

Rigorous QCD predictions for decays of P -wave quarkonia

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Rigorous QCD predictions for the decay rates of the P -wave states of heavy quarkonia are presented. They are based on a new factorization theorem which is valid to leading order in the heavy-quark velocity and to all orders in the running coupling constant of QCD. The decay rates for all four P states into light-hadronic or electromagnetic final states are expressed in terms of two phenomenological parameters, whose coefficients are perturbatively calculable. Logarithms of the binding energy encountered in previous perturbative calculations of P -wave decays are factored into a phenomenological parameter that is related to the probability for the heavy-quark-antiquark pair to be in a color-octet S -wave state. Applying these predictions to charmonium, we use measured decay rates for the χ_{c1} and χ_{c2} to predict the decay rates of the χ_{c0} and h_c .

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One of the earliest applications of perturbative QCD was the calculation of the light hadronic and electromagnetic decay rates of S -wave quarkonia [1]. That calculation was based on the assumption that the annihilation of the heavy quark and antiquark is a short-distance process which, because of the asymptotic freedom of QCD, can be computed in perturbation theory. It was assumed that nonperturbative effects could be factored into $R_{nS}(0)$, the nonrelativistic wave function at the origin. This assumption has been supported by subsequent calculations beyond leading order [2, 3]. Taking a similar approach in the case of P -wave states [4], one might expect to be able to calculate their decay rates into light hadrons in terms of a single nonperturbative input $R'_{nP}(0)$, the derivative of the nonrelativistic wave function at the origin. Unfortunately explicit calculations at order α_s^3 (leading order for 3P_1 and 1P_1 (Ref. [5]) and next-to-leading order for 3P_0 and 3P_2 (Ref. [6])) reveal infrared divergences — a clear indication of sensitivity to nonperturbative effects beyond those contained in $R'_{nP}(0)$. In previous phenomenological applications, the divergence has been replaced by a logarithm of the binding energy or the confinement radius [6] or the radius of the bound state [7]. None of these prescriptions has any fundamental justification.

A rigorous treatment of the P -wave decays requires a clean separation between short-distance effects, which can be calculated as a perturbation series in the running coupling constant of QCD, and long-distance effects. The long-distance effects must either be calculated with some nonperturbative method, such as lattice QCD, or else absorbed into a small set of parameters that can be determined phenomenologically. In this paper, we present predictions for P -wave decays based on a rigorous QCD analysis. The QCD predictions are based on factoriza-

tion theorems [8] that will only be justified at an intuitive level in this paper, since our focus will be on their phenomenological implications. The decay rates of the four P states into light hadrons, as well as the decay rates of the 3P_0 and 3P_2 states into two photons, are all predicted in terms of two phenomenological parameters. In addition to $R'_{nP}(0)$, there is a second parameter associated with the probability for the heavy quark and antiquark to be in a color-octet S state. When these predictions are combined with equally rigorous relations between the rates for radiative transitions between P and S states, their predictive power is quite remarkable. From the decay rates of the 3P_2 and 3P_1 states of the first radial excitation of quarkonium, we are able to predict the inclusive decay rates of the 3P_0 and 1P_1 states. For charmonium, some of our predictions differ significantly from the presently accepted results.

The n th radial excitation of heavy quarkonium is split into angular-momentum states $n(^{2S+1}L_J)$ with parity $(-1)^{L+1}$ and charge conjugation $(-1)^{L+S}$. Our basic results are factorization formulas for the decay rates of these states into light hadronic or electromagnetic final states. The factorization formulas are valid to leading order in v^2 , where v is the typical velocity of the heavy quark, and to all orders in the QCD coupling constant $\alpha_s(M_Q)$. For S -wave ($L = 0$) and P -wave ($L = 1$) states, they have the schematic form

$$\Gamma(n(^{2S+1}S) \rightarrow X) = G_1(n) \widehat{\Gamma}_1(Q\bar{Q}(^{2S+1}S) \rightarrow X), \quad (1)$$

$$\Gamma(n(^{2S+1}P) \rightarrow X) = H_1(n) \widehat{\Gamma}_1(Q\bar{Q}(^{2S+1}P) \rightarrow X) + H_8(n) \widehat{\Gamma}_8(Q\bar{Q}(^{2S+1}S) \rightarrow X). \quad (2)$$

