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Quark mass matrices presented by a power series expansion in $|V_{us}|$

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In a general three-family Hermitian quark mass matrix model (M_u, M_d), if we take a quark basis on which M_u takes a diagonal form D_u, the structure of $M_d \equiv \widehat{M}$ is almost determined by three down-quark masses (m_d, m_s, m_b) and three Kobayashi-Maskawa matrix parameters $(|V_{us}|, |V_{cb}|, |V_{ub}|)$, except for phases of the matrix elements $\widehat{M}_{ij}$. By using the experimental facts $|V_{us}| \sim \lambda$, $|V_{cb}| \sim \lambda^2$, $|V_{ub}| \sim \lambda^3$, and $m_d/m_s \sim m_s/m_b \sim \lambda^2$, the mass matrices $(D_u, \widehat{M})$ are presented in terms of a power series in λ.

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Recent remarkable progress in Z and B decay experiments has provided rich and fruitful data for studying quark mass matrices, at least, as far as the three families are concerned. At present, we roughly know the values of their masses and Kobayashi-Maskawa (KM) matrix [1] parameters except for the top-quark mass and CP-violation phase parameter. If we can find out more beautiful and simpler relations behind the observed quark mass spectra and family mixing, they will offer us a fruitful clue to the origin of quark mass generation, to the origin of families, and so on. At present, the investigation of the quark mass matrix structure from the phenomenological point of view is a timely endeavor.

In such a phenomenological study of quark mass matrices, we know that any model (M'_u, M'_d) which is related to a model (M_u, M_d) by $M'_u = U_0^\dagger M_u U_0$ and $M'_d = U_0^\dagger M_d U_0$ with an arbitrary unitary matrix U_0 provides the same predictions as the model (M_u, M_d), as far as physically observable quantities are concerned. In general, the number of independent parameters in up- and down-quark mass matrices M_u and M_d is bigger than that of independent observable quantities. For example, the 3×3 Hermitian quark mass matrices (M_u, M_d) have, in general, 18 independent parameters (however, two of 18 can easily be seen as these are phase parameters with no physical meaning), while the number of independent observable quantities is 10. However, the number 18 is the maximal number in the most general case. If we choose a special quark basis, we can decrease this number [2].

In this paper, we choose a quark basis where M_u takes a diagonal form D_u. Then, we have seven independent parameters in M_d whose number is the same as the observable quantities (for simplicity, we consider a Hermitian quark mass matrix model):

$$M_d = \widehat{M} \equiv V D_d V^\dagger \equiv \begin{pmatrix} \widehat{M}_{11} & |\widehat{M}_{12}|e^{i\phi_{12}} & |\widehat{M}_{31}|e^{-i\phi_{31}} \\ |\widehat{M}_{12}|e^{-i\phi_{12}} & \widehat{M}_{22} & |\widehat{M}_{23}|e^{i\phi_{23}} \\ |\widehat{M}_{31}|e^{i\phi_{31}} & |\widehat{M}_{23}|e^{-i\phi_{23}} & \widehat{M}_{33} \end{pmatrix} . \quad (1)$$

The seven independent parameters in the down-quark mass matrix $\widehat{M}$ can completely be determined by the values of the three down-quark masses and four independent parameters of the KM matrix V. At present, of the seven parameters, we do not yet know the value of the CP-violation phase parameter. Nevertheless, as we show later, without knowing the value of the CP-violation phase parameter, we can almost determine the structure of the mass matrix $\widehat{M}$, (1), except for the phases ϕ_{ij}.

