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Predictions for neutral K and B meson physics

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Using supersymmetric grand unified theories, we have recently invented a framework which allows the prediction of three quark masses, two of the parameters of the Kobayashi-Maskawa matrix, and $\tan\beta$, the ratio of the two electroweak vacuum expectation values. These predictions are used to calculate ϵ and ϵ' in the kaon system, the mass mixing in the B_d^0 and B_s^0 systems, and the size of CP asymmetries in the decays of neutral B mesons to explicit final states of given CP.

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In previous papers [1] we have invented a predictive framework for quark and lepton masses and mixings based on the Georgi-Jarlskog ansatz [2] for the form of the mass matrices in supersymmetric grand unified theories [3]. In this paper we use this scheme to make predictions for parameters of the neutral K and B meson systems. In particular we show that the CP asymmetries in neutral B meson decays are large and will provide a powerful test of the scheme. We begin by reviewing the predictions for quark masses and mixings.

The top-quark mass is predicted to be heavy:

$$m_t=(179\text{ GeV})\left[\frac{m_b}{4.15\text{ GeV}}\right]\left[\frac{m_c}{1.22\text{ GeV}}\right]\times\left[\frac{0.053}{V_{cb}}\right]^2\left[\frac{1.46}{\eta_b}\right]\left[\frac{1.84}{\eta_c}\right], \tag{1}$$

where η_i is the QCD enhancement of a quark mass scaled from m_t to m_i. Perturbativity of the top-quark Yukawa coupling also requires that $m_t < 187$ GeV. In this paper the central values of η_i quoted correspond to a complete two-loop QCD calculation with $\alpha_s(M_Z)=0.109$, whereas in Ref. [1] an approximate two-loop result was given.

The value of $\alpha_s(M_Z)=0.109$ comes from a one-loop analysis of the unification of gauge couplings which for simplicity *ignored threshold corrections at the grand unified and supersymmetry-breaking scales* [1]. However, these threshold corrections will be present at some level in all grand unified models [4], and would not have to be very large for our predicted value of the QCD coupling to range over all values allowed by the data at the CERN e^+e^- collider LEP: 0.115 ± 0.008. Whatever the threshold corrections are, they must give an acceptable value for strong coupling. Hence it is important to consider the range of top-quark masses allowed by the LEP range of α_s. Larger values of α_s lead to larger η_i reducing m_t. Increasing $\alpha_s(M_z)$ from 0.109 to 0.123 decreases m_t from 179 to 150 GeV. Alternatively, $\alpha_s(M_Z)=0.123$ allows the top-quark mass to be near the fixed-point value of 187 GeV with $V_{cb}=0.047$. The above numbers refer to the running mass parameter. The pole mass, which is to be compared with experimental observations, is 4.5% larger.

The particular form of the quark mass matrices leads to an unusual form for the Kobayashi-Maskawa matrix:

$$V=\begin{pmatrix} c_1-s_1s_2e^{-i\phi} & s_1+c_1s_2e^{-i\phi} & s_2s_3 \\ -c_1s_2-s_1e^{-i\phi} & c_1e^{-i\phi}-s_1s_2 & s_3 \\ s_1s_3 & -c_2s_3 & e^{i\phi} \end{pmatrix}, \quad (2)$$

where $s_1=\sin\theta_1$, etc., and we have set $c_2=c_3=1$.[1] We do not lose any generality by choosing the phases of quark fields such that θ_1, θ_2, and θ_3 all lie in the first quadrant. We have predicted [1]

$$s_1=0.196, \quad s_2=0.053\chi \quad (3)$$

where

$$\chi=\left[\frac{m_u/m_d}{0.6}\frac{1.22\text{ GeV}}{m_c}\frac{\eta_c}{1.84}\right]^{1/2}, \quad (4)$$

while the input V_{cb} determines s_3 which must be chosen quite large in view of (1). The ratio m_s/m_d, which we predict to be 25.15, strongly prefers $m_u/m_d<0.8$, while the present value of $|V_{ub}/V_{cb}|=s_2$ prefers m_u/m_d larger than 0.4 In all of our predictions the largest uncertainty lies in m_u/m_d, which we will display through the parameter χ.

