

## New method for determination of the $D \rightarrow \bar{K}^* + e^+ + \nu$ axial-vector form factors without resorting to angular distributions

Xuan-Yem Pham

*Laboratoire de Physique Théorique et Hautes Energies, Paris, France\**

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Together with the rate  $\Gamma(P \rightarrow V + e + \nu)$  where  $P$  and  $V$  stand for pseudoscalar and vector mesons, respectively, measurement of the slope  $d\Gamma/dq^2$  at  $q^2=0$  is sufficient to determine the two axial-vector form factors  $A_1(0)$  and  $A_2(0)$  and, hence, to bypass the complicated fourfold angular distributions fitting method used by the experimentalists. Examples of  $D \rightarrow \bar{K}^* + e^+ + \nu$  and  $B \rightarrow \rho + e^+ + \nu$  are given.

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The knowledge of the form factors in the semileptonic decays of mesons containing heavy flavors is of great interest for at least two reasons: the extraction from data of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element involving charm and  $b$ -flavored quarks, on the one hand, and the calculations of exclusive hadronic decays using the factorization approach, on the other hand. These two problems can only be solved if the form factors are unambiguously known. Furthermore, they are intimately connected to the domain of  $CP$  violation in  $b$ -flavored physics, since the latter depends on the CKM mixing angles and on the hadronic  $B$  decays that in turn rest on form factors by factorization.

The quark model has been generally regarded as providing a good description of semileptonic processes since its predictions agree quite well with the rate  $D \rightarrow K + e + \nu$  and with both the rate and the polarization of the final vector meson  $D^*$  in  $B \rightarrow D^* + e + \nu$ . The model then was a serious candidate for use in calculating the form factors in all other semileptonic modes so as to extract the CKM matrix from data. However in these last two years, a serious discrepancy between the quark model and experiments arises on the rate and presumably also on the polarization of the  $K^*$  in the mode  $D \rightarrow K^* + e + \nu$ . While consensus seems to be settled among six experimental groups [1] which show that the rate  $\Gamma(D \rightarrow K^* + e + \nu)$  is only about one-half the  $\Gamma(D \rightarrow K + e + \nu)$  (quark model prediction indicates a rather opposite trend), the experimental situation on the  $K^*$  polarization in  $D \rightarrow K^* + e + \nu$  is still confused, its longitudinal versus transverse ratio  $R$  is quoted as  $0.47^{+0.95+0.09}_{-0.12-0.15}$  [2],  $1.18 \pm 0.18 \pm 0.08$  [3],  $1.8^{+0.6}_{-0.3} \pm 0.3$  [4] while the quark model predicts  $R$  around 1. The discrepancy then stimulated the E691 [4] Collaboration and very recently the E653 [3] Collaboration to go to the

source of the problem by directly measuring the three form factors. Their method is based on the fourfold likelihood fit. Following [5,6] let us introduce three angles: the "weak" one  $\theta$  between the  $K^*$  and the  $e$  momenta, the "strong" one  $\theta_v$  in the cascade decay of  $K^*$  into  $K + \pi$ ,  $\theta_v$  is the angle between the  $K^*$  and  $K$  momenta and the azimuthal  $\chi$  angle between the weak and the strong decay planes. While the single angular  $\theta_v$  distribution measures only the ratio  $R$  (longitudinal versus transverse  $K^*$  polarization), the triple angular  $\theta_v, \theta, \chi$  correlation (together with  $q^2$ ) directly separates the form factors. This fourfold fitting method which requires very high Monte Carlo statistics is used by both the E691 and E653 groups; their results [3,4] for the ratio  $A_2/A_1$  are however at variance with one another, thus giving different results for  $R$  mentioned earlier. Facing such a situation, we suggest that instead of looking at the angular distributions (either single  $\theta_v$  or triple  $\theta_v, \theta, \chi$ ), the measurement of the slope  $d\Gamma/dq^2$  at  $q^2=0$  provides a much simpler alternative that can be used to extract the two principal axial form factors  $A_1, A_2$ .

