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Gravitational memory?

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We discuss the formation of primordial black holes during the early stages of the Universe if the effective gravitational "constant" evolves with time. We describe two possible courses of cosmological evolution: first, one in which the black-hole horizon evolves on the Hubble time scale, and a second, in which the long-range scalar field from which the gravitational coupling is derived remains constant over the horizon scale while evolving in the cosmological background. The second scenario leads to significant changes to the standard picture of black-hole explosions because black holes retain "memory" of the value of the gravitational constant at the time of their formation. There are significant implications for observational searches for black-hole explosions if the gravitation "constant" has changed with time since the Universe was about 10^{-25} s old.

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There are a variety of motivations for considering the formation of small black holes during the early moments of the expansion of the Universe. If variations in the gravitational potential were sufficiently large then black-hole formation is inevitable in the early Universe [1]. Such variations can arise from inhomogeneous initial conditions [2], phase transitions [3], vacuum bubble collisions [4], or the gravitational decay of cosmic-string loops [5]. Any black holes formed in this manner can exert a significant influence upon the future evolution of the Universe. Indeed, primordial black holes which are massive enough to avoid Hawking evaporation by the present are an ideal cold-dark-matter candidate. However, interest in such relics of the very early Universe was first aroused by the possibility of observing the final stages of the Hawking evaporation of a black hole with a lifetime of order the Hubble age of the Universe [6]. This would present astronomers with direct information about a local quantum gravitational event not dissimilar to the big bang itself. And even the nonobservation of such effects would provide important information about the smoothness of the early Universe. Recently there has been interest in the ways in which the Hawking evaporation time and temperature of black holes might be modified by higher-order curvature corrections to the Einstein-Hilbert Lagrangian of general relativity [7]. Although these considerations have significant implications for the existence of relics in the present Universe, because black-hole evaporation can terminate in stable Planck mass relics (see also [8]), the observable consequences of the explosive evaporation are not significantly affected. Here, we discuss some possible consequences of the time variation of the gravitational coupling constant G for the scenario of black-hole evaporation. In this case there are potentially very strong changes in the observable aspects of black-hole explosions to be expected. We shall assume that if any period of inflation occurred during the early Universe then it was completed before the formation of black holes with Hawking lifetime equal to the age of the Universe [7].

In order to illustrate the effects of a varying-G cosmology it is most transparent to use a scalar-tensor gravity theory [9,10] with gravitational action ($c=\hbar=1$):

$$S=(16\pi)^{-1}\int[\varphi R-\varphi^{-1}\omega(\varphi)g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}+2\varphi\lambda(\varphi)](-g)^{1/2}d^4x \ . \tag{1}$$

Variation of S and the matter field action with respect to the metric $g_{\mu\nu}$ and the scalar field φ gives the field equations, linking the space-time Ricci tensor $R_{\mu\nu}$ to the covariantly conserved energy-momentum tensor of the matter fields, $T_{\mu\nu}$:

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R-\lambda(\varphi)g_{\mu\nu}=8\pi\varphi^{-1}T_{\mu\nu}+\varphi^{-2}\omega(\varphi)(\varphi_{,\mu}\varphi_{,\nu}-\tfrac{1}{2}g_{\mu\nu}\varphi_{,\lambda}\varphi^{,\lambda})+\varphi^{-1}(\varphi_{;\mu\nu}-g_{\mu\nu}\Box\varphi) \ , \tag{2}$$

$$\Box\varphi+\tfrac{1}{2}\varphi_{,\mu}\varphi^{,\mu}\frac{d}{d\varphi}\{\ln[\omega(\varphi)\varphi^{-1}]\}+\tfrac{1}{2}\varphi[\omega(\varphi)]^{-1}\{R+2d[\varphi\lambda(\varphi)]/d\varphi\}=0 \ . \tag{3}$$

Substituting the trace of (2) in (3) we obtain

$$[3+2\omega(\varphi)]\Box\varphi+[2\varphi^2\lambda'(\varphi)-2\varphi\lambda(\varphi)]=8\pi T-\omega'(\varphi)\varphi_{,\mu}\varphi^{,\mu} \ . \tag{4}$$