The angle ϕ is determined by the requirement that $|V_{us}|=\sin\theta_C$:

$$\begin{aligned} c_\phi&=\frac{1}{\chi}[0.51(1\pm0.11)-0.13\chi^2]=0.38^{+0.21}_{-0.14}, \\ s_\phi&\simeq0.92\left[1.15-\frac{0.15}{\chi^2}(1\pm0.22)\right]=0.92^{-0.11}_{+0.05}. \end{aligned} \quad (5)$$

In the first expressions the χ dependence is shown explicitly, together with the uncertainty from the measured value of the Cabibbo angle $\sin\theta_C=0.221\pm0.003$. Note that the $O(1\%)$ uncertainties in θ_C become greatly magnified in ϕ. For this reason we keep track of the θ_C dependence in our predictions. For $\cos\phi$ the expression is exact, while for $\sin\phi$ it is good to better than 1%. The final numerical expressions correspond to the limits $\chi^2=1\mp\frac{1}{3}$ and $\sin\theta_C=0.221\pm0.003$, which are used for all numerical predictions in this paper. Notice that c_ϕ is determined to be positive and the experimental data on $\text{Re}\epsilon$ in the kaon system forces s_ϕ positive. Hence there is no quadrant ambiguity: choosing $\theta_{1,2,3}$ all in the first quadrant means that ϕ is also in the first quadrant. The rephase-invariant measure of CP violation [5] is given in our model by

$$\begin{aligned} J&=\text{Im}V_{ud}V_{tb}V^*_{ub}V^*_{td}=c_1c_2c_3s_1s_2s_3^2s_\phi \\ &=2.6\times10^{-5}\left[\frac{V_{cb}}{0.053}\right]^2 f(\chi), \end{aligned} \quad (6)$$

where

$$f(\chi)=\chi\left[1.15-\frac{0.15}{\chi^2}(1\pm0.22)\right]=1^{-0.29}_{+0.23}. \quad (7)$$

The scheme which leads to these predictions involves mass matrices at the unification sale with seven unknown real parameters. Six of these are needed to describe the eigenvalues, $m_u \ll m_c \ll m_t$ and $m_d \ll m_s \ll m_b$, while the seventh is the CP-violating phase. Hence a more predictive theory, having fewer than seven input parameters, must either relate the up-quark mass matrix to that of the down quark, or must have an intrinsic understanding of the family mass hierarchy. Without solving these problems the most predictive possible theory will involve seven Yukawa parameters. Such a predictive scheme can only be obtained by relating the parameters of the lepton mass matrix to those of the down-quark mass matrix. To our knowledge the only way of doing this while maintaining predictivity is to use the ansatz invented by Georgi and Jarlskog [2]. The crucial point about our scheme for fermion masses is that *it is the unique scheme which incorporates the grand unified theory (GUT) scale mass relations $m_b=m_\tau$, $m_s=m_\mu/3$ and $m_d=3m_e$ with seven or less Yukawa parameters and completely independent up- and down-quark matrices.* The factors of 3 result from there being three quark colors.

It is because of this uniqueness that the detailed confrontation of this model with experiment is important. If the model is excluded, for example, by improving measurements of m_t, V_{cb}, or V_{ub}/V_{cb}, then the whole approach of searching for a maximally predictive grand unified scheme may well be incorrect. Alternatively it may mean that there is a very predictive scheme, but it involves relations between the up- and down-quark matrices in an important way. It is important to calculate the observable parameters of the neutral K and B meson systems as accurately as possible, so that future experiments and lattice gauge theory calculations will allow precision tests of this scheme.

Beneath the scale of grand unification our effective theory is just that of the minimal supersymmetric standard model (MSSM). Hence in the rest of this paper we wish to give the predictions for ϵ, ϵ', x_d, x_s, and the CP violating angles α, β, γ in the minimal supersymmetric standard model, with the Kobayashi-Maskawa (KM) matrix given by Eq. (2), and with our predicted values for quark masses.

