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Leptonic decay of light vector mesons in an independent quark model

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Leptonic decay widths of light vector mesons are calculated in a framework based on the independent quark model with a scalar-vector harmonic potential. Assuming a strong correlation to exist between the quark-antiquark momenta inside the meson, so as to make their total momentum identically zero in the center-of-mass frame of the meson, we extract the quark and antiquark momentum distribution amplitudes from the bound quark eigenmode. Using the model parameters determined from earlier studies, we arrive at the leptonic decay widths of (ρ,ω,ϕ) as (6.26 keV, 0.67 keV, 1.58 keV) which are in very good agreement with the respective experimental data (6.77 ± 0.32 keV, 0.6 ± 0.02 keV, 1.37 ± 0.05 keV).

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I. INTRODUCTION

Though quantum chromodynamics (QCD) is considered to be the underlying theory of strong interaction between quarks and gluons at the structural levels of hadrons, many low-energy phenomena such as spectroscopy, static electromagnetic properties, as well as weak and electromagnetic decay, etc., cannot be explained in a straightforward manner from first-principles QCD. Therefore one needs to resort to phenomenological models. Out of many such successful models developed so far, a chiral potential model with an equally mixed scalar-vector harmonic potential [1] of independent quarks in a relativistic Dirac framework has been used successfully to study several low-energy phenomena in the baryonic sector such as octet-baryon masses [2], magnetic moments [3], weak electric form factors [4], nucleon electromagnetic form factors, and charge radii [5]. This model has also been quite successful in explaining pion mass, its decay constant [6], $(\rho-\pi)$ as well as $(\rho-\omega)$-mass splittings [7] and the radiative decay [8] of ordinary light mesons. In view of this wide ranging application of the model to both baryons and mesons in the light flavor sector, it has proved to be rather a simple and successful alternative to the cloudy bag model (CBM) [9], the modern hybrid version of the bag model endowed with chiral symmetry. The purpose of the present work is to extend its applicability to the study of the leptonic decay of vector mesons in a light flavor sector such as ρ, ω, and ϕ. The leptonic decay width of heavier vector mesons in the charm and bottom flavor sector has been extensively studied in the nonrelativistic approach through the Van Royen–Weisskopf formula with appropriate radiative corrections [10]. However the same approach is not suitable for ordinary vector mesons in the light flavor sector, where the constituent quark dynamics is more relativistic. The present model, which is based on the ansatz of the dominant confining interaction phenomenologically taken in the form of an equally mixed scalar-vector harmonic potential, can most suitably be applied to vector mesons such as ρ, ω, and ϕ where the short-range Coulomb-like vector interaction can be believed to have a less prominent role.

Our approach here is quite similar to that of Margolis and Mendel [11] in the bag model where we follow the usual method of positronium annihilation [12]. Here we assume that the constituent quark-antiquark pair inside the meson annihilates mainly to a single virtual photon which subsequently gives rise to a lepton pair. We further assume that the center-of-mass motion does not play any important role in the dynamics of the system during decay. In that case one can consider a strong correlation to exist between the quark-antiquark momentum so as to have the total momentum identically zero in the center-of-mass frame of the vector meson. With such a consideration the ground state of the decaying vector meson can suitably be represented with the appropriate momentum distribution of the bound quark-antiquark pair in the corresponding SU(6) spin-flavor configuration. Then the transition probability amplitude for the leptonic decay, calculated from the appropriate Feynman diagram, can be expressed effectively as the free quark-antiquark pair-annihilation amplitude integrated over the model momentum distribution. There is of course an obvious difficulty relating to the energy conservation at the quark-photon vertex since the sum total of the kinetic energy alone carried by the annihilating quark-antiquark in this process is not equal to the mass energy of the decaying meson at its rest frame. This is a common feature with all phenomenological models based on leading-order calculations in the absence of a rigorous field-theoretic formulation of bound quark-antiquark annihilation inside the meson. Therefore we content ourselves with accepting the usual assumption that the differential amount of energy is somehow made available to the photon when quark-antiquark annihilation occurs with the disappearance of the meson bound state. In Sec. II, we briefly describe the framework of our model determining the quark-antiquark momentum distribution amplitude in the ground state of the vector meson. We obtain the transition matrix element for the leptonic decay of light vector mesons in Sec. III, where we derive also the corresponding expression for the decay widths. Finally Sec. IV provides the results and discussions.