In (1) and (2), all nonperturbative effects are factored into the parameters $G_1(n)$, $H_1(n)$, and $H_8(n)$. They depend on the radial quantum number n , but are independent of the total spin S and total angular momentum J to leading order in v^2 . The spin-independence follows from the fact that the QCD interactions of heavy quarks are independent of the spin of the quark, up to relativistic corrections. The factors $\widehat{\Gamma}_1$ and $\widehat{\Gamma}_8$ are hard subprocess rates for the annihilation of the heavy quark and antiquark at threshold, with the $Q\bar{Q}$ in a $2^{2S+1}L$ angular-momentum state and in a color-singlet state for $\widehat{\Gamma}_1$ and a color-octet state for $\widehat{\Gamma}_8$. The subprocess rates are calculable as a perturbation series in $\alpha_s(M_Q)$, the QCD running coupling constant evaluated at the heavy-quark mass. The parameters G_1 , H_1 , and H_8 are proportional to the probabilities for the bound state to contain a $Q\bar{Q}$ pair in a color-singlet S -wave, a color-singlet P -wave, and a color-octet S -wave state, respectively, with separation $r \rightarrow 0$. They may simply be taken as phenomenological parameters to be determined by experiment, but they can also be given rigorous nonperturbative definitions [8] and can therefore be measured using lattice simulations of QCD. In the factorization formula (2), H_8 and $\widehat{\Gamma}_1$ depend on an arbitrary factorization scale μ in such a way that the complete decay rate is independent of μ . We have implicitly set $\mu = M_Q$.

The color-octet contributions to P -wave decays ($H_8\widehat{\Gamma}_8$) may seem peculiar if one is used to thinking of mesons as color-singlet $Q\bar{Q}$ states. However, any meson is a superposition of many components, involving any number of quarks and gluons:

$$|M\rangle = \psi_{Q\bar{Q}}|Q\bar{Q}\rangle + \psi_{Q\bar{Q}g}|Q\bar{Q}g\rangle + \dots \quad (3)$$

In some of these components, notably $|Q\bar{Q}g\rangle$, the heavy quark and antiquark are in a color-octet state. The color-singlet and color-octet pieces of the factored decay rates represent contributions coming from $Q\bar{Q}$ and $Q\bar{Q}g$ components of the meson respectively. The probability carried by the $Q\bar{Q}g$ state (and higher states) is of order v^2 — heavy quarks do not easily radiate a gluon. Consequently such components can be neglected for many applications, including S -wave annihilations. However, the $|Q\bar{Q}\rangle$ contribution to annihilations of P -wave quarkonium is suppressed by v^2 , owing to the angular-momentum barrier, which pushes the quarks apart. Furthermore, in the $Q\bar{Q}g$ component of P -wave quarkonium, the quark and antiquark can be in an S -wave state, with no angular-momentum barrier to hinder their annihilation. The $Q\bar{Q}g$ contribution to the annihilation therefore competes with, and in some cases even dominates, that coming from $Q\bar{Q}$. The enhanced role of the $Q\bar{Q}g$ component makes the decays of P states particularly interesting: they are among the very few processes in quarkonium physics that give us a glimpse of physics beyond the simple quark-potential model.

To leading order in v^2 , H_1 is directly related to the nonrelativistic wave function $\psi_{Q\bar{Q}} = R_{nP}(r)Y_{1m}(\hat{r})$:

$$H_1(n) \approx \frac{9}{2\pi} \frac{|R'_{nP}(0)|^2}{M_Q^4}. \quad (4)$$

There is no simple formula for H_8 in terms of R_{nP} since H_8 is determined by the $Q\bar{Q}g$ wave function $\psi_{Q\bar{Q}g}$ rather than by $\psi_{Q\bar{Q}}$. However, in the limit of very large quark mass, H_8 is dominated by $Q\bar{Q}g$ configurations in which the gluon has large momentum, and an approximate perturbative relation [8] can be obtained between H_1 and H_8 :

$$H_8(n) \approx \frac{16}{27\beta_0} \ln\left(\frac{\alpha_s(E_n)}{\alpha_s(M_Q)}\right) H_1(n), \quad (5)$$

where E_n is the binding energy, $\beta_0 = (33 - 2n_f)/6$, and n_f is the number of light flavors. The constant accompanying the logarithm in (5) cannot be computed perturbatively and is probably quite important for charmonium and bottomonium.