The updated values of running quark masses at the energy scale 1 GeV are

$$\begin{aligned} &m_u = 0.0056 \pm 0.0011 \text{ GeV}, && m_d = 0.0099 \pm 0.0011 \text{ GeV}, \\ &m_c = 1.45 \pm 0.02 \text{ GeV}, && m_s = 0.199 \pm 0.033 \text{ GeV}, \\ &m_t = 349^{+68}_{-75} \text{ GeV}, && m_b = 7.07 \pm 0.08 \text{ GeV}. \end{aligned} \quad (2)$$

Here, the values of light-quark masses m_u, m_d, and m_s have been quoted from the estimates of Dominguez and de Rafael [3]. The present experimental value [4] of $\Lambda_{\overline{\rm MS}}$ is $\Lambda^{(4)}_{\overline{\rm MS}} = 260^{+54}_{-40}$ MeV, where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme. The heavy-quark masses m_c and m_b have been evaluated from $m_c(m_c) = 1.27 \pm 0.02$ GeV and $m_b(m_b) = 4.20 \pm 0.05$ GeV, according to the prescription of Gasser and Leutwyler [5] and Narison [6], but by using the new value $\Lambda^{(4)}_{\overline{\rm MS}} = 0.26$ GeV. The top-quark mass m_t has been obtained from $m_t(m_t) = 124^{+24}_{-27}$ GeV, by using $\Lambda^{(4)}_{\overline{\rm MS}} = 0.26$ GeV and the physical top-quark mass value [7] $m_t^{\rm phys} = 130^{+25}_{-28}$ GeV, which has been estimated from the radiative corrections based on the standard electroweak theory (assuming a Higgs-boson mass $m_H \simeq m_Z$). [The error values in (2) do not include the error from $\Lambda^{(4)}_{\overline{\rm MS}}$ [8].] Note that these heavy-quark mass values at $\mu = 1$ GeV are highly dependent on the value of $\Lambda_{\overline{\rm MS}}$, and, in addition, the value of m_t relies on the validity of the standard minimal Higgs model.

On the other hand, experimental values of the KM matrix elements are [4]

$$\begin{aligned} &|V_{us}| = 0.2205 \pm 0.0018 \ , \\ &|V_{cb}| = 0.043 \pm 0.007 \ , \\ &|V_{ub}/V_{cb}| = 0.10 \pm 0.03 \ . \end{aligned} \tag{3}$$

Note that the values of $|V_{cb}|$ and $|V_{ub}|$ are dependent [9] on the choices of models and quark mass values.

At present, the values (2) and (3) should not be taken rigidly. Therefore, although we can numerically evaluate each matrix element of $\widehat{M}$, (1), by using the values (2) and (3), it will not be so useful. Rather, we think that it is useful to express $\widehat{M}$ in terms of m_d, m_s, m_b, $|V_{us}|$, $|V_{cb}|$, and $|V_{ub}|$, but not numerically.

In the present paper, by assuming that $|V_{us}| \sim \lambda$, $|V_{cb}| \sim \lambda^2$, $|V_{ub}| \sim \lambda^3$, and $m_d/m_s \sim m_s/m_b \sim \lambda^2$, we express the down-quark mass matrix $\widehat{M}$ in terms of a power series expansion in λ. The present data can determine all matrix elements of $\widehat{M}$ except for the phases ϕ_{ij} of $\widehat{M}_{ij}$, where only the phase parameter $\phi \equiv \phi_{12} + \phi_{23} + \phi_{31}$ can affect CP violation effects. Then, the general form of the phenomenologically favorable mass matrix form (M_u, M_d) is given by $(U_0^\dagger D_u U_0, U_0^\dagger \widehat{M} U_0)$. With the help of the resultant mass matrix form $(D_u, \widehat{M})$, we will give an example of phenomenological study of (M_u, M_d).

First, let us express the seven parameters of $\widehat{M}$ in terms of three down-quark masses (d_1, d_2, d_3) and four KM matrix parameters. We take $\widehat{M}_{11}$, $\widehat{M}_{22}$, $\widehat{M}_{33}$, $|\widehat{M}_{12}|$, $|\widehat{M}_{23}|$, $|\widehat{M}_{31}|$, and

$$\phi \equiv \phi_{12} + \phi_{23} + \phi_{31} \ , \tag{4}$$

as the seven mass matrix parameters in (1). Note that the phase parameter ϕ is observable, but two of the three phase factors ϕ_{ij} are not observable quantities; i.e., CP-nonconservation effects appear only through the phase parameter ϕ. On the other hand, as four independent parameters of the KM matrix V, it is convenient to choose [10]

