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Bounds on minicharged neutrinos in the minimal standard model

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In the minimal standard model (MSM) with three generations of quarks and leptons, neutrinos can have tiny charges consistent with electromagnetic gauge invariance. There are three types of nonstandard electric charge given by $Q_{\rm st} + \epsilon(L_i - L_j)$, where $i, j = e, \mu, \tau$ $(i \neq j)$, $Q_{\rm st}$ is the standard electric charge, L_i is a family-lepton number, and ϵ is an arbitrary parameter which is put equal to zero in the usual incarnation of the MSM. These three nonstandard electric charges are of considerable theoretical interest because they are compatible with the MSM Lagrangian and $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes U(1)_Y$ gauge anomaly cancellation. The two most conspicuous implications of such nonstandard electric charges are the presence of two generations of massless charged neutrinos and a breakdown in electromagnetic universality for e, μ, and τ. We use results from (i) charge conservation in β decay, (ii) the physical consequences of charged atoms in various contexts, (iii) the anomalous magnetic moments of charged leptons, (iv) neutrino-electron scattering, (v) energy loss in red giant and white dwarf stars, and (vi) limits on a cosmologically induced thermal photon mass, to place bounds on ϵ. While the constraints derived for ϵ are rather severe in the $L_e - L_{\mu,\tau}$ cases ($|\epsilon| < 10^{-17} - 10^{-21}$), the $L_\mu - L_\tau$ case allows $|\epsilon|$ to be as large as about 10^{-14}.

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The study of electromagnetism is one of the most fundamental activities of both theoretical and experimental physics. In the relativistic quantum domain germane to particle physics, electromagnetism is very successfully described through the direct coupling of massless photons to electrically charged particles via the familiar vector current interaction. In the minimal standard model (MSM), one genus of fermion, the neutrino, is taken to have no direct coupling with photons. However, it is not actually mandatory within the structure of the MSM for neutrinos to possess exactly zero electric charge. The purpose of this paper is to investigate the dramatic consequences of not having neutrinos with precisely zero electric charge in the MSM.

In order to understand how charged neutrinos can arise in the MSM, it is necessary to study the global symmetries of the theory. The MSM exhibits five U(1) invariances which commute with its non-Abelian gauge symmetry group $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L$. One of these is the Abelian gauge symmetry $\mathrm{U}(1)_Y$ where Y is the generator of weak hypercharge, while the other four are the symmetries $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{L_{e,\mu,\tau}}$, where B and $L_{e,\mu,\tau}$ are baryon number and the family-lepton numbers, respectively. The usual version of the MSM is constructed so that these last four groups are automatic global symmetries of the classical Lagrangian, having no associated gauge fields.

An interesting, nontrivial constraint on gauge models is anomaly cancellation. This is often imposed so that the standard proof of the renormalizability of gauge theories applies. Alternatively, one may simply demand as an aesthetic principle that quantum effects not spoil the naive gauge invariance of a model, leading also to gauge anomaly cancellation. However one motivates it, it is striking that in the MSM all gauge anomalies from $\mathrm{SU}(3)_c\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$ cancel within each fermion family. In model building one usually finds that anomaly cancellation imposes severe constraints on the allowed U(1) charges.

It is interesting to note, therefore, that $\mathrm{U}(1)_Y$ is not the only anomaly-free Abelian invariance of the MSM. A simple calculation demonstrates that differences in family-lepton numbers are also completely anomaly-free[1] with respect to $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. For a three family model, there are thus three of these anomaly-free combinations, given by

$$L_{e\mu} \equiv L_e - L_\mu \;\text{ or }\; L_{e\tau} \equiv L_e - L_\tau \;\text{ or }\; L_{\mu\tau} \equiv L_\mu - L_\tau. \tag{1}$$

It is important to understand that although each of these differences is individually anomaly-free, no two are anomaly-free with respect to each other.[2]

This interesting observation immediately leads one to ask whether or not these particular subsets of the global symmetries of the MSM have associated gauge fields. A possibility is that one of $\mathrm{U}(1)_{L_{e\mu}}$, $\mathrm{U}(1)_{L_{e\tau}}$, or $\mathrm{U}(1)_{L_{\mu\tau}}$

[1]Note that the cancellation of the mixed gauge-gravitational anomaly is required in order to derive these invariances as the unique anomaly-free set.