The coupling parameter $\omega(\varphi)$ and the cosmological function $\lambda(\varphi)$ must be specified to complete the theory. In the case where $\lambda=0$ and $\omega=\omega_0=\text{const}$ we obtain the Brans-Dicke theory [11]. In general, the weak field corrections to general relativity tend to zero when $\omega\to\infty$ so long as $\omega'(\varphi)\omega^{-3}\to 0$. We shall cite the Brans-Dicke case as the simplest example of a cosmology in which

$G \propto \varphi^{-1}$ in what follows. We know that the observational limits, approximately $\omega_0 > 500$, from solar system tests and cosmological nucleosynthesis are very strong in this case, but there exist many other scalar-tensor theories $\omega'(\varphi) > 0$, which are compatible with observational constraints. In such theories $\omega(\varphi)$ can be very small in the early Universe, thus producing significant effects from the time evolution of $G(\varphi)$; however, if $\omega(\varphi)$ becomes larger than about 500 by the nucleosynthesis epoch ($t \sim 1-10^3$ s) then observable deviations from general relativistic Friedmann models will be negligible [10]. Since we shall be interested in variations in $G(\varphi)$ which occur during the first 10^{-20} s of the expansion, when black holes small enough to evaporate can form, there can always be significant evolution of G during this very early phase without adverse solar system or nucleosynthesis effects and our arguments are not constrained to operate within the observation restrictions of Brans-Dicke models alone.

It was shown by Hawking [12] that static vacuum black-hole solutions of the Brans-Dicke equations are identical to those of general relativity. We can easily generalize this result to all scalar-tensor theories governed by (1)–(4). This is evident from Eqs. (2)–(4). If φ is constant then the field equations reduce to those of general relativity. The same conclusions hold for cosmological solutions of Einstein's equations with φ constant [hence $\lambda(\varphi)$ is constant and equivalent to the usual cosmological constant] so long as $\varphi\lambda(\varphi)=4\pi T$. If $\lambda \neq 0$ this requires T to be constant and hence the source must be a massless scalar field (or a $p=-\rho$ perfect fluid) plus any trace-free stress. If $\lambda=0$ then the equivalence with Einstein's theory requires $T=0$, that is, a vacuum or a radiation field.

If primordial black-hole formation occurred during a phase of the very early Universe in which the equation of state was that of radiation then black holes could form exactly as in general relativity. For simplicity we shall assume these holes to be of Schwarzschild type. It is possible that the equation of state deviates from that of blackbody radiation during the very early stages of the Universe and in such periods φ would not remain constant. In fact, even for radiation-dominated cosmologies there are general solutions with time-varying $G(\varphi)$ in which $G(\varphi)$ tends to a constant with increasing time.

We now pose the following problem: what happens to black holes during the subsequent evolution of such a universe if the gravitational "constant" evolves in time? In general, the scalar field evolution ensures that the present value of the gravitation "constant" $G(t_0)$ will differ from its value $G(t_f)$ at the time t_f when primordial black holes formed. The problem may be stated in this form irrespective of the precise scalar-tensor coupling or the details of the cosmological evolution between t_f and t_0. Of course, given particular models for this evolution (of which the Brans-Dicke theory is the simplest), one can use other observational constraints derived from nucleosynthesis and solar-system observations to constrain the magnitude of the ratio $G(t_f)/G(t_0)$. Here, we shall confine our attention to general points of principle.

If black holes form at t_f and $G(t_f) \neq G(t_0)$ then we consider the following subsequent histories.

(a) A Schwarzschild black hole of size $R_f=2G(t_f)M$ forms when a mass M enters the cosmological horizon at time t_f. If there is cosmological evolution of $G(t)$ with time then the black hole adjusts its size quasistatically, evolving through a sequence of Schwarzschild states approximated by $R=2G(t)M$.

