CP violation in the decays $B^0 \to \Psi K_S$ and $B^0 \to \pi^+\pi^-$: A probe for new physics

João M. Soares and Lincoln Wolfenstein
Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
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We point out the strong correlation between the CP-violating asymmetries in the decays $B^0 \to \Psi K_s$ and $B^0 \to \pi^+\pi^-$ that is predicted by the Kobayashi-Maskawa model. This results in a very restricted allowed region for the asymmetries, and so their measurement will provide a powerful test of the standard model. To show how new physics may give very different results, we look at the predictions of a simple model where both the standard model and superweak contributions to the K^0 and B^0 mass matrices coexist. We find that, for possible values of the parameters, practically any values are allowed for the asymmetries.

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The most striking prediction of the standard model of CP violation (the Kobayashi-Maskawa model) is the large asymmetries in B decays [1]. A major goal of proposed B factories is the discovery of these quantities. Here we look at how such experiments could test the standard model, and the ways in which new physics may show up [2].

The CP-violating asymmetry in the decay $B^0 \to f$, where f is a CP eigenstate, is parametrized by

$$a(f) \equiv \frac{\Gamma(B^0\to f)-\Gamma(\bar{B}^0\to f)}{\Gamma(B^0\to f)+\Gamma(\bar{B}^0\to f)} = \tilde{a}(f)\eta_{CP}(f)\frac{x}{1+x^2} , \quad (1)$$

where $\eta_{CP}(f)$ is the CP phase of the final state f, and $x \equiv \Delta M/\Gamma$. In the standard model, this and all other CP-violating quantities originate in the complex nature of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In our parametrization [3], to order λ^3, the only complex matrix elements are V_{td} and V_{ub}, given by

$$V_{td} = |V_{td}|e^{-i\beta} = A\lambda^3(1-\rho-i\eta) , \quad (2)$$

$$V_{ub} = |V_{ub}|e^{-i\gamma} = A\lambda^3(\rho-i\eta) , \quad (3)$$

where $\lambda=0.22$ and $A=0.9\pm0.1$ (corresponding to $|V_{cb}|=0.045\pm0.005$). The parameters ρ and η are constrained by the measurement of $|V_{ub}|$ (in semileptonic B decays), x_d (in $B_d^0-\bar{B}_d^0$ oscillations), and $|\varepsilon_K|$ (in CP-violating neutral K decays) [4]. For illustrative purposes we have set $m_t = 140$ GeV and used the constraints

$$\rho^2+\eta^2=(0.50\pm0.09)^2 , \quad (4)$$

$$(1-\rho)^2+\eta^2=1.25^{+1.62}_{-0.64} , \quad (5)$$

$$\eta=\frac{1}{0.7+2.63A^2(1-\rho)}(0.87^{+0.79}_{-0.34}) . \quad (6)$$

These correspond to $B_K=\frac{3}{4}\pm\frac{1}{4}$, $f_B\sqrt{B_B}\eta_{QCD}=(150\pm50)$ MeV, $\tau_B=(1.18\pm0.14)\times10^{-12}$ sec, $|V_{ub}|/|V_{cb}|=0.11\pm0.02$, and $x_d=0.66\pm0.11$. The curves that correspond to Eqs. (4)–(6) are plotted in Fig. 1, showing the allowed region in the (ρ,η) plane.

The prototype experiment, and likely the first to be carried out, is the measurement of the asymmetry in $B^0 \to \Psi K_S$, or other decays associated with the quark transition $b \to c+\bar{c}+s$. For these decays, in the standard model,

$$\tilde{a}(\Psi K_S) = \sin2\beta = \frac{2\eta(1-\rho)}{\eta^2+(1-\rho)^2} , \quad (7)$$

and given the constraints on the CKM parameters, any value between 0.1 and 1.0 is allowed.

