Flavor-changing radiative B decays and flavor-conserving radiative decays of ground-state mesons

El hassan El aaoud and Riazuddin
Physics Department, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia
(Received 9 September 1991)

A relativistic quark model appropriate to a universal one-gluon exchange plus a screening confining potential motivated by lattice gauge theory is used in evaluating the exclusive rate for $B \to K^*(892)\gamma$ arising from the quark subprocess $b \to s\gamma$ with a view to test the sensitivity of this rate to the screening length. Relativistic as well as recoil corrections are included in numerical evaluations of the form factors. This procedure is extended to flavor-conserving radiative decays of ground-state mesons: $V(1^-) \to P(0^-)\gamma$. It is found that the screening length increases as one moves to the light sector, thereby providing information about quark-antiquark interaction at large distances.

PACS number(s): 13.40.Hq, 12.40.Aa, 12.40.Qq

I. INTRODUCTION

The flavor-changing weak decays of mesons have always been a rich source of information about basic interaction in particle physics. They play a crucial role in determining the fundamental parameters of the standard model (quark mixing angles, quark masses, etc.). In particular, the mass of the charmed quark was predicted [1] from the flavor-changing processes $K_L \to \mu\mu$, $K \to \pi e^+ e^-$. Recently, radiative B decays $B \to K^*(892)\gamma$ received intensive theoretical studies. The presence of heavy b quark permits the use of a spectator model in evaluating the relevant hadronic matrix elements where the relevance of the use of a potential model comes in.

Since the proposal of the nonrelativistic quark model (NRQM), many attempts have been made to apply it to various processes involving elementary particles. While the NRQM gives satisfactory results for hadrons containing heavy quarks, it gives overestimated decay rates for radiative decays of ground-state mesons. Many improvements taking into account different possible corrections have been made to reduce the discrepancies between theory and experiment. Calculations show that the effects of relativistic corrections become important as the quark mass decreases and considerably reduces the overestimated predictions for decay widths in the NRQM especially for mesons in the light sector. In particular, Godfrey and Isgur [2] proposed a unified quark model, using a universal one-gluon exchange plus linear confining potential motivated by quantum chromodynamics and attempted to identify possible relativistic effects. Another approach which includes relativistic corrections was followed by Faustov and Galkin [3,4], where they used a relativistic quark model based on a quasipotential formalism and found that relativistic corrections play an important role in meson decays.

In this paper we use a relativistic constituent quark model, with the screening confining potential to study flavor-changing radiative decays of the (heavy) B meson to a hard photon and a strange meson. We show that the form factors are strongly damped by the recoil effect and that the branching ratio is considerably reduced by introducing a screening of the linear confining potential suggested by lattice gauge theory [5] with a screening length $\mu^{-1} = 0.8 \pm 0.2$ fm for flavor bound states of heavy and light quarks. Thus, in principle, $B \to K^*(892)\gamma$ decay can test the above modification of a linearly rising potential at large distances as discussed in Sec. II. We have extended the same approach to evaluate the decay rates of flavor-conserving radiative decays of ground-state mesons. Here the relativistic corrections are more important than the recoil effects, while in B decays the opposite was the case. As expected, the fit to such decays indicates that the screening length increases as we move from the heavy to light sector. Thus these decays give some information on the quark-antiquark interaction at large distances.

II. FLAVOR-CHANGING DECAYS

The decay $B \to K^*(892)\gamma$ at the quark level is described by $b \to s\gamma$ and is dominated by short-distance physics. It gives rise to an effective weak Hamiltonian written in terms of quark and gluon fields [6]:

$$H_{\text{eff}} = C[m_b \bar{s}\sigma_{\mu\nu} q^\nu b_R + m_s \bar{s}\sigma_{\mu\nu} q^\nu b_L]\varepsilon^\mu(q) , \tag{1}$$

where the constant C contains all short-distance QCD corrections arising from gluon exchange between internal and external lines of the loop [7]. The hadronic matrix element is evaluated by expressing the matrix elements of H_{eff} between $|B\rangle$ and $|K^*\rangle$ states in terms of two form factors. The branching ratio is given by [8,9]

