

***P*-wave charmonium production in *B*-meson decays**

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We calculate the decay rates of *B* mesons into *P*-wave charmonium states using new factorization formulas that are valid to leading order in the relative velocity of the charmed quark and antiquark and to all orders in the running coupling constant of QCD. We express the production rates for all four *P* states in terms of two nonperturbative parameters: the derivative of the wave function at the origin and another parameter related to the probability for a charmed-quark-antiquark pair in a color-octet *S*-wave state to radiate a soft gluon and form a *P*-wave bound state. Using existing data on *B* meson decays into χ_{c1} to estimate the color-octet parameter, we find that the color-octet mechanism may account for a significant fraction of the χ_{c1} production rate and that *B* mesons should decay into χ_{c2} at a similar rate.

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The production rate of quarkonium states in various high-energy physics processes can provide valuable insight, not only into the interactions between a heavy quark and antiquark, but also into the elementary processes that produce the $Q\bar{Q}$ pair. The spin-1 *S*-wave resonances, such as the J/ψ of charmonium, are of special experimental significance because they have extremely clean signatures through their leptonic decay modes. Because a significant fraction of the ψ 's come from the decays of the *P*-wave χ_{cJ} states, an understanding of the production of *P*-wave resonances is necessary in order to understand inclusive ψ production. The *P* states are also important in their own right because they probe a qualitatively different aspect of the $Q\bar{Q}$ production process. While the *S* states probe only the production at short distances of a $Q\bar{Q}$ pair in a color-singlet state, the *P* states, as we shall show, also probe the production of a $Q\bar{Q}$ pair in a color-octet state.

One of the simplest production processes for charmonium states is the decay of a *B* meson or baryon. For B^- , \bar{B}^0 , \bar{B}_s , and Λ_b (but not for \bar{B}_c), the *c* and the \bar{c} that form the charmonium bound state must both be produced by the decay or annihilation of the *b* quark. Since this is a short-distance process which occurs on a length scale of order $1/M_b$, where M_b is the mass of the *b* quark, it should be possible to apply perturbative QCD to calculate the inclusive decay rate into a particular charmonium state. If we neglect contributions that are suppressed by powers of Λ/M_b , where Λ is a typical momentum scale for light quarks, the decay rate of a *B* hadron is given by the decay rate of the *b* quark, with the light antiquark in the *B* meson and the light quarks in the *B* baryon treated as noninteracting spectators. For the hadronic and the semileptonic decay rates, the leading corrections are suppressed by two powers of Λ/M_b (Ref. [1]). We expect perturbative QCD calculations to yield predictions for the inclusive decay rates of *B* hadrons into charmonium states that are comparable in accuracy to the predictions

for their semileptonic decay rates.

Most previous calculations of the rate for charmonium production in *B* meson decays [2–5] have been carried out under the assumption that the production mechanism is the decay at short distances of a *b* quark into a color-singlet $c\bar{c}$ pair plus other quarks and gluons, with the *c* and \bar{c} having almost equal momenta and residing in the appropriate angular-momentum state. We will refer to this as the “color-singlet mechanism.” It was assumed that the only nonperturbative input required in the calculation is the nonrelativistic $c\bar{c}$ wave function (for *S* states) or its derivative (for *P* states) at the origin. In this paper we point out that this assumption fails for *P* states, the failure being signaled by the presence of infrared divergences in the QCD radiative corrections. The divergences appear because there is a second production mechanism for *P* states that also contributes at leading order in α_s , namely the decay at short distances of the *b* quark into a color-octet $c\bar{c}$ pair in an *S* state, plus other partons. We will call this the “color-octet mechanism.” In addition to the derivative of the nonrelativistic wave function at the origin, calculations of *P*-wave production require a second nonperturbative input parameter, the probability for the color-octet $c\bar{c}$ pair to radiate a soft gluon and form a color-singlet *P*-wave bound state. The two production mechanisms are summarized below by factorization formulas which are accurate to leading order in v^2 , the square of the relative velocity of the charmed quark and antiquark, and to all orders in the strong coupling constant $\alpha_s(M_b)$. The hard subprocess rates appearing in these formulas are calculated to leading order in the QCD coupling constant. Estimates of the nonperturbative parameter associated with the color-octet mechanism are obtained from experimental data on the production rate of χ_c states in *B* meson decays. We find that the color-octet mechanism may account for a significant fraction of the observed decay rate into χ_{c1} and that *B* mesons should decay into χ_{c2} at a similar rate.