$$\alpha \equiv |V_{us}| \ , \quad \beta \equiv |V_{cb}| \ , \quad \gamma \equiv |V_{ub}| \ , \quad \omega \equiv |V_{cd}|^2 - |V_{us}|^2 \ , \tag{5}$$

because we can confine present experimentally unknown quantities on the KM matrix into the only one parameter ω. Then, by putting

$$|V_{ij}|^2 = \begin{pmatrix} 1-\alpha^2-\gamma^2 & \alpha^2 & \gamma^2 \\ \alpha^2+\omega & 1-\alpha^2-\beta^2-\omega & \beta^2 \\ \gamma^2-\omega & \beta^2+\omega & 1-\beta^2-\gamma^2 \end{pmatrix} \tag{6}$$

into the formulas

$$\widehat{M}_{ii} = |V_{i1}|^2 d_1 + |V_{i2}|^2 d_2 + |V_{i3}|^2 d_3 \ , \tag{7}$$

$$\begin{aligned} |\widehat{M}_{ij}|^2 = \ & |V_{i1}|^2 |V_{j1}|^2 (d_1 - d_2)(d_1 - d_3) \\ & + |V_{i2}|^2 |V_{j2}|^2 (d_2 - d_3)(d_2 - d_1) \\ & + |V_{i3}|^2 |V_{j3}|^2 (d_3 - d_1)(d_3 - d_2) \quad (i \neq j) \ , \end{aligned} \tag{8}$$

we can completely determine all matrix elements $\widehat{M}_{ij}$ of $\widehat{M}$ except for the phases ϕ_{ij} in terms of the observable parameters, down-quark masses (d_1, d_2, d_3) and the four KM matrix parameters α, β, γ, and ω:

$$\begin{aligned} \widehat{M}_{11} &= d_1 + \alpha^2(d_2 - d_1) + \gamma^2(d_3 - d_1) \ , \\ \widehat{M}_{22} &= d_2 - \alpha^2(d_2 - d_1) + \beta^2(d_3 - d_2) - \omega(d_2 - d_1) \ , \\ \widehat{M}_{33} &= d_3 - \beta^2(d_3 - d_2) - \gamma^2(d_3 - d_1) + \omega(d_2 - d_1) \ , \\ |\widehat{M}_{12}|^2 &= \alpha^2(1-\alpha^2)(d_2 - d_1)^2 + \alpha^2\beta^2(d_3 - d_2)(d_2 - d_1) - \alpha^2\gamma^2(d_3 - d_1)(d_2 - d_1) \\ &\quad + \beta^2\gamma^2(d_3 - d_1)(d_3 - d_2) + \left[(1-\gamma^2)(d_3 - d_1)(d_2 - d_1) - \alpha^2(d_2 - d_1)^2\right]\omega \ , \\ |\widehat{M}_{23}|^2 &= \beta^2(1-\beta^2)(d_3 - d_2)^2 + \alpha^2\beta^2(d_3 - d_2)(d_2 - d_1) + \alpha^2\gamma^2(d_3 - d_1)(d_2 - d_1) - \beta^2\gamma^2(d_3 - d_1)(d_3 - d_2) \\ &\quad - \left[(1-2\beta^2)(d_3 - d_2)(d_2 - d_1) + \alpha^2(d_2 - d_1)^2 - \gamma^2(d_3 - d_1)(d_2 - d_1)\right]\omega - (d_2 - d_1)^2\omega^2 \ , \\ |\widehat{M}_{31}|^2 &= \gamma^2(1-\gamma^2)(d_3 - d_1)^2 - \alpha^2\beta^2(d_3 - d_2)(d_2 - d_1) - \alpha^2\gamma^2(d_3 - d_1)(d_2 - d_1) \\ &\quad - \beta^2\gamma^2(d_3 - d_1)(d_3 - d_2) - \left[(1-\gamma^2)(d_3 - d_1)(d_2 - d_1) - \alpha^2(d_2 - d_1)^2\right]\omega \ . \end{aligned} \tag{9}$$