The physical reason for measuring the slope of  $D \rightarrow \bar{K}^* + e^+ + \nu$  at  $q^2=0$  can be understood in the following way: in the electron massless limit (the case of the muon will be discussed later), and in the rest frame of the  $D$  meson, when  $q^2=0$ , the positron  $e^+$  and the neutrino  $\nu$  have momenta exactly parallel and opposite to that of  $\bar{K}^*$ . Because of the  $V-A$  property of the weak leptonic current, the neutrino and positron helicities are opposite; hence, the helicity of the lepton pair is zero. By total angular momentum conservation, and since  $D$  is spinless,  $\bar{K}^*$  must be in the zero helicity state; i.e., it must be longitudinally polarized. At  $q^2=0$ , the transverse decay rate vanishes identically; then measurement of the width at  $q^2=0$  (denoted as  $\gamma_0$ ) is nothing more than measuring its longitudinal component. It turns out [see Eqs. (3), (7)] that the latter is a linear combination of the two form factors  $A_1$  and  $A_2$ ; then, together with the integrated rate  $\Gamma(D \rightarrow \bar{K}^* + e^+ + \nu)$  which is essentially a bilinear combination of  $A_1$  and  $A_2$ ,  $\gamma_0$  can be used to separate them.

\*Postal address: Université Pierre et Marie Curie, Tour 16, 1er étage, 4 Place Jussieu, 75252 Paris CEDEX 05, France.

Putting this argument into formulas, let us first remind ourselves of the definition of the dimensionless form factors<sup>1</sup> as directly measured (through the fourfold fitting method) by the E691 and E653 groups [3,4]:

$$\begin{aligned} \langle V(p_1), \epsilon^*(p_1) | V_\mu + A_\mu | P(p) \rangle = & -i \frac{2V(q^2)}{M+m} \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p^\rho p_1^\sigma \\ & + (M+m) A_1(q^2) \epsilon_\mu^* - 2 \frac{(\epsilon^* \cdot p)}{M+m} A_2(q^2) p_\mu - 2m \frac{(\epsilon^* \cdot p)}{q^2} A_3(q^2) q_\mu . \end{aligned} \quad (1)$$

Here  $M$  and  $m$  are the pseudoscalar  $P$  and the vector  $V$  mesons masses with four-momentum  $p$  and  $p_1$ , respectively,  $q = (p - p_1)$  is the momentum transfer. The last form factor  $A_3(q^2)$  proportional to  $q_\mu$  does not contribute to the decay in the lepton massless limit.

As suggested a long time ago [7], instead of these form factors, it is more convenient to introduce the three spacelike  $F_\lambda$  and one timelike  $T$  helicity amplitudes which have an obvious physical meaning: they correspond to the decay of the pseudoscalar  $P$  meson into the three helicity states  $\lambda = \pm 1, 0$  of the vector  $V$  meson through the three spacelike and one timelike components of the weak current. These helicity decay amplitudes can be obtained by projecting out [5] the covariant form factors Eq. (1) into the three-polarization vector of the  $V$  and the four-polarization vector of the virtual off-shell  $W$  (three from its spin-1 component and one from its spin-zero component, which is also the time component of the weak current). We then get

$$F_\pm(q^2) = (M+m) \left[ A_1(q^2) \mp \frac{2MK(q^2)}{(M+m)^2} V(q^2) \right], \quad (2)$$

$$F_0(q^2) = \frac{M+m}{2m} \frac{1}{\sqrt{q^2}} \left[ (M^2 - m^2 - q^2) A_1(q^2) - \frac{4M^2 K^2(q^2)}{(M+m)^2} A_2(q^2) \right], \quad (3)$$

$$T(q^2) = \frac{MK(q^2)}{m\sqrt{q^2}} \left[ (M+m) A_1(q^2) - \frac{M^2 - m^2 + q^2}{M+m} A_2(q^2) - 2m A_3(q^2) \right]. \quad (4)$$