In Ref. [6] it will be shown that in the MSSM the supersymmetric contributions to ϵ, x_d, x_s and to the CP violating angles α, β, and γ in B meson decay are small. Here we will simply give a simplified discussion of why these contributions are negligible for quark masses and mixings of interest to us. In the standard model with a heavy top quark the quantities ϵ, x_d, and x_s are dominated by box diagrams with two internal top quarks. The amplitude of these standard model box diagrams can be written as

$$B_{ij}=A_{\text{SM}}(V_{ti}V^*_{tj})^2, \quad (8)$$

where $i,j=d,s,b$ label the relevant external mass eigen-

[1]The angle θ_3 used in this paper corresponds to $\theta_3-\theta_4$ used in Ref. [1].

state quark flavors of the diagram and the dependence on the Kobayashi-Maskawa matrix elements is shown explicitly. The leading supersymmetric contributions to these three quantities come from box diagrams with internal squarks and gluinos. In this case the flavor changes occur through off-diagonal squark masses: M_{ij}^2. The amplitude for these box diagrams can be written as

$$B'_{ij} = A_{\rm MSSM}(M_{ij}^2)^2 , \tag{9}$$

where again only the relevant flavor structure has been shown explicitly. It has been assumed that squarks of flavor i and j are degenerate.

In the MSSM the squarks are all taken to be degenerate at the grand unified scale. The squark mass matrices evolve according to renormalization group equations which generate nondegeneracies and flavor-changing entries. When all quarks are light a very simple approximation for the flavor-changing entries of the mass matrices results [7]. Since we predict a top-quark Yukawa coupling close to unity, this is not good enough for our purposes. We use the analytic solutions of the renormalization group equations valid to one loop order in the top-quark Yukawa coupling, but with out Yukawa couplings neglected [8]. This is the same approximation used to obtain our quark mass and mixing predictions [1] and is sufficient, provided $\tan\beta$ is not so large as to make the bottom-quark Yukawa coupling large. In this approximation only SU(2)-doublet squarks have flavor-changing masses. We are able to find a very convenient approximation for the induced flavor changing mass squared matrix elements for the down-type doublet squarks:

$$\frac{M_{ij}^2}{M^2} \simeq 0.4 V_{ti} V_{tj}^* \left[\frac{1+3\xi^2}{1+5.5\xi^2} \right] , \tag{10}$$

where M is the mass of the (nearly) degenerate squarks and ξ is the ratio of the gluino to squark mass at the grand unified scale. For values of the top-quark Yukawa coupling consistent with the prediction of Eq. (1), this approximation is good to better than a factor of 2. The exact result has only a slight sensitivity to the trilinear scalar coupling A of the MSSM, which we have neglected.

Comparing the standard model box amplitude [Eq. (8)] to that of the MSSM superbox amplitude [Eqs. (9) and (10)] it is apparent that the dependence on the Kobayashi-Maskawa matrix elements is identical. Hence the ratio of box diagrams is independent of the external flavors i,j:

$$\left[\frac{B'_{ij}}{B_{ij}} \right] = I(x) \left[\frac{100 \text{ GeV}}{M} \right]^2 \left[\frac{1+3\xi^2}{1+5.5\xi^2} \right]^2 , \tag{11}$$

where $x = m_{\tilde g}/M$ and $m_{\tilde g}$ is the gluino mass. The monotonic function I results from the momentum integral of the superbox diagram [7] and varies from $I(1)=1/30$ to $I(0)=1/3$. If the squarks are taken to be light (e.g., 150 GeV) in an attempt to enhance the superbox amplitude, then $x \geq 1$ to avoid an unacceptably light gluino, resulting in $I \leq 1/30$. To increase I therefore requires an increase in M, but this rapidly decreases the importance of the superbox diagram. The ξ-dependent factor in Eq. (11) is always less than unity. We conclude that the supersymmetric contributions to ϵ, x_d, and x_s are unimportant in our scheme.

We have shown that, in the MSSM with degenerate squarks, the superbox diagrams have the same Kobayashi-Maskawa phases as in the standard-model box diagrams. This implies that the supersymmetric diagrams do not affect the CP asymmetry parameters α, β, and γ in B^0 decay, as is well known [9].