[2]Note that the quark analogues of Eq. (1) are explicitly broken in the MSM Lagrangian (this is manifested through a nondiagonal Kobayashi-Maskawa matrix), so the only anomaly-free Abelian invariance acting on quarks is $\mathrm{U}(1)_Y$.

is gauged as a local symmetry that has no role to play in electroweak physics. The Z' model which ensues has recently been studied in the literature [1].

Another fascinating possibility, which will be the focus of this paper, is for the definition of the weak hypercharge in the MSM to be altered in one of three ways:

$$Y_{e\mu} = Y_{\rm st} + 2\epsilon L_{e\mu} \quad \text{or } Y_{e\tau} = Y_{\rm st} + 2\epsilon L_{e\tau} \quad \text{or } Y_{\mu\tau} = Y_{st} + 2\epsilon L_{\mu\tau}, \tag{2}$$

where $Y_{\rm st}$ is the standard hypercharge of the MSM and ϵ is a free parameter. After electroweak symmetry breaking, this leads to nonstandard unbroken electric charges given by

$$Q_{ij} = Q_{\rm st} + \epsilon L_{ij}, \tag{3}$$

where $Q_{\rm st} = I_3 + Y_{\rm st}/2$ is standard electric charge and $i, j = e, \mu, \tau$ $(i \neq j)$. Equation (3) defines the precise ways in which electric-charge quantization can fail in the multifamily MSM[3] [5]. Note that electromagnetic gauge invariance is still exact, and the photon as usual has no zero-temperature mass (thermal masses will be considered later).

The electric-charge generators Q_{ij} alter the physical electric charges for two out of the three families of leptons. For instance, under $Q_{\mu\tau}$,

$$Q_e = -1, \quad Q_\mu = -1 + \epsilon, \quad Q_\tau = -1 - \epsilon, \quad Q_{\nu_e} = 0, \quad Q_{\nu_\mu} = \epsilon, \quad Q_{\nu_\tau} = -\epsilon, \tag{4}$$

while the quark charges assume their standard values, of course. The two observable consequences of this are that e, μ, and τ do not have identical charges, and two neutrino flavors have equal and opposite charges. The purpose of this paper is to derive phenomenological bounds on ϵ for each of the three nonstandard MSM's.[4] Our phenomenological constraints come either from physics which would be sensitive to (small) violations of electromagnetic universality for e, μ, and τ, or from limits connected with the existence of minicharged massless neutrinos.

Several phenomenological analyses on minicharged particles have recently been published [2, 6–11]. Reference [2] deals specifically with another form of electric-charge dequantization featuring electrically charged neutrinos (see footnote 3 above), while Ref. [6] is discussed in footnote 4 above. The papers in Refs. [7, 8] deal with minicharged particles in models where electric-charge conservation is violated, while Refs. [9–11], on the other hand, examine constraints on completely new and exotic particles of tiny electric charge. Some of the constraints derived in these papers are immediately applicable to the models of charge dequantization considered here, while others are irrelevant. It is important to determine the specific phenomenological constraints on the parameter ϵ in Eq. (3) because of the strong theoretical underpinning it has from the structure of the MSM.

The parameter ϵ for the $\mathrm{U}(1)_{Y_{e\mu}}$ and $\mathrm{U}(1)_{Y_{e\tau}}$ cases [see Eq. (2)] can be directly and severely constrained from a variety of experiments. By assuming electric-charge conservation (which is exact in the models under consideration) in β decay, Zorn, Chamberlain, and Hughes [12] were able to constrain the charge of the electron neutrino, which in our notation leads to $|\epsilon| < 4\times10^{-17}$. A bound of $|\epsilon| < 10^{-19}$ is obtained from the observation of electron neutrinos from supernova 1987A [13]. Also, since electrons now have a charge of $Q_e = -1 + \epsilon$, atoms are no longer electrically neutral (which is a classic signature of electric-charge dequantization). Reference [14] provides a useful summary of experiments on the neutrality of matter performed to date. These authors obtain a bound on the electron-proton charge magnitude difference, which translates into $|\epsilon| < 1.6 \times 10^{-21}$.

