RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 46, NUMBER 6 15 SEPTEMBER 1992

New test of the equivalence principle for the antiproton

R. J. Hughes
University of California, Los Alamos National Laboratory, Physics Division, P-15, Los Alamos, New Mexico 87545
(Received 3 June 1992)

New tests of the weak equivalence principle for the antiproton and other hadrons are derived from a recent test of *CPT* symmetry in the $K^0\bar{K}^0$ system using the MIT bag model.

PACS number(s): 04.80.+z, 12.40.Aa, 14.20.Dh, 14.40.Aq

The question of how to test the weak equivalence principle (WEP) for antiparticles [1–3] has recently been the subject of revived attention [4–8], but the measurement of the gravitational acceleration of an elementary particle such as the antiproton is very difficult [4]. However, an anomalous gravitational acceleration of the antiproton would be caused by different gravitational couplings to quarks and antiquarks, which might also be apparent in the neutral kaon system because of its remarkable sensitivity to small energy differences between the K^0 and $\bar{K}^0$. Indeed, the action of gravity on this system has been studied by Good [9], Bell and Perring [10], Thirring [11], Kenyon [12], and others [13], although not from the point of view of relating it to the gravitational coupling of the antiproton. In this paper an improved test of the WEP for the antiproton will be deduced from a recent test of *CPT* for the masses of the neutral kaons [14].

We start by reviewing the influence of a possibly anomalous gravitational interaction on neutral kaons [9–13]. In the absence of gravity, the neutral kaon wave function at time t, corresponding to a K^0 created at $t=0$, may be described by [11]

$$\psi(t)=\exp[iS_{\rm cl}(t)/\hbar]\psi_{K^0}\ , \tag{1}$$

where the K^0 wave function is $\psi_{K^0}=\binom{1}{0}$, the $\bar{K}^0$'s is $\psi_{\bar{K}^0}=\binom{0}{1}$, and the action is

$$S_{\rm cl}(t)=-\int^t dt'\mathbf{M}c^2/\gamma\ . \tag{2}$$

In Eq. (2) $\gamma=(1-v^2/c^2)^{-1/2}$, where v is the speed of the kaon, the mass matrix is

$$\mathbf{M}=\begin{bmatrix} m & \Delta m/2 \\ \Delta m/2 & m \end{bmatrix}\ , \tag{3}$$

where m is the mass of the K^0 and $\bar{K}^0$ [$m_{1,2}\equiv m\mp\Delta m/2$ are the masses of the $K_{1,2}=2^{-1/2}(K^0\pm\bar{K}^0)$ propagation eigenstates, respectively], and the integration is performed along the particle's trajectory. We ignore the possible *CP*- or *CPT*-violating terms, and the imaginary parts in $\mathbf{M}$ because these will not be relevant in what follows.

The influence of gravity on a system that obeys WEP can be introduced through the interaction Lagrangian

$$L_{\rm int}=-\tfrac{1}{2}h_{\mu\nu}T^{\mu\nu}\ , \tag{4}$$

between the (weak) background tensor gravitational field

$$h_{\mu\nu}=\frac{2U}{c^2}{\rm diag}(1,1,1,1)\ , \tag{5}$$

and the system's energy-momentum tensor $T^{\mu\nu}$. In Eq. (5) U is the Newtonian gravitational potential and we use the Minkowski metric $\eta_{\mu\nu}={\rm diag}(1,-1,-1,-1)$. For a system for which we wish to derive a test of the WEP, an adjustable coupling parameter α may be introduced between its energy-momentum tensor and the gravitational field (5), by using the interaction Lagrangian

$$L'_{\rm int}=-\tfrac{1}{2}\alpha h_{\mu\nu}T^{\mu\nu}\ , \tag{6}$$

so that the WEP is obeyed when $\alpha=1$ [2,6,7].