Here  $K(q^2)$  is the three-momentum of the vector meson  $V$  in the rest frame of  $P$ :

$$2MK(q^2) = \sqrt{(M^2 + m^2 - q^2)^2 - 4M^2 m^2}.$$

The  $\mp$  sign in the large square brackets of Eq. (2) reflects the  $V - A$  property of the weak current as previously noted [7]. The last term  $T(q^2)$  comes from the time component and its contribution to the decay width is proportional to  $\mu^2$ , the charged lepton mass squared. Fully detailed fourfold distributions in  $Q^2$ ,  $\theta$ ,  $\theta_\nu$ , and  $\chi$  are given in Ref. [5] and in the lepton massless limit in Ref. [6]. Integration over  $\theta_\nu$  and  $\chi$  of the Ref. [5] formula, one recovers the old result on the twofold  $q^2, \theta$  distributions [7]:

$$\begin{aligned} \frac{d\Gamma}{dq^2 d\cos\theta} = & \frac{G^2 |V_{ij}|^2 K(q^2) q^2}{256\pi^3 M^2} \left[ 1 - \frac{\mu^2}{q^2} \right]^2 \left\{ (1 - \cos\theta)^2 F_+^2(q^2) + (1 + \cos\theta)^2 F_-^2(q^2) + s \sin^2\theta F_0^2(q^2) \right. \\ & \left. + \frac{\mu^2}{q^2} [\sin^2\theta (F_+^2 + F_-^2) + 2(T + F_0 \cos\theta)^2] \right\}. \end{aligned} \quad (5)$$

Again integrating over  $\theta$ , we get [7]

$$\frac{d\Gamma}{dq^2} = \frac{G^2 |V_{ij}|^2 K(q^2) q^2}{96\pi^3 M^2} \left[ 1 - \frac{\mu^2}{q^2} \right] \left\{ (F_+^2 + F_-^2 + F_0^2) \left[ 1 + \frac{\mu^2}{2q^2} \right] + \frac{3}{2} \frac{\mu^2}{q^2} T^2 \right\} \quad (6)$$

with the transverse  $d\Gamma_T/dq^2$ , longitudinal  $d\Gamma_L/dq^2$ , and scalar  $d\Gamma_S/dq^2$  decay rates, respectively, given by

$$\frac{d\Gamma_{T,L,S}}{dq^2} = \frac{G^2 |V_{ij}|^2 K(q^2)}{96\pi^3 M^2} \left[ 1 - \frac{\mu^2}{q^2} \right] \left[ \left[ q^2 + \frac{\mu^2}{2} \right] (F_+^2 + F_-^2), \left[ q^2 + \frac{\mu^2}{2} \right] F_0^2, \frac{3}{2} \mu^2 T^2 \right]. \quad (7)$$

<sup>1</sup>The  $V, A_{1,2,3}$  form factors are related to the ones of Ref. [5] as

$$A_1 = \frac{F_1^A}{(M+m)}, \quad A_2 = -\frac{1}{2}(M+m)F_2^A, \quad A_3 = -\frac{q^2}{2m}F_3^A, \quad V = -\frac{M+m}{2}F^\nu$$

and the ones of Ref. [6] as

$$A_1 = \frac{f}{M+m}, \quad A_2 = -(M+m)a_+, \quad A_3 = \frac{q^2}{2m}(a_+ - a_-), \quad V = (M+m)g.$$

