in the diquark sector is prohibited by conservation of angular momentum. Indeed, the interpolating-field method gives

$$\begin{aligned}\langle B_{\bar 3}(v',s')|j_\mu|B_{\bar 3}(v,s)\rangle&\\&=\langle 0|\bar u(v',s')\phi_{v'}h_{v'}j_\mu\bar h_v\phi_v^\dagger u(v,s)|0\rangle\\&=\bar u(v',s')M_\mu u(v,s) ,\end{aligned} \tag{3.20}$$

with

$$M_\mu=\langle 0|\phi_{v'}j_\mu\phi_v^\dagger|0\rangle . \tag{3.21}$$

Now Lorentz invariance implies that

$$M_\mu=a(v+v')_\mu+bk_\mu . \tag{3.22}$$

Since $k^\mu(v+v')_\mu=0$, it is clear that conservation of the electromagnetic current implies $b=0$. Consequently,

$$\langle B_{\bar 3}(v',s')|j_\mu|B_{\bar 3}(v,s)\rangle=a\bar u(v',s')(v+v')_\mu u(v,s) , \tag{3.23}$$

which is nothing but the usual convection current due to the charge. We thus conclude that

$$a_6=0 , \tag{3.24}$$

in the heavy-quark limit.

We now turn to the matrix element of the B_6-B_6 transition. We have

$$\langle B_6(v',s')|j_\mu|B_6(v,s)\rangle=\bar B^{\,\alpha}(v',s')M_{\mu\alpha\beta}B^\beta(v,s) , \tag{3.25}$$

where

$$M^{\mu\alpha\beta}=\langle 0|\phi_{v'}^\alpha j^\mu\phi_v^{\beta\dagger}|0\rangle . \tag{3.26}$$

The general expression of $M_{\mu\alpha\beta}$ linear in k is

$$M_{\mu\alpha\beta}=f_1 g_{\alpha\beta}(v+v')_\mu+f_2 g_{\alpha\beta}k_\mu+f_3 g_{\mu\alpha}k_\beta+f_4 g_{\mu\beta}k_\alpha+f_5 v_\alpha v'_\beta(v+v')_\mu\ , \tag{3.27}$$

where we do not display form factors proportional to v'_α or v_β because of $\bar{B}_\alpha v'^\alpha=0$ and $v^\beta B_\beta=0$. Conservation of the electromagnetic vector current then indicates that

$$f_2=0,\quad f_3+f_4=0\ , \tag{3.28}$$

and hence

$$M_{\mu\alpha\beta}=f_1 g_{\alpha\beta}(v+v')_\mu+f_3(g_{\mu\alpha}k_\beta-g_{\mu\beta}k_\alpha)+f_5 v_\alpha v'_\beta(v+v')_\mu\ . \tag{3.29}$$

Using the interpolating field (3.17), we find

$$\begin{aligned}\langle B_6(v',s')|j_\mu|B_6(v,s)\rangle &=-\tfrac{1}{3}\bar{u}(v',s')\gamma_5(\gamma^\alpha+v'^\alpha)M_{\mu\alpha\beta}(\gamma^\beta+v^\beta)u(v,s)\\ &=-\tfrac{1}{3}\bar{u}(v',s')(\{f_1(2+v\cdot v')-f_5[1-(v\cdot v')^2]\}(v+v')_\mu+f_3(\gamma_\mu \not{k}-\not{k}\gamma_\mu))u(v,s)\ .\end{aligned} \tag{3.30}$$

This leads to

$$\langle B_6(v',s')\gamma(k,\epsilon)|j_\mu A^\mu|B_6(v,s)\rangle=-\tfrac{1}{3}f_3\bar{u}(v',s')\sigma\cdot F u(v,s)\ , \tag{3.31}$$

with $\sigma\cdot F=\sigma_{\mu\nu}F^{\mu\nu}$, $F^{\mu\nu}=i(k^\mu\epsilon^\nu-k^\nu\epsilon^\mu)$, and

$$a_1=-\tfrac{1}{3}f_3\ , \tag{3.32}$$

where we have dropped a convection current term. Likewise, for the magnetic $B_6^*-B_6$ coupling, we get

$$\begin{aligned}\langle B_6^*(v',s')|j_\mu|B_6(v,s)\rangle &=-\frac{1}{\sqrt{3}}\bar{u}^\alpha(v',s')M_{\mu\alpha\beta}(v^\beta+\gamma^\beta)\gamma_5 u(v,s)\ ,\\ &=-\frac{1}{\sqrt{3}}f_3\bar{u}^\alpha(v',s')(g_{\mu\alpha}\not{k}-k_\alpha\gamma_\mu)\gamma_5 u(v,s)\end{aligned} \tag{3.33}$$

and, hence,

$$\langle B_6^*(v',s')\gamma(k,\epsilon)|j_\mu A^\mu|B_6(v,s)\rangle=-i\frac{f_3}{\sqrt{3}}\bar{u}^\mu(v',s')\gamma^\nu\gamma_5 F_{\mu\nu}u(v,s)\ , \tag{3.34}$$

where only the magnetic-type terms contribute. Comparing this with (3.9) and applying the relation (3.12) yields

$$a_3=\frac{1}{2\sqrt{3}}f_3\ . \tag{3.35}$$

Similarly,

$$\begin{aligned}\langle B_6^*(v',s')|j_\mu|B_6^*(v,s)\rangle &=\bar{u}^\alpha(v',s')M_{\mu\alpha\beta}u^\beta(v,s)\\ &=f_3\bar{u}^\alpha(v',s')(g_{\mu\alpha}k_\beta-g_{\mu\beta}k_\alpha)u^\beta(v,s)\end{aligned} \tag{3.36}$$

and

$$\langle B_6^*(v',s')\gamma(k,\epsilon)|j_\mu A^\mu|B_6^*(v,s)\rangle=if_3\bar{u}^\alpha(v',s')F_{\alpha\beta}u^\beta(v,s)\ . \tag{3.37}$$