Some interesting terrestrial effects are possible if ϵ is nonzero because the Earth may be charged. We will assume first of all that the number of protons in the Earth is equal to the number of electrons. It is certainly possible for this assumption to be wrong, and we will comment on this issue again a little later on.

If $\epsilon \neq 0$, then atoms are charged, and so mutually repulsive forces will exist between our assumed charged Earth and laboratory samples of ordinary matter. If $|\epsilon|$ is large enough, then experiments should already have been sensitive to this. Given that no evidence of such an effect exists, we derive upper "bounds" on $|\epsilon|$ below from a couple of considerations. Note that these limits are not bounds in the rigorous sense of the word, because our assumption that the number of protons equals the number of electrons in the Earth need not be correct.

Eötvös experiments measuring the differential attraction or repulsion of Earth with samples of material A and material B (both taken to be pure elements), lead to an upper "bound" on ϵ given by

$$\epsilon^2 < 10^{-12}\frac{Gm_N}{\alpha_{\rm em}}\left(\frac{Z_A}{M_A} - \frac{Z_B}{M_B}\right)^{-1}, \tag{5}$$

where G is Newton's constant, m_N is the mass of a nucleon, $\alpha_{\rm em}$ is the electromagnetic fine-structure constant, $Z_{A,B}$ are atomic numbers of material A and B, respectively, while $M_{A,B}$ are the masses of atoms of A and B. For typical materials (for instance, copper and lead, see Ref. [15]) this yields

$$|\epsilon| < 10^{-23} \tag{6}$$

[3]If right-handed neutrinos are added to the MSM fermion spectrum, and only Dirac neutrino masses are induced after electroweak symmetry breaking, then the family-lepton numbers are in general explicitly broken and the above form of electric-charge dequantization is excluded. In this case, however, $B-L$ generates an anomaly-free U(1) symmetry, and so charge dequantization can ensue through $Q = Q_{\rm st} + \epsilon(B-L)$ [2, 3] (for bounds on ϵ in this model see Ref. [2]). If bare Majorana masses are included for the right-handed neutrinos, then $B-L$ is also explicitly broken, and no electric-charge dequantization at all is allowed [4].

[4]While this paper was being written up, we came across a paper (Ref. [6]) which quotes some bounds on charge dequantization in the MSM, but it is our intention to do a much more thorough analysis here.

or so. Although this "bound" is a couple of orders of magnitude better than the limits quoted above, it should not be taken too seriously given the electron and/or proton number assumption.

Experiments near the Earth's surface indicate that the Earth has a radial electric field of less than about 100 V/m [8]. With equal proton and electron numbers, we then obtain that

$$|\epsilon| < 10^{-27}. \tag{7}$$

We emphasize, however, that the assumption of equal electron and proton numbers in the Earth is important, and so this limit cannot be regarded as a rigorous bound.

How different do the electron and proton numbers need to be to invalidate these "bounds"? Let us examine the radial electric field limit in more detail. A rigorous constraint can actually be derived if the numbers of protons and electrons are allowed to vary. It is

$$N_p - N_e + \epsilon N_e < 10^{24}, \tag{8}$$

where $N_p (N_e)$ is the number of protons (electrons) in the Earth. The proton number of the Earth is about 10^{51}, so with $N_p = N_e$ we recover the result of Eq. (7). We can ask what $\Delta \equiv N_p - N_e$ needs to be to make the limit on $|\epsilon|$ as weak as the bounds of 10^{-21} and 10^{-17} from atomic neutrality and charge conservation in β decay, respectively. Given that $N_p \sim N_e \sim 10^{51}$, we see from Eq. (8) that

$$\begin{aligned} |\epsilon| \sim 10^{-21} \;&\Rightarrow\; |\Delta| \sim 10^{30}, \\ |\epsilon| \sim 10^{-17} \;&\Rightarrow\; |\Delta| \sim 10^{34}. \end{aligned} \tag{9}$$

If we assume that a nonzero Δ is due to excess electrons, then this amounts to between 1–10^4 kg of electrons. If it is due to the presumably less mobile protons, then this is a mass in the range 10^3–10^7 kg. By way of comparison, a cubic meter of Earth has a mass of about 5500 kg. Another way of looking at this is that it corresponds to a number density of excess electrons or protons of about 1–10^4 particles per cubic millimeter.