We have assumed that at the grand unified scale the squark mass matrices are proportional to the unit matrix. In supergravity theories this proportionality is expected only at the Planck scale. In general it is possible that large interactions of the quarks with superheavy fields could introduce large flavor changing effects from renormalization group scaling between Planck and grand scales [10]. We assume that this does not happen, as would be the case if the only large Yukawa coupling in the grand unified theory is that which generates the top-quark mass.

We now proceed to our predictions. Since the K_L-K_S mass difference receives large long distance contributions we do not think it provides a useful test of our theory. On the other hand, all observed CP violation is described by the single parameter ϵ, which is reliably calculated from short-distance physics. To calculate ϵ we use a manifestly phase-invariant formula for ϵ in terms of J [11]. We find that the box diagram with internal top quarks dominates. Including a 20% contribution from the diagrams with one top and one charm we find

$$|\epsilon| = 7.2\times 10^{-3} B_K \left[\frac{m_t}{176 \text{ GeV}} \right]^2 \times \left[\frac{J}{2.79\times 10^{-5}} \right] \left[\frac{V_{cb}}{0.053} \right]^2 \frac{s_1^2}{s_c^2} . \tag{12}$$

The parameter B_K describes the large uncertainty in the matrix element of a four-quark operator between kaon states. We use experiment for $|\epsilon|$ and make a prediction for B_K:

$$B_K = 0.40(1\pm 0.01\pm 0.03) \left[\frac{4.15 \text{ GeV}}{m_b} \right]^2 \left[\frac{1.22 \text{ GeV}}{m_c} \right]^2 \times \left[\frac{\eta_b}{1.46} \right]^2 \left[\frac{\eta_c}{1.84} \right]^2 f(\chi)^{-1} , \tag{13}$$

where we have used Eqs. (1) and (6) for m_t and J. The first uncertainty shown comes from the experimental value of $|\epsilon| = (2.26\pm 0.02)\times 10^{-3}$ while the second comes from $\sin\theta_C$. This prediction for B_K is strikingly successful. We stress that m_u/m_d cannot vary too much in our theory: values larger than 0.6 are strongly disfavored by the fact that m_s/m_d is 25, while the present experimental values of V_{ub}/V_{cb} disfavor m_u/m_d lower than 0.6. Allowing $\chi^2 = 1\pm 1/3$, $\sin\theta_C = 0.221\pm 0.003$ gives the range $B_K = 0.31-0.57$ (for $m_b = 4.15$ GeV and $m_c = 1.22$ GeV). This should be compared with recent lattice results $B_K = 0.7\pm 0.2$ in the quenched approximation [12].

In the standard model, predictions for ϵ'/ϵ are very uncertain because they depend sensitively on (i) various strong interaction matrix elements, (ii) the value of Λ_{QCD}, (iii) the value of the strange-quark mass m_s, and (iv) the value of the top-quark mass. We use our central predictions for Λ_{QCD} and m_s, and rely on the $1/N$ approximation for the QCD matrix elements [13]. Using the analytic expression given in Ref. [13] we are able to derive our prediction

$$\frac{\epsilon'}{\epsilon} \simeq 3.9\times 10^{-4}(2.7m_t^{-0.5}+0.5m_t^{0.1} -2.2m_t^{0.4})\chi^{0.4}\left[\frac{0.4}{B_K}\right]^{0.8}, \tag{14}$$

where m_t is to be given in units of 179 GeV and is predicted in Eq. (1), and B_K is predicted in Eq. (13). This small result is not unexpected, given the large value of m_t [14], and we stress that it is uncertain because we do not know how well to trust the $1/N$ matrix elements.