The matrix analogue of the energy-momentum tensor for the neutral kaons is

$$T^{\mu\nu}=\mathbf{M}\gamma\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}\ , \tag{7}$$

where $x^\mu=(ct,\mathbf{x})$ are the space-time coordinates along the kaon's world line, so a generalization of (6) to this system is

$$L_{\rm int}(K)=-\tfrac{1}{2}h_{\mu\nu}\bar{T}^{\mu\nu}\ , \tag{8}$$

where

$$\bar{T}^{\mu\nu}=\begin{bmatrix} \alpha m & \Delta m/2 \\ \Delta m/2 & \bar{\alpha}m \end{bmatrix}\gamma\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}\ , \tag{9}$$

and hence gravity can be introduced into Eq. (1) with the replacement

$$S_{\rm cl}\to S=S_{\rm cl}+\int dt\, L_{\rm int}(K)\ .$$

The parameters $\alpha,\bar{\alpha}$ allow one possible violation of WEP, which corresponds to giving the K^0 and $\bar{K}^0$ gravitational accelerations of αg and $\bar{\alpha}g$, respectively, in a gravitational field where matter experiences an acceleration $g=|\nabla U|$ [6]. This model does not allow a violation of the WEP in the off-diagonal elements of $\mathbf{M}$, but this omission will not affect our results because the experimental data that we shall use relate to the diagonal elements alone.

For a horizontally directed kaon beam the potential U can be regarded as constant because the change in gravitational potential energy during one K_L lifetime is very much smaller than the off-diagonal terms in the mass matrix $\mathbf{M}$. Hence, the gravitational terms can be absorbed into redefinitions of the elements of the mass matrix, which then, apparently, become potential and velocity dependent. However, the influence of gravity on the units of length and time must be included before physical inferences can be drawn [15–17]. We shall assume that physical length and time measurements are made by instruments that obey the WEP.

From Eq. (4) the effect of gravity on a time standard ("clock") with the Hamiltonian $H_{\rm clock}$ is given by [16]

$$H_{\rm clock} \to H_{\rm clock} + \tfrac{1}{2} h_{\mu\nu} \int d^3x\, T^{\mu\nu}_{\rm clock} \ , \tag{10}$$

where we have assumed that the gravitational potential does not vary significantly over the dimensions of the clock. From (10) and von Laue's relativistic virial theorem [18] $\int d^3x\, T^i_i = 0$ for a closed system; we obtain $H_{\rm clock} \to (1+U/c^2) H_{\rm clock}$ [16,17], and so the clock frequency, based on transitions between different energy levels of $H_{\rm clock}$, is redshifted [16]. Therefore, we may introduce local (physical) time units $\mathcal{T} = (1+U/c^2)T$ to replace the flat space-time ones, T [15].

Local length units can be obtained from the local time unit and the local speed of light, the influence of gravity on which is obtained from the gravitationally modified action for the electromagnetic field [15]

$$S_{\rm em} = \frac{1}{8\pi c} \int d^4x [(1-2U/c^2)\mathbf{E}^2 - (1+2U/c^2)\mathbf{B}^2] \ . \tag{11}$$

Treating the potential U as constant over the propagation distances of interest, we find that the speed of light becomes $c \to c(1+2U/c^2)$, and so local length units $\mathcal{L} = (1-U/c^2)L$ can be introduced to replace the flat space-time ones L [15].

Using the physical time and length units, the kaon action in the presence of gravity can be written as

$$S(t) = S_{\rm cl}(t) - \int^t dt' \begin{bmatrix} (\alpha-1)m & 0 \\ 0 & (\bar\alpha-1)m \end{bmatrix} U\gamma(1+v^2/c^2) \ , \tag{12}$$

where v is now the kaon's speed in physical units. In particular, note that when the WEP is obeyed ($\alpha=\bar\alpha=1$) physical quantities have their flat space-time values [15]. However, if WEP is violated there would be a K^0-$\bar K^0$ energy difference, which can equivalently be regarded as giving the K^0 and $\bar K^0$ effective (*CPT*-violating [16]) masses of

$$m_{\rm eff}(K^0) = m \left[1 + (\alpha-1)\gamma^2 \frac{U}{c^2}(1+v^2/c^2) \right] \tag{13}$$

and

$$m_{\rm eff}(\mathbf{K}^0) = m \left[1 + (\bar\alpha-1)\gamma^2 \frac{U}{c^2}(1+v^2/c^2) \right] \ , \tag{14}$$

respectively, in the mass matrix.

This potential dependence of physical quantities conflicts with the freedom to shift the potential by a constant that is implied by a linear, massless field equation for U, but it is an inescapable consequence of a violation of WEP and the conservation of energy [7,9,19]. Therefore, we must augment the field equation with the physically motivated boundary condition that the potential vanishes at "infinity" (far from all sources of gravitational field), so that special relativistic results are recovered in the absence of gravity [20]. The value of the potential increases with increasing distance scale [9,16], and recently Kenyon has pointed out [12] that the largest value of the potential that can be reliably estimated is that of our supercluster: $|U_{\rm supercluster}/c^2| \approx 3\times10^{-5}$.