This together with Eq. (3.15) leads to

$$a_5=\frac{f_3}{2}\ . \tag{3.38}$$

It follows from Eqs. (3.32), (3.35), and (3.38) that the coupling constants a_1, a_3, and a_5 are related via heavy-quark symmetry.

We next turn to the a_2 term and get

$$\langle B_6(v',s')|j_\mu|B_{\bar{3}}(v,s)\rangle=\bar{B}^\nu(v',s')M_{\mu\nu}u(v,s)\ , \tag{3.39}$$

with

$$M^{\mu\nu}=\langle 0|\phi_{v'}^\nu j^\mu\phi_v^\dagger|0\rangle\ . \tag{3.40}$$

Setting

$$M_{\mu\nu}=i\delta\epsilon_{\mu\nu\alpha\beta}k^\alpha v^\beta\ , \tag{3.41}$$

we obtain

$$\langle B_6(v',s')|j_\mu|B_{\bar{3}}(v,s)\rangle=-\frac{\delta}{2\sqrt{3}}\bar{u}(v',s')(\not{k}\gamma_\mu-\gamma_\mu\not{k})u(v,s)\ . \tag{3.42}$$

It then follows that

$$\langle B_6(v',s')\gamma(k,\epsilon)|j_\mu A^\mu|B_{\bar{3}}(v,s)\rangle=\frac{\delta}{2\sqrt{3}}\bar{u}(v',s')\sigma\cdot F u(v,s) \tag{3.43}$$

and

$$a_2=\frac{1}{2\sqrt{3}}\delta\ . \tag{3.44}$$

Likewise, for the a_4 coupling we have

$$\langle B_6^*(v',s')\gamma(k,\epsilon)|j_\mu A^\mu|B_{\bar{3}}(v,s)\rangle=\frac{\delta}{2}\epsilon_{\mu\nu\alpha\beta}\bar{u}^\mu(v',s')\gamma^\nu F^{\alpha\beta}u(v,s) \tag{3.45}$$

and

$$a_4=\frac{\delta}{2}\ . \tag{3.46}$$

Equations (3.24), (3.32), (3.35), (3.38), (3.44), and (3.46) together give

$$a_3=-\frac{\sqrt{3}}{2}a_1,\quad a_5=-\frac{3}{2}a_1,\quad a_4=\sqrt{3}a_2,\quad a_6=0\ . \tag{3.47}$$

Consequently, only two of the six couplings $a_1,\ldots,a_6$ are independent. Furthermore, these two couplings are independent of the heavy masses.

There are two corrections which we would like to incorporate in the Lagrangian (3.9). First, when the heavy-quark mass is not infinite, i.e., $m_Q\neq\infty$, we may take into account the effects of the couplings a_i' induced by heavy quarks and of the Dirac magnetic moments μ_B of heavy baryons. Second, as in the meson case, SU(3)-breaking effects due to light-quark mass differences can be incorporated by replacing the charge matrix $\mathcal{Q}$ by

$$\mathcal{Q}\to\widetilde{\mathcal{Q}}=\begin{pmatrix}\frac{2}{3} & 0 & 0\\ 0 & -\dfrac{\alpha}{3} & 0\\ 0 & 0 & -\dfrac{\beta}{3}\end{pmatrix}, \tag{3.48}$$

where

$$\alpha=\frac{m_u}{m_d},\quad \beta=\frac{m_u}{m_s}\ . \tag{3.49}$$

We now use the nonrelativistic quark model to calculate the coupling constants a_i and a_i'. We choose the magnetic field along the z direction so that

$$\sigma_{\mu\nu}F^{\mu\nu}=-2\boldsymbol{\sigma}\cdot\mathbf{H}=-2\sigma_z H\ . \tag{3.50}$$

Note that in the rest frame of the heavy baryon

$$\epsilon_{\mu\nu\alpha\beta}\bar{B}_6^{*\mu}\widetilde{\mathcal{Q}}\gamma^{\nu}F^{\alpha\beta}B_6=2(\mathbf{B}_6^*)_z\widetilde{\mathcal{Q}}B_6H\ , \tag{3.51}$$

$$\bar{B}_6^{*\mu}\widetilde{\mathcal{Q}}F_{\mu\nu}B_6^{*\nu}=-(\mathbf{B}_6^*\widetilde{\mathcal{Q}}\times\mathbf{B}_6^*)_zH\ , \tag{3.52}$$

and the wave function of the $\mathbf{B}_6^*$ is given by

$$\begin{aligned}&\mathbf{B}_6^*(\tfrac{3}{2})=\boldsymbol{\varepsilon}_1u_\uparrow\quad\text{for } s_z=\tfrac{3}{2}\ ,\\ &\mathbf{B}_6^*(\tfrac{1}{2})=\frac{1}{\sqrt{3}}\boldsymbol{\varepsilon}_1u_\downarrow+(\tfrac{2}{3})^{1/2}\boldsymbol{\varepsilon}_3u_\uparrow\quad\text{for } s_z=\tfrac{1}{2}\ ,\end{aligned} \tag{3.53}$$

where

$$\boldsymbol{\varepsilon}_1=-\frac{1}{\sqrt{2}}(1,i,0),\quad \boldsymbol{\varepsilon}_3=(0,0,1)\ . \tag{3.54}$$