$$\eta = \frac{\Gamma(B \to K^*\gamma)}{\Gamma(b \to s\gamma)} = \left[\frac{m_b(m_B^2 - m_{K^*}^2)}{m_B(m_b^2 - m_s^2)} \right]^3 \left[1 + \frac{m_s^2}{m_b^2} \right]^{-1} \frac{1}{2}(f_1^2 + 4f_2^2) , \tag{2a}$$

where f_1 and f_2 are Lorentz scalar functions (form factors) related to the form factors $F(q^2)$ and $\tilde{F}(q^2)$ in the quark model by [8, 10]

$$f_1(q^2)=\left[\frac{1}{2}\left[1+\frac{m_{K^*}^2}{m_B^2}\right]\right]^{1/2}[F(q^2)+q_0\tilde{F}(q^2)]\ ,\quad q_0=\frac{m_B^2-m_{K^*}^2}{2m_B}\ ,\quad f_2(q^2)=\frac{1}{2}f_1(q^2)\ . \tag{2b}$$

The form factors $F(q^2)$ and $\tilde{F}(q^2)$ are expressible in terms of overlap integrals involving S-wave momentum-space wave functions of B and K^*. We may mention that the emitted photon is real so that $q^2=0$, i.e., $\mathbf{K}\neq 0$, and the recoil and relativistic corrections are included in the evaluation of the form factors. With the approximations $E\sim|\mathbf{K}|$ and $E_b\sim m_b$, one can show that

$$F(0)=\frac{1}{\sqrt{2}}\left[\frac{2\beta_B\beta_K}{\beta_B^2+\beta_K^2}\right]^{3/2}\exp\left[-\frac{m_d^2K^2}{2M_K^2(\beta_B^2+\beta_K^2)}\right]\left\{1+\frac{\beta_B^2\beta_K^2}{m_b^2(\beta_B^2+\beta_K^2)}\frac{m_b}{2K}\left[1-\frac{m_dK^2}{3M_K\beta_K^2}\left[1-\frac{m_d\beta_B^2}{M_K(\beta_B^2+\beta_K^2)}\right]\right]\right\} \tag{3}$$

and

$$\tilde{F}(0)=\frac{1}{\sqrt{2}}\left[\frac{2\beta_B\beta_K}{\beta_B^2+\beta_K^2}\right]^{3/2}\left\{\exp\left[-\frac{m_d^2K^2}{2M_K^2(\beta_B^2+\beta_K^2)}\right]\right\}\frac{1}{K}\left\{1-\frac{2m_b+K}{2m_bK}\frac{m_dK}{M_K}\frac{m_d\beta_B^2}{(\beta_B^2+\beta_K^2)}\right\}, \tag{4}$$

where $K=|\mathbf{K}|$ and $M_K=m_s+m_d$. Here the β's are the variational parameters which appear in the Gaussian wave functions chosen for the above-mentioned momentum-space wave functions. For numerical calculations we use $\alpha_s=0.5$, $m_b=5.12$ GeV, $m_s=0.55$ GeV, $m_u=m_d=0.33$ GeV, $m_{K^*}=0.892$ GeV, and $m_B=5.27$ GeV, and the variational parameters β_B and β_{K^*} are fixed by introducing in the Schrödinger equation a screening confining potential [5] instead of linear one:

$$V(r)=-\frac{4}{3}\frac{\alpha_s}{r}+\sigma r\frac{1-e^{-\mu r}}{\mu r}\ ,$$
$$\mu^{-1}=0.8\pm0.2 \text{ fm and } \sigma=0.18\pm0.02 \text{ GeV}^2\ . \tag{5}$$

As the numerical calculations show, different values of $f_1(0)$ and the branching ratio η are obtained for the recoil momentum K having the relativistic $[K=(m_B^2-m_{K^*}^2)/2m_B]$ or the nonrelativistic $[K=\sqrt{(m_{K^*}/m_B)}(m_B-m_{K^*})]$ values. The results are summarized in Table I for both linear and screening potentials. The form factors are strongly damped, especially when $\mathbf{K}$ is relativistic, by the exponential term which describes the main recoil effects.