Note also that an interesting effect can occur at the level of galaxies. Naively, a limit on $|\epsilon|$ may be obtained from the observed stability of galaxies by requiring that electrostatic repulsion not exceed the gravitational attraction. This yields $|\epsilon| < (G m_N^2/10)^{1/2} = 10^{-20}$ where equal numbers of protons and electrons are again assumed. However, the relic neutrino cloud from the big bang will act as a polarizable medium at the galactic level, and so any galactic charge will be screened to some extent. A simple order of magnitude estimate for the screening length is $(\epsilon e T_\nu)^{-1}$ where $T_\nu \simeq 2K$ is the temperature of the relic neutrinos. For $|\epsilon|$ of the order of 10^{-20} the screening length is therefore expected to be less than typical galactic radii. Therefore, galactic charges for reasonable values of $|\epsilon|$ should be rendered unobservable.

Since all of the bounds on the $U(1)_{Y_{e\mu}}$ and $U(1)_{Y_{e\tau}}$ models are quite severe, the main interest of this paper is to derive bounds on the significantly less constrained model defined by $U(1)_{Y_{\mu\tau}}$. We will examine several phenomenological constraints on ϵ for this case.

The first bound is derived by comparing the anomalous magnetic moments a_μ and a_e of the muon and electron, respectively. (Since the tau anomalous moment is not as precisely measured as the other two we do not need to consider it.) The dominant contribution which ϵ makes to the anomalous moment of the muon comes from the 1-loop Schwinger correction, yielding

$$a_\mu^{(1\ \mathrm{loop})} = (\epsilon - 1)^3 \frac{\alpha_{\mathrm{em}}}{2\pi} \frac{e}{2m_\mu} \tag{10}$$

compared with the electron result $a_e^{(1\ \mathrm{loop})} = (\alpha_{\mathrm{em}}/2\pi)(e/2m_e)$. Keeping only linear terms in ϵ we therefore find that the muon anomalous moment is shifted from its standard value by an amount δa_μ given approximately by

$$\delta a_\mu \simeq -3\epsilon \frac{\alpha_{\mathrm{em}}}{2\pi} \frac{e}{2m_\mu}. \tag{11}$$

We obtain a bound by simply demanding that this shift be less than the experimental uncertainty in a_μ. This approach is justified because of the impressive agreement between the measured anomalous moments and the standard theoretical calculations. The best measurement of a_μ [16] has an error of $\pm 9 \times 10^{-9}(e/2m_\mu)$ yielding

$$|\epsilon| < 10^{-6}. \tag{12}$$

This bound is many orders of magnitude less than the bounds on the gauged $U(1)_{Y_{e\mu}}$ and $U(1)_{Y_{e\tau}}$ models. Quite apart from the specific models we are considering in this paper, it is also interesting to note that this is the most stringent *model-independent* bound on the difference in the electric charges of the electron and muon.

The second constraint we will analyze comes from the measured ν_μ-e scattering cross section $\sigma(\nu_\mu e)$. When $\epsilon = 0$, this process is well described by the exchange of a Z^0 gauge boson in the t channel. For nonzero ϵ there is an additional contribution coming from t-channel photon exchange. We will obtain our bound by demanding that the photon contribution to the cross section lie within experimental errors.

The exact expression for $\sigma(\nu_\mu e)$ includes direct Z^o, direct photon and interference terms, and is rather complicated. The complication arises because of the need to keep the electron mass finite when calculating the t-channel photon-exchange diagram. However, a useful approximate expression is obtained by keeping only those terms which diverge in the massless electron limit. The result for the ϵ-dependent contribution to the cross section is

$$\delta\sigma(\nu_\mu e) \simeq \left[\frac{2\pi\alpha_{\mathrm{em}}^2}{m_e^2} - \left(\frac{2\pi\alpha_{\mathrm{em}}^2}{m_e E_\nu} - 2\sqrt{2}\alpha_{\mathrm{em}} G_F x(1-4x) \right) \ln\left(\frac{E_\nu}{m_e}\right) \right] \epsilon^2 - 2\sqrt{2}\alpha_{em} G_F (1-4x) \ln\left(\frac{E_\nu}{m_e}\right) \epsilon, \tag{13}$$

where $x \equiv \sin^2\theta_W$, G_F is the Fermi constant, m_e is the electron mass, and E_ν is the incident neutrino energy.