II. QUARK-ANTIQUARK MOMENTUM DISTRIBUTION

The study of leptonic decay of vector mesons using a field-theoretic calculation requires an appropriate representation of the initial state of the decaying vector mesons in terms of the constituent quark and antiquark with their respective momenta and spin. But the bound constituent quark and antiquark inside the meson are in definite energy states having no definite momenta. Nevertheless one can find out the momentum distribution amplitude for the constituent quark and antiquark inside the meson immediately before their annihilation to a lepton pair. This can be done by a suitable momentum space projection of the corresponding bound quark orbital derivable in a model, for which one may have to rely on certain simplifying assumptions. In view of this it is worthwhile to present briefly the outline and certain conventions of the model adopted here for our calculations. According to this model a light hadron in general is pictured as a color-singlet assembly of a quark and an antiquark independently confined by an average flavor-independent potential of the form [1]

$$U(r)=\tfrac{1}{2}(1+\gamma^0)(ar^2+V_0)\ . \tag{1}$$

This potential form is taken in the model as a phenomenological representation of the confining interaction which is expected to be generated by a nonperturbative multigluon mechanism. The quark-gluon interaction at short distance originating from one-gluon exchange and the quark-pion interaction required in the nonstrange flavor sector to preserve chiral symmetry are presumed here to be residual interactions compared to the dominant confining interaction. Although these residual interactions treated perturbatively in the model are crucial in determining the mass splittings [2,7] in light hadron spectroscopy, their role in hadronic decay processes is considered less significant. Therefore to a first approximation it is believed that the zeroth-order quark dynamics inside the meson core generated by the confining part of the interaction phenomenologically represented by $U(r)$ in Eq. (1) can provide an adequate description of the leptonic decay of vector mesons.

The quark Lagrangian density in zeroth order in such a picture is

$$\mathcal{L}_q^0(x)=\bar{\psi}_q(x)[(i/2)\gamma^\mu\overleftrightarrow{\partial}_\mu-m_{q2}-U(r)]\psi_q(x)\ . \tag{2}$$

Then the ensuing Dirac equation with

$$E_q'-(E_q-V_0/2),\quad m_q'=(m_q+V_0/2)\ ,$$
$$\lambda_q=(E_q'+m_q'),\quad r_{0q}=(a\lambda_q)^{-1/4} \tag{3}$$

admits static solutions of positive and negative energy in zeroth order which for the ground state of the meson can be obtained in the form

$$\Phi_{q\lambda}^{(+)}(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} ig_q(r)/r \\ \sigma\cdot\hat{\mathbf{r}}f_q(r)/r\end{bmatrix}\chi_\lambda\ ,$$
$$\Phi_{q\lambda}^{(-)}(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} \sigma\cdot\hat{r}f_q(r)/r \\ -ig_q(r)/r\end{bmatrix}\tilde{\chi}_\lambda\ . \tag{4}$$

The two-component spinors χ_λ and $\tilde{\chi}_\lambda$ here stand for

$$\chi_\uparrow=\begin{bmatrix}1\\0\end{bmatrix},\quad \chi_\downarrow=\begin{bmatrix}0\\1\end{bmatrix}\ ,$$

and

$$\tilde{\chi}_\uparrow=\begin{bmatrix}0\\-i\end{bmatrix},\quad \tilde{\chi}_\downarrow=\begin{bmatrix}i\\0\end{bmatrix}\ ,$$

respectively. The reduced radial parts in the upper and lower component solutions corresponding to a quark flavor q are

$$g_q(r)=\mathcal{N}_q(r/r_{0q})\exp(-r^2/2r_{0q}^2)\ ,$$
$$f_q(r)=-(\mathcal{N}_q/\lambda_q r_{0q})(r/r_{0q})^2\exp(-r^2/2r_{0q}^2)\ , \tag{5}$$

where the normalization factor $\mathcal{N}_q$ is given by the expression

$$\mathcal{N}_q^2=\frac{8\lambda_q}{\sqrt{\pi}r_{0q}}\Big/(3E_q'+m_q')\ . \tag{6}$$