In a recent CERN experiment (NA31), with kaons of energies ~ 100 GeV, a test of *CPT* for the real parts of the diagonal terms in the neutral kaon mass matrix was obtained that can be expressed as [14]

$$|[m(K^0) - m(\bar K^0)]/m(K^0)| \le 5\times10^{-18} \ .$$

So, following Kenyon [12], and applying this constraint to the masses (13) and (14) and using the potential of the supercluster, we obtain the constraint

$$|\alpha - \bar\alpha| \le 2.5\times10^{-18} \ . \tag{15}$$

However, this constraint is stronger than Kenyon's [12] because we have included the large velocity-dependent enhancement of gravity that is present with a tensor interaction [10,11,13,16]. [Note that (15) does not demonstrate WEP for neutral kaons, because it only shows that gravity couples with essentially equal strength to the K^0 and $\bar K^0$ [11], not that the strength is the same as for ordinary matter [2].]

We shall now relate the constraint (15) to the gravitational acceleration of the antiproton by using a quark model, with the additional input of the tests of WEP for matter from the Eötvös-Dicke experiments [20], from which we can infer that the proton and neutron obey WEP [8]. We shall use the cavity approximation [21] to the MIT bag model [22], in which a hadron at rest is modeled as a spherical volume with the energy-momentum tensor $T^{\mu\nu}_{\rm hadron} = T^{\mu\nu}_{\rm constituents} + T^{\mu\nu}_{\rm bag}$ where

$$T^{\mu\nu}_{\rm constituents} = \sum_i \frac{i}{2} \bar\psi_i \gamma^\mu \overleftrightarrow{\partial}^\nu \psi_i, \ \text{and}\ T^{\mu\nu}_{\rm bag} = \eta^{\mu\nu} B \ . \tag{16}$$

In Eq. (16) the summation extends over the wave functions ψ_i of the color-singlet combination of valence quarks and antiquarks inside the bag, which for mass m_i obey the Dirac equation $(i\partial\!\!\!/ - m_i c/\hbar)\psi_i = 0$, and the boundary condition $i\hat{\mathbf{r}}\cdot\gamma\psi_i = \psi_i$ on the surface, which has outwards unit normal $\hat{\mathbf{r}}$. The constant B in Eq. (16) is the confinement energy density. We shall assume *CPT* symmetry for the quark and antiquark masses.

In the low-mass hadrons the quarks or antiquarks occupy the lowest cavity mode wave function, which for radius R is characterized by a dimensionless frequency ω_i that satisfies the equation [21]

$$\tan(kR)=\frac{kR}{1+m_i'R+R\sqrt{k^2+m_i'^2}}\ , \tag{17}$$

where $k=\sqrt{\omega_i^2/R^2-m_i^2c^2/\hbar^2}$, and $m_i'=m_ic/\hbar$. For the u and d quarks, which we take to be massless [21], the frequency is $\omega_0\approx 2.04$, and for a strange quark mass of $m_s\approx 300$ MeV/c^2 it is $\omega_S\approx 2.60$ [21]. The rest energy E_0 of the hadron is determined by minimizing the total energy [21]

$$E(R)=\sum_i\frac{\hbar c|\omega_i|}{R}+\frac{4}{3}\pi R^3B\ , \tag{18}$$

with respect to the radius, giving

$$E_0=\frac{4}{3}\left[4\pi B\left[\sum_i\hbar c|\omega_i|\right]^3\right]^{1/4}\ . \tag{19}$$