Another class of experiments on CP violation corresponds to the suppressed decays of the type $b \to u+\bar{u}+d$, such as $B^0 \to \pi^+\pi^-$. In the standard model, and neglecting penguin diagrams,

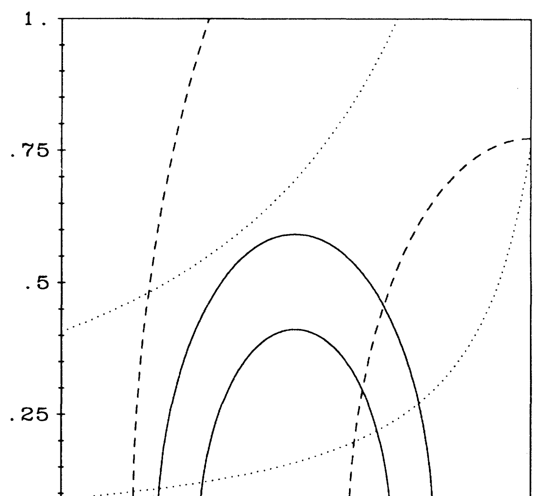

FIG. 1. The standard model constraints on the CKM parameters ρ and η, from the measurement of $|V_{ub}|$ (full line), x_d (dashed line), and $|\varepsilon_K|$ (dotted line), for $m_t = 140$ GeV.

$$\tilde{a}(\pi^+\pi^-) = \sin 2(\beta+\gamma) = \frac{2\eta[\rho-\rho^2-\eta^2]}{(\rho^2+\eta^2)[(1-\rho)^2+\eta^2]} . \quad (8)$$

Combining Eqs. (7) and (8), we find

$$\frac{\tilde{a}(\pi^+\pi^-)}{\tilde{a}(\Psi K_S)} = \frac{\rho-K}{K(1-\rho)} , \quad (9)$$

with $K=\rho^2+\eta^2$. Using the constraints on the CKM parameters, we show in Fig. 2 the allowed values of $(\tilde{a}(\Psi K_S), \tilde{a}(\pi^+\pi^-))$, in the standard model, for $m_t=140$ GeV. The region is slightly expanded if a penguin contribution of the expected magnitude [5] is added to the $B^0 \to \pi^+\pi^-$ decay amplitude. The first really significant test of the standard model that is likely to be made is the correlation between two asymmetries such as $\tilde{a}(\Psi K_S)$ and $\tilde{a}(\pi^+\pi^-)$. It is this correlation that is illustrated in Fig. 2 for $m_t=140$ GeV. For fixed K (determined by $|V_{ub}|$) the correlation is given in parametric form by Eq. (9) with ρ as a parameter. The width of the band in Fig. 2 is due to the uncertainty in K; as K increases one moves across the band to the right. The ends of the band are determined by the extreme values of ρ, which depends on f_B, B_K, and m_t. For larger values of m_t the extreme right-handed part of the band is narrower and the upper left-hand boundary extends further upward. For smaller values of m_t, the upper boundary is much lower and the allowed region is drastically reduced. The dashed line in Fig. 2 shows the prediction of the superweak theory [6], that the asymmetry parameter $\tilde{a}$ is the same for all final states.

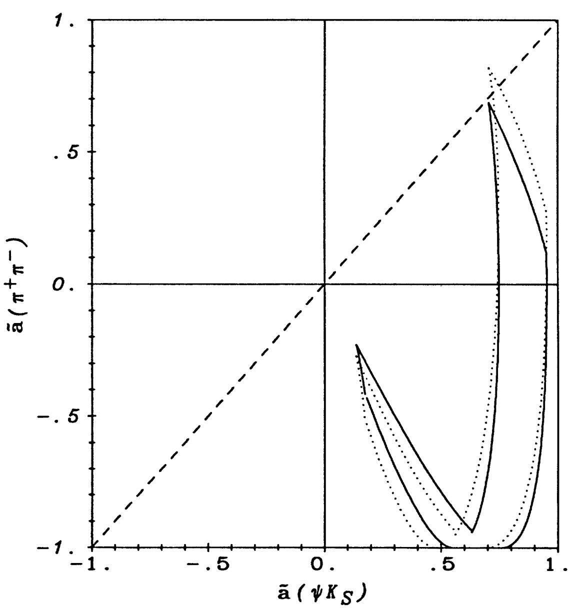

FIG. 2. The allowed region on the asymmetries plane, in the standard model, corresponds to the area enclosed by the full line. The dotted line is the result obtained when the penguin contribution to $\tilde{a}(\pi^+\pi^-)$ is considered. The dashed line gives the superweak prediction.

It is clear from Fig. 2 that four possibilities may emerge when measurements of $\tilde{a}(\Psi K_S)$ and $\tilde{a}(\pi^+\pi^-)$ have been made, with reasonable errors.