By working out the trace terms $\mathrm{tr}(\bar{B}'\widetilde{\mathcal{Q}}B)$ for $B=B_{\bar{3}}, B_6$, and B_6^*, we obtain

$$\begin{aligned}&\langle\Sigma_Q^{+1}\uparrow|\mathcal{L}_B^{(2)}|\Sigma_Q^{+1}\uparrow\rangle=-2(\tfrac{2}{3}a_1+e_Qa_1')\ ,\\ &\langle\Sigma_Q^{0}\uparrow|\mathcal{L}_B^{(2)}|\Lambda_Q\uparrow\rangle=-\sqrt{2}a_2\left[\frac{2}{3}+\frac{\alpha}{3}\right],\\ &\langle\Sigma_Q^{*+1}(\tfrac{1}{2})|\mathcal{L}_B^{(2)}|\Sigma_Q^{+1}\uparrow\rangle=2(\tfrac{2}{3})^{1/2}(\tfrac{2}{3}a_3+e_Qa_3')\ ,\\ &\langle\Sigma_Q^{*0}(\tfrac{1}{2})|\mathcal{L}_B^{(2)}|\Lambda_Q\uparrow\rangle=2(\tfrac{2}{3})^{1/2}\left[\frac{1}{\sqrt{2}}\left[\frac{2}{3}+\frac{\alpha}{3}\right]a_4\right],\\ &\langle\Sigma_Q^{*+1}(\tfrac{3}{2})|\mathcal{L}_B^{(2)}|\Sigma_Q^{*+1}(\tfrac{3}{2})\rangle=2(\tfrac{2}{3}a_5+e_Qa_5')\ ,\\ &\langle\Lambda_Q\uparrow|\mathcal{L}_B^{(2)}|\Lambda_Q\uparrow\rangle=-2\left[\left[\frac{2}{3}-\frac{\alpha}{3}\right]a_6+2e_Qa_6'\right],\end{aligned} \tag{3.55}$$

where we have dropped the magnetic field H for convenience. The number in parentheses after a B_6^* state indicates the value of s_z.

In the quark model, the spin-flip magnetic interaction has the form

$$\mathcal{L}_{\rm em}=\boldsymbol{\mu}\cdot\mathbf{H}\ , \tag{3.56}$$

with

$$\boldsymbol{\mu}=\sum_q\mu_q\boldsymbol{\sigma}_q\ , \tag{3.57}$$

where $\mu_q=e_qe/2m_q$ is the magnetic moment of the quark q with its electric charge e_q in units of e. Next, the flavor-spin wave functions of heavy baryons needed are

$$\begin{aligned}|\Sigma_Q^{+1}\uparrow\rangle&=\frac{1}{\sqrt{6}}[2|Q\downarrow\rangle|u\uparrow u\uparrow\rangle\\ &\qquad-|Q\uparrow\rangle(|u\uparrow u\downarrow\rangle+|u\downarrow u\uparrow\rangle)]\ ,\\ |\Sigma_Q^{0}\uparrow\rangle&=\frac{1}{\sqrt{12}}[2|Q\downarrow\rangle(|u\uparrow d\uparrow\rangle+|d\uparrow u\uparrow\rangle)\\ &\qquad-|Q\uparrow\rangle(|u\uparrow d\downarrow\rangle+|d\uparrow u\downarrow\rangle\\ &\qquad+|u\downarrow d\uparrow\rangle+|d\downarrow u\uparrow\rangle)]\ ,\\ |\Lambda_Q\uparrow\rangle&=\tfrac{1}{2}|Q\uparrow\rangle(|u\uparrow d\downarrow\rangle-|u\downarrow d\uparrow\rangle\\ &\qquad-|d\uparrow u\downarrow\rangle+|d\downarrow u\uparrow\rangle)\ ,\\ |\Sigma^{+1*}(\tfrac{1}{2})\rangle&=\frac{1}{\sqrt{3}}[|Q\downarrow\rangle|u\uparrow u\uparrow\rangle\\ &\qquad+|Q\uparrow\rangle(|u\uparrow u\downarrow\rangle+|u\downarrow u\uparrow\rangle)]\ ,\\ |\Sigma_Q^{+1*}(\tfrac{3}{2})\rangle&=|Q\uparrow\rangle|u\uparrow u\uparrow\rangle\ ,\\ |\Sigma_Q^{0*}(\tfrac{1}{2})\rangle&=\frac{1}{\sqrt{6}}[|Q\downarrow\rangle(|u\uparrow d\uparrow\rangle+|d\uparrow u\uparrow\rangle)\\ &\qquad+|Q\uparrow\rangle(|u\uparrow d\downarrow\rangle+|u\downarrow d\uparrow\rangle\\ &\qquad+|d\uparrow u\downarrow\rangle+|d\downarrow u\uparrow\rangle)]\ .\end{aligned} \tag{3.58}$$