If, in calculating the gluon emission process that gives (5), one neglects the running of the coupling constant, then the perturbative expression for H_8 reduces to

$$H_8(n) \sim \frac{16}{27\pi} \alpha_s \ln\left(\frac{M_Q}{E_n}\right) H_1(n). \quad (6)$$

This logarithm of the binding energy is precisely the infrared divergence that was found in previous (nonrigorous) perturbative analyses of P -wave decays [5, 6]. Infrared divergences arose in this earlier work because the $Q\bar{Q}g$ component of the meson was neglected. In our analysis the infrared sensitivity is factored into the nonperturbative parameter H_8 , so the subprocess rate $\widehat{\Gamma}_1$ involves only hard contributions.

For the decay rates of the P states into light hadrons, the factorization formula (2) in more explicit form is

$$\begin{aligned} \Gamma(n({}^3P_J) \rightarrow \text{LH}) &= H_1(n) \widehat{\Gamma}_1(Q\bar{Q}({}^3P_J) \rightarrow \text{partons}) \\ &\quad + H_8(n) \widehat{\Gamma}_8(Q\bar{Q}({}^3S_1) \rightarrow \text{partons}), \\ &\quad J = 0, 1, 2, \end{aligned} \quad (7)$$

$$\begin{aligned} \Gamma(n({}^1P_1) \rightarrow \text{LH}) &= H_1(n) \widehat{\Gamma}_1(Q\bar{Q}({}^1P_1) \rightarrow \text{partons}) \\ &\quad + H_8(n) \widehat{\Gamma}_8(Q\bar{Q}({}^1S_0) \rightarrow \text{partons}), \end{aligned} \quad (8)$$

where “LH” on the left side of (7) or (8) represents all final states consisting of light hadrons and “partons” on the right side represents perturbative final states consisting of gluons and light-quark-antiquark pairs. Note that, for the 3P_J decays, the second term on the right side of (7) is independent of J . For the decay rates of the 3P_0 and 3P_2 states into two photons, the factorization formula (7) simplifies to the color-singlet term only, because color conservation forbids the annihilation of the $Q\bar{Q}$ in a color-octet state:

$$\Gamma(n({}^3P_J) \rightarrow \gamma\gamma) = H_1(n) \widehat{\Gamma}_1(Q\bar{Q}({}^3P_J) \rightarrow \gamma\gamma), \quad J = 0, 2. \quad (9)$$

When applied to decays into a hard photon plus light hadrons, the factorization formula (8) has remarkable implications. The color-octet term allows the 1P_1 state to decay into a hard photon plus light hadrons at order $\alpha\alpha_s(M_Q)$ through the subprocess $Q\bar{Q}({}^1S_0) \rightarrow \gamma g$. This

TABLE I. Subprocess rates $\widehat{\Gamma}_1$ and $\widehat{\Gamma}_8$ for decays of P -wave quarkonium states.

Decay mode	$\widehat{\Gamma}_1$	Color-singlet subprocess	$\widehat{\Gamma}_8$	Color-octet subprocess
${}^3P_0 \rightarrow \text{LH}$	$(4\pi/3)\alpha_s^2$	${}^3P_0 \rightarrow gg$	$(\pi n_f/3)\alpha_s^2$	${}^3S_1 \rightarrow q\bar{q}$
${}^3P_1 \rightarrow \text{LH}$	$O(\alpha_s^3)$		$(\pi n_f/3)\alpha_s^2$	${}^3S_1 \rightarrow q\bar{q}$
${}^3P_2 \rightarrow \text{LH}$	$(16\pi/45)\alpha_s^2$	${}^3P_2 \rightarrow gg$	$(\pi n_f/3)\alpha_s^2$	${}^3S_1 \rightarrow q\bar{q}$
${}^1P_1 \rightarrow \text{LH}$	$O(\alpha_s^3)$		$(5\pi/6)\alpha_s^2$	${}^1S_0 \rightarrow gg$
${}^3P_J \rightarrow \gamma + \text{LH}$	$O(\alpha\alpha_s^2)$		$O(\alpha\alpha_s^2)$	
${}^1P_1 \rightarrow \gamma + \text{LH}$	$O(\alpha\alpha_s^2)$		$2\pi e_Q^2 \alpha\alpha_s$	${}^1S_0 \rightarrow \gamma g$
${}^3P_0 \rightarrow \gamma\gamma$	$6\pi e_Q^4 \alpha^2$	${}^3P_0 \rightarrow \gamma\gamma$	0	
${}^3P_2 \rightarrow \gamma\gamma$	$(8\pi/5)e_Q^4 \alpha^2$	${}^3P_2 \rightarrow \gamma\gamma$	0	