Our calculations for $f_1(0)$ and η for the linear confining potential as given in Table I agree more or less with those obtained in Refs. [8–10]. A comparison of our results for the linear and screening potentials presented in Table I shows that the ratio η is considerably reduced by introducing the screening potential. The main corrections come from the recoil effects, which is more important than in the case of linear potential. The results in Table I correspond to $\mu^{-1}=0.8$ fm and $\sigma=0.18$ GeV^2, the central values given in Ref. [5]. We wish to discuss the sensitivity of η to the above parameters. The results are as follows:

	K relativistic η	K nonrelativistic η
$\mu^{-1}=0.8$ fm	$(0.24^{+0.13}_{-0.09})\times10^{-2}$	$(6.8\pm1.5)\times10^{-2}$
$\mu^{-1}=1.0$ fm	$(0.5\pm0.2)\times10^{-2}$	$(10\pm2)\times10^{-2}$

where the uncertainity indicated corresponds to uncertainties in $\sigma(\sigma=0.18\pm0.02$ GeV$^2)$. We have not included the values for $\mu^{-1}=0.6$ fm since it gives negligible values of η for K relativistic.

The above results show that the branching ratio η is rather sensitive to the screening length μ^{-1} and the normalization parameter σ. For relativistic K the maximum we can get for η is 0.7%. But for nonrelativistic K, the values of η range from 5% to 12%. By contrast, the corresponding values for the linear confining potential and those obtained from QCD sum rules and symmetry properties range from 5% to 40%. Thus, if η is found to be larger than 12%, one may doubt the screened confining potential. However, if it is found to be less than 1%, then it may support the screened confining potential.

TABLE I. Numerical results for linear and screening potential.

	K relativistic		K nonrelativistic	
	Linear	Screening	Linear	Screening
β_{K^*}	0.34 GeV	0.24 GeV	0.34 GeV	0.24 GeV
β_B	0.41 GeV	0.32 GeV	0.41 GeV	0.32 GeV
$f_1(0)$	0.17	0.045	0.46	0.24
η	0.03	0.0023	0.23	0.06

III. FLAVOR-CONSERVING DECAYS

Since the parities of final and initial states are identical in radiative decays of ground-state mesons, the contribution to the matrix elements is given by the magnetic dipole transition. In general, the Lorentz invariance allows one to write

$$M_\mu\equiv\langle P(K)|{:}J_\mu^{e\cdot m}(0){:}|V(P,e)\rangle$$
$$\propto\mu_{PV}\varepsilon_{\mu\nu\rho\sigma}e^\nu K^\rho P^\sigma\ , \tag{6}$$

where $|P(K)\rangle$ and $|V(P,e)\rangle$ are the pseudoscalar- and vector-meson states (with momentum P and K, respectively), which in the weak binding limit can be expressed as so-called mock-meson states [2,8,11]. In Eq. (6) e is the polarization of the vector meson, μ_{PV} is the magnetic transition matrix element and is given in Table II for the various processes. The decay rate is

$$\Gamma=4\alpha\mu_{PV}^2\omega^3\begin{bmatrix}\frac{1}{3}\\1\end{bmatrix} \text{ for } \begin{bmatrix}V\to P+\gamma\\P\to V+\gamma\end{bmatrix}, \tag{7}$$

where ω is the photon energy and α is the fine-structure constant.