Experiments on ν_μ-e scattering use incident neutrino energies E_ν of a few GeV's [17, 18]. Therefore the ratio E_ν/m_e is large (> 3000), which illustrates why the approximate cross section of Eq. (13) is useful. By inputting the values of the various quantities appearing in this expression, we see that the first and third terms dominate over the second. To obtain a bound on ϵ we use the result of the BBKOPST Collaboration [18]:

$$\sigma(\nu_\mu e)/E_\nu = (1.85 \pm 0.25 \pm 0.27) \times 10^{-42}\ \text{cm}^2\ \text{GeV}^{-1} \quad \text{with } E_\nu = 1.5\ \text{GeV}. \tag{14}$$

By adding the statistical and systematic errors in Eq. (14) in quadrature, we find that

$$|\epsilon| < 10^{-9} \tag{15}$$

with both the first and third terms in Eq. (13) of roughly equal importance. Note that this bound is three orders of magnitude more stringent than that from using anomalous magnetic moments.

Both of the above bounds on the gauged $U(1)_{Y_{\mu\tau}}$ version of the MSM were derived from considerations that were purely within the ambit of particle physics. We will now present two bounds which also require the use of astrophysics and cosmology, and so our faith in their veracity will be as solid or weak as our belief in the required astrophysical and cosmological models.

It is well known that bounds on weakly coupled particles can be obtained by requiring that their production in stars be not so strong as to cause premature (and unobserved) cooling. In our case, the decay in red giant stars of massive plasmon states into charged $\nu_\mu\bar{\nu}_\mu$ and $\nu_\tau\bar{\nu}_\tau$ pairs can occur. These very weakly interacting neutrinos and antineutrinos can then escape from the star, thus cooling it. The authors of Ref. [10] have (effectively) calculated a bound on ϵ from red giant cooling by demanding that the rate of energy loss per unit volume to minicharged neutrino-antineutrino pairs not exceed the nuclear energy generation rate per unit volume. They estimate that the former quantity is given by

$$\left(\frac{d^2E}{dV\,dt}\right)_{\nu\bar{\nu}} \simeq 10^{34} \times \epsilon^2\ \text{ergs}\,\text{cm}^{-3}\ \text{sec}^{-1} \tag{16}$$

and requiring that this not exceed about 10^6 ergs cm^{-3} sec^{-1} yields the bound

$$|\epsilon| < 10^{-14}. \tag{17}$$

This result is interesting because it is five orders of magnitude more stringent than the limit obtained from ν_μ-e scattering. The authors of Ref. [11] were also able to derive an astrophysical bound by looking at the cooling of white dwarf stars, obtaining

$$|\epsilon| < 10^{-13} \tag{18}$$

which is an order of magnitude less severe than Eq. (17). Most astrophysicists are confident that the stellar structure and evolution of red giants and white dwarfs are sufficiently well understood that these bounds are to be taken very seriously. It may nevertheless be wise to caution that, due to the very nature of the subject matter, one cannot ascribe as much confidence on these bounds as one can on bounds of purely particle physics origin.

We now turn to an interesting cosmological consequence of having minicharged neutrinos. The standard hot big bang model of cosmology predicts the existence of a thermal background of each flavor of neutrino. The temperature of this bath of thermal neutrinos is found to be slightly less than the 3K temperature of the microwave photon background, $T_\nu \simeq 2$K. Because μ and τ neutrinos are charged in the $U(1)_{Y_{\mu\tau}}$ model, they form a background "cosmic plasma" which permeates the entire Universe. All particles, and in particular photons, have to propagate through this thermal heat bath of neutrinos. Photons will therefore acquire a nonzero "electric mass" from interacting with this medium (in a similar manner to the aforementioned acquisition by photons of a nonzero plasmon mass in stellar interiors). Known bounds on photon electric masses[5] will therefore constrain ϵ, since it is impossible for photons to avoid propagating through the neutrino background plasma.