decay produces a final state consisting of a hard photon recoiling against a hard gluon jet and against the hadrons from the fragmentation of the gluon in the $Q\bar{Q}g$ state. It is possible that this dramatic decay mode of the 1P_1 state could serve as an experimental signature for this particle.

The leading subprocess rates $\widehat{\Gamma}_1$ and $\widehat{\Gamma}_8$ for light-hadronic and electromagnetic decays of P states are listed in Table I. They can be extracted from previous calculations of P -wave decays [4–6] by using the expressions (4) and (6) for H_1 and H_8 . Since next-to-leading-order corrections in α_s have not been computed for the $\widehat{\Gamma}_8$'s, we work only to leading order throughout. However, we note that, since H_1 and H_8 are independent parameters, a higher-order contribution involving H_1 could, in principle, be numerically important compared to a leading-order contribution involving H_8 .

The two nonperturbative parameters H_1 and H_8 can be obtained by measuring the decay rates into light hadrons of the 3P_1 and 3P_2 states, which are the two P states most accessible to experiment. At leading order in v^2 and $\alpha_s(M_Q)$, they are

$$H_1 \approx \frac{45}{16\pi} \frac{\Gamma({}^3P_2 \rightarrow \text{LH}) - \Gamma({}^3P_1 \rightarrow \text{LH})}{\alpha_s^2(M_Q)}, \quad (10)$$

$$H_8 \approx \frac{3}{\pi n_f} \frac{\Gamma({}^3P_1 \rightarrow \text{LH})}{\alpha_s^2(M_Q)}. \quad (11)$$

For applications at leading order in $\alpha_s(M_Q)$, it is convenient to eliminate H_1 and H_8 to obtain direct relations between decay rates. From (7) and (9), we get the following relations between the decay rates of the spin-triplet states, valid to leading order in v^2 and in $\alpha_s(M_Q)$:

$$\frac{\Gamma({}^3P_0 \rightarrow \text{LH}) - \Gamma({}^3P_1 \rightarrow \text{LH})}{\Gamma({}^3P_2 \rightarrow \text{LH}) - \Gamma({}^3P_1 \rightarrow \text{LH})} \approx \frac{15}{4}, \quad (12)$$

$$\frac{\Gamma({}^3P_0 \rightarrow \gamma\gamma)}{\Gamma({}^3P_2 \rightarrow \text{LH}) - \Gamma({}^3P_1 \rightarrow \text{LH})} \approx \frac{135}{8} e_Q^4 \left(\frac{\alpha}{\alpha_s(M_Q)} \right)^2, \quad (13)$$

$$\frac{\Gamma({}^3P_2 \rightarrow \gamma\gamma)}{\Gamma({}^3P_2 \rightarrow \text{LH}) - \Gamma({}^3P_1 \rightarrow \text{LH})} \approx \frac{9}{2} e_Q^4 \left(\frac{\alpha}{\alpha_s(M_Q)} \right)^2. \quad (14)$$

These differ from previous predictions [4], which can be obtained by setting $\Gamma({}^3P_1 \rightarrow \text{LH}) = 0$. At leading order, the decays of the two spin-1 states involve only the color-octet term, so the ratios of the decay rates are simply the ratios of the subprocess rates $\widehat{\Gamma}_8$ in Table I:

$$\frac{\Gamma({}^1P_1 \rightarrow \text{LH})}{\Gamma({}^3P_1 \rightarrow \text{LH})} \approx \frac{5}{2n_f}, \quad (15)$$

$$\frac{\Gamma({}^1P_1 \rightarrow \gamma + \text{LH})}{\Gamma({}^3P_1 \rightarrow \text{LH})} \approx \frac{6}{n_f} e_Q^2 \frac{\alpha}{\alpha_s(M_Q)}. \quad (16)$$

The prediction (15) was first made by Barbieri *et al.* [5].