It is then straightforward to show that

$$\begin{aligned}&\langle\Sigma_Q^{+1}\uparrow|\mathcal{L}_{\rm em}|\Sigma_Q^{+1}\uparrow\rangle=\tfrac{4}{3}\mu_u-\tfrac{1}{3}\mu_Q\ ,\\ &\langle\Sigma_Q^{0}\uparrow|\mathcal{L}_{\rm em}|\Lambda_Q\uparrow\rangle=-\frac{1}{2\sqrt{3}}\mu_u(2+\alpha)\ ,\\ &\langle\Lambda_Q\uparrow|\mathcal{L}_{\rm em}|\Lambda_Q\uparrow\rangle=\mu_Q\ ,\\ &\langle\Sigma_Q^{+1*}(\tfrac{1}{2})|\mathcal{L}_{\rm em}|\Sigma_Q^{+1}\uparrow\rangle=\frac{2\sqrt{2}}{3}(\mu_u-\mu_Q)\ ,\\ &\langle\Sigma_Q^{0*}(\tfrac{1}{2})|\mathcal{L}_{\rm em}|\Lambda_Q\uparrow\rangle=\frac{1}{\sqrt{6}}\mu_u(2+\alpha)\ ,\\ &\langle\Sigma_Q^{+1*}(\tfrac{3}{2})|\mathcal{L}_{\rm em}|\Sigma_Q^{+1*}(\tfrac{3}{2})\rangle=2\mu_u+\mu_Q\ ,\end{aligned} \tag{3.59}$$

where we have dropped the magnetic field as before. Comparing this with Eq. (3.55) leads to

$$\begin{aligned}&a_1=-\mu_u,\quad a_2=\frac{\sqrt{6}}{4}\mu_u,\quad a_3=\frac{\sqrt{3}}{2}\mu_u\ ,\\ &a_4=\frac{3}{2\sqrt{2}}\mu_u,\quad a_5=\tfrac{3}{2}\mu_u,\quad a_6=0\ ,\end{aligned} \tag{3.60}$$

and

$$a_1'=\frac{1}{6}\frac{\mu_Q}{e_Q},\quad a_2'=a_4'=0,\quad a_3'=-\frac{1}{\sqrt{3}}\frac{\mu_Q}{e_Q},$$
$$a_5'=\frac{1}{2}\frac{\mu_Q}{e_Q},\quad a_6'=-\frac{1}{4}\frac{\mu_Q}{e_Q}. \tag{3.61}$$

It is evident that the relations (3.47) for the couplings $a_1,\ldots,a_6$ predicted by heavy-quark symmetry are satisfied in the quark-model calculation, as they should be.

IV. APPLICATIONS

In this section we apply our results obtained so far to the electromagnetic decays of heavy hadrons. As we recall, there are eight unknown coupling constants in the meson and baryon sectors, but they are reduced to three via the use of heavy-quark symmetry. The nonrelativistic quark model is then applied to compute them. Consequently, the dynamics of the radiative transitions for emission of soft photons and pions is completely determined by heavy-quark and chiral symmetries, supplemented by the quark model.

As an application, we first focus on the two-body radiative decays such as $P^*\to P\gamma$, $\Sigma_Q\to\Lambda_Q\gamma$, and $\Xi_Q'\to\Xi_Q\gamma$. Since the heavy hadrons, e.g., B^*,Ξ_c',Ξ_b, are dominated by electromagnetic decays, the decay widths of these heavy particles can be directly calculated. When combining with our previous results [3] on the strong decays of D^* and Σ_c, we can also predict the total widths and branching ratios of these particles. We shall also consider the radiative decays involving one-pion emission. Some examples of kinematically allowed modes are $\Sigma_c\to\Lambda_c\pi\gamma$, $\Xi_c^*\to\Xi_c\pi\gamma$, etc. We shall see later that the decay $\Sigma_c^+\to\Lambda_c^+\pi^0\gamma$ provides a nice test on the chiral structure of the electromagnetic gauge-invariant Lagrangian $\mathcal{L}_B^{(2)}$, whereas the four-particle contact interaction dictated by $\mathcal{L}_B^{(1)}$ can be tested by the other channel $\Sigma_c^0\to\Lambda_c^+\pi^-\gamma$.

We begin with the $P^*\to P\gamma$ decays. The decay width corresponding to the general amplitude

$$A[P^*(v,\epsilon^*)\to P\gamma(k,\epsilon)]=-i\rho\epsilon_{\mu\nu\alpha\beta}k^\mu\epsilon^\nu v^\alpha\epsilon^{*\beta} \tag{4.1}$$

is

$$\Gamma(P^*\to P\gamma)=\frac{\rho^2}{12\pi M^{*2}}k^3, \tag{4.2}$$

where k is the photon momentum in the c.m. system. From Eqs. (2.19), (2.41), and (2.42), we obtain the couplings

$$\rho(P^{*1/2})=2\sqrt{M_PM_{P^*}}\left[-\frac{1}{3}\frac{e}{2m_d}+e_Q\frac{e}{2m_Q}\right],$$
$$\rho(P^{*-1/2})=2\sqrt{M_PM_{P^*}}\left[\frac{2}{3}\frac{e}{2m_u}+e_Q\frac{e}{2m_Q}\right], \tag{4.3}$$
$$\rho(P^{*0})=2\sqrt{M_PM_{P^*}}\left[-\frac{1}{3}\frac{e}{2m_s}+e_Q\frac{e}{2m_Q}\right].$$