The quark binding energy of zeroth order in the meson ground state is derivable from the bound-state condition

$$\sqrt{\lambda_q/a}\,(E_q'-m_q')=3\ . \tag{7}$$

Thus knowing the quark-antiquark eigenmodes in the ground state of the meson, it is possible to obtain their corresponding momentum distribution amplitude. If $G_q(\mathbf{p},\lambda,\lambda')$ is the amplitude for finding a bound quark of flavor q in its eigenmode $\Phi_{q\lambda}^{(+)}(\mathbf{r})$ in a state of definite momentum $\mathbf{p}$ and spin projection λ', then it can be given by

$$\Phi_{q\lambda}^{(+)}(\mathbf{r})=\frac{1}{(2\pi)^{3/2}}\sum_{\lambda'}\int d\mathbf{p}\,G_q(\mathbf{p},\lambda,\lambda')\sqrt{m_q/E_p}\,U_q(\mathbf{p},\lambda')\times\exp(i\mathbf{p}\cdot\mathbf{r})\ , \tag{8}$$

where $U_q(\mathbf{p},\lambda')$ is the usual free Dirac spinor which is normalized according to the relations

$$U_q^\dagger(\mathbf{p},\lambda_1)U_q(\mathbf{p},\lambda_2)=(E_p/m_q)\delta_{\lambda_1\lambda_2}=(\sqrt{\mathbf{p}^2+m_q^2}/m_q)\delta_{\lambda_1\lambda_2}\ ,$$
$$\bar{U}_q(\mathbf{p},\lambda_1)U_q(\mathbf{p},\lambda_2)=\delta_{\lambda_1\lambda_2}\ . \tag{9}$$

Now Eq. (8) can easily be inverted to give

$$G_q(\mathbf{p},\lambda,\lambda')=\frac{1}{(2\pi)^{3/2}}\sqrt{m_q/E_p}\,U^\dagger(\mathbf{p},\lambda')\times\int d\mathbf{r}\,\Phi_{q\lambda}^{(+)}(\mathbf{r})\exp(-i\mathbf{p}\cdot\mathbf{r}) \ . \tag{10}$$

Taking $\Phi_{q\lambda}^{(+)}(\mathbf{r})$ as provided in Eqs. (4)–(7), with $\alpha=1/2r_{0q}^2$, one can obtain

$$\int d\mathbf{r}\,\Phi_{q\lambda}^{(+)}(\mathbf{r})\exp(-i\mathbf{p}\cdot\mathbf{r}) =\frac{i\pi\mathcal{N}_q}{\sqrt{2}\alpha}\exp(-p^2/4\alpha)\begin{bmatrix}1\\ \sigma\cdot\mathbf{p}/\lambda_q\end{bmatrix}\chi_\lambda \ , \tag{11}$$

which when substituted into Eq. (10) leads to

$$G_q(\mathbf{p},\lambda,\lambda')=G_q(p)\delta_{\lambda\lambda'} \ , \tag{12}$$

where

$$G_q(p)=\frac{1}{(2\pi)^{3/2}}\frac{i\pi\mathcal{N}_q}{2\alpha\lambda_q}(E_p+E_q)\times\sqrt{(1+m_q/E_p)}\exp(-p^2/4\alpha) \ . \tag{13}$$

Now following Margolis and Mendel [11], we make a crucial assumption regarding the existence of a strong correlation between momenta of the quark and antiquark inside the meson so as to have their total momenta identically zero in the center-of-mass frame of the vector meson. In that case the momentum distribution amplitude $\tilde{G}_V(\mathbf{p}_1,\mathbf{p}_2)$ for finding a quark of momentum $\mathbf{p}_1$ together with its antiquark of momentum $\mathbf{p}_2=-\mathbf{p}_1$, must be equal to the momentum distribution amplitude $G_q(p_1)$ of finding a quark of momentum $\mathbf{p}_1$. Then one can represent the ground state of a neutral vector meson such as (ρ,ω,ϕ) with a particular spin projection S_V as

$$|V,S_V\rangle=\sqrt{3}\sum_{(q,\lambda_1,\lambda_2)\in(V,S_V)}\int d\mathbf{p}_1 d\mathbf{p}_2\delta^{(3)}(\mathbf{p}_1+\mathbf{p}_2)\tilde{G}_V(\mathbf{p}_1,\mathbf{p}_2)C_{\lambda_1\lambda_2}^{S_V}\zeta_q^V b_q^\dagger(\mathbf{p}_1,\lambda_1)\tilde{b}_q^\dagger(\mathbf{p}_2,\lambda_2)|0\rangle \ ,$$

which reduces to

$$|V,S_V\rangle=\sqrt{3}\sum_{(q,\lambda_1,\lambda_2)\in(V,S_V)}\int d\mathbf{p}_1 G_q(\mathbf{p}_1)C_{\lambda_1\lambda_2}^{S_V}\zeta_q^V b_q^\dagger(\mathbf{p}_1,\lambda_1)\tilde{b}_q^\dagger(-\mathbf{p}_1,\lambda_2)|0\rangle \ . \tag{14}$$