We shall assume for definiteness that gravity couples to all quarks with the common strength α_q, to antiquarks with a common strength $\bar{\alpha}_q$, and to the bag energy momentum with a possibly different coupling parameter α_g. Hence, a positive energy (quark) wave function of type i now satisfies the equation

$$\left[i\partial\!\!\!/-m_ic/\hbar+\frac{i}{2}\alpha_q h_{\mu\nu}\gamma^\mu\partial^\nu\right]\psi_i=0$$

inside the bag, and so the effect of gravity on the quark frequency is given by $\omega_i\to\omega_i(1+2\alpha_qU/c^2)$, where we have treated U as a constant over the bag dimensions. Similarly, the antiquark (negative energy) wave functions and frequencies are obtained by the replacement $\alpha_q\to\bar{\alpha}_q$ in the above expressions. The total energy of a hadron at rest in a gravitational field is now given by

$$E_0\to E_0\left\{1+\frac{U}{c^2}\left[\frac{3\left[\alpha_q\sum_{\text{quarks}}\omega_i+\bar{\alpha}_q\sum_{\text{antiquarks}}|\omega_i|\right]}{2\left[\sum_{\text{quarks}}\omega_i+\sum_{\text{antiquarks}}|\omega_i|\right]}-\frac{\alpha_g}{2}\right]\right\}\ , \tag{20}$$

in this model, where we see that, if quarks, antiquarks, and the confinement energy obey the WEP, $\alpha_q=\bar{\alpha}_q=\alpha_g=1$, then so do hadrons, because (20) becomes $E_0\to E_0(1+U/c^2)$.

From Eq. (20) the WEP-violating neutral kaon parameters satisfy

$$\alpha-\bar{\alpha}=\frac{3}{2}\frac{(\alpha_q-\bar{\alpha}_q)(\omega_0-\omega_S)}{\omega_0+\omega_S}\ , \tag{21}$$

in this model, and so we find

$$|\alpha_q-\bar{\alpha}_q|\le 1.5\times 10^{-17}\ , \tag{22}$$

from (15) and (22). Therefore, gravity is required to couple to quarks and antiquarks with essentially equal strength.

The total energy of a nucleon at rest in the gravitational field is

$$E(N)=E_0(N)\left[1+\frac{3\alpha_q-\alpha_g}{2}\frac{U}{c^2}\right]\ , \tag{23}$$

whereas an antiproton's is

$$E(\bar{p})=E_0(N)\left[1+\frac{3\bar{\alpha}_q-\alpha_g}{2}\frac{U}{c^2}\right]\ , \tag{24}$$

by virtue of (20). [In Eqs. (23) and (24) $E_0(N)$ is the nucleon's rest energy in the absence of gravity.] Hence, the gravitational accelerations of the nucleon and antiproton are given by $g(N)/g=(3\alpha_q-\alpha_g)/2$, and $g(\bar{p})/g=(3\bar{\alpha}_q-\alpha_g)/2$, respectively. The Eötvös-Dicke experiments provide the constraint $|[g(N)-g]/g|<2\times 10^{-8}$, from which we may deduce that

$$|(3\alpha_q-\alpha_g)/2-1|<2\times 10^{-8}\ . \tag{25}$$

(Note that there is no requirement that the quarks and confinement energy actually obey WEP.) Therefore, from (22) and (25) the gravitational acceleration of the antiproton satisfies the constraint

$$|g(\bar{p})-g|/g<2\times 10^{-8}\ . \tag{26}$$

Hence, we conclude that, on the basis of the assumptions that we have made, the antiproton obeys WEP to the same precision as the neutron or proton. The constraint (26) is four orders of magnitude better than the previous limit [6], and exceeds the precision of a 1% measurement of the antiproton's gravitational acceleration [4] by 6 orders of magnitude.

The test of WEP, (26), that we have derived for the antiproton is based on the conventional gravitational interaction that is mediated by a single, essentially infinite-range tensor field, which couples to the energy-momentum tensor. Although this is the simplest model that is consistent with other weak-field tests of gravity, the possibility that an anomalous gravitational acceleration of the antiproton might be caused by hypothetical vector and scalar interactions [23] has also been suggested. However, such an anomaly is now constrained to be much smaller than a one percent effect on antiprotons, and a single vector field coupled to baryon number (which would have no effect on kaons) is even more strongly constrained (less than 2×10^{-6} g) [5,8].

We have assumed *CPT* symmetry for the quark and antiquark masses. But even if *CPT* violation was present, it would not provide a plausible exception to the limit (26), because it would have to be fine-tuned to 17 significant figures to cancel the effects of a larger quark-antiquark gravitational coupling difference at precisely the kaon energies in the NA31 measurement, and in the particular gravitational environment of the Earth.