As suggested from Wolfenstein's parametrization [11] the present data (3) roughly show $\alpha \sim \lambda$, $\beta \sim \lambda^2$, and $\gamma \sim \lambda^3$. Therefore, it is convenient to define the following parameters with values of the order of one:

$$v_1 \equiv \alpha/\lambda \ , \quad v_2 \equiv \beta/\lambda^2 \ , \quad v_3 \equiv \gamma/\lambda^3 \ , \quad w \equiv \omega/\lambda^6 \ . \tag{10}$$

On the other hand, the present data (2) roughly show $m_d/m_s \sim m_s/m_b \sim \lambda^2$. In fact, the Weinberg-Fritzsch empirical relation [12,13] $|V_{us}| \simeq \sqrt{m_d/m_s}$ is well satisfied with the exprimental values. Therefore, it is convenient to define the following parameters with values of the order of one:

$$r_{1d} \equiv (d_1/d_3)/\lambda^4 \ , \quad r_{2d} \equiv (d_2/d_3)/\lambda^2 \ . \tag{11}$$

By using these parameters with the order of one, which are defined by (10) and (11), we can express $\widehat{M}$ as a power series expansion with respect to λ:

$$\widehat{M} \simeq d_3 \begin{pmatrix} (r_{1d}+r_{2d}v_1^2)\lambda^4 & r_{2d}v_1 e^{i\phi_{12}}\lambda^3 & v_3 e^{-i\phi_{31}}\lambda^3 \\ r_{2d}v_1 e^{-i\phi_{12}}\lambda^3 & r_{2d}\lambda^2 & v_2 e^{i\phi_{23}}\lambda^2 \\ v_3 e^{i\phi_{31}}\lambda^3 & v_2 e^{-i\phi_{23}}\lambda^2 & 1 \end{pmatrix} \ . \tag{12}$$

Here, we have shown only the first leading term of the λ-power series in each matrix element. Each matrix element of $\widehat{M}$ up to λ^6 is

$$\widehat{M}_{11} = (r_{1d}+r_{2d}v_1^2)\lambda^4 + (v_3^2 - r_{1d}v_1^2)\lambda^6 + O(\lambda^{10}) \ ,$$

$$\widehat{M}_{22} = r_{2d}\lambda^2 + (v_2^2 - r_{2d}v_1^2)\lambda^4 + (r_{1d}v_1^2 - r_{2d}v_2^2)\lambda^6 + O(\lambda^8) \ ,$$

$$\widehat{M}_{33} = 1 - v_2^2\lambda^4 + (r_{2d}v_2^2 - v_3^2)\lambda^6 + O(\lambda^8) \ , \tag{13}$$

$$|\widehat{M}_{12}| = r_{2d}v_1\lambda^3 - \frac{1}{2v_1}[(2r_{1d}+r_{2d}v_1^2)v_1^2 - a]\lambda^5 + O(\lambda^7) \ ,$$

$$|\widehat{M}_{31}| = v_3\lambda^3 - \frac{1}{2v_3}r_{2d}a\lambda^5 + O(\lambda^7) \ ,$$

$$|\widehat{M}_{23}| = v_2\lambda^2 - r_{2d}v_2\lambda^4 + \frac{1}{2v_2}[(2r_{2d} - v_1^2 - v_2^2)v_2^2 - r_{2d}a]\lambda^6 + O(\lambda^8) \ ,$$

where the parameter a is defined by

$$a \equiv v_1^2 v_2^2 + w \ , \tag{14}$$

and as seen from (15) below, the limit of $a \simeq 0$ corresponds to a case of "maximal" CP violation ($|\sin\phi| = 1$).