There are also relations among the radiative transitions between P and S states that are accurate up to relativistic corrections of order v^2 (Ref. [9]). The predictions for the decay rates of P states into S states plus a photon are

$$\frac{\Gamma({}^1P_1 \rightarrow \gamma {}^1S_0)}{E_\gamma^3} \approx \frac{\Gamma({}^3P_J \rightarrow \gamma {}^3S_1)}{E_\gamma^3}, \quad J = 0, 1, 2, \quad (17)$$

where E_γ in the denominator is understood to be the energy of the photon for the transition in the numerator. In terms of the masses M_P and M_S of the bound states, $E_\gamma = (M_P^2 - M_S^2)/(2M_P)$.

We now apply our QCD predictions to the charmonium system. For the first radial excitation, the 3P_J states are called χ_{cJ} and the 1P_1 state is called the h_c . We use measured decay rates of the χ_{c1} and χ_{c2} to predict the inclusive decay rates of the χ_{c0} and h_c . It is important to have reasonable estimates for the theoretical errors in our predictions if they are to be compared with experimental results. The two main sources of theoretical error are relativistic corrections and higher-order perturbative corrections. In potential models of charmonium, the average value of v^2 is found to be about 0.23 (Ref. [10]). Since our factorization formulas are valid only to leading order in v^2 , we expect an error on the order of 20% due to relativistic effects. Similarly, we expect deviations on the order of 20% from the equalities (17) involving the radiative transition rates. We can estimate the perturbative error from the size of the perturbative corrections in other bound-state calculations. Based on a number of next-to-leading-order calculations for S -wave bound states [7], we

TABLE II. Predictions for total and partial decay rates of P -wave charmonium states.

	Γ in MeV	$\Gamma(\text{LH})$	$\Gamma(\gamma J/\psi), \Gamma(\gamma \eta_c)$
χ_{c0}	4.8 ± 0.7 ($\pm 35\%$)	4.7 ± 0.7 ($\pm 36\%$)	0.099 ± 0.010 ($\pm 20\%$)
χ_{c1}	Input: 0.88 ± 0.14	0.64 ± 0.10	0.240 ± 0.041
χ_{c2}	Input: 1.98 ± 0.18	1.71 ± 0.16	0.267 ± 0.033
h_c	0.98 ± 0.09 ($\pm 22\%$)	0.53 ± 0.08 ($\pm 36\%$)	0.45 ± 0.05 ($\pm 20\%$)

estimate the perturbative error to be $4\alpha_s(M_c)/\pi$, where M_c is the mass of the charm quark. To avoid the ambiguity in the value of M_c , we determine $\alpha_s(M_c)$ by taking the coupling constant $\alpha_s(M_b) = 0.179 \pm 0.009$ extracted from bottomonium decays [7] and evolving it down to the scale M_c . This does not require that we know M_c and M_b separately, but only their ratio, for which we use the ratio of the J/ψ and Υ masses: $M_c/M_b \approx 0.33$. The resulting value of the coupling constant is $\alpha_s(M_c) = 0.25 \pm 0.02$. Our estimate of the perturbative error is therefore 30%. We treat the 8% error in the value of $\alpha_s(M_c)$ itself as an experimental error. We take the QED coupling at the scale M_c to be $\alpha = 1/133.3$.

We assume in our analysis that the decays into light hadrons and the radiative transitions to J/ψ or η_c are the only decay modes which contribute appreciably to the total decay rates of χ_{cJ} and h_c . In particular, we neglect pionic transitions of the P states to the S states, of which the most important decay modes should be $J/\psi + \pi\pi$ and $\eta_c + \pi\pi$. The rate for the particular decay $h_c \rightarrow J/\psi + \pi\pi$ has been estimated within a well-developed phenomenological framework [11] to be on the order of 6 keV, and the other two-pion transition rates should be of the same order of magnitude. The errors due to neglecting these contributions to the total decay rates are negligible compared to other errors.

Precision measurements of the total decay rates of the 3P_1 state χ_{c1} and the 3P_2 state χ_{c2} have recently been carried out at Fermilab by the E760 collaboration [12]. Their results, with statistical and systematic errors added in quadrature, are listed as input values in the first column of Table II. Previous experiments have measured the branching fractions for the radiative transitions of the χ_{c1} and χ_{c2} into the J/ψ [13], and they are listed as input values in the first column of Table III. We use the radiative branching fractions and the total decay rates to obtain the partial rates given in Table II for light-hadronic and radiative decays of the χ_{c1} and χ_{c2} .