(1) The results may be consistent with both the standard model and the superweak theory, as noted by Winstein [7]. For our illustrative example this corresponds to a narrow region where both asymmetries are about 0.7. For larger values of m_t this is a somewhat larger region.

(2) The results are clearly inconsistent with the superweak theory but are consistent with the region allowed by the standard model in Fig. 2.

(3) The results are consistent with the superweak theory (dashed line in Fig. 2) but not with the standard model.

(4) The results are not consistent with either the standard model or the superweak theory.

The major purpose of this work is to explore this last possibility. The simplest model is a combination of the standard model and superweak interactions. We assume that the decay amplitudes are correctly given by the standard model but that there may be new physics contributing to the off-diagonal entries M_{12} in the $K^0-\bar{K}^0$ and $B^0-\bar{B}^0$ mass matrices.

We first consider the original superweak interaction, with a CP-violating term in M_{12} of the K^0 system, but no superweak contribution to the B^0 system. For the K^0 mass matrix, the imaginary part of $-M_{12}(K)$ is

$$m'_K = m'_K(\mathrm{SM}) + m'_K(\mathrm{SW}) , \quad (10)$$

where $m'_K = \sqrt{2}\Delta M_K |\varepsilon_K| \simeq 1.7\times 10^7$ sec^{-1} is the experimental value, and

$$m'_k(\mathrm{SM}) = \left[\frac{A}{0.9}\right]^2 \frac{B_K}{0.75}\eta \times \left[0.7+2.1\left[\frac{A}{0.9}\right]^2(1-\rho)\right] 1.95\times 10^7 \ \mathrm{sec}^{-1} . \quad (11)$$

Because of the additional term in Eq. (10), the curves in the (ρ,η) plane, corresponding to the $|\varepsilon_K|$ constraint, are obtained multiplying those in Fig. 1 by $z=m'_K(\mathrm{SM})/m'_K$. Using the $|V_{ub}|$ constraint, which is not modified by the superweak addition, it follows that

$$|z| \le 4.1 . \quad (12)$$

As z is allowed to vary, the constraints on η from $|\varepsilon_K|$ are lost and the only constraints come from $|V_{ub}|$ and x_d. It is sufficient to allow z to vary from -1 to 1, corresponding to $m'_K(\mathrm{SW})$ varying from twice the experimental value to zero. The result of relaxing the $|\varepsilon_K|$ constraints is that the allowed region is given by Fig. 3. The most striking effect is that the region with opposite signs for the asymmetry parameters is allowed.

We now turn to M_{12} for the B^0 system. For the standard model, M_{12}^{SM} has a phase 2β and modulus

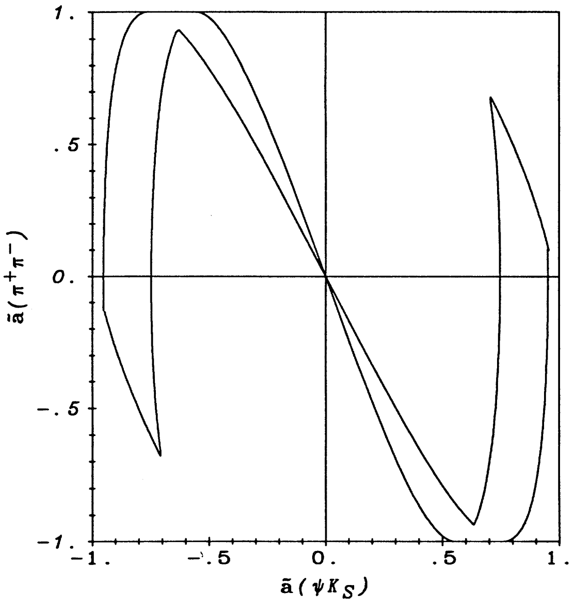

FIG. 3. The asymmetries plane of Fig. 2 when the $|\varepsilon_K|$ constraint is removed.