Another assumption in our argument was that there are only two gravitational couplings: one for quarks, and another for antiquarks. A more general scheme would allow different couplings for each quark and antiquark. However, unless these couplings are fine-tuned to 17 significant figures, the phenomenological considerations in this paper require equal gravitational couplings to the u, d, $\bar{d}$, s, and $\bar{s}$ quarks (but no constraint on the gravitational coupling to the $\bar{u}$ antiquark). On symmetry grounds we expect that gravity couples with the same strength to the $\bar{u}$ and $\bar{d}$, which is supported at the level of five parts in ten thousand when the existing test of WEP for the antiproton [6] is included. [Even if fine-tuning of the couplings to evade (22) and (26) did occur, a 1% measurement of the gravitational acceleration of the antiproton would still be a factor of 20 less sensitive than the existing limit [6].] Therefore, rejecting fine-tuning, our improved test of WEP, (26), for the antiproton holds in even this more general scenario. (Also, all hadrons that contain u, d, and s quarks obey WEP to the same precision.)

The strength of the constraint (22) would be reduced in quark models in which less of a hadron's energy is carried by the valence quarks. Some reduction is likely because, in the infinite-momentum frame, all of the nucleon's momentum is carried by the valence quarks in this version of the bag model [24], whereas experimentally one-half is carried by the "sea." Nevertheless, this change would be unlikely to alter the constraint (26) because of the corresponding reduction in the antiproton's energy fraction carried by the valence antiquarks.

The hadronic mass fractions carried by confinement energy (25%), and in valence quark energy (75%) are the same for all hadrons in the quark model that we have used. If the split should be different for different hadrons we would still be able to derive the test (26) of WEP for the antinucleon, because the confinement-kinetic energy division is required to be the same as the nucleon's by *CPT* symmetry.

We conclude that, although a 1% measurement of the antiproton's gravitational acceleration would be a direct, assumption-free test of WEP, it would not be competitive with the tests inferred here, elsewhere [5,6], or those potentially possible with antihydrogen [7,25], if the gravitational force is mediated by a single, essentially infinite-range tensor interaction.

[1] E. Segrè, Annu. Rev. Nucl. Sci. **8**, 127 (1958).
[2] E. F. Beall, Phys. Rev. D **1**, 961 (1970).
[3] T. Goldman and M. M. Nieto, Phys. Lett. **112B**, 437 (1982).
[4] R. E. Brown, J. B. Camp, and T. W. Darling, Nucl. Instrum. Methods **B56/57**, 480 (1991).
[5] E. G. Adelberger *et al.*, Phys. Rev. Lett. **66**, 850 (1991).
[6] R. J. Hughes and M. H. Holzscheiter, Phys. Rev. Lett. **66**, 854 (1991).
[7] R. J. Hughes and M. H. Holzscheiter, J. Mod. Opt. **39**, 263 (1992).
[8] T. E. O. Ericson and A. Richter, Europhys. Lett. **11**, 295 (1990).
[9] M. L. Good, Phys. Rev. **121**, 311 (1961).
[10] J. S. Bell and J. K. Perring, Phys. Rev. Lett. **13**, 348 (1964).
[11] W. E. Thirring, Essays Phys. **4**, 125 (1972).
[12] I. R. Kenyon, Phys. Lett. B **237**, 274 (1990).
[13] O. Nachtmann, Acta Phys. Aust. Suppl. **VI**, 485 (1969); S. H. Aronson *et al.*, Phys. Rev. D **28**, 495 (1983).
[14] R. Carosi *et al.*, Phys. Lett. B **237**, 303 (1990).
[15] W. E. Thirring, Ann. Phys. (N.Y.) **16**, 96 (1961).
[16] J. S. Bell, in *Fundamental Symmetries,* edited by P. Bloch *et al.* (Plenum, New York, 1987), p. 1.
[17] R. J. Hughes, Phys. Rev. D **41**, 2367 (1990).
[18] M. von Laue, Ann. Phys. (Leipzig) **35**, 524 (1911).
[19] P. Morrison, Am. J. Phys. **26**, 358 (1958).
[20] C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1981).
[21] T. DeGrand *et al.*, Phys. Rev. D **12**, 2060 (1975).
[22] A. Chodos *et al.*, Phys. Rev. D **9**, 3471 (1974).
[23] T. Goldman, R. J. Hughes, and M. M. Nieto, Phys. Lett. B **171**, 217 (1986).
[24] R. J. Hughes, Nucl. Phys. **B138**, 319 (1978).
[25] G. Morpurgo, Phys. Rev. Lett. **67**, 1047 (1991).