The color-singlet and color-octet mechanisms for the production of P -wave quarkonium states have analogues in the decays of these states. For S -wave resonances, the electromagnetic and light-hadronic decays proceed through the annihilation at short distances of the heavy quark and antiquark in a color-singlet S -wave state. The decay rates are then proportional to the square of $R_S(0)$, the nonrelativistic wave function at the origin, with coefficients that can be calculated perturbatively as a series in $\alpha_s(M_Q)$, where M_Q is the heavy-quark mass. In the case of P -waves, one might expect that the decay should also proceed through the annihilation at short distances of the $Q\bar{Q}$ pair in a color-singlet P -wave state. If this were the case, the decay rates would be calculable in terms of a single nonperturbative factor $R'_P(0)$, the derivative of the nonrelativistic wave function at the origin. While this expectation holds true for decay rates into two photons, explicit calculations of the light hadronic decay rates reveal infrared divergences [6]. It has recently been shown that these infrared divergences can be systematically factored into a second nonperturbative parameter, which is proportional to the probability for the quark and antiquark to be in a color-octet S -wave state at the origin [7]. These results can be summarized by rigorous factorization formulas for the decay rates of quarkonium states into light-hadronic or electromagnetic final states [7, 8]. The factorization formulas are valid to leading order in v^2 , where v is the typical velocity of the heavy quark, and to all orders in the QCD coupling constant $\alpha_s(M_Q)$.

The inclusive production rates for charmonium states also obey factorization formulas. To leading order in v^2 , where v is the typical velocity of the charmed quark relative to the charmed antiquark, and to all orders in $\alpha_s(M_b)$, the inclusive decay rate into ψ satisfies the simple factorization formula assumed in [2–5]:

$$\Gamma(b \rightarrow \psi + X) = G_1 \hat{\Gamma}_1(b \rightarrow c\bar{c}(^3S_1) + X), \quad (1)$$

where $\hat{\Gamma}_1$ is the hard subprocess rate for producing a color-singlet $c\bar{c}$ pair with equal momenta and in the appropriate angular-momentum state $^{2S+1}L_J = ^3S_1$. It can be calculated perturbatively as a series in the QCD running coupling constant $\alpha_s(M_b)$. The nonperturbative parameter G_1 in (1) is proportional to the probability for the $c\bar{c}$ pair to form a bound state and is related to the nonrelativistic wave function at the origin $R_S(0)$:

$$G_1 \approx (3/2\pi)|R_S(0)|^2/M_c^2. \quad (2)$$

A phenomenological determination that makes use of the electronic decay rate of the ψ gives $G_1 \approx 108$ MeV. It can also be given a rigorous nonperturbative definition, so that it can be measured in lattice simulations of QCD [8]. The production rate of the radial excitation ψ' obeys the same factorization formula (1), but with a parameter G_1 that is smaller by the ratio of the electronic decay rates of ψ and ψ' : $G_1 \approx 43$ MeV.

In previous work [4], the decays of b quarks into P -wave charmonium states were assumed to satisfy simple one-term factorization formulas such as (1). Because of the color-octet production mechanism, such formulas are incomplete, and their breakdown is signaled by the ap-

pearance of infrared divergences in order α_s . The correct factorization formulas for P -wave production rates have two terms:

$$\begin{aligned} \Gamma(b \rightarrow h_c + X) &= H_1 \hat{\Gamma}_1(b \rightarrow c\bar{c}(^1P_1) + X; \mu) \\ &\quad + 3 H'_8(\mu) \hat{\Gamma}_8(b \rightarrow c\bar{c}(^1S_0) + X), \end{aligned} \quad (3)$$

$$\begin{aligned} \Gamma(b \rightarrow \chi_{cJ} + X) &= H_1 \hat{\Gamma}_1(b \rightarrow c\bar{c}(^3P_J) + X; \mu) \\ &\quad + (2J + 1) H'_8(\mu) \hat{\Gamma}_8(b \rightarrow c\bar{c}(^3S_1) + X). \end{aligned} \quad (4)$$