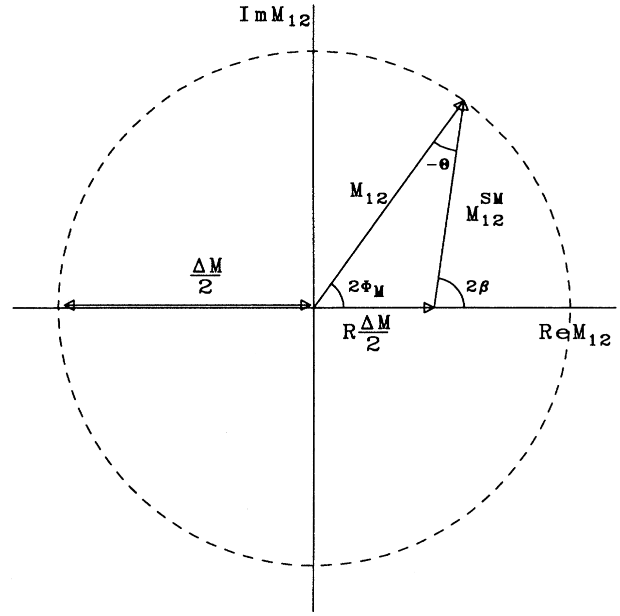

FIG. 4. Representation of $M_{12} = M_{12}^{\rm SM} + R\Delta M/2$ on the complex plane.

$$|M_{12}^{\rm SM}| = [(1-\rho)^2 + \eta^2]\left[\frac{A}{0.9}\right]^2 \times \left[\frac{\eta_{\rm QCD} B_B f_B^2}{(150\ {\rm MeV})^2}\right] 2.24 \times 10^{11}\ {\rm sec}^{-1}\ . \quad (13)$$

We consider, in addition to m'_K(SW) for the K^0 system, a superweak contribution $R\Delta M/2$ to the real part of M_{12}, and assume that the contribution $I\Delta M/2$, to the imaginary part, is negligible. This possibility was proposed by Liu and Wolfenstein [8] on the basis of scaling arguments from the K^0 mass matrix. The distinction between R and I depends on the phase convention of the CKM matrix; we use the phase convention indicated by Eqs. (2) and (3).

In Fig. 4, the two contributions to M_{12} are represented graphically in the complex plane. The phase of M_{12} is

$$2\phi_M = 2\beta + \theta\ . \quad (14)$$

Because of the superweak contribution θ, the asymmetry parameter becomes

$$\tilde{a} = \tilde{a}_{\rm SM} \cos\theta + b \sin\theta\ , \quad (15)$$

with $\tilde{a}_{\rm SM}$ given in Eqs. (7) and (8), and

$$b(\Psi K_S) = \cos 2\beta = \frac{(1-\rho)^2 - \eta^2}{(1-\rho)^2 + \eta^2}\ , \quad (16)$$

$$b(\pi^+\pi^-) = \cos 2(\beta+\gamma) = \frac{\eta^4 - \eta^2[1 + 2\rho(1-\rho)] + \rho^2(1-\rho)^2}{(\rho^2+\eta^2)[(1-\rho)^2+\eta^2]}\ . \quad (17)$$

Because of the superweak contribution to the modulus of M_{12}, it is necessary to find the new curves in the (ρ,η) plane that correspond to the x_d constraint. They are given by

$$\frac{\Delta M}{2} = \left| |M_{12}^{\rm SM}| e^{2i\beta} + R\frac{\Delta M}{2} \right|\ , \quad (18)$$

where $\Delta M \equiv x_d \Gamma = 5.6 \times 10^{11}\ {\rm sec}^{-1}$.

In the following, we choose arbitrary values of R, positive or negative. The asymmetry is given by Eq. (15), with

$$\sin\theta \simeq -R \sin 2\beta \quad (19)$$

and $\cos\theta = \pm\sqrt{1-\sin^2\theta}$. The $\pm$ sign corresponds to the same sign in the equation for the x_d constraint:

$$|M_{12}^{\rm SM}| = \frac{\Delta M}{2}\{-R \cos 2\beta \pm [1 - (R \sin 2\beta)^2]^{1/2}\} \quad (20)$$

that follows from Eq. (18). Using the graphic construction of Fig. 4, and fixing β, it can be seen that Eq. (20) has no solutions if $R > 1$ or if $R < -1/|\sin 2\beta|$, only one solution, corresponding to $\cos\theta > 0$ if $|R| \le 1$, two solutions otherwise. These solutions are subject to the constraint that $M_{12}^{\rm SM}$ is consistent with Eq. (13) for some acceptable choice of parameters. Because of the $|V_{ub}|$ constraint, the range of interest turns out to be restricted to $-5.3 \le R \le 0.95$, for the entire allowed range of (ρ,η).