The terms which include the deviation parameter from the symmetric KM matrix [14] ω appear in λ^5 in $\widehat{M}_{12}$ and $\widehat{M}_{31}$ and in λ^6 in $\widehat{M}_{23}$. Since $\det\widehat{M} = d_1 d_2 d_3$, the phase parameter ϕ and the parameter ω must satisfy the relation

$$\cos\phi \simeq \frac{v_1^2 v_2^2 + w}{2v_1 v_2 v_3} \simeq \frac{a}{2v_1 v_2 v_3} \ . \tag{15}$$

Then, the rephasing-invariant quantity [15] J, which is a measure of CP nonconservation, is given by

$$J = \frac{|\widehat{M}_{12}||\widehat{M}_{23}||\widehat{M}_{31}|}{(d_3-d_1)(d_3-d_2)(d_2-d_1)} \sin\phi \simeq \lambda^6 v_1 v_2 v_3 \sin\phi \ . \tag{16}$$

From (15), the limit of $a = 0$ means $\phi = \pm\pi/2$, so that the case provides a maximal J for v_1, v_2, and v_3 fixed.

On the other hand, correspondingly to (12), we denote the up-quark mass matrix D_u as

$$D_u = u_3 \begin{pmatrix} r_{1u}\lambda^6 & 0 & 0 \\ 0 & r_{2u}\lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \ . \tag{17}$$

Here, although the up-quark mass ratio $m_t : m_c : m_u$ is nearer to $1 : \alpha^3 : \alpha^7$ rather than $1 : \alpha^2 : \alpha^6$, we have defined the up-quark mass ratio parameters r_{1u} and r_{2u} by $r_{1u} \equiv (u_1/u_3)/\lambda^6$ and $r_{2u} \equiv (u_2/u_3)/\lambda^2$, because, as seen later, it is convenient to express the diagonal elements in terms of even powers of λ for a model which requires $(M_u)_{11} = (M_d)_{11} = 0$ [otherwise we have half-integer powers of λ for the rotation angles θ_2 and θ_3 defined in (21)]. Hereafter, in the demonstrative discussion of the usefulness of using $(D_u, \widehat{M})$, we use the definition $r_{1u} \equiv (u_1/u_3)/\lambda^6$ and $r_{2u} \equiv (u_2/u_3)/\lambda^2$ in consideration of the correspondence between M_u and M_d. However, another definition $r_{1u} \equiv (u_1/u_3)/\lambda^8$ and $r_{2u} \equiv (u_2/u_3)/\lambda^4$ is also attractive because of the empirical relation $u_1 u_3 \sim u_2^2$.

Now, a general quark mass matrix form which is consistent with experiments is given by

$$M_u = U_0^\dagger D_u U_0 \ , \quad M_d = U_0^\dagger \widehat{M} U_0 \ , \tag{18}$$

where U_0 is an arbitrary unitary matrix. Parameters included in U_0 and two of three phase parameters ϕ_{12}, ϕ_{31}, and ϕ_{23} (for example, δ_1 and δ_2 defined by $\phi_{12} = \delta_1+\delta_2$, $\phi_{31} = \delta_1-\delta_2$, and $\phi_{23} = \phi-2\delta_1$) are unobservable quantities.

Our next task is to seek for the quark basis (in other words, U_0) by which we can obtain a beautiful description of mass matrices. We do not need to pay attention to what U_0 we should choose as far as we are interested in the observable quantities, but the choice of U_0 is essential for model building of mass matrices.

In general, matrix elements of D_u and $\widehat{M}$ are mixed by choosing a mixing matrix U_0 with sizable mixing angles. Then, it should be noted that in the resultant matrix elements of M_u and M_d, for example, the value $r_{1u}\lambda^6$ from $(D_u)_{11}$ cannot be neglected, while the term $-v_2^2\lambda^4$ from $\widehat{M}_{33}$ is not so essential. Our prescription of the mass matrix phenomenology is useful for careful treatment of such small terms. If we were to express $(D_u, \widehat{M})$ with numerical values, we would not be able to distinguish whether a numerical value comes from $(D_u)_{11}$ or from a higher λ^n term of $\widehat{M}_{33}$.