The factors $\hat{\Gamma}_1$ and $\hat{\Gamma}_8$ are hard subprocess rates for the decay of the b quark into a $c\bar{c}$ pair in the appropriate angular-momentum state with vanishing relative momentum, the $c\bar{c}$ being in a color-singlet state for $\hat{\Gamma}_1$ and in a color-octet state for $\hat{\Gamma}_8$. They can be calculated perturbatively as series in $\alpha_s(M_b)$, with coefficients that are free of infrared divergences. Note that in (4) the only dependence on the total angular momentum quantum number J in the second term on the right side lies in the coefficient $2J + 1$. The nonperturbative parameters H_1 and H'_8 are proportional to the probabilities for a $c\bar{c}$ pair in a color-singlet P -wave and a color-octet S -wave state, respectively, to fragment into a color-singlet P -wave bound state. They can be given rigorous nonperturbative definitions [8] in terms of matrix elements in nonrelativistic QCD. The parameter H_1 is directly related to the nonrelativistic wave function for the heavy quark and antiquark:

$$H_1 \approx (9/2\pi)|R'_P(0)|^2/M_c^4. \quad (5)$$

Its value has been determined phenomenologically from the light hadronic decay rates of the χ_{c1} and χ_{c2} to be $H_1 \approx 15$ MeV [7]. There is no rigorous perturbative expression for H'_8 in terms of the wave function $R_P(r)$, since a $c\bar{c}$ pair in a color-octet S -wave state can make a transition to a color-singlet P -wave state through the radiation of a soft gluon, which is a nonperturbative process. The parameter H'_8 is not related in any simple way to the analogous parameter H_8 that appears in P -wave decays [7], so it cannot be determined from data on the decays of P states. A phenomenological determination of H'_8 can come only from a production process. Below we will use experimental data on χ_c production in B meson decays to obtain a rough determination of H'_8 .

In the factorization formulas (3) and (4), H'_8 and $\hat{\Gamma}_1$ depend on an arbitrary factorization scale μ in such a way that the complete decay rate is independent of μ . In order to avoid large logarithms of μ/M_b in $\hat{\Gamma}_1$, the factorization scale μ should be chosen on the order of M_b . The scale dependence of $H'_8(\mu)$ is governed by a renormalization-group equation, which to leading order in $\alpha_s(\mu)$ is [8]

$$\mu(d/d\mu)H'_8(\mu) \approx (16/27\pi)\alpha_s(\mu) H_1. \quad (6)$$

The matrix element H_1 is independent of the factorization scale μ . The solution to the renormalization-group equation is therefore elementary. For example, for $\mu < M_c$, the solution is

$$H'_8(M_b) = H'_8(\mu) + \left[\frac{16}{27\beta_3} \ln \left(\frac{\alpha_s(\mu)}{\alpha_s(M_c)} \right) + \frac{16}{27\beta_4} \ln \left(\frac{\alpha_s(M_c)}{\alpha_s(M_b)} \right) \right] H_1, \quad (7)$$

where $\beta_n = (33 - 2n)/6$ is the first coefficient in the QCD β function for n flavors of massless quarks. In the limit in which M_b is very large, the contribution to H'_8 from the perturbative evolution dominates, and one can estimate $H'_8(M_b)$ by setting $\alpha_s(\mu) \sim 1$ and neglecting the constant

$$\mathcal{L}_{\text{weak}} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left(\frac{2C_+ - C_-}{3} \bar{c} \gamma_\mu (1 - \gamma_5) c \bar{s} \gamma^\mu (1 - \gamma_5) b + (C_+ + C_-) \bar{c} \gamma_\mu (1 - \gamma_5) T^a c \bar{s} \gamma^\mu (1 - \gamma_5) T^a b \right), \quad (8)$$

where G_F is the Fermi constant and the V_{ij} 's are elements of the Kobayashi-Maskawa mixing matrix. The weak interaction that gives the Cabibbo-suppressed transition $b \rightarrow c\bar{c}d$ is obtained by replacing \bar{s} by \bar{d} and V_{cs}^* by V_{cd}^* in (8). The coefficients C_+ and C_- in (8) are Wilson coefficients that arise from evolving the effective 4-quark interaction mediated by the W boson from the scale M_W down to the scale M_b . To leading order in $\alpha_s(M_b)$ and to all orders in $\alpha_s(M_b) \ln(M_W/M_b)$, they are