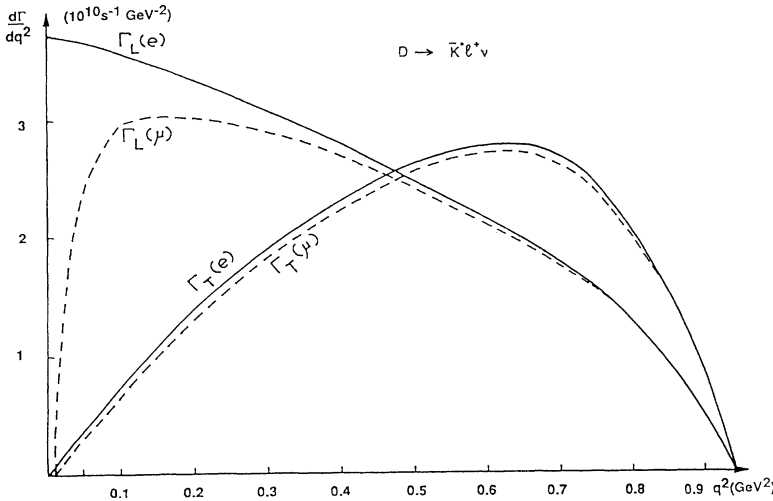


FIG. 1.  $q^2$  distribution of the longitudinal  $\Gamma_L$  and transverse  $\Gamma_T$  rates in  $D \rightarrow \bar{K}^* + l^+ + \nu$  for both electron and muon. Solid line: electron. Broken line: muon. We use  $A_1(0)=0.5$ ,  $A_2(0)=0.4$ ,  $V(0)=1$  for illustration.

Putting Eqs. (2), (3) into Eq. (7), the shapes of  $d\Gamma_L/dq^2$  and  $d\Gamma_T/dq^2$  are then illustrated in Fig. 1 for both  $D \rightarrow \bar{K}^* + e^+ + \nu$  and  $D \rightarrow \bar{K}^* + \mu^+ + \nu$ . Compared to  $\Gamma_L$  and  $\Gamma_T$ , the scalar ones  $\Gamma_S$  [because of the  $\mu^2$  term in Eq. (7)] are completely negligible for both electron and muon cases. For the electron mode, in principle, the threshold factor  $(1-m_e^2/q^2)^2$  in Eq. (7) leads to the *very abrupt vanishing* of  $d\Gamma_L/dq^2$  at  $q^2=m_e^2$ ; however, this abrupt vanishing is *invisible* even for a very tiny  $q^2$  of only few  $m_e^2$ , as can be seen in Fig. 1. This steplike behavior of  $d\Gamma_L/dq^2$  for the electron stands out in contrast with the muon case for which the vanishing of  $d\Gamma_L/dq^2$  at  $q^2=m_\mu^2$  is smooth. The distant behavior of  $d\Gamma_L/dq^2$  between the electron and muon as  $q^2 \rightarrow \mu^2$ , due to the kinematical  $(1-\mu^2/q^2)^2$  term, is general, independent of the form factors and simply reflects the large difference between the electron and muon masses. Taking advantage of this invisible and abrupt threshold behavior at  $q^2 \rightarrow 0$  of  $d\Gamma_L/dq^2$ , one can safely neglect the electron mass (and not the muon one) in Eq. (7). This means that the measurement of the  $D \rightarrow \bar{K}^* + e^+ + \nu$  rate at  $q^2$  as small as possible is nothing more than measuring its longitudinal component. Moreover, even at relatively large  $q^2 \simeq 0.01, 0.02$  GeV<sup>2</sup>, the longitudinal rate completely dominates the negligible transverse one. On the contrary for  $D \rightarrow \bar{K}^* + \mu^+ + \nu$ , even at small  $q^2$  of few  $m_\mu^2$ , the

measured rate already mixes the longitudinal  $\Gamma_L$  with the transverse  $\Gamma_T$  ones; they cannot be separated by the  $q^2$  distribution. We conclude that the method of measuring the slope  $d\Gamma/dq^2$  at ( $q^2 \rightarrow \mu^2$ ) to single out its longitudinal component is only applicable to the electron mode and not to the muon one; hence, the E653 experiment with  $D \rightarrow \bar{K}^* + \mu^+ + \nu$  cannot be straightforwardly applied for the slope.