As an example, the computed results for $D^*\to D+\gamma$ are exhibited in Table I for the constituent quark masses

$$m_u=338\text{ MeV},\quad m_d=322\text{ MeV},\quad m_s=510\text{ MeV}, \tag{4.4}$$

given by the Particle Data Group [10], and $m_c=1.6$ GeV. To determine the D^* branching ratios, we have included the partial widths of $D^*\to D\pi$ predicted in Ref. [3] with the axial quark coupling $g_A^{ud}=0.75$. It is evident that the agreement between theory and the most recent experimental measurement of CLEO II [9] is excellent. In particular, the observed small branching ratio of $D^{*+}\to D^+\gamma$ by CLEO II is consistent with our theoretical expectation, contrary to the large PDG [10] average value. This also means that it is not necessary to invoke a large anomalous magnetic moment for the charmed quark as previously conjectured. The total widths of the D^* [12] are

$$\Gamma_{\rm tot}(D^{*+})=141\text{ keV},$$
$$\Gamma_{\rm tot}(D^{*0})=102\text{ keV}, \tag{4.5}$$
$$\Gamma_{\rm tot}(D_s^{*+})=0.3\text{ keV},$$

which are also listed in Table I. The $\Gamma_{\rm tot}(D^{*+})$ predicted here is very close to the upper limit $\Gamma_{\rm tot}(D^{*+})<131$ keV (90% C.L.) published by the ACCMOR Collaboration [11]. We urge the experimentalists to perform more precision measurements of $\Gamma_{\rm tot}(D^*)$.

Before proceeding, we should stress that it is important to include the corrections due to the magnetic moment of the charmed quark as its mass is not too large compared to the light quarks, $m_s/m_c\approx\frac{1}{3}$, and its charge is $\frac{2}{3}e$. It is clear from Eq. (4.3) that the charmed quark contribution is largely destructive in the radiative decays of D^{*+} and D_s^{*+}. Had we worked in the heavy-quark limit, we would have obtained

$$\Gamma(D^{*0}\to D^0\gamma)=23\text{ keV},$$
$$\Gamma(D^{*+}\to D^+\gamma)=6\text{ keV}, \tag{4.6}$$
$$\Gamma(D_s^{*+}\to D_s^+\gamma)=2.4\text{ keV},$$

TABLE I. Predicted branching ratios of the D^* mesons. The predicted partial widths of $D^*\to D+\pi$ are taken from Ref. [3] for $g_A^{ud}=0.75$. For comparison, the experimental results of CLEO II [9] and PDG [10] average values are given in the last two columns.

Decay mode	Γ (keV)	B (%)$_{\rm theory}$	B (%)$_{\rm CLEO}$	B (%)$_{\rm PDG}$
$D^{*+}\to D^0\pi^+$	95	67.3	68.1±1.0±1.3	55±4
$D^{*+}\to D^+\pi^0$	44	31.2	30.8±0.4±0.8	27.2±2.5
$D^{*+}\to D^+\gamma$	2	1.5	1.1±1.4±1.6	18±4
$D^{*+}\to$all	141			
$D^{*0}\to D^0\pi^0$	68	66.7	63.6±2.3±3.3	55±6
$D^{*0}\to D^0\gamma$	34	33.3	36.4±2.3±3.3	45±6
$D^{*0}\to$all	102			
$D_s^{*+}\to D_s^+\gamma$	0.3	~100		

which are significantly different from those presented in Table I. Finally, for completeness we also give the results for the radiative decays of B^*:

$$\Gamma(B_u^{*+}\to B_u^+\gamma)=0.84\ \text{keV}\ ,$$
$$\Gamma(B_d^{*0}\to B_d^0\gamma)=0.28\ \text{keV}\ , \tag{4.7}$$

where we have used the mass values $m_{B^*}=5324.6$ MeV and $m_b=5$ GeV.

We next turn to the baryon sector and consider the following two-body radiative decays with some specific examples:

$$\begin{aligned} &B_6\to B_{\bar 3}+\gamma:\ \Sigma_Q\to\Lambda_Q+\gamma,\ \ \Xi'_Q\to\Xi_Q+\gamma\ ,\\ &B_6^*\to B_{\bar 3}+\gamma:\ \Sigma_Q^*\to\Lambda_Q+\gamma,\ \ \Xi'^*_Q\to\Xi_Q+\gamma\ ,\\ &B_6^*\to B_6+\gamma:\ \Sigma_Q^*\to\Sigma_Q+\gamma,\ \ \Xi'^*_Q\to\Xi'_Q+\gamma\ ,\\ &\qquad\qquad\Omega_Q^*\to\Omega_Q+\gamma\ .\end{aligned} \tag{4.8}$$

The electromagnetic decay of a sextet baryon B_6 into a $B_{\bar 3}$ plus a photon is described by the amplitude

$$M[B_6\to B_{\bar 3}+\gamma(k)]=i\eta_1\bar u_{\bar 3}\sigma_{\mu\nu}k^\mu\epsilon^\nu u_6\ . \tag{4.9}$$