The resultant mass matrix structure $(D_u, \widehat{M})$ readily suggests the following empirical sum rule: If we suppose that the matrix elements $\widehat{M}_{11}$ must, at least, be smaller than the order of λ^6 since the corresponding $(D_u)_{11}$ is of the order of λ^6, we obtain a sum rule

$$r_{1d} + r_{2d} v_1^2 \simeq 0 \ , \tag{19}$$

i.e., the Weinberg-Fritzsch sum rule [12,13]

$$\alpha \equiv |V_{us}| \simeq \sqrt{|d_1/d_2|} \simeq 0.223 \ . \tag{20}$$

When we require that $(M_d)_{11} = 0$, it is natural that we also require $(M_u)_{11} = 0$. For example, a model in which M_u and M_d have a similar structure is provided by a rotation $R \equiv R(\theta_1, \theta_2, \theta_3)$ with $\theta_1 \sim \lambda^2$, $\theta_2 \sim \lambda^4$, and $\theta_3 \sim \lambda^2$, where $R(\theta_1, \theta_2, \theta_3)$ is defined by

$$R(\theta_1,\theta_2,\theta_3) = \begin{pmatrix} c_3c_2 & s_3c_2 & s_2 \\ -s_3c_1 - c_3s_1s_2 & c_3c_1 - s_3s_1s_2 & s_1c_2 \\ s_3s_1 - c_3c_1s_2 & -c_3s_1 - s_3c_1s_2 & c_1c_2 \end{pmatrix} , \tag{21}$$

($c_i = \cos\theta_i$ and $s_i = \sin\theta_i$). Then, we obtain

$$M_u = R^\dagger D_u R = \begin{pmatrix} 0 & -r_{2u}t_3\lambda^4 & (-t_2+t_3t_1)\lambda^4 \\ -r_{2u}t_3\lambda^4 & r_{2u}\lambda^2 & -t_1\lambda^2 \\ (-t_2+t_3t_1)\lambda^4 & -t_1\lambda^2 & 1 \end{pmatrix} , \tag{22}$$

$$M_d = R^\dagger \widehat{M} R = \begin{pmatrix} 0 & r_{2d}v_1e^{i\phi_{12}}\lambda^3 & v_3e^{-i\phi_{31}}\lambda^3 \\ r_{2d}v_1e^{-i\phi_{12}}\lambda^3 & r_{2d}\lambda^2 & (v_2e^{i\phi_{23}} - t_1)\lambda^2 \\ v_3e^{i\phi_{31}}\lambda^3 & (v_2e^{-i\phi_{23}} - t_1)\lambda^2 & 1 \end{pmatrix} , \tag{23}$$

where $t_1 = \theta_1/\lambda^2$, $t_2 = \theta_2/\lambda^4$, and $t_3 = \theta_3/\lambda^2$, and we have shown only the first leading term in each matrix element of M_u and M_d. The more detailed expressions of $(M_u)_{11}$ and $(M_d)_{11}$ are

$$(M_u)_{11} = (r_{1u} + r_{2u}t_3^2)\lambda^6 + (-t_2 + t_3t_1)^2\lambda^8 + \cdots \ , \tag{24}$$

$$(M_d)_{11} = (r_{1d} + r_{2d}v_1^2)\lambda^4 + 2\cos\phi_{12} r_{2d} v_1 t_3 \lambda^5 + \cdots \ . \tag{25}$$

The requirement $(M_u)_{11} = 0$ leads to

$$t_3 \simeq \pm\sqrt{-r_{1u}/r_{2u}} \ . \tag{26}$$

On the other hand, since the second term (λ^5 term) in (25) cannot be neglected, the requirement $(M_d)_{11} = 0$ leads to [13]

$$|V_{us}| \simeq \sqrt{-\frac{d_1}{d_2}} \pm \cos\phi_{12}\sqrt{-\frac{u_1}{u_2}} \ , \tag{27}$$

instead of (19) [(20)]. Since the experimental values of quark masses, (2), give $\sqrt{-d_1/d_2} \simeq 0.223$ and $\sqrt{-u_1/u_2} \simeq 0.062$, the factor $\cos\phi_{12}$ must be vanishingly small, i.e., $\phi_{12} \simeq \pm\pi/2$, in this model. If we consider a model with $\phi_{31} + \phi_{23} = 0$ (and also $= \pm\pi$), the model gives

$$|\sin\phi| = |\sin(\phi_{12} + \phi_{31} + \phi_{23})| = |\sin\phi_{12}| \simeq 1 \ , \tag{28}$$

which provides the "maximal" CP violation in the meaning stated in the sentence after (16) and is in reasonable agreement [16] with experiment.