For  $D \rightarrow \bar{K}^* + e^+ + \nu$ , we put  $m_e=0$  and define the slope at  $y=0$  ( $y \equiv q^2/M^2$ ) as  $\gamma_0 \equiv d\Gamma/dy|_{y=0}$ . From Eqs. (2), (3), (6), (7) we get

$$\gamma_0 = \frac{G^2 |V_{cs}|^2 M^5}{192\pi^3} \left[ 1 - \frac{m^2}{M^2} \right]^3 \left[ \frac{M+m}{2m} \right]^2 \times \left[ A_1(0) - \frac{M-m}{M+m} A_2(0) \right]^2. \quad (8)$$

Equation (8), together with the integrated width  $\Gamma(D \rightarrow \bar{K}^* + e^+ + \nu)$  considered in Eq. (9) below, can be used to separate the  $A_1$  and  $A_2$  form factors. Indeed, integration over  $q^2$  of Eq. (6), using the monopole form factors  $f(q^2) = f(0)/(1-q^2/\Lambda^2)$  where  $f(q^2)$  stands for  $V(q^2)$ ,  $A_1(q^2)$ ,  $A_2(q^2)$  with  $\Lambda=2.1$  GeV for  $V(q^2)$  and  $\Lambda=2.5$  GeV for  $A_{1,2}(q^2)$ , has been previously performed [8,9] and the result is [9]

$$\Gamma = \frac{G^2 |V_{cs}|^2 M^5}{192\pi^3} [0.238 A_1^2(0) + 0.0105 A_2^2(0) - 0.078 A_1(0) A_2(0) + 0.0039 V^2(0)] \\ = 17.69 A_1^2(0) [1 + 0.044 \rho_A^2 - 0.327 \rho_A + 0.016 \rho_V^2] 10^{10} \text{ s}^{-1} \quad (9)$$

with  $\rho_A = A_2(0)/A_1(0)$  and  $\rho_V = V(0)/A_1(0)$ .

From Eq. (9), we notice that for  $\rho_V \simeq 2$  as indicated by experiments [3,4] the contribution to the rate of the vector form factor  $V(q^2)$  is negligible, such that Eq. (9) can be used as a constraint contour represented by an ellipse in the  $(A_1, A_2)$  plane previously considered [9]. The slope  $\gamma_0$  is not yet measured and we strongly advocate that it should be. Its simplicity is to be contrasted with the fourfold angular fitting method used until now. Once

done,  $\gamma_0$  can be considered as a second constraint represented by a straight line in the  $(A_1, A_2)$  plane. The intersection of the latter with the ellipse then determines  $A_1(0)$  and  $A_2(0)$ . Let us write  $\gamma_0$  (in units of  $10^{10} \text{ s}^{-1}$ ) as  $\gamma_0 = a \times 10^{10} \text{ s}^{-1}$  where  $a$  is a pure number to be measured by experiment; then Eq. (8) can be numerically rewritten as

$$A_1(0) - 0.35 A_2(0) - 0.11 \sqrt{a}. \quad (10)$$

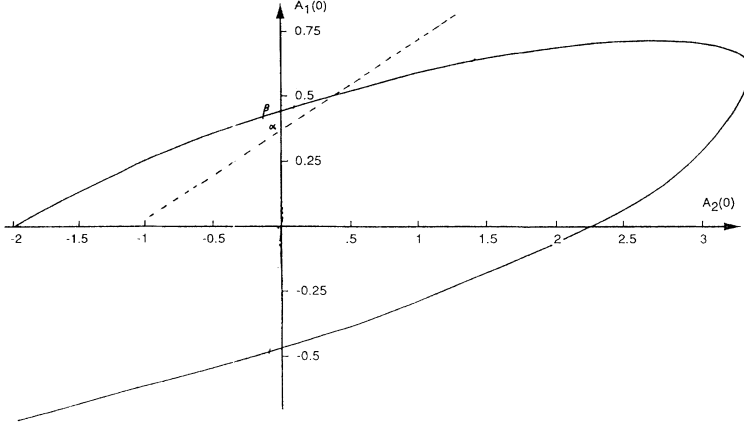


FIG. 2. Constraints in the  $(A_1, A_2)$  plane: Ellipse (solid line) from Eq. (9) for the rate  $\Gamma \rightarrow (K^* + e + \nu)$ . Straight line (broken line) from Eq. (10) for the slope  $\gamma_0 = M^2(d\Gamma/dq^2)|_{q^2=0}$ . The two curves cut the ordinate  $A_1(0)$  axis at the points  $\beta$  and  $\alpha$ , respectively (see the text).