The leading supersymmetric contribution to ϵ'/ϵ comes from a diagram with an internal gluino and an insertion of the flavor-changing squark mass of Eq. (10). We find that this superpenguin amplitude is small compared to the ordinary penguin amplitude:

$$\frac{A(\text{superpenguin})}{A(\text{penguin})} \simeq 0.04\left[\frac{150\text{ GeV}}{M}\right]^2\left[\frac{1+3\xi^2}{1+5.5\xi^2}\right] \times\left[5-4\left[\frac{m_t}{180\text{ GeV}}\right]^2\right]^{-1} \tag{15}$$

for the case of degenerate squarks and gluino of mass M. This is partly because the loop is numerically smaller, but is also because the ordinary penguin diagram is enhanced by an order of magnitude by a large $\ln m_t$ factor. Even though these supersymmetric contributions are negligible, the uncertainties in the QCD matrix elements still imply that ϵ'/ϵ cannot be considered a precision test of our scheme.

The dominant standard model contribution to $B^0\bar{B}^0$ mass mixing arises from the box diagram with internal top quarks. We find that, for the B_d^0,

$$x_d=\frac{\Delta m}{\Gamma}=0.25\left[\frac{\sqrt{B}f_B}{150\text{ MeV}}\right]^2\left[\frac{m_t}{\text{GeV}}\right]^2|V_{td}|^2 . \tag{16}$$

Using Eq. (1) for $m_t, V_{td}=s_1s_3$ and the experimental value for x_d of 0.67 ± 0.10 we predict

$$\sqrt{B}f_B=(167\text{ MeV})\left[\frac{4.15\text{ GeV}}{m_b}\right]\left[\frac{1.22\text{ GeV}}{m_c}\right] \times\left[\left[\frac{\eta_b}{1.46}\right]\left[\frac{\eta_c}{1.84}\right]\frac{|V_{cb}|}{0.053}\left[\frac{0.55}{\eta}\right]\right]^{1/2}. \tag{17}$$

We note that if x_d, m_t, and V_{td} are allowed to range over their experimentally allowed values, the prediction of the box diagram [Eq. (16)] implies that $\sqrt{B}f_B$ will have to range over an order of magnitude. It is therefore a nontrivial success for our theory that it gives a prediction for $\sqrt{B}f_B$ which is close to the quoted values for $f_B\sqrt{B}$. Notice that we have used the QCD factor $\eta=0.55$, rather than 0.85, since this gives values of $\sqrt{B}f_B$ in a scheme appropriate for comparison with lattice results [15]. Our prediction should be compared with recent lattice results $f_B=205\pm40$ MeV and $\sqrt{B}f_B=220\pm40$ MeV [16]. Our predictions for m_t (1), B_K (13), and f_B (17) all depend on η_i which depend on α_s. The numbers quoted are for the fairly low value of $\alpha_s=0.109$. Threshold corrections at the grand unified scale could increase this, easily resulting in a 20% increase in $\eta_b\eta_c$. This not only reduces the top-quark mass, but gives improved agreement with lattice calculations for both B_K and f_B. At any rate, once the top-quark mass is measured our predictions for B_K and f_B will be sharpened considerably. Our results for B_K and f_B agree with those of Ref. [17].

The standard model box diagram relates the mass mixing in B_s^0 to that in B_d^0 by

$$\frac{x_s}{x_d}=\frac{|V_{ts}|^2}{|V_{td}|^2}\left[\frac{B_sf_{B_s}^2}{B_df_{B_d}^2}\right]=25\left[\frac{B_sf_{B_s}^2}{B_df_{B_d}^2}\right], \tag{18}$$

where we used our result for the ratio of Kobayashi-Maskawa factor: $c_1^2/s_1^2=25$ with negligible uncertainty. If the ratio of B meson decay constants could be accurately calculated, and if large values of x_s (say 15 to 25) could be measured, then (18) provides a precision test of our theory.

Finally we consider CP asymmetries which result when B^0 and $\bar{B}^0$ can decay to the same CP eigenstate f [18]. Unitarity of the KM matrix implies that the first and third columns are orthogonal: $V_{ud}V_{ub}^*+V_{cd}V_{cb}^*+V_{td}V_{tb}^*=0$. This can be represented as a triangle since the sum of three vectors is zero. Labeling the angles opposite these three vectors as β, α, and γ, respectively, one finds that the CP asymmetries are proportional to $\sin 2\beta$ for ($B_d\to\psi K_S$, etc.), $\sin 2\alpha$ (for $B_d\to\pi^+\pi^-$, etc.), or $\sin 2\gamma$ for ($B_s\to\phi K_s$, etc.).