Here $b_q^\dagger(\mathbf{p}_1,\lambda)$ and $\tilde{b}_q^\dagger(-\mathbf{p}_1,\lambda)$ operating on the vacuum state are quark and antiquark creation operators, respectively. The summation with the flavor coefficient ζ_q^V and the spin-configuration coefficient $C_{\lambda_1\lambda_2}^{S_V}$ represents the appropriate SU(6) spin-flavor structure of the particular vector meson V with its spin projection S_V. The factor $\sqrt{3}$ is effectively due to the color-singlet configuration. Thus the initial vector-meson state represented in the model by the expression in Eq. (14) with the momentum distribution amplitude as given in Eq. (13) can enable one to determine the transition probability amplitude for the leptonic decay.

III. LEPTONIC DECAY WIDTH

Assuming that the main contribution to the leptonic decay process of neutral vector mesons such as (ρ,ω,ϕ) comes from single virtual-phonon annihilation of the bound quark-antiquark pair inside the meson, we can illustrate it by the corresponding Feynman diagram in Fig. 1. Then we can write the S-matrix element in configuration space as

$$S_{fi}=\langle e^-(k_1,\delta_1)e^+(k_2,\delta_2)|(-ie^2)\int d^4x_1 d^4x_2\left[\bar{\psi}_e^{(-)}(x_2)\gamma^\mu\psi_e^{(-)}(x_2)\mathcal{D}_{\mu\nu}(x_2-x_1)\sum_q e_q\bar{\psi}_q^{(+)}(x_1)\gamma^\nu\psi_q^{(+)}(x_1)\right]|V,S_V\rangle \ , \tag{15}$$

where $\mathcal{D}_{\mu\nu}(x_2-x_1)$ is the photon propagator. The quark and lepton field expansions are taken as

$$\psi_q(x)=\sum_{\lambda'}\frac{1}{(2\pi)^{3/2}}\int d\mathbf{p}'\sqrt{m_q/E_{p'}}[b_q(\mathbf{p}',\lambda')U_q(\mathbf{p}',\lambda')\exp(-ip'x)+\tilde{b}_q^\dagger(\mathbf{p}',\lambda')V_q(\mathbf{p}',\lambda')\exp(ip'x)] \tag{16}$$

and

$$\psi_e(x)=\sum_{\delta'}\frac{1}{(2\pi)^{3/2}}\int d\mathbf{k}'\sqrt{m_e/E_{k'}}[d_e(\mathbf{k}',\delta')U_e(\mathbf{k}',\delta')\exp(-ik'x)+\tilde{d}_e^\dagger(\mathbf{k}',\delta')V_e(\mathbf{k}',\delta')\exp(ik'x)] \ . \tag{17}$$

Now simplifying the leptonic and hadronic parts separately by a vacuum-insertion technique and using the initial vector-meson state as per Eq. (14), one can obtain, with $\hat{O}\equiv(1,0,0,0)$ and $(E_{p_1}+E_{p_2})\simeq M_V$,

$$S_{fi}=-i(2\pi)^4\delta^{(4)}(k_1+k_2-\hat{O}M_V)\times\mathcal{M}_{S_V}(k_1,k_2,\delta_1,\delta_2) \ , \tag{18}$$

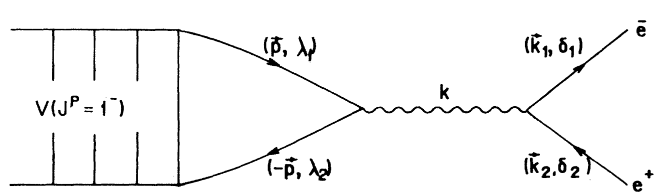

FIG. 1. One-photon contribution to the leptonic decay of vector mesons.