If the E691 Collaboration measured the slope  $\gamma_0$ , they would give its central value  $\gamma_0 = 17.5 \times 10^{10} \text{ s}^{-1}$  and the E653 group would give  $\gamma_0 = 8.9 \times 10^{10} \text{ s}^{-1}$  (provided that the electron replaces the muon in this experiment). These numbers can be considered as an independent check for consistency of these experiments. Equations (9), (10) are plotted in Fig. 2, taking  $\Gamma(D \rightarrow \bar{K}^* + e + \nu) = 4.3 \times 10^{10} \text{ s}^{-1}$  as given by the world average. A very intriguing possibility for a negative  $A_2(0)$  form factor, completely at variance, not only in magnitude but also in sign, with the quark model has been envisaged [6] to explain the large ratio  $R \simeq 2.4$  found earlier by the E691 group from their  $\theta_\nu$  distribution [10]. This unlikely possibility can be tested by measuring the slope  $\gamma_0$  as can be seen in Fig. 2. In the  $(x, y)$  plane [ $x$  stands for  $A_2(0)$  and  $y$  for  $A_1(0)$ ], the ellipse Eq. (9) and the straight line Eq. (10), respectively, cut the ordinate  $y$  axis at the points  $\beta$  and  $\alpha$ . Depending on their relative positions, i.e.,  $\beta$  higher or lower than  $\alpha$ ,  $A_2(0)$  is positive or negative, respectively. The fact can be translated numerically in the following way. Suppose that in an experiment, one measures both the slope  $\gamma_0$  and the rate  $\Gamma$ , with  $\gamma_0 = a \times 10^{10} \text{ s}^{-1}$  and  $\Gamma = b \times 10^{10} \text{ s}^{-1}$  (the world average central value of  $b$  is 4.3 and the E691 group [4] gives  $b = 4.04$ ). From Eq. (9) and taking [3,4]  $\rho_V = 2$ , we obtain  $y_\beta^2 = b/18.84$  and Eq. (10) gives  $y_\alpha^2 = (0.11)^2 a$ , from that we get the following result: de-

pending on  $y_\beta \lesseqgtr y_\alpha$ , i.e., the ratio  $a/b$  smaller or greater than 4.38,  $A_2(0)$  is positive or negative, respectively, the case  $a/b = 4.38$  corresponds obviously to  $A_2(0) = 0$ . We are eager to learn from experiments the measurement of the slope  $\gamma_0$ .

The case of  $V_{bu}$  and  $B \rightarrow \rho + e + \nu$ . The reason why the quark model fails in the description of  $D \rightarrow K^* + e + \nu$  may be traced back to the argument of Ref. [11] suggesting that, when both the initial and final quarks are heavy and the fractional energy release is small [ $\Lambda_{\text{QCD}}/M \ll (M^2 - m^2)/2M^2 \ll 1$ ], then the quark model should be good not only in the inclusive decay but also point by point in the Dalitz plot of the exclusive modes. This is not the case for  $D$  decays: even if the initial charm quark might be considered as heavy, the final strange quark is not. The same argument *a fortiori* must be applied to  $B \rightarrow \rho + e + \nu$ ; here, the quark model calculations of form factors are even more suspicious; the extraction of the matrix element  $V_{bu}$  cannot be trusted by using only the rate recently measured [12] due to theoretical uncertainties of the form factors. It is more confident to use experimental information on the rate (already measured) and the slope (as suggested here) than to rely on theoretical models of form factors.