Its decay rate is simply given by

$$\Gamma(B_6\to B_{\bar 3}+\gamma)=\frac{1}{\pi}\eta_1^2k^3\ , \tag{4.10}$$

where k is the photon momentum in the c.m. system. For completeness, we give here the results of the radiative decay of a spin-$\frac{3}{2}$ heavy baryon, though none of these heavy baryons have been found yet. The amplitude of the transition $B_6^*\to B_{\bar 3}+\gamma$ reads

$$M(B_6^*\to B_{\bar 3}+\gamma)=i\eta_2\epsilon_{\mu\nu\alpha\beta}\bar u\gamma^\nu k^\alpha\epsilon^\beta u^\mu\ . \tag{4.11}$$

The evaluation of the corresponding decay width involves the use of the projection operator

$$\begin{aligned} P_{\mu\nu}(v)\equiv\sum_s u_\mu(p,s)\bar u_\nu(p,s)&=\frac{\not p+m}{2m}\left[-g_{\mu\nu}+\frac{1}{3}\gamma_\mu\gamma_\nu+\frac{1}{3m}(\gamma_\mu p_\nu-\gamma_\nu p_\mu)+\frac{2}{3m^2}p_\mu p_\nu\right]\\ &=\left[-g_{\mu\nu}+\frac{1}{3}\gamma_\mu\gamma_\nu-\frac{1}{3m}(\gamma_\mu p_\nu-\gamma_\nu p_\mu)+\frac{2}{3m^2}p_\mu p_\nu\right]\frac{\not p+m}{2m}\ . \end{aligned} \tag{4.12}$$

The final result is

$$\Gamma(B_6^*\to B_{\bar 3}+\gamma)=\frac{k}{48\pi}\eta_2^2\left[1-\frac{m_f^2}{m_i^2}\right]^2(3m_i^2+m_f^2)\ , \tag{4.13}$$

where m_i (m_f) is the mass of the initial (final) baryon in the decay. Except for a different coupling constant, a similar formula holds for the decay $B_6^*\to B_6+\gamma$.

We are ready to elaborate on the above results by some examples. The first example is $\Sigma_c^+\to\Lambda_c^++\gamma$. From Eqs. (3.9), (3.48), (3.49), and (3.60), we find

$$\begin{aligned}\eta_1(\Sigma_c^+-\Lambda_c^+)&=\sqrt{2}\left[\frac{2}{3}+\frac{\alpha}{3}\right]a_2\\ &=\frac{1}{\sqrt{3}}\frac{e}{2m_u}\left[\frac{2}{3}+\frac{1}{3}\frac{m_u}{m_d}\right]\ ,\end{aligned} \tag{4.14}$$

which in turn implies that

$$\Gamma(\Sigma_c^+\to\Lambda_c^++\gamma)=93\ \text{keV}\ . \tag{4.15}$$

This together with the partial rate $\Gamma(\Sigma_c^+\to\Lambda_c^+\pi^0)=2.43$ MeV (for $g_A^{ud}=0.75$) obtained in Ref. [3] yields the total decay width of Σ_c^+,

$$\Gamma_{\text{tot}}(\Sigma_c^+)=2.54\ \text{MeV}\ , \tag{4.16}$$

and the branching ratio of $\Sigma_c^+\to\Lambda_c^++\gamma$,

$$B(\Sigma_c^+\to\Lambda_c^++\gamma)=3.8\%\ . \tag{4.17}$$

The second example is $\Xi'_c\to\Xi_c+\gamma$. The coupling η_1 is given by

$$\begin{aligned}\eta_1(\Xi_c'^+-\Xi_c^+)&=\sqrt{2}\left[\frac{2}{3}+\frac{\beta}{3}\right]a_2\\ &=\frac{1}{\sqrt{3}}\frac{e}{2m_u}\left[\frac{2}{3}+\frac{m_u}{3m_s}\right]\ ,\\ \eta_1(\Xi_c'^0-\Xi_c^0)&=\sqrt{2}\left[-\frac{\alpha}{3}+\frac{\beta}{3}\right]a_2\\ &=\frac{e}{6\sqrt{3}}\left[\frac{1}{m_s}-\frac{1}{m_d}\right]\ ,\end{aligned} \tag{4.18}$$

for $\Xi_c'^+-\Xi_c^+$ and $\Xi_c'^0-\Xi_c^0$ transitions, respectively. We get

$$\Gamma(\Xi_c'^+\to\Xi_c^++\gamma)=16\ \text{keV}\ ,$$
$$\Gamma(\Xi_c'^0\to\Xi_c^0+\gamma)=0.3\ \text{keV}\ . \tag{4.19}$$

In the above we have used the mass $m_{\Xi_c}=2470$ MeV from PDG [10] and the mass difference $m_{\Xi'_c}-m_{\Xi_c}\simeq 100$ MeV from a theoretical estimate [13]. We also assume no mixing between Ξ'_c and Ξ_c. If the mass difference turns out to be this small, there will be no strong decays for Ξ'_c. We thus have a prediction for the total width of Ξ'_c:

$$\Gamma_{\text{tot}}(\Xi_c'^+)=16\ \text{keV},\quad \Gamma_{\text{tot}}(\Xi_c'^0)=0.3\ \text{keV}\ . \tag{4.20}$$