(b) A Schwarzschild black hole of size R_f forms at t_f, but subsequently the scalar field remains constant over the length scale R_f while it evolves on larger scales at the cosmological rate. At present the black hole will have a size that is determined by the value of $G(t_f)$ at the time of its formation while the background universe is characterized by the value $G(t_0)$. In effect, the scalar field φ is a function of space and time which does not vary within the black-hole horizon on small scales, but varies in time over larger scales.

Both of these scenarios have striking features. The first (a) requires that there be no static black holes during any period when G changes. In the absence of an exact solution it is difficult to assess the ramifications of this. For some small interval $(t-t_f)$ after formation we would expand $G(t)=G(t_f)+(t-t_f)\dot{G}(t_f)+\cdots$ to leading order; thus, the black-hole area changes as $A(t)=A(t_f)[1+2(t-t_f)\{\dot{G}/G\}_f+\cdots]$ and the Hawking temperature becomes $T_{\rm BH} \propto (GM)^{-1} \propto \{G(t_f)\}^{-1}[1-(t-t_f)\{\dot{G}/G\}_f+\cdots]$. Black-hole area decrease occurs in the absence of evaporation if $\dot{G}<0$. If the hole still radiates as a blackbody then the usual Hawking lifetime $\tau_{\rm BH}=\alpha G^2M^3$ for complete evaporation of a black hole of mass M formed at $t_f=0$ will be modified to

$$\alpha G^2(0)M^3=\tau-\tau^2\{\dot{G}(0)/G(0)\}+O(\tau^3)\ , \tag{5}$$

where α gives the number of spin states evaporated ($\alpha=2$ when only photons are emitted).

If $G(t) \propto t^{-n}$ then $M^3 \propto t^{2n+1}$ for $n \neq -\frac{1}{2}$ and $M^3 \propto \ln(t)$ for $n=-\frac{1}{2}$. Thus, for $n \neq -\frac{1}{2}$ and the small time intervals over which the approximation (5) holds, the mass-lifetime relation becomes $\tau \propto M^{3/(2n+1)}$ and, when $n<-\frac{1}{2}$, large holes start to evaporate faster than small ones, whereas small holes begin to evaporate faster when $n>-\frac{1}{2}$ (which includes the standard case of general relativity with $n=0$). In the first case the evaporation slows down whereas in the second it is an explosive instability.

Another consideration for this scenario might be to ask whether a black hole ever forms. In Brans-Dicke theory there exist static spherically symmetric vacuum solutions [11] which are naked singularities rather than Schwarzschild black holes; however, it is not clear that they could be the end point of gravitational collapse of matter in the early Universe. It is also worth recalling that general relativistic cosmological models which are scalar field dominated make primordial black-hole formation very difficult. Since the scalar field contributes a pressure $p=\rho$, so the Jeans length equals the horizon size, overdensities need to collapse almost as soon as they enter the particle horizon or pressure forces will support and dissipate them. However, the probability of black-hole formation from bubble collisions or cosmic string decay can still be large in the scalar-field-dominated case. It was once thought that $p=\rho$ states in the early Universe admitted the possibility of similarity solutions growing at the same rate as the horizon [13] but subsequent investigation revealed that solutions that grow in this way are pathological [14] and will not arise in practice.

The second scenario (b) assumes that there is no nonquantum evolution of the black holes because the φ field remains constant over the scale of the event horizon. Hence, the hole behaves like a small gravitationally bound structure in an expanding universe. The novelty of this possibility is that a black hole carries with it a "gravitational memory" of the value of the gravitational coupling $G(t_f)$ at the time of its formation. The Hawking temperature and lifetime $\tau_{\rm BH}$ of a black hole formed in the early Universe at time t_f will be determined by the value of $G(t_f)$, *at the time of its formation,* not by the value of $G(t_0=\tau_{\rm BH})$ today. The lifetime of a black hole formed when $G=G(t_f)$ is $\tau_{\rm BH}=\alpha G^2(t_f)M^3 \sim 3\times 10^{-27}\alpha\{G^2(t_f)/G^2(t_0)\}(M/1g)^3$ s. This lifetime equals the present age of the Universe, t_0 for holes with initial mass