If we require a further condition $(M_u)_{31} = (M_d)_{31} = 0$ according to Fritzsch's ansatz [13], from the relations $(M_u)_{31} = (-t_2 + t_3t_1)\lambda^4 + O(\lambda^5)$ and

$$(M_d)_{31} = v_3e^{i\phi_{31}}\lambda^3 - (v_2e^{-i\phi_{23}}t_3 + t_2 - t_3t_1)\lambda^4 + O(\lambda^5) \ , \tag{29}$$

we obtain

$$\sin(\phi - \phi_{12}) = \sin(\phi_{31} + \phi_{23}) = 0 \tag{30}$$

and [17]

$$|V_{ub}/V_{cb}| \simeq \sqrt{-u_1/u_2} \ . \tag{31}$$

The relation (30) is favorable to the case of "maximal" CP violation, (28), because (30) means $\sin\phi = \sin\phi_{12}$. However, the prediction $\gamma/\beta = \sqrt{-u_1/u_2} \simeq 0.062$ is somewhat small compared with the experimental value 0.10 ± 0.03.

Concerning the prediction of $|V_{ub}|$, an alternative model, where $(M_u)_{12} = (M_d)_{31} = 0$ instead of $(M_u)_{31} = (M_d)_{31} = 0$, is also interesting: In order to give $(M_u)_{12} = 0$, we consider a rotation $R(\theta_1, \theta_2, 0)$ with $\theta_1 \simeq t_1\lambda^2$ and $\theta_2 \simeq t_2\lambda^3$. Then $(M_u)_{ij}$ and $(M_d)_{ij}$ are given by replacing $t_2\lambda^4$ and $t_3\lambda^2$ in (23)–(26) with $t_2\lambda^3$ and $t_3 = 0$, respectively. The requirements $(M_u)_{11} = 0$ and $(M_d)_{11} = 0$ lead to $t_2 \simeq \pm\sqrt{-r_{1u}}$ and $|V_{us}| \simeq \sqrt{-d_1/d_2}$, (20). The requirement $(M_d)_{31} = (v_3e^{i\phi_{31}} - t_2)\lambda^3 + \cdots = 0$ leads to $\sin\phi_{31} = 0$ and

$$|V_{ub}| \simeq \sqrt{-u_1/u_3} \ . \tag{32}$$

The predicted value $\sqrt{-u_1/u_3} \simeq 0.0040$ is in good agreement with the experimental value $|V_{ub}| = 0.0043 \pm 0.0019$.

Thus, the matrix form $(D_u, \widehat{M})$ is very useful for seeking empirical sum rules on quark masses and mixings. Although here we have discussed only an "extended" form of the Fritzsch-type mass matrix [13], the form $(D_u, \widehat{M})$ will also be useful for a phenomenological study of the "democratic"-type mass matrices [18]. However, the study of the democratic-type mass matrices with the help of the mass matrix form $(D_u, \widehat{M})$ is too specialized to be treated in the present paper, so it will be given elsewhere.

In conclusion, we have given the down-quark mass matrix $M_d \equiv \widehat{M}$ at the $M_u = D_u$ frame by using only seven observable quantities (m_d, m_s, m_b, $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, and ω), and have determined the mass matrix structure $(D_u, \widehat{M})$ by expressing in terms of λ power series expansion. The general form of the experimentally favorable quark mass matrix (M_u, M_d) is given by $M_u = U_0^\dagger D_u U_0$ and $M_d = U_0^\dagger \widehat{M} U_0$. In other words, a mass matrix model (M_u, M_d) inconsistent with the mass matrix form $(D_u, \widehat{M})$ will be ruled out. The mass matrix form $(D_u, \widehat{M})$ is also useful for seeking empirical sum rules on quark masses and mixings. Thus the mass matrix form $(D_u, \widehat{M})$ given by (12) and (17) can offer a useful tool for model building of the mass matrices from the phenomenological point of view.

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