$$C_+(M_b) \approx [\alpha_s(M_b)/\alpha_s(M_W)]^{-6/23}, \quad (9)$$

$$C_-(M_b) \approx [\alpha_s(M_b)/\alpha_s(M_W)]^{12/23}. \quad (10)$$

Taking $\alpha_s(M_W) = 0.116$ and $\alpha_s(M_b) = 0.20$, we find that $C_+(M_b) \approx 0.87$ and $C_-(M_b) \approx 1.34$. When a b quark decays through the interaction (8), the first term produces a $c\bar{c}$ pair in a color-singlet state, while the second produces a $c\bar{c}$ pair in a color-octet state. The color-singlet coefficient $2C_+ - C_-$ is decreased dramatically by renormalization-group evolution, from 1 at the scale M_W , to $2C_+ - C_- \approx 0.40$ at the scale M_b . The color-octet coefficient $(C_+ + C_-)/2$ is increased slightly, from 1 at the scale M_W , to $(C_+ + C_-)/2 \approx 1.10$ at the scale M_b . Since the dramatic suppression of $2C_+ - C_-$ is due to a cancellation between $2C_+$ and C_- , it is sensitive to both the choice M_b for the scale and to higher-order perturbative corrections to the Wilson coefficients. This sensitivity can be removed only by calculations beyond leading order. The Wilson coefficients C_+ and C_- have been calculated at next-to-leading order [10], but the calculations are meaningful only when combined with decay rates that are also calculated beyond leading order.

It is convenient to express all the subprocess rates appearing in the factorization formulas (1), (3), and (4) in terms of

$$\widehat{\Gamma}_0 = |V_{cb}|^2 (G_F^2/144\pi) M_b^3 M_c (1 - 4M_c^2/M_b^2)^2. \quad (11)$$

To leading order in $\alpha_s(M_b)$, the color-singlet subprocess rates $\widehat{\Gamma}_1$ can be extracted from previous calculations [2–4]. The sum of the two subprocess rates that contribute to the decay into the ψ at leading order is

$$\widehat{\Gamma}_1(b \rightarrow c\bar{c}(^3S_1) + s, d) = (2C_+ - C_-)^2 (1 + 8M_c^2/M_b^2) \widehat{\Gamma}_0, \quad (12)$$

where we have used $|V_{cs}|^2 + |V_{cd}|^2 \approx 1$. The subprocess

$H'_8(\mu)$ in (7). Taking $\alpha_s(M_b) \approx 0.20$ and $\alpha_s(M_c) \approx 0.31$, we obtain $H'_8(M_b) \approx 3$ MeV. This should be regarded as only a rough estimate, since the physical value of M_b is probably not large enough for the constant $H'_8(\mu)$ to be negligible.

The subprocesses in the factorization formulas (3) and (4) are decays of the b quark into $c\bar{c}$ plus other quarks and gluons. The dominant contributions involve the coupling of the b quark to $c\bar{c}s$ via an effective 4-quark weak interaction [9], which can, by Fierz rearrangement, be put into the form

rate that contributes to the decay into χ_{c1} is [4]

$$\widehat{\Gamma}_1(b \rightarrow c\bar{c}(^3P_1) + s, d) = 2(2C_+ - C_-)^2 (1 + 8M_c^2/M_b^2) \widehat{\Gamma}_0. \quad (13)$$

The color-singlet subprocess rates that contribute to the production of h_c , χ_{c0} , and χ_{c2} (the 1P_1 , 3P_0 , and 3P_2 states, respectively) vanish to leading order in α_s because of the J^{PC} quantum numbers of these charmonium states.