In the approximation that $c_2=c_3=1$ and that $s_1s_2\ll 1$, we calculate $\sin 2\alpha$, $\sin 2\beta$, and $\sin 2\gamma$ to an accuracy of $1+0(\lambda^3)$, i.e., to 1% accuracy:

$$\sin 2\alpha=-2c_\phi s_\phi ,$$
$$\sin 2\beta=\frac{2c_1s_1s_2s_\phi}{s_c^2}\left[1+\frac{c_1s_2c_\phi}{s_1}\right], \tag{19}$$
$$\sin 2\gamma=2c_\phi s_\phi\frac{s_1^2}{s_c^2}\left[1+\frac{c_1s_2}{c_\phi s_1}\right].$$

It is interesting to note that s_3 does not appear anywhere in these results. This is because all lengths of the unitarity triangle are simply proportional to s_3. This is similar to the well known result that in the Wolfenstein approximation all lengths of the triangle are proportional to A. For us this lack of sensitivity to s^3 is an essentially exact result. Given that we know s_1 precisely, and that ϕ is extracted from s_1, s_2, and the Cabibbo angle s_c, the only uncertainties in numerically evaluating α, β, and γ come

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from experimental uncertainties in $\sin\theta_C$ and in the dependence of s_2 on m_u/m_d, m_c, and η_c via χ shown in Eqs. (3) and (4). We calculate $\sin2\alpha$, $\sin2\beta$, and $\sin2\gamma$ in terms of χ for $\sin\theta_C = 0.221 \pm 0.03$. The results are shown in Fig. 1. The solid line is for $\sin\theta_C = 0.21$ while the long (short) dashed lines are for $\sin\theta_C = 0.24(0.218)$. Present experiments allow very wide ranges of α, β, γ: $-1 < \sin2\alpha, \sin2\gamma < 1$ and $0.1 < \sin2\beta < 1$ [19] so that our predictions are in a sufficiently narrow range that measurements of these CP asymmetries will provide a precision test of our model. Our predictions are very positive for experimentalists: $\sin2\beta$ is not near its lower bound, and for the two most experimentally challenging cases, $\sin2\alpha$ and $\sin2\gamma$, the asymmetries are close to being maximal.

How well can our model be tested with an asymmetric B factory at the $\Upsilon(4S)$ with luminosity of 3.10^{33} cm^{-2} s^{-1} [20]? We assume a total integrated luminosity of 10^{41} cm^{-2}, and find, using the numbers in [19], that for decay to a final state of branching ratio B, the quantity $\sin2\alpha$ (or $\sin2\beta$ or $\sin2\gamma$) will be measured with an error bar $\pm\delta$:

$$\delta = 0.05 \left[\frac{B}{4.10^{-5}}\right]^{1/2} \left[\frac{10^{41}\ \mathrm{cm}^{-2}}{\int \mathcal{L}\, dt}\right]^{1/2}, \tag{20}$$

where $B = (4,3,2) \times 10^{-5}$ for $B_d \to \phi K_s$, $B_d \to \pi^+\pi^-$, $B_s \to \rho K_s$ relevant for measuring $\sin2\beta$, $\sin2\alpha$, and $\sin2\gamma$, respectively. Measuring all three quantities to ± 0.05 will provide a spectacular precision test of our model. The values of $\sin2\alpha$, $\sin2\beta$, and $\sin2\gamma$ must be fit by a single value of m_u/m_d which will be determined at the ± 0.1 level.