Inserting the partial rates into light hadrons into (10) and (11), we determine the nonperturbative parameters for P -wave decays of charmonium to be

$$H_1 \approx 15.3 \pm 3.7 \text{ MeV} (\pm 36\%), \quad (18)$$

$$H_8 \approx 3.26 \pm 0.73 \text{ MeV} (\pm 36\%). \quad (19)$$

Here, and throughout the remainder of this paper, the first error is from the uncertainties in our experimental inputs and the second error is our estimate of the theoretical uncertainty. The theoretical error is computed by combining the relativistic error of 20% and the perturbative error of 30%, using the standard formulas for propagating independent errors. The ratio of the two nonperturbative parameters in (18) and (19) is $H_8/H_1 \approx 0.21$. This is roughly consistent with the value that one would obtain from the leading-log perturbative expression (5) by arbitrarily setting $\alpha_s(E_n) \sim 1$. However, there is no *a priori* justification for using the leading-log approximation at such small values of the heavy-quark mass.

We proceed to calculate the total decay rates of the χ_{c0} and h_c . The experimental values for the radiative transition rates of χ_{c1} and χ_{c2} in Table II are in good agreement with the theoretical prediction (17). This gives us confidence in our predictions in Table II for the radiative transition rates of χ_{c0} and h_c . In calculating the radiative transition rate of the h_c , we have assumed that its mass is at the center of gravity of the nine spin states of χ_{c0} , χ_{c1} , and χ_{c2} : $M_{h_c} = 3525 \text{ MeV}$. The predictions in Table II for the decay rates of χ_{c0} and h_c into light hadrons are obtained by using (12) and (15). Adding the light-hadronic and radiative decay rates, we obtain the total decay rates given in Table II. The prediction for the χ_{c0} differs from the presently accepted value [13] of $(14 \pm 5) \text{ MeV}$ by several standard deviations. Our calculations suggest a real discrepancy between this measurement of the χ_{c0} decay rate and the experimental data on the χ_{c1} and χ_{c2} that we have used as our input.

In Table III, we present our predictions for the branching fractions for decays of the charmonium P states into final states containing photons. The branching fractions for χ_{c0} and χ_{c2} into two photons are calculated from (13) and (14). The branching fraction for the h_c to decay into a hard photon plus light hadrons is calculated using (16). Our prediction for the radiative branching fraction of the χ_{c0} is significantly larger than the accepted value [13] of 0.0066 ± 0.0018 . Our prediction for the branching fraction of the χ_{c2} into two photons is considerably smaller

TABLE III. Predictions for branching fractions of P -wave charmonium states.

	$B(\gamma J/\psi), B(\gamma \eta_c)$	$B(\gamma + \text{LH})$	$B(\gamma\gamma)$
χ_{c0}	0.021 ± 0.004 ($\pm 40\%$)		$(6.8 \pm 1.9) \times 10^{-4}$ ($\pm 50\%$)
χ_{c1}	Input: 0.273 ± 0.016		0
χ_{c2}	Input: 0.135 ± 0.011		$(4.1 \pm 1.1) \times 10^{-4}$ ($\pm 36\%$)
h_c	0.46 ± 0.05 ($\pm 22\%$)	0.017 ± 0.003 ($\pm 42\%$)	0

than the accepted value [13] of $(11 \pm 6) \times 10^{-4}$. A rather tantalizing prediction is that the branching fraction for the h_c to decay into a hard photon plus light hadrons is about 2%. This is large enough that it may be possible to detect the decay of the h_c by observing the resulting hard photon.

P -wave annihilations provide unique information on the dynamical role of the gluon in determining hadron structure. They challenge us to go beyond the quark-potential picture in modeling hadrons. In this paper we have outlined the first rigorous formalism for describing these processes in QCD. We have applied the formalism in a detailed analysis of charmonium P -wave decays. Within our formalism it is possible to refine system-

atically the theoretical predictions for the decay rates, both by computing higher-order perturbative contributions, and also by using lattice simulations to compute the nonperturbative parameters. The promise of significant improvements in both theory and experiment make the P -wave decays important testing grounds for ideas about both perturbative and nonperturbative QCD.

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