The overlap integral appearing in Table II takes, in the rest frame of the decaying meson, the form

$$I_f=\int d^3\mathbf{p}'\,\phi_P\left[\mathbf{p}'-\frac{m_{\rm sp}}{M}\mathbf{K}\right]\phi_V(\mathbf{p}')m_f \times\left[\frac{E+m_f}{EE'(E'+m_f)}\right]^{1/2}, \tag{8}$$

$$E=(\mathbf{p}'^2+m_f^2)^{1/2}\,,\quad E'=[(\mathbf{K}-\mathbf{p}')^2+m_f^2]^{1/2}\,.$$

Here $M=m_f+m_{\rm sp}$, where m_f and $m_{\rm sp}$ are the masses of decaying and spectator quarks, respectively. It is clear from Eq. (8) that both recoil and relativistic corrections are included in our calculations. They are described by the momentum mismatch $m_{\rm sp}\mathbf{K}/M$ and the factor $m_f[(E+m_f)/EE'(E'+m_f)]^{1/2}$, respectively. In the NRQM, the I_f's are equal to 1. To proceed further with the calculation, we choose the Gaussian-momentum-state trial wave functions

$$\phi_X(\mathbf{p})=(\pi\beta^2)^{-3/4}\exp(-\mathbf{p}^2/2\beta^2)\,,\quad X=P,V\,. \tag{9}$$

One cannot perform the integration analytically. However, a numerical estimation is possible. To estimate the overlap integral, we choose the parameters of [2]. We vary the screening length μ^{-1} in the potential in Eq. (5) in such a way that the expectation value of the energy is minimum for the variational parameters β having the particular values required to obtain a best fit for the decay rates for $\rho\to\pi\gamma$ and $K^*\to K\gamma$. In this way we determine the screening length $\mu^{-1}=2.7$ and 3.6 fm for K^* and (ρ,ω) mesons, respectively. Numerical calculations show that the overlap integrals are strongly damped by the relativistic corrections. For example, taking recoil corrections alone, for kaons, decays I_s and $I_{u,d}$ are reduced to 0.96 and 0.87, while the relativistic corrections damped them to 0.72 and 0.48, respectively. This shows the importance of relativistic corrections for the light sector of hadrons. Our results are summarized in Table II, where a comparison with the experimental results is also given. A comparison of our results with those of [2] shows that our results are more or less in agreement with theirs. However, this reference uses an *ad hoc* smearing factor of the type $(m/E)^f$, where the exponent f is chosen to fit $\rho\to\pi+\gamma$, without any theoretical justification. On the other hand, our calculation has a theoretical basis in the sense of using a screening confining potential where the screening length becomes more important for the light sector. A different approach, in evaluating the relativistic corrections, was followed in Ref. [4]. However, the results are again more or less in agreement with those obtained here.

TABLE II. Transition magnetic moments and predicted and experimental decay rates. Ideal mixing for ω and ϕ is assumed, while $\varphi=\arctan\sqrt{2}+\theta$, $\theta=-11°$.

Process	μ_{PV}	$\Gamma_{\rm predicted}$ (keV)	$\Gamma_{\rm expt}$ (keV) [12]
$\rho^\pm\to\pi^\pm\gamma$	$-\frac{1}{6}\left[\frac{I_d}{m_d}-\frac{2I_u}{m_u}\right]$	64	67.1 ± 8.8
$K^{0*}\to K^0\gamma$	$\frac{1}{6}\left[\frac{I_s}{m_s}+\frac{I_d}{m_d}\right]$	122	115 ± 12
$K^{\pm*}\to K^\pm\gamma$	$\frac{1}{6}\left[\frac{I_s}{m_s}-\frac{2I_u}{m_u}\right]$	56	50 ± 5
$\omega^0\to\pi^0\gamma$	$-\frac{1}{6}\left[\frac{I_d}{m_d}+\frac{2I_u}{m_u}\right]$	579	717 ± 51
$\rho\to\eta\gamma$	$\frac{I_u}{2m_u}\cos\varphi$	62	57 ± 12
$\omega\to\eta\gamma$	$\frac{I_u}{6m_u}\cos\varphi$	7.9	4 ± 2
$\phi\to\eta\gamma$	$\frac{I_s}{3m_s}\sin\varphi$	59	56 ± 4
$\eta'\to\omega\gamma$	$\frac{I_u}{6m_u}\sin\varphi$	12	6 ± 1
$\eta'\to\rho\gamma$	$\frac{I_u}{2m_u}\sin\varphi$	132	62 ± 9

IV. CONCLUSIONS

In studying the flavor-changing and conserving radiative decays $P\to V\gamma$ or $V\to P\gamma$, a relativistic quark model has been used in the following sense.