The rate  $B^0 \rightarrow \rho^- + e^+ + \nu$  is calculated to be

$$\Gamma(B^0 \rightarrow \rho^- + e^+ + \nu) = \frac{G^2 |V_{bu}|^2 M_B^5}{192\pi^3} [c_1 A_1^2(0) + c_2 A_2^2(0) - c_{12} A_1(0) A_2(0) + c_3 V^2(0)] \quad (11)$$

with

$$\begin{aligned} c_1 &= 3.73(6.08), \quad c_2 = 1.23(1.62), \\ c_{12} &= 4.01(5.59), \quad c_3 = 0.057(0.13). \end{aligned} \quad (12)$$

In Eqs. (12), the first numbers correspond to the  $q^2$  independent (constant) form factors while the parentheses correspond to the use of monopole form factors  $f(q^2) = f(0)/(1 - q^2/\Lambda^2)$  with  $\Lambda_\nu = 5.33 \text{ GeV}$  for  $V(q^2)$  and  $\Lambda_a = 5.75 \text{ GeV}$  for  $A_1(q^2)$  and  $A_2(q^2)$ . From Eqs. (12) we remark that the monopole  $q^2$  dependence form factors enhance the integrated width by roughly 50%, although the poles  $\Lambda$  (which are very close to the edge of the physical region) change the form factors from  $q^2 = 0$

to  $q^2 = q_{\text{max}}^2 = (M_B - m_\rho)^2$  by a much larger factor around 3. This behavior is to be contrasted with the charm case where the poles are far from the physical region. In the following, we use the monopole form factors and the numbers in parentheses of Eqs. (12) for the width. The rate  $\Gamma(B^+ \rightarrow \rho^0 + e^+ + \nu)$  is half of  $\Gamma(B^0 \rightarrow \rho^- + e^+ + \nu)$  by isospin and has been recently measured by ARGUS [12]. Numerically the rate  $\Gamma(B^+ \rightarrow \rho^0 + e^+ + \nu)$  is given by

$$\begin{aligned} \Gamma(B^+ \rightarrow \rho^0 + e^+ + \nu) &= 43.31 |V_{bu}|^2 A_1^2(0) [1 + 0.26\rho_A^2 - 0.92\rho_A \\ &\quad + 0.021\rho_V^2] \times 10^{13} \text{ s}^{-1}. \end{aligned} \quad (13)$$

Taking [12]  $B(B^+ \rightarrow \rho^0 + e^+ + \nu) = (1.13 \pm 0.36 \pm 0.27) \times 10^{-3}$  and the  $B^+$  lifetime as  $(11.8 \pm 1.1) \times 10^{-13}$  s, experiment [12] gives  $\Gamma_{\text{expt}}(B^+ \rightarrow \rho^0 + e^+ + \nu) = (9.57 \pm 3.05 \pm 1.28 \mp 0.89) \times 10^8 \text{ s}^{-1}$  where the last  $\mp$  error comes from the error in the lifetime. It turns out that for  $\rho_A$  and  $\rho_V$  ranging between 0 and 2, which is plausible, the factor in square brackets in Eq. (13) is always  $\leq 1.08$  such that taking the central value  $9.57 \times 10^8 \text{ s}^{-1}$  for  $\Gamma_{\text{expt}}$  we get

$$A_1(0) |V_{bu}| \geq 1.42 \times 10^{-3}. \quad (14)$$

On the other hand, the slope  $\gamma_0 \equiv M_B^2 (d\Gamma/dq^2)(B^+ \rightarrow \rho^0 + e^+ + \nu)$  at  $q^2=0$  is given by Eq. (8) with  $M$  and  $m$  the  $B$  and  $\rho$  mesons masses and the factor  $\frac{1}{2}$  from isospin taken into account. We get

$$\begin{aligned} \gamma_0 &= \frac{1}{2} \frac{G^2 |V_{bu}|^2 M_B^5}{192\pi^3} \left[ 1 - \frac{m_\rho^2}{M_B^2} \right]^3 \left[ \frac{M_B + 2m_\rho}{2m_\rho} \right]^2 \\ &\times \left[ A_1(0) - \frac{M_B - m_\rho}{M_B + m_\rho} A_2(0) \right]^2 \\ &= 103 |V_{bu}|^2 (A_1 - 0.74 A_2)^2 \times 10^{13} \text{ s}^{-1}. \end{aligned} \quad (15)$$