$$M=\left[\frac{t_0}{\alpha G^2(t_0)}\right]^{1/3}\left[\frac{G(t_0)}{G(t_f)}\right]^{2/3}; \tag{6}$$

that is,

$$M\sim 4.4\times 10^{14}\ \mathrm{g}\times\left[\frac{G(t_0)}{G(t_f)}\right]^{2/3}, \tag{7}$$

where the first term on the right-hand side of (7) gives the standard Hawking mass assuming the Hubble constant is $100\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ and the density of the Universe is equal to the critical density [15]. The Hawking temperature of these black holes become

$$T_{\rm BH}=24\ \mathrm{MeV}\{4.4\times 10^{14}\ \mathrm{g}/M\}\{G(t_0)/G(t_f)\}^{1/3}. \tag{8}$$

In the standard evaporation picture [6], with constant G, black holes of this temperature initially emit photons, light neutrinos, electron-positron pairs, and gravitons together with Boltzmann-suppressed abundances of more massive particles such as μ's, τ's, and more massive hadrons. The detailed spectrum has been analyzed by MacGibbon and Carr [15] taking into account the detailed behavior of quarks and gluons emitted in jetlike events at energies close to the QCD confinement scale $\Lambda_{\rm QCD}\sim 250-300$ MeV. They discuss the detailed observational features expected of the emission over the 50 MeV–1 GeV range.

However, we see that the effect of a "gravitational memory" upon the evaporation process is to alter the mass and temperature of primordial black holes that will be undergoing explosive evaporation today. The change in $T_{\rm BH}$ by the factor $\{G(t_0)/g(t_f)\}^{1/3}$ means that the observable effects of black-hole evaporations may be very different to those normally envisaged even if there is only a very short period of $G(\varphi(t))$ evolution during the first 10^{-25} s of the Universe's history. For example, if $G(t_f)>10^6G(t_0)$, where $t_f\sim 10^{-21}$ s, then the black-hole evaporation temperature is reduced to less than 0.24 MeV, below the electron rest mass, and there no longer exists any possibility of detecting black-hole explosions via the observation of radio or γ-ray bursts created by relativistic electrons and positrons evaporated from the hole spiralling in the Galactic magnetic field [15,16]. Likewise the limits deduced from the x-ray and γ-ray backgrounds [15] are significantly affected. If $G(t_f)>10^{12}G(t_0)$ then the black-hole temperature is less than 2.4 keV and the photon emission is primarily in the x-ray band with massive particle emission restricted to very light weakly interacting particles. A more detailed analysis of these changes and the restrictions must be given elsewhere. Here our purpose is simply to point out that the possibility of time variation of the gravitation constant during the very early stages of the Universe may completely change the manifestations of black-hole evaporation in the present day Universe. Conversely, the quoted limits on the possible abundance of black-hole explosions depend crucially upon the history of $G(\varphi)$ at very early times. As a corollary, the observation of black-hole explosions would allow us to draw conclusions about the gravitational Lagrangian at very high energies.

In summary, we have displayed a simple scenario in which the gravitational coupling constant is derived from a time-dependent scalar field. Such a possibility exists in a wide variety of scalar-tensor gravity theories. Assuming primordial black holes form after inflation during the early stages of the Universe, we discuss two possible consequences of the subsequent time variation of G for the quantum evolution of a primordial black hole. In the first, where quasistatic evolution of the hole on a Hubble time scale is assumed to occur, there are no longer any time-independent black holes. In the other, more likely scenario, the scalar field is constant over the horizon scale but varies in time on larger scales. As a result the black-hole horizon retains memory of the value of the gravitational "constant" on the horizon scale at the time when the black hole formed in the very early Universe. Consequently, the mass and temperature of black holes that completely evaporate by the present time can differ significantly from the standard values first derived by Hawking [6] for the case of unchanging G. In addition to offering a new probe of the strength of gravity during the first 10^{-25} s of the Universe's history, and the evolution of extra space dimensions [17], this possibility has considerable implications for observational searches for exploding black holes and the conclusions that can legitimately be drawn about their possible cosmic abundance from existing astronomical observations of radiation backgrounds and cosmic rays.

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