In Figs. 5(a) and 5(b), we plot the plane of the asymmetries for $R = 0.65$, -0.5, and -2.0. Notice that, as $|R|$ increases, the ΨK_S asymmetry decreases if R is positive, and increases if R is negative, until it gets close to -1 where the behavior is more complicated. Again, this can be easily seen from the graphic representation of M_{12}.

In Fig. 6, we give the allowed region obtained by letting R and z take values across the range of interest. The plane of the asymmetries is almost entirely covered. This is to be compared with the standard model prediction in Fig. 2, which strongly constrains the outcome of the future experiments.

We have included so far two types of superweak interactions, a CP-violating term in the $K^0-\bar{K}^0$ mass matrix and a CP-conserving term in the $B^0-\bar{B}^0$. While this seems strange at first, it is quite logical. It is possible that the CP-conserving term in the $K^0-\bar{K}^0$ mass matrix is much larger than the imaginary part; indeed it may be roughly as large as the experimental value of ΔM_K, because of the uncertainty in the standard model long-distance contribution to ΔM_K. However, this term turns out to have no effect on the calculation of CP-violating parameters so we have not made use of it. In going from the K^0 system to the B^0 system, it is natural to consider that the superweak interaction scales [8,9] as $(m_b/m_s)^n$ where n might be 0, 1, or 2. Even with $n=2$ it is hard to get a large enough value of I for the $B^0-\bar{B}^0$ mass matrix to produce a large effect. On the other hand, if we allow for a large CP-conserving part of the K^0 case, one can easily obtain values of R in the range we have considered.

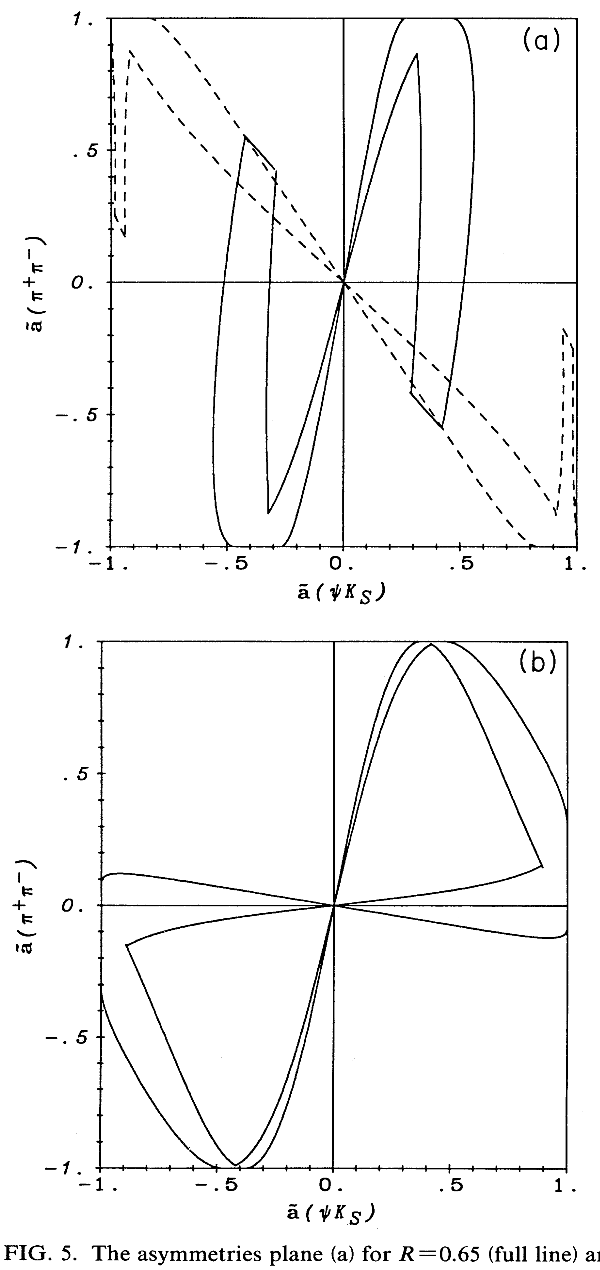

FIG. 5. The asymmetries plane (a) for $R=0.65$ (full line) and $R=-0.5$ (dashed line), (b) for $R=-2.0$.