where $\mathcal{M}(k_1,k_2,\delta_1,\delta_2)$ is the transition matrix element for the decay process. We must mention here that in extracting the correct δ function relating to the energy conservation at the photon-hadron vertex there is an obvious difficulty. This is due to the fact that, in a zeroth-order description such as the present one, the total energy available to the lepton pair produced comes out to be the sum total kinetic energies $(E_{p_1}+E_{p_2})$ of the annihilating quark-antiquark pair, which is not equal to the rest energy of the decaying meson. This being a common feature with such leading-order calculation one usually assumes that the differential amount of energy is somehow made available to the photon when quark-antiquark annihilation occurs with the vanishing of the meson bound state. With this consideration, $(E_{p_1}+E_{p_2})$ in the argument of the delta function in Eq. (18) has been replaced by the meson mass M_V. If we write

$$l_\mu(k_1,k_2,\delta_1,\delta_2)=m_e\overline{U}_e(\mathbf{k}_1,\delta_1)\gamma_\mu V_e(\mathbf{k}_2,\delta_2)/\sqrt{E_{k_1}E_{k_2}}\ , \tag{19}$$

$$h^\mu_{S_V}=\sum_{(\lambda_1,\lambda_2)\in S_V}\int d\mathbf{p}\, G_q(p)(m_q/E_p)C^{S_V}_{\lambda_1\lambda_2}\times\overline{V}_q(-\mathbf{p},\lambda_2)\gamma^\mu U_q(\mathbf{p},\lambda_1)\ ,$$

then the transition matrix element in Eq. (18) can be expressed as

$$\mathcal{M}_{S_V}(k_1,k_2,\delta_1,\delta_2)=\frac{i}{(2\pi)^3}\sqrt{3}e^2\langle e_q\rangle_V\times l_\mu(k_1,k_2,\delta_1,\delta_2)h^\mu_{S_V}/(k_1+k_2)^2\ , \tag{20}$$

where $\langle e_q\rangle_V$ arising out of the specific flavor configuration of individual vector mesons stands for specific values such as

$$\langle e_q\rangle_{\rho,\omega,\phi}\equiv\left[\frac{1}{\sqrt{2}},\frac{1}{3\sqrt{2}},\frac{1}{\sqrt{3}}\right]\ .$$

In fact the timelike component of $h^\mu_{S_V}$ in Eq. (19) vanishes identically for all S_V, since $V_q^\dagger(-\mathbf{p},\lambda_2)U_q(\mathbf{p},\lambda_1)=0$. So the transition matrix element effectively becomes

$$\mathcal{M}_{S_V}(k_1,k_2,\delta_1,\delta_2)=\frac{-i}{(2\pi)^3}e^2\sqrt{3}\langle e_q\rangle_V\mathbf{l}(k_1,k_2,\delta_1,\delta_2)\cdot\mathbf{h}_{S_V}/(k_1+k_2)^2\ . \tag{21}$$

The leptonic decay width of a vector meson can now be calculated from the expression

$$\Gamma(V\to e^+e^-)=\frac{1}{(2\pi)^2}\int d\mathbf{k}_1 d\mathbf{k}_2\delta^4(k_1+k_2-\hat{O}M_V)\times\overline{\sum_{S_V,\delta_1,\delta_2}}|\mathcal{M}_{S_V}(k_1,k_2,\delta_1,\delta_2)|^2\ , \tag{22}$$

where $\overline{\sum}_{S_V,\delta_1,\delta_2}$ stands for the sum over the final-state lepton spins (δ_1,δ_2) and the average over the initial vector-meson spin. In fact it can be found from the explicit calculations that the contributions of $\mathcal{M}_{S_V}$ to $\Gamma(V\to e^+e^-)$ for $S_V=\pm1,0$ separately are all equal in magnitude. Therefore the initial-state spin averaging would be equivalent to taking the contribution of $S_V=+1$ (for example) only for which the lone nonvanishing spin-configuration coefficient $C^{S_V}_{\lambda_1\lambda_2}\equiv C^{S_V=+1}_{\uparrow\uparrow}=+1$. With such a consideration, one can write

$$\Gamma(V\to e^+e^-)=\frac{12\alpha^2_{\rm em}\langle e_q\rangle^2_V}{(2\pi)^6}L^{ij}H_{ij}(S_V=+1) \tag{23}$$

when

$$L^{ij}=\int\frac{d\mathbf{k}_1 d\mathbf{k}_2}{4E_{k_1}E_{k_2}}\delta^{(4)}(k_1+k_2-\hat{O}M_V)\times{\rm Tr}[(\not{k}_1-m_e)\gamma^i(\not{k}_2+m_e)\gamma^j](k_1+k_2)^4\ , \tag{24}$$

$$H_{ij}(S_V=+1)=\int\frac{d\mathbf{p}\,d\mathbf{p}'}{4E_pE_{p'}}G_q(p)G^*_q(p')\times[\overline{V}_q(-\mathbf{p}\uparrow)\gamma_iU_q(\mathbf{p}\uparrow)\times\overline{U}_q(\mathbf{p}'\uparrow)\gamma_jV_q(-\mathbf{p}'\uparrow)]\ . \tag{25}$$