(i) In writing the relevant operator responsible for the above decays in terms of a two-component Pauli form, no relativistic approximation has been made.

(ii) Recoil effects due to the large mass differences between the initial and final meson states are fully taken into account in calculating the overlap integrals in the evaluation of the matrix element of the relevant operator.

(iii) the Schrödinger equation appropriate to a short-range Coulomb-type potential arising due to one-gluon exchange and a QCD-inspired long-range confining potential is used, but only to the extent in fixing the variational parameters in a trial wave function which is used in calculating the overlap integrals.

(iv) The new element in our calculation is the use of a screened long-range confining potential instead of a pure linear term.

For the flavor-changing radiative decays of the B meson, the results are found to be very sensitive to the screening parameter introduced previously by lattice-gauge-theory calculations in the study of the spectroscopy of a heavy quark (antiquark) and light antiquark (quark) system. We extended the above model to study radiative decays of ground-state mesons, which are flavor conserving. It is found that the relativistic corrections are much more pronounced for the light sector. The recoil as well as relativistic corrections are taken into account in evaluating the decay rates. Here the screening length is used as a free parameter to be fixed by making a best fit to the decay rates. In fact, one obtains a good agreement with the experimental data (except possibly for ω, the experimental value of which has fluctuated over the years) for mesons involving strange quarks with a screening length μ^{-1}, which is about 3 times as large as for the B meson, while it is about 4 times larger for the case of nonstrange mesons. As one would expect, we find that the screening length $1/\mu$ increases as we move from the heavy to the light sector. Thus radiative decays are a source of information on the quark-antiquark interaction at large distances, particularly when lighter quarks are involved.

ACKNOWLEDGMENTS

The authors would like to acknowledge the support of King Fahd University of Petroleum and Minerals for this work, in particular, the computational facilities made available to them. One of us (R.) would like to thank Professor Patrick J. O'Donnell for pointing out an error in Ref. [8].

[1] M. K. Gaillard and B. W. Lee, Phys Rev. D **10**, 897 (1974); E. Ma, *ibid.* **9**, 3103 (1974).

[2] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[3] V. O. Galkin and R. N. Faustov, Yad. Fiz. **44**, 1575 (1986) [Sov. J. Nucl. Phys. **44**, 1023 (1986)].

[4] V. O. Galkin and R. N. Faustov, in *Quarks '88* Proceedings of the International Seminar, Tbilisi, USSR, edited by A. N. Tavkhelidze, V. A. Matveev, S. A. Larin, and S. Yu. Khlelinikov (World Scientific, Singapore, 1989), p. 624.

[5] E. Laerman, F. Longhamer, I. Schmitt, and P. M. Zerwas, Phys. Lett. B **173**, 437 (1986).

[6] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981); N. G. Deshpande and G. Eilam, Phys. Rev. D **26**, 2463 (1982); N. G. Deshpande and M. Nazerimonfared, Nucl. Phys. **B213**, 390 (1983).

[7] S. Bertolini, F. Borzumati, and A. Masiero, Phys. Rev. Lett. **59**, 180 (1987).

[8] N. G. Deshpande, P. Lo, and J. Trampetic, Z. Phys. C **40**, 369 (1988) (there is a missing term in the expression of $f_1(0)$ given in this reference as pointed out in Ref. [10] below).

[9] T. Altomari, Phys. Rev. D **37**, 677 (1988).

[10] P. J. O'Donnell and H. K. K. Tung, Phys. Rev. D **44**, 741 (1991).

[11] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989); C. Hayne and N. Isgur, *ibid.* **25**, 1944 (1982); N. Isgur, *ibid.* **12**, 3666 (1975).

[12] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).