The rate for direct production of ψ in B meson decay has been calculated to next-to-leading order in α_s [5]. Unfortunately, renormalization group effects were treated incorrectly in that calculation. The treatment of [5] was equivalent to using $2C_+ - C_- = [\alpha_s(M_b)/\alpha_s(M_W)]^{-24/23}$ and $(C_+ + C_-)/2 = [\alpha_s(M_b)/\alpha_s(M_W)]^{3/23}$ for the color-singlet and color-octet coefficients at the scale M_b . This treatment correctly reproduces the term proportional to $\alpha_s \ln(M_W/M_b)$ in the order- α_s correction, but it fails to reproduce the leading logarithms at order α_s^2 and higher. The results of [5] were presented only in graphical form, which prevents us from extracting the correct order- α_s contribution to the subprocess rate for ψ production. In using the leading-order result (12), one should keep in mind that the next-to-leading correction proportional to $\alpha_s(C_+ + C_-)^2$ may be as important numerically as the leading term, which is proportional to $(2C_+ - C_-)^2$.

The color-octet subprocess rates $\widehat{\Gamma}_8$ appearing in (3) and (4) require a new calculation. The most straightforward method (although not the simplest) is to calculate the infrared divergent part of the rate for the decay $b \rightarrow c\bar{c}s$, with the $c\bar{c}$ in a color-singlet P -wave state after having emitted the soft gluon. This rate can be calculated in terms of $R'_p(0)$ by making use of a covariant formalism [12]. The infrared divergences come from the region of phase space in which the momentum of the radiated gluon is soft. In this region, the matrix element can be factored into the amplitude for $b \rightarrow c\bar{c}s$, with the $c\bar{c}$ projected onto an S state, and a term that depends on the gluon momentum. The divergence comes from integrating over the phase space of the gluon. Imposing an infrared cutoff μ on the energy of the soft gluon, one finds that the divergence is proportional to $\ln(M_b/\mu) H_1$. The final results for the infrared divergent terms in the decay rates are

$$\Gamma(b \rightarrow h_c + sg) \sim \frac{12}{\pi^2} |V_{cs}|^2 (C_+ + C_-)^2 \alpha_s \times \ln \frac{M_b}{\mu} \frac{|R'_P(0)|^2}{M_c^4} \widehat{\Gamma}_0, \quad (14)$$

$$\Gamma(b \rightarrow \chi_{cJ} + sg) \sim (2J+1) \frac{4}{3\pi^2} |V_{cs}|^2 (C_+ + C_-)^2 \alpha_s \times \ln \frac{M_b}{\mu} \frac{|R'_P(0)|^2}{M_c^4} \left(1 + 8 \frac{M_c^2}{M_b^2}\right) \widehat{\Gamma}_0. \quad (15)$$

One can identify the corresponding infrared divergence in the perturbative expression for H'_8 by neglecting the running of the coupling constant in (6). The resulting expression for H'_8 is

$$H'_8(M_b) \sim (16/27\pi) \alpha_s \ln(M_b/\mu) H_1. \quad (16)$$

Using this identification together with the expression (5) for H_1 , we find that the color-octet subprocess rates defined in (3) and (4) are

$$\widehat{\Gamma}_8(b \rightarrow c\bar{c}(^1S_0) + s, d) = \frac{3}{2} (C_+ + C_-)^2 \widehat{\Gamma}_0, \quad (17)$$

$$\widehat{\Gamma}_8(b \rightarrow c\bar{c}(^3S_1) + s, d) = \frac{1}{2} (C_+ + C_-)^2 (1 + 8 M_c^2/M_b^2) \widehat{\Gamma}_0. \quad (18)$$

We now turn to the phenomenological applications of our results. Among the corrections to the factorization formulas for B hadron decays are the effects of spectator quarks and antiquarks, which are suppressed by powers of Λ/M_b (Ref. [1]). It is clear from decays of D mesons that spectator effects are much more important for total decay rates than for semileptonic decays. Our predictions for the inclusive decay rates into charmonium states should therefore be more reliable if they are normalized to the semileptonic decay rate, instead of the total decay rate. These branching ratios should be identical for B^- , \bar{B}^0 , \bar{B}_s , and Λ_b , even if their lifetimes differ substantially. To leading order in α_s , the semileptonic decay rate is [11]

$$\Gamma(b \rightarrow e^- \bar{\nu}_e + X) = |V_{cb}|^2 \frac{G_F^2}{192\pi^3} M_b^5 F(M_c/M_b), \quad (19)$$

where $F(x) = 1 - 8x^2 - 24x^4 \ln x + 8x^6 - x^8$. Approximating the ratio M_c/M_b of the heavy-quark masses by the ratio $M_D/M_B \approx 0.35$ of the corresponding meson masses, we find that the phase space factor is $F(M_c/M_b) \approx 0.41$ and that the semileptonic decay rate (19) is $|V_{cb}|^2 (M_b/5.3 \text{ GeV})^5 (3.9 \times 10^{-11} \text{ GeV})$. The Kobayashi-Maskawa factor $|V_{cb}|^2$, as well as the extreme sensitivity to the value of the bottom-quark mass M_b , cancels in the ratio between the charmonium and semileptonic decay rates.