So far, the examples of radiative decays considered do not test critically heavy-quark or chiral symmetry. The results follow simply from the quark model. We now offer examples in which both heavy-quark and chiral symmetries enter in a crucial way. These are the radiative decays of heavy baryons involving an emitted pion. Some examples which are kinematically allowed are

$$\Sigma_c \to \Lambda_c \pi\gamma, \quad \Sigma_c^* \to \Lambda_c \pi\gamma, \quad \Sigma_c^* \to \Sigma_c \pi\gamma, \quad \Xi_c^* \to \Xi_c \pi\gamma \ .$$

To be specific, we focus on the decay $\Sigma_c \to \Lambda_c \pi\gamma$. The Feynman diagrams for the decay follow from the Lagrangian $\mathcal{L}_B^{(1)}$ and $\mathcal{L}_B^{(2)}$ given in the last section. There are a total of eight possible diagrams as depicted in Fig. 1: Six of them arise from baryon poles, one from the meson pole, and one from the four-point contact term. For the present discussion, we will limit ourselves to the situation in the heavy-quark limit as to bring out the simplifications that occur in the symmetry limit. Thus the pion and photon are both soft, and we will neglect terms of order q/m_Q and/or k/m_Q with q and k being the pion and photon momenta, respectively. It turns out that the contact interaction dictated by the Lagrangian $\mathcal{L}_B^{(1)}$ can be nicely tested by the decay $\Sigma_c^0 \to \Lambda_c^+ \pi^- \gamma$, whereas a test on the chiral structure of $\mathcal{L}_B^{(2)}$ is provided by the process $\Sigma_c^+ \to \Lambda_c^+ \pi^0 \gamma$. Let us discuss the latter first.

It is interesting to see that only Figs. 1(d) and 1(f) survive in the heavy-quark limit. Figures 1(b) and 1(c) vanish because of isospin conservation. Figures 1(g) and 1(h) do not exist for a neutral pion. Figure 1(e) is prohibited owing to the absence of the $B_{\bar{3}}B_{\bar{3}}\pi$ coupling. The $\Lambda_c\Lambda_c\gamma$ coupling of Fig. 1(a) is of the convection current type only [cf. Eq. (3.24)], and in the heavy-quark limit it is canceled out by a similar convection current $\Sigma_c\Sigma_c\gamma$ coupling of Fig. 1(d). (This cancellation is also required by gauge invariance.) Consequently, we only have to consider Fig. 1(f) and the magnetic coupling of Fig. 1(d). The amplitudes are

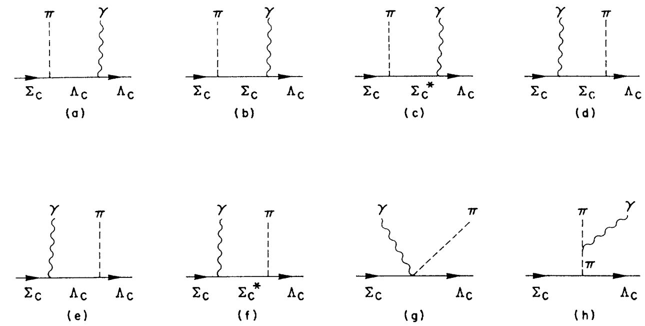

FIG. 1. Possible Feynman diagrams for the decays $\Sigma_c^+ \to \Lambda_c^+ \pi^0 \gamma$ and $\Sigma_c^0 \to \Lambda_c^+ \pi^- \gamma$.

$$\begin{aligned} A(\Sigma_c^+ \to \Lambda_c^+ \pi^0 \gamma) &= A_d + A_f \ , \\ A_d &= i\frac{4\sqrt{2}a_1 g_2}{3f_\pi}\frac{1}{v\cdot k}\bar{u}_{\Lambda_c}(v',s')(\not{q} - q\cdot v)\sigma_{\mu\nu}k^\mu \epsilon^\nu u_{\Sigma_c}(v,s) \ , \\ A_f &= -i\frac{a_3 g_4}{\sqrt{2}f_\pi}\left[\frac{2}{3}-\frac{\alpha}{3}\right]\frac{1}{-v\cdot k + M_{\Sigma_c} - M_{\Sigma_c^*}}\bar{u}_{\Lambda_c}(v',s')q_\sigma P^{\sigma\lambda}(v')\epsilon_{\lambda\nu\alpha\beta}\gamma^\nu k^\alpha \epsilon^\beta u_{\Sigma_c}(v,s) \ , \end{aligned} \tag{4.21}$$

where $P^{\sigma\lambda}(v')$ is the projection operator given by (4.12). Recall that

$$g_4 = -\sqrt{3}g_2, \quad g_2 = -0.75(\tfrac{2}{3})^{1/2} \ , \tag{4.22}$$

for $g_A^{ud} = 0.75$. Beyond the heavy-quark limit, obvious $1/m_Q$ corrections arise from the magnetic moment μ_c of the charmed quark and μ_B of the charmed baryons.