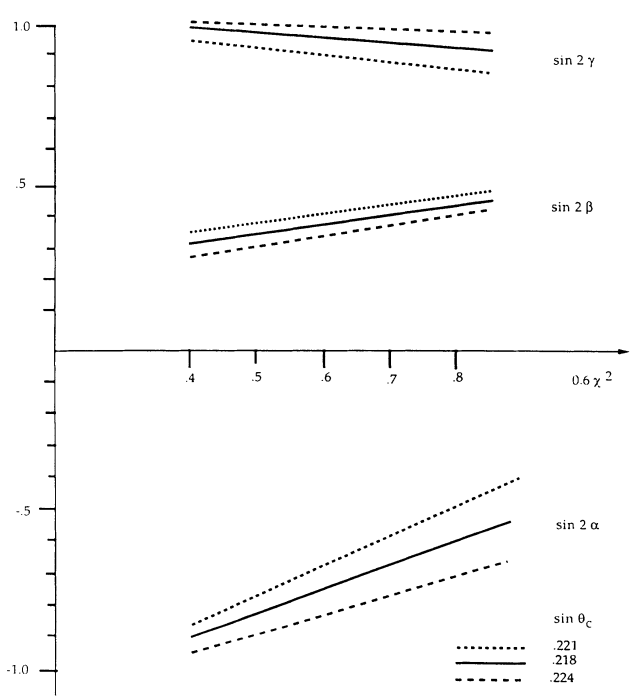

FIG. 1. Predictions for $\sin2\alpha$, $\sin2\beta$, and $\sin2\gamma$. χ is defined in Eq. (4).

We stress two important features of our predictions for these CP asymmetry parameters. Firstly, as in the standard model, they are relatively insensitive to unknown QCD matrix elements. Secondly, they test the Georgi-Jarlskog ansatz in a deep way. For example the only dependence on the renormalization of gauge couplings beneath the grand scale comes from uncertainties in η_c. These uncertainties could be removed completely by taking the strange-quark mass as input. Taking $m_s = 180 \pm 60$ MeV only leads to a 15% uncertainty in χ.

In this paper we have made accurate predictions for parameters in the neutral K and B systems, in the belief that the scheme of Ref. [1] will be decisively tested in the future. It is worth stressing that the predictions for B_K and $\sqrt{B} f_B$ are close to central quoted theoretical values, and thus are already strikingly successful. We have followed the consequences of the only framework incorporating the Georgi-Jarlskog mechanism which uses the minimal number of Yukawa couplings and has independent up- and down-quark mass matrices: there is absolutely no guarantee that ϵ or x_d will be predicted correctly. Consider for example the case when ϵ is dominated by the top-quark contribution which is proportional to $m_t^2 J B_K \,\mathrm{Re}(V_{td} V_{ts}^* V_{us} V_{ud}^*)$. Even though a theory may give successful predictions for m_t(100–200 GeV), V_{td}(0.003–0.018), and V_{ts}(0.030–0.054), it is not guaranteed that the prediction for ϵ will be anywhere close to experiment. The quality $m_t^2 \,\mathrm{Re}(V_{td} V_{ts}^* V_{us} V_{ud}^*)$ has a spread of a factor of 50, and the quantity J which is proportional to $s_1 s_2 s_3^2 s_\phi$ could vary over a very wide range. In particular recall that the phase ϕ is determined by the requirement that $|V_{us}| = 0.221 \pm 0.003$. We think that it is extremely nontrivial that the prediction of our theory for $m_t^2 \,\mathrm{Re}(V_{td} V_{ts}^* V_{us} V_{ud}^*) J$ is such that the central prediction for B_K is 0.4.

The essential results of this paper are given in Eq. (13) for B_K (from ϵ), Eq. (17) for $\sqrt{B} f_B$ (from x_d), Eq. (18) for x_s and the figure for $\sin2\alpha$, $\sin2\beta$, $\sin2\gamma$. The prediction for ϵ'/ϵ in Eq. (14) is less important as it involves uncertainties from the matrix elements. Once the top-quark mass is accurately known, the range of predicted values for B_K and $\sqrt{B} f_B$ will narrow. The largest uncertainty in B_K comes from m_u/m_d. CP asymmetries in decays of neutral B mesons offer the hope of a precision test of our theory which is free of strong interaction uncertainties. An asymmetric B factory operating at the $\Upsilon(4S)$ with an integrated luminosity of 10^{41} cm^{-2} can determine $\sin2\alpha$, $\sin2\beta$, $\sin2\gamma$ to an accuracy of ± 0.05, and this will lead to a determination of m_u/m_d to within ± 0.1.

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