The thermal electric mass of the photon is calculated through the 1-loop contribution of the charged neutrinos to the photon vacuum polarization tensor, where the internal neutrino propagators are taken at finite temperature. Since a similar calculation is performed in Ref. [20], we will omit the technical details of how this computation is done. The result is

$$[m_\gamma^{\text{el}}]^2 = N_\nu \frac{2\pi}{3}\epsilon^2 \alpha_{\text{em}} (kT_\nu)^2, \tag{19}$$

where $N_\nu = 2$ is the number of charged neutrino flavors and k is Boltzmann's constant. The best bound on the photon electric mass comes from a test of Gauss's law (or, equivalently, Coulomb's law), and is [21]

$$m_\gamma^{el} < 10^{-25}\ \text{GeV}. \tag{20}$$

The resulting bound for ϵ is therefore

$$\epsilon < 10^{-12}. \tag{21}$$

It is interesting that this limit is stronger than those obtained from particle physics measurements, but less severe than those obtained from energy loss in stellar objects.

We should remark here that the derivation of the astrophysical bound [Eqs. (17) and (18)] and the cosmological bound [Eq. (21)] on ϵ assumes that ν_μ and ν_τ have masses less than about 10 keV. Otherwise, (a) the plasmon decay into $\nu\bar{\nu}$ will be forbidden kinematically inside

[5]In principle, a photon can also have a "magnetic mass." However, a nonzero magnetic mass cannot arise from thermal effects [19], so it is irrelevant to the present discussion. Note that the most stringent bounds on the photon mass are derived from knowledge of magnetic fields, and are thus constraints on the magnetic rather than the electric mass of the photon.

red giants and white dwarfs where the typical temperature is of order 10 keV and (b) the cosmological mass density constraint requires that the keV neutrinos decay or annihiliate in the early stages of the evolution of the Universe, so that they will not be around today to give a thermal mass to the photon. While it is true that in the MSM the neutrinos have no zero-temperature masses (as the photon, the neutrinos also acquire a thermal mass of order $\epsilon^2 T$ from the background photons), by slightly modifying the Higgs sector (e.g., adding a Higgs triplet), it is possible to give a small "Dirac" mass for ν_μ and ν_τ without violating charge conservation. The present experimental limits on the masses of ν_μ and ν_τ are 270 keV and 35 MeV respectively, so it is not impossible to invalidate the bounds in Eqs. (17) and (18) and in Eq. (21).

As noted above, by some minor modifications to the Higgs sector neutrinos can acquire tiny masses without violating electromagnetic gauge invariance. However, there will be no neutrino mixing and hence no neutrino oscillations in this case. Therefore, the MSM with minicharged neutrinos cannot account for the apparent deficit in the flux of neutrinos coming from the Sun through any form of neutrino oscillation mechanism. On the other hand, by utilizing another class of extensions to the basic model, the neutrino deficit may be explained by endowing ν_e with a transition magnetic moment with either $(\nu_\mu)^c$ or $(\nu_\tau)^c$, depending on whether the $U(1)_{Y_{e\mu}}$ or $U(1)_{Y_{e\tau}}$ case is considered. This mechansim would also have the advantage of explaining the possible anticorrelation of the solar neutrino flux with sunspot activity.

Another important cosmological question to consider is whether charged relic neutrinos can induce an overall charge for the Universe. If they can then electrostatic repulsion will contribute to the expansion of the Universe. The simple answer to this question is that no overall charge for the Universe will be generated because electric-charge conservation is still exact in our models. This will follow provided, of course, that a neutral universe is posited as an initial condition for the big bang. Charged neutrinos are therefore no more problematic in this regard than any other stable charged particles.

In summary then, we have discovered that experiments on the neutrality of atoms places a bound given by $|\epsilon| < 10^{-21}$ on the allowed nonstandard electric charges $Q_{\rm st} + \epsilon(L_e - L_\mu)$ and $Q_{\rm st} + \epsilon(L_e - L_\tau)$. A direct bound on the electron-neutrino charge of $|\epsilon| < 4\times10^{-17}$ is obtained from similar experiments where charge conservation in β decay is assumed. However, one of the main points of our paper is that the other allowed nonstandard charge $Q_{\rm st} + \epsilon(L_\mu - L_\tau)$ is constrained far less profoundly. Upper bounds on ϵ of 10^{-14}, 10^{-13}, 10^{-12}, 10^{-9}, and 10^{-6} were derived from, respectively, energy loss in red gaint stars, energy loss in white dwarf stars, the thermal electric mass of the photon, ν_μ-e scattering and the anomalous magentic moment of the muon.

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