Unlike the charm case  $D \rightarrow K^* + e + \nu$  where  $V_{cs}$  is relatively known, and the form factors can be extracted from data (using either the triple angular fit or the slope  $\gamma_0$  advocated here), for the  $b$ -flavored case  $B \rightarrow \rho + e + \nu$  there are three unknowns  $V_{bu}$ ,  $A_1(0)$ ,  $A_2(0)$  [the vector form factor  $V(q^2)$  contributes negligibly to  $\Gamma$  as in the charm case] and two equations (13) and (15); then only the products  $|V_{bu}| A_1(0)$  and  $|V_{bu}| A_2(0)$  can be obtained, once  $\gamma_0$  is measured. However the ratio  $\gamma_0/\Gamma$  like the ratio  $\rho_A = A_2(0)/A_1(0)$  is free from  $|V_{bu}|$  and this ratio  $\rho_A$  extracted from data through Eqs. (13), (15) can be confronted with theoretical models; only those having the predicted  $\rho_A$  in agreement with data could be used confidently for the determination of  $|V_{bu}|$ .

We conclude by urging measurements of the slope  $\gamma_0$  in  $D \rightarrow K^* e \nu$  and  $B \rightarrow \rho e \nu$  for which the rates are already known. The method which is simpler than the fourfold fit can be used to extract the axial-vector form factors

and distinguish between different theoretical models.

Finally, we remark that within the factorization approach for hadronic decays of heavy flavors, the rate  $\Gamma(P \rightarrow V + \pi)$ , such as  $D \rightarrow K^* \pi$ ,  $B \rightarrow \rho \pi$ ,  $B \rightarrow D^* \pi$ , is directly connected to the slope  $\gamma_0$  by the formula

$$\Gamma(P \rightarrow V \pi) = 6\pi^2 a_1^2 |V_{ud}|^2 \left[ \frac{f_\pi}{M} \right]^2 \gamma_0, \quad (16)$$

where  $a_1$  is the hard gluonic QCD-corrected coefficient ( $a_1 \simeq 1.25$  for charm and  $\simeq 1.12$  for  $b$ -flavored quarks in the  $1/N_C$  expansion) and  $f_\pi \simeq 132 \text{ MeV}$  is the pion decay constant. Equation (16) can be easily derived when one matches the left-hand side  $\Gamma(P \rightarrow V + \pi)$  given in Ref. [9] with Eq. (8).

As a bonus, the measurement of the slope  $\gamma_0$  provides therefore a test, via Eq. (16), of the factorization approach. Moreover if we take the latter for granted, and using  $a_1 = 1.25$ ,  $\Gamma_{\text{expt}}(D^0 \rightarrow K^* \pi^+) = (10.92 \pm 0.95) \times 10^{10} \text{ s}^{-1}$ , then Eqs. (10) and (16) give

$$A_1(0) - 0.35 A_2(0) = 0.55 \pm 0.05. \quad (17)$$

Since  $A_1(0)$ , is derived from the rate  $D \rightarrow \bar{K}^* e \nu$ , is known [4,8,9] to be around 0.5, then Eq. (17) which is a direct consequence of factorization favors the negligible  $A_2(0) \simeq 0$  of the E691 Collaboration. Caution must be taken, however, since factorization is only an approximation [9] for charm decay; since consensus on the form factors by direct measurement is not yet settled, any conclusion seems to be premature.

Lastly, measurements of the rate  $\Gamma$  and the slope  $\gamma_0$  can only separate the two axial-vector form factors  $A_1(q^2)$  and  $A_2(q^2)$ ; for the vector form factor  $V(q^2)$ , its determination can be done by looking at the "weak" angle  $\theta$  forward-backward distribution Eq. (5) that isolates the  $F_\pm$  helicity amplitudes; hence  $V(q^2)$  via Eq. (2), and we are rejoining previous [7] discussions. Its contribution to the decay rate is however negligible as we already know; the two axial-vector form factors  $A_1(q^2)$  and  $A_2(q^2)$  considered in this paper are important.

LPTHE is Laboratoire Associé au CNRS, V.A. 280.

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