The leading-order QCD prediction for the inclusive decay rate into ψ is obtained by inserting (12) into (1). To take into account phase space restrictions as accurately as possible, we approximate the ratio M_c/M_b of quark masses by $M_\psi/(2M_B) \approx 0.29$. Normalizing to the semileptonic decay rate, we obtain the ratio

$$R(\psi) \equiv \frac{\Gamma(b \rightarrow \psi + X)}{\Gamma(b \rightarrow e^- \bar{\nu}_e + X)} \approx 6.8 (2C_+ - C_-)^2 \frac{G_1}{M_b}. \quad (20)$$

Multiplying by the observed semileptonic branching fraction for B mesons of 10.7%, we find that the pre-

dicted inclusive branching fraction for ψ is 0.23%. There are large theoretical uncertainties in this result because order- α_s corrections to the color-singlet Wilson coefficient $2C_+ - C_-$ may be significant and because the perturbative corrections to the subprocess rate (12) proportional to $\alpha_s(C_+ + C_-)^2$ need not be small compared to the leading-order term, which is proportional to $(2C_+ - C_-)^2$. These uncertainties can be removed only by a complete calculation of the subprocess rate $\widehat{\Gamma}_1(b \rightarrow \psi X)$ to order α_s . There are additional theoretical errors due to uncertainties in the quark masses M_c and M_b , relativistic corrections of order v^2 , and corrections of order Λ^2/M_b^2 . However these uncertainties are probably small compared to the perturbative errors.

For the P states, we define ratios of the inclusive charmonium and semileptonic decay rates, analogous to the ratio $R(\psi)$ defined in (20). We take into account the phase-space restrictions in the decay rate into charmonium as accurately as possible by using half the bound-state mass for M_c and the B meson mass for M_b , so that M_c/M_b varies from 0.32 to 0.34 for the various P states. For the mass of the h_c , we have taken the center of gravity of the χ_{cJ} states. The resulting leading-order QCD predictions for the ratios are then

$$R(h_c) \approx 14.7 (C_+ + C_-)^2 H'_8(M_b)/M_b, \quad (21)$$

$$R(\chi_{c0}) \approx 3.2 (C_+ + C_-)^2 H'_8(M_b)/M_b, \quad (22)$$

$$R(\chi_{c1}) \approx 12.4 (2C_+ - C_-)^2 H_1/M_b + 9.3 (C_+ + C_-)^2 H'_8(M_b)/M_b, \quad (23)$$

$$R(\chi_{c2}) \approx 15.3 (C_+ + C_-)^2 H'_8(M_b)/M_b. \quad (24)$$

These predictions are subject to the same uncertainties as the prediction for ψ given in (20).

The ratios predicted above are for the direct production of charmonium states in the decay of the B hadron. The branching fractions that are measured directly by experiment include indirect production from cascade decays of higher charmonium states. The branching fractions for direct production can only be obtained by making assumptions about the cascade decays. The inclusive branching fractions that have been measured are $(1.12 \pm 0.16)\%$ for ψ [13], $(0.46 \pm 0.20)\%$ for ψ' [13], $(0.54 \pm 0.21)\%$ for χ_{c1} [14], and an upper bound of 0.4% for χ_{c2} [14]. The branching fractions for the cascade processes $\psi' \rightarrow \psi + X$, $\psi' \rightarrow \chi_{c1} + \gamma$, and $\chi_{c1} \rightarrow \psi + \gamma$ are approximately 57%, 9%, and 27%, respectively [13]. If one assumes that there are no other cascade processes that give significant contributions, then the branching fractions for direct production by B meson decay are $(0.71 \pm 0.20)\%$ for ψ , $(0.46 \pm 0.20)\%$ for ψ' , and $(0.50 \pm 0.21)\%$ for χ_{c1} . The ratio of the direct production rates of ψ' and ψ is consistent, within the large error bars, with the ratio of their electronic widths, which is 0.40 [13]. However, the result for ψ is several standard deviations larger than the prediction of (20). The discrepancy could be due to order- α_s corrections to the Wilson coefficient $2C_+ - C_-$ or to corrections to the subprocess rate (12) proportional to $\alpha_s(C_+ + C_-)^2$. Alternatively, it could be due to contributions from cascade decays into ψ and ψ' from higher charmonium states. Studies of the momentum distribution [15] and the polarization [2, 4] of