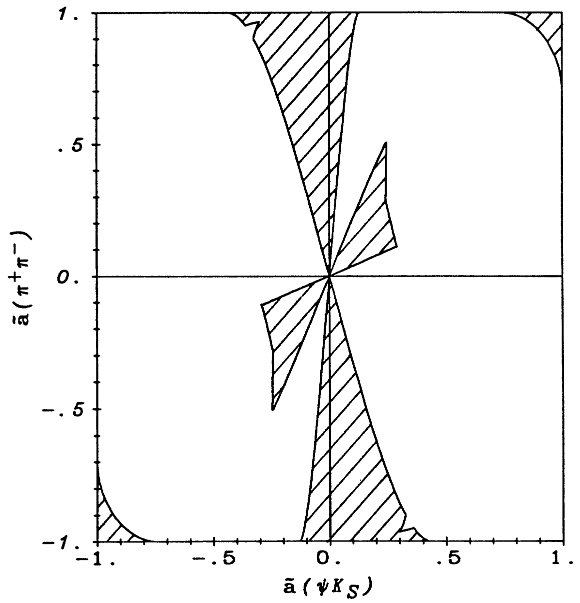

FIG. 6. The blank area is the allowed region on the asymmetries plane combining different values of R across the range $-5.3 \leq R \leq 0.95$.

The assumption that I/R is small depends on the phase convention. We believe that it could be significant in the phase convention we use. The point is that one can imagine requiring the superweak interaction to have a small value of I/R in a quark basis defined before the CKM mixing is introduced. In our phase convention the CP-violating part of the CKM matrix is of order $\eta\lambda^3$, so that it is reasonable that a small value of I/R would only be changed by a small amount as a result of the CKM mixing.

On the other hand, note that the angle θ in Eq. (14) is a rephasing invariant, and a measurable quantity. The unitarity triangle can be constructed, in principle, by measuring its sides from the CP-conserving decays $b \to u l^- \bar{\nu}$ and $K \to \pi\nu\bar{\nu}$ [10], except for an ambiguity as to the sign of η. The parameter θ is then determined from the asymmetry $\tilde{a}(\Psi K_S)$ using Eq. (15), up to a fourfold ambiguity. The ambiguity is removed (except for some special cases), and the sign of η is determined, by the asymmetry $\tilde{a}(\pi^+\pi^-)$. The other rephasing invariant is R^2+I^2; it could be determined from ΔM if the large uncertainty in $\eta_{QCD} B_B f_B^2$ were to be reduced.

We have defined ΔM to be positive by labeling the two eigenstates according to their masses. It is assumed that it is a good approximation to neglect $\Delta\Gamma$, and so write the asymmetry as in Eq. (1). Then, the sign of the asymmetry depends only on the sign of $\tilde{a}$. For $B^0 \to \Psi K_S$, it is that of η. In fact, for any value of R, $\sin(2\beta+\theta)$ has the same sign as $\sin(2\beta)$ (see Fig. 4). Then, if the only superweak effect is R for the B^0 system ($z=1$), $\tilde{a}(\Psi K_S)$ is always positive. Negative values require a large superweak effect in the K^0 system ($z<0$) or a large contribution from I.

It is at first somewhat surprising that the allowed range of asymmetries is expanded so much by varying R. The reason is that for the standard model the asymmetries are almost completely determined by the CKM parameters (ρ,η) and so the different asymmetries are strongly correlated. This correlation is lost when one introduces the extra parameter R, thus leaving only the small nonallowed region in Fig. 6. If we allow for a significant value of I in addition, then nearly all of this region will be filled.

To summarize, the measurement of two asymmetries in B^0 decays, one for the quark transition $b \to c + \bar{c} + s$ ($B^0 \to \Psi K_S$) and one for $b \to u + \bar{u} + d$ ($B^0 \to \pi^+\pi^-$), can provide an important test for the standard model as shown in Fig. 2. While this curve depends on our choice of constraints (see Fig. 1), our major point is independent of this particular illustration. It is clear that there is a large range of values for these measurements that could provide signals of new superweak interactions.

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