After some standard algebra in evaluating the trace followed by the subsequent integration in Eq. (24), one can easily find

$$L^{ij}=\frac{2\pi}{3M_V^2}\delta^{ij}\ , \tag{26}$$

which leads to

$$\Gamma(V\to e^+e^-)=\frac{4\alpha^2_{\rm em}\langle e_q\rangle^2_V}{(2\pi)^5M_V^2}\sum_{i=1}^{3}H_{ii}(S_V=+1)\ . \tag{27}$$

In this form it is easier to calculate the hadronic component $\sum_i H_{ii}(S_V=+1)$ in a straightforward manner to obtain

$$\sum_{i=1}^{3}H_{ii}(S_V=+1)=\frac{32\pi^2}{9}\left|\int_0^\infty dp\,p^2G_q(p)(2+m_q/E_p)\right|^2\ . \tag{28}$$

Now using the quark-antiquark momentum distribution function of the model given in Eq. (13) and then substituting Eq. (28) into Eq. (27), one can obtain the final form of the leptonic decay width in the present model as

$$\Gamma(V\to e^+e^-)=\frac{\alpha^2_{\rm em}}{9\pi M_V^2}\left[\frac{\mathcal{N}_q}{\alpha\lambda_q}\right]^2\langle e_q\rangle^2_VI_V^2\ , \tag{29}$$

where

$$I_V=\int_0^\infty dp\,p^2\sqrt{(1+m_q/E_p)}(2+m_q/E_p)\times(E_q+E_p)\exp(-p^2/4\alpha)\ . \tag{30}$$

The integral I_V can be evaluated numerically by the standard quadrature technique.

We can also derive an expression for the electromagnetic decay constant f_V of the vector mesons in this model from the usual defining relation

$$\frac{1}{M_V^2}\left\langle 0\left|\sum_q e_q\bar{\psi}_q^{(+)}(0)\gamma^\mu\psi_q^{(+)}(0)\right|V,S_V\right\rangle=\frac{1}{\sqrt{2M_V}}\epsilon^\mu f_V \; . \tag{31}$$

Now an explicit evaluation of the left-hand side can enable one to express f_V in terms of $\Gamma(V\to e^+e^-)$ as

$$f_V=[3\Gamma(V\to e^+e^-)/(4\pi\alpha_{\rm em}^2 M_V)]^{1/2} \; . \tag{32}$$

Hence the model predictions regarding the leptonic decay of ordinary vector mesons of the light flavor sector can either be given in terms of $\Gamma(V\to e^+e^-)$ or f_V using expressions in Eqs. (29) and (32).

IV. RESULTS AND DISCUSSION

In this section we evaluate the leptonic decay widths of ordinary neutral vector mesons ρ, ω, and ϕ as well as their electromagnetic decay constants using the expressions in Eqs. (29), (30), and (32) derived in Sec. III. The calculations involve primarily the potential parameters of the model (a,V_0), the quark masses (m_u,m_d,m_s). The meson masses appearing in the calculations are in fact taken as the observed ones.

Our purpose here is, in a way, to make a parameter-free calculation of the decay widths by using the potential parameters obtained earlier from the applications of the present model to baryon and meson sectors [1–10]. Thus we have

$$(a,V_0)=(0.017\,166\ \text{GeV}^3,\ -0.1375\ \text{GeV}) \; . \tag{33}$$

Our choice for the quark masses $m_u=m_d$ and m_s are however somewhat different from those of Ref. [2] used in the baryon sector. They are

$$(m_u=m_d,m_s)=(0.01\ \text{GeV},\ 0.024\ \text{GeV}). \tag{34}$$

Such a choice of the quark mass parameters and the potential parameters in this model has satisfactorily explained the partial decay widths of twelve possible $M1$ transitions such as $V\to P\gamma$ and $P\to V\gamma$ within the traditional picture of photon emission by a confined quark and/or antiquark in a "static" calculation [8]. Then the model dynamics described in Sec. II provides the ground-state confined quark energy E_q, the scale factor r_{0q}, and the normalization constant $\mathcal{N}_q$ relevant for the present calculation in the following manner:

$$\begin{aligned}&(E_u=E_d,E_s)\equiv(0.451\ \text{GeV},\ 0.546\ \text{GeV}),\\&(r_{0u}=r_{0d},r_{0s})\equiv(3.352\ \text{GeV}^{-1},\ 2.934\ \text{GeV}^{-1}),\\&(\mathcal{N}_u=\mathcal{N}_d,\mathcal{N}_s)\equiv(0.643\ \text{GeV}^{1/2},\ 0.775\ \text{GeV}^{1/2}) \; .\end{aligned} \tag{35}$$

Then the integral I_V in Eq. (30) is evaluated numerically with the help of standard Gaussian quadrature technique to yield

$$(I_\rho=I_\omega,I_\phi)\equiv(0.063\ \text{GeV}^4,\ 0.165\ \text{GeV}^4) \; . \tag{36}$$

Now using the results in Eqs. (35) and (36), we can calculate the leptonic decay widths $\Gamma(V\to e^+e^-)$ and the electromagnetic decay constants f_V for the vector mesons ρ, ω, and ϕ. The results are provided in Table I in comparison with the corresponding experimental values [15] showing very good agreement to within 10% or better.

If, however, we take the quark mass parameters along with the potential parameters entirely according to Ref. [2] such that

$$(m_u=m_d,m_s)=(78.75\ \text{MeV},\ 315.75\ \text{MeV}) \; , \tag{37}$$

then all the relevant quantities necessary for the calculation of the leptonic decay widths in this model would become

$$\begin{aligned}&(E_u=E_d,E_s)\equiv(471\ \text{MeV},\ 591\ \text{MeV}) \; ,\\&(r_{0u}=r_{0d},r_{0s})\equiv(3.208\ \text{GeV}^{-1},\ 2.831\ \text{GeV}^{-1}) \; ,\\&(\mathcal{N}_u^2=\mathcal{N}_d^2,\mathcal{N}_s^2)\equiv(0.498\ \text{GeV},\ 0.649\ \text{GeV}) \; ,\\&(I_\rho=I_\omega,I_\phi)\equiv(0.087\ \text{GeV}^4,\ 0.212\ \text{GeV}^4).\end{aligned} \tag{38}$$

The leptonic decay widths $\Gamma(V\to e^+e^-)$ and the corresponding decay constants calculated with this set of parameters are provided within parentheses in Table I. We observe that the decay widths which are 15–40 % higher than the corresponding experimental values are nevertheless quite comparable with those obtained by Margolis and Mendel [11] in the bag model in a similar calculation with completely correlated quark-antiquark momenta. Bag model calculations [13] with completely uncorrelated quark-antiquark momenta provide results 30–40 % lower with respect to experimental values.

TABLE I. Leptonic decay widths $\Gamma(V\to e^+e^-)$ and the decay constant f_V in comparison with the results of Refs. [11] and [14], respectively, together with the experiment.

Physical quantity	V	Present calculation	Ref. [11]/Ref. [14]	Experiment [15]
$\Gamma(V\to e^+e^-)$	ρ	6.26(8.10)	7.80	6.77 ± 0.32
	ω	0.67(0.87)	0.84	0.60 ± 0.02
(keV)	ϕ	1.58(1.84)	1.69	1.37 ± 0.05
f_V	ρ	0.19(0.22)	0.21	0.20 ± 0.04
	ω	0.06(0.07)	0.07	0.06 ± 0.01
	ϕ	0.08(0.09)	0.07	0.08 ± 0.01

Hayne and Isgur [14] had extended the nonrelativistic quark model calculations beyond the static approximation developing a formalism based on a quark-antiquark momentum distribution in Gaussian form chosen in an *ad hoc* manner. We find that the electromagnetic decay constants f_V obtained in our calculations are in good agreement with those of Ref. [14]. Thus our approach, which is quite similar to that of Hayne and Isgur, has the distinction of realizing the quark-antiquark momentum distribution in Gaussian form directly from our model dynamics.

Thus within the working approximations adopted here, the model provides a simple calculational framework to explain successfully the leptonic decay of vector mesons in the ordinary light flavor sector.

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