We now come back to the decay $\Sigma_c^0 \to \Lambda_c^+ \pi^- \gamma$. The main contribution comes from the convection current coupling of Figs. 1(a), 1(g), and 1(h). Other diagrams due to the magnetic-type couplings are suppressed by factors of k/m_u, which should be small since $m_{\Sigma_c} - m_{\Lambda_c} - m_\pi \sim 30$ MeV and $m_u \sim 330$ MeV. The contact-term Lagrangian for Fig. 1(g) can be read off from Eqs. (3.8) and (2.11b),

$$\mathcal{L}' = -\frac{ieg_2}{\sqrt{2}f_\pi}A_\mu \bar{\Lambda}_c^+ \gamma^\mu \gamma_5 \pi^+ \Sigma_c^0 \ . \tag{4.23}$$

The amplitudes are given by

$$A(\Sigma_c^0 \to \Lambda_c^+ \pi^- \gamma) \cong A_a + A_g + A_h \ , \tag{4.24}$$

with

$$\begin{aligned} A_a &= \frac{eg_2}{\sqrt{2}f_\pi}\bar{u}_{\Lambda_c} v\cdot\epsilon \frac{1}{v\cdot k}\not{q}\gamma_5 u_{\Sigma_c} \ , \\ A_g &= \frac{eg_2}{\sqrt{2}f_\pi}\bar{u}_{\Lambda_c}\not{\epsilon}\gamma_5 u_{\Sigma_c} \ , \\ A_h &= -\frac{eg_2}{\sqrt{2}f_\pi}\frac{q\cdot\epsilon}{q\cdot k}\bar{u}_{\Lambda_c}(\not{q}+\not{k})\gamma_5 u_{\Sigma_c} \ . \end{aligned} \tag{4.25}$$

It is easily seen that gauge invariance is respected. It will be interesting to work out the energy and angular distributions of the pion and Λ_c. A detailed analysis of this is planned to be presented in a future publication.

Aside from the decay rates for $B^* \to B\gamma$ given by (4.7), we have not calculated any of the radiative decay rates for baryons containing a b quark. This is only because there is scarcely any data on the masses and mass differences of these baryons. Once they are known, the same equations (3.59)–(3.61) and (4.9)–(4.13) can be applied to obtain the decay rates.

V. CONCLUSIONS

Heavy-quark and chiral symmetries together provide an ideal framework for studying the low-energy dynamics of heavy mesons and heavy baryons. Symmetry considerations reduce to a minimum the number of free parameters in the theory, and symmetry-breaking corrections can be estimated in principle. Yet few if any quantitative predictions can be made in strong and electromagnetic interactions without further assumptions. It is here that the nonrelativistic quark model comes to the rescue. All the free parameters needed for the low-energy dynamics of ground-state heavy hadrons are calculable in the nonrelativistic quark model. Moreover, these calculations depend only on the spin-flavor wave functions of the quarks and are independent of the details of the spatial wave functions. Therefore simplicity and (almost) uniqueness characterize these quark-model predictions. We regard them as a theoretical benchmark to be compared with experiments as well as other theoretical models.

In Refs. [3] and [7] and the present work, we have explored in detail the predictions of this theoretical formalism on strong decays, heavy-flavor-conserving nonleptonic decays, and radiative decays of heavy hadrons. These results may now be combined to obtain predictions for the total widths and branching ratios of certain heavy particles. In particular, the branching ratios obtained for D^{*+} and D^{*0} agree very well with the most recent measurements of CLEO II. This excellent agreement between theory and experiment makes it ever more urgent to study and understand the various symmetry-breaking corrections to the strong and radiative decays. This is particularly so should the upper limit for $\Gamma_{\rm tot}(D^{*+})$ [11] be confirmed by future experiments. We would like to know if it is possible to incorporate these corrections to improve the quark-model calculations. We have begun an investigation to answer these questions. The $1/m_Q$ corrections due to the heavy-quark magnetic moment that we have included in Secs. II and III are exact as a result of the normalization conditions of the Isgur-Wise functions at $v=v'$. Other $1/m_Q$ corrections including those to the light-quark electromagnetic currents and axial-vector currents require a more careful discussion. We plan to communicate these results in a future publication.

There are many other weak-radiative-decay modes of great interest such as

$$B \to D(D^*)\gamma, \quad \Lambda_b \to \Sigma_c\gamma, \quad \Xi_b \to \Xi_c\gamma \ .$$

Unfortunately, the effective heavy-quark theory developed thus far cannot be applied to these processes. The intermediate states in the relevant pole diagrams are very far from their mass shell. For example, the four-momentum squared of the D pole in the decay $B \to D^*\gamma$ is m_B^2. This means that the residual momentum of the D meson defined by $P_\mu = m_D v_\mu + k_\mu$ must be of order m_B so that the approximation $k/m_D \ll 1$ required by the effective heavy-quark theory is no longer valid. Nevertheless, there is a special class of weak radiative decays in which heavy flavor is conserved that deserves a detailed study. Some examples are $\Xi_Q \to \Lambda_Q\gamma$ and $\Omega_Q \to \Xi_Q\gamma$. In these decays, weak radiative transitions arise from the diquark sector of the heavy baryon, whereas the heavy quark behaves as a "spectator." However, the dynamics of these radiative decays is more complicated than their counterpart in nonleptonic weak decays, e.g., $\Xi_Q \to \Lambda_Q\pi$, which have been studied in Ref. [7]. We hope to study in the future these heavy-flavor-conserving weak radiative decays.

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