the ψ 's could help to determine whether cascades from states other than ψ' and χ_{c1} are important.

The measurement of the rate for B -meson decay into χ_{c1} (Ref. [14]) makes it possible to extract a phenomenological value for the unknown matrix element H'_8 . Dividing the branching fraction for direct decay of the B meson into χ_{c1} by the semileptonic branching fraction $(10.7 \pm 0.5)\%$ [13], we find the branching ratio (23) to be 0.047 ± 0.020 . With the choice $M_b = 5.3$ GeV, the color-singlet term on the right side of (23) is 0.006. Attributing the remainder to the color-octet term, we find that $H'_8(M_b) \approx (4.8 \pm 2.3)$ MeV. The quoted error is due to the experimental error in the branching ratio only, and does not include the theoretical error due to the potentially large perturbative corrections to the Wilson coefficient $2C_+ - C_-$ and to the color-singlet subprocess rate $\hat{\Gamma}_1$. One can also use (24) together with the upper bound on the branching fraction into χ_{c2} to obtain the upper bound $H'_8(M_b) < 2.7$ MeV. This bound is consistent with the determination from χ_{c1} production, given the large error bar. The large theoretical uncertainty in our determination of H'_8 could be reduced dramatically by calculating the order- α_s corrections to the P -wave subprocess rates. In the absence of these calculations, one should regard our determination of H'_8 as only a rough estimate.

The color-octet mechanism has a dramatic effect on the pattern of the production rates for P -wave charmonium states. Taking the value $H'_8(M_b) \approx 2.5$ MeV, which is consistent with both the determination from decays into χ_{c1} and with the upper bound from decays into χ_{c2} , we find that the relative production rates predicted by (21)–(24) are $h_c : \chi_{c0} : \chi_{c1} : \chi_{c2} = 1.3 : 0.3 : 1 : 1.3$. A previous calculation of the decay rates of B mesons into charmonium states [4], which did not take into account the color-octet mechanism, gave the relative rates $h_c : \chi_{c0} : \chi_{c1} : \chi_{c2} = 0 : 0 : 1 : 0$. The observation of χ_{c2} production at a rate comparable to that for χ_{c1} production would be a dramatic confirmation of the color-octet production mechanism and would provide a direct mea-

surement of the matrix element $H'_8(M_b)$.

P -wave production provides unique information on the production of heavy-quark pairs and on their binding into quarkonium. In this paper we have outlined a consistent factorization formalism for describing these processes in QCD. In addition to the familiar color-singlet production mechanism, the factorization formulas take into account the color-octet mechanism, in which a heavy quark-antiquark pair is created at short distances in a color-octet S -wave state and subsequently fragments by a non-perturbative process into a color-singlet P -wave bound state. The inclusive production rates for all four P states can be calculated in terms of two nonperturbative inputs: a parameter H_1 that is related to the derivative of the wave function at the origin and a second parameter H'_8 that gives the probability for the fragmentation from the color-octet S -wave state. We have applied this formalism to the production of P -wave charmonium in decays of B hadrons and have used experimental data to obtain a crude estimate of the parameter H'_8 . An accurate determination of H'_8 requires perturbative calculations beyond leading order as well as more accurate experimental data on χ_c production in B meson decays. The accurate determination of this parameter would be very useful because the production rates for P -wave charmonium states in photoproduction, leptonproduction, hadron collisions, and other high-energy physics processes satisfy factorization formulas involving the same two nonperturbative parameters H_1 and H'_8 that appear in B meson decay. Knowledge of these two parameters would, in principle, allow one to compute all of the inclusive P -wave charmonium production rates.

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