RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 46, NUMBER 5 1 SEPTEMBER 1992

Generation nonuniversality and precision electroweak measurements

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(Received 27 January 1992; revised manuscript received 18 February 1992)

A gauge model of generation nonuniversality proposed 10 years ago predicted that the τ lifetime should be longer than what it would be in the standard model. This has been confirmed by recent e^+e^- to Z data, although the statistical significance is only 2.3σ. We show that this model is also consistent with other precision electroweak measurements, and in particular it contributes *negatively* to the ρ parameter measured at the Z peak.

PACS number(s): 12.15.Cc, 12.15.Ji, 13.35.+s, 13.38.+c

In 1981, a gauge model of generation nonuniversality was proposed [1] with three main predictions: (1) a longer τ lifetime, (2) an observable B-$\bar{B}$ mixing, and (3) heavier W and Z masses. In 1987, the world average of the τ lifetime was reported [2] to be indeed longer than was expected from the standard model. In the same year, B-$\bar{B}$ mixing was also observed [3] at a level much higher than was expected. An updated analysis of our model was then given [4] with special emphasis on the possible rare decay $b \to sl^+l^-$. However, this was based on the assumption that the mass of the t quark was *not* the dominant contribution to the observed B-$\bar{B}$ mixing. At present, with the knowledge that $m_t > 91$ GeV from $p\bar{p}$ data [5], this original prediction of our model can no longer be used to distinguish it from the standard model.

In 1991, an enormous amount of precision data at the Z peak has become available from the e^+e^- collider LEP at CERN. The standard model can now be tested at the level of its radiative corrections and it does very well indeed [6]. However, there are two possible discrepancies. (1) New measurements of the τ lifetime confirm [7] the old world average that it is longer than what the standard model predicts by a few percent. (2) The ρ parameter measured at the Z peak tends to be less than 1, which goes against the prediction of the standard model, i.e.,

$$\rho \simeq 1 + \frac{3\sqrt{2} G_F m_t^2}{16\pi^2} , \tag{1}$$

where the small logarithmic contribution of the Higgs boson has been omitted. Of course, the errors are still large enough so that ρ slightly greater than 1 is allowed and as a result a strong upper bound on m_t is obtained. Given that our prediction of a longer τ lifetime has acquired further experimental support, we now consider what our model has to say about ρ. We will show in the following that its contribution to ρ is necessarily negative and that our third original prediction that the observed W and Z masses should be greater than would be expected from low-energy data is also becoming true.

Our model is based on the group $U(1) \times SU(2)_1 \times SU(2)_2 \times SU(2)_3$ with gauge couplings g_0, g_1, g_2, g_3, respectively. The Higgs bosons are doublets under $U(1) \times SU(2)_i$ and self-dual quartets under $SU(2)_j \times SU(2)_k$ with vacuum expectation values v_{0i} and v_{jk}. The left-handed fermions are doublets under $U(1) \times SU(2)_i$, with each generation coupling to a separate SU(2). The right-handed fermions are singlets coupling only to U(1). Assuming the hierarchy

$$v_{01}^2 + v_{02}^2 \ll v_{03}^2 \ll v_{13}^2 + v_{23}^2 \ll v_{12}^2 , \tag{2}$$

we find [1] that e-μ universality holds very well but e-μ-τ universality fails to the extent that the parameter

$$\xi = 1 + v_{03}^2 / (v_{13}^2 + v_{23}^2) \tag{3}$$

may differ from 1. This is analogous to the case of strong isospin, i.e., flavor SU(2), which holds very well because the light-quark masses m_u and m_d are much less than the interaction energy scale of quantum chromodynamics, whereas m_s is not as negligible so that flavor SU(3) is only an approximate symmetry. The electromagnetic coupling is given by

$$e^{-2} = g_0^{-2} + g_1^{-2} + g_2^{-2} + g_3^{-2} , \tag{4}$$

and the Fermi weak coupling is generalized to a matrix

$$\left[\frac{4G_F}{\sqrt{2}} \right]_{ij} = \begin{cases} v_{03}^{-2}, & i=3 \text{ or } j=3 \text{ or both} , \\ v_{03}^{-2} + (v_{13}^2 + v_{23}^2)^{-1}, & \text{otherwise} , \end{cases} \tag{5}$$

so that any low-energy weak interaction involving the third generation has its effective strength reduced by ξ^{-1}. If we consider only the first two generations, then the effective low-energy weak neutral-current interaction is given by

$$(4G_F/\sqrt{2})[(j^{(3)} - \sin^2\theta_W j^{\text{em}})^2 + C(j^{\text{em}})^2] , \tag{6}$$

where

$$\sin^2\theta_W = (1 - e^2/g_0^2) - (e^2/g_3^2)(1 - \xi^{-1}) \tag{7}$$

and

$$C=(e^4/g_3^4)(1-\xi^{-1})\xi^{-1} . \tag{8}$$

Note that the same G_F appears in the above as in the charged-current interaction without any adjustment of parameters. This remarkable feature of our model is of course also shared by the standard model. Using Eqs. (4), (7), and (8), we see that

$$0<C<\sin^4\theta_W(\xi-1) \tag{9}$$

and the standard model is recovered in the limit $\xi=1$.

In τ decay, its effective coupling is reduced by ξ^{-1} so that its lifetime is longer by a factor of ξ^2 as compared with the prediction of e-μ-τ universality. The new world average including the recent LEP measurements is [7]

$$\xi^{-2}=0.948\pm0.022 . \tag{10}$$

Although the statistical significance here is only 2.3σ, we note that this effect (if true) does require ξ to be greater than 1, as predicted by Eq. (3). We also note that m_τ^5 enters into the derivation of Eq. (10) and there has only been one precise measurement of m_τ; hence, a second precise measurement from the new e^+e^- machine Beijing Electron-Positron Collider (BEPC) will be most welcome.

Assuming that $\xi-1$ is indeed nonzero and positive, the observed W and Z bosons will be slightly different from those of the standard model. There are in fact three sets of W's and Z's in our model. All contribute to the effective four-fermion weak interaction at low energy through their virtual exchange. However, once a particular gauge boson is produced, we are of course only looking at the interactions of that specific particle. Hence we expect a number of deviations from the standard model if we scrutinize the various precise measurements now available at the Z peak. The physical gauge bosons of our model have been identified in detail previously [8]. We note only that e-μ universality implies that one set of W and Z bosons are very heavy and do not mix with the other two sets. The observed Z boson is then a linear combination of two neutral gauge bosons Z_1 and Z_2, with interactions given by [8]

$$H_{\rm int}=-(g_0^2+g_{123}^2)^{1/2}Z_1\left[j_L^{(3)}-\frac{e^2}{g_{123}^2}j^{\rm em}\right]+\frac{g_{12}^2}{(g_{12}^2+g_3^2)^{1/2}}Z_2j_L^{(3)}-(g_{12}^2+g_3^2)^{1/2}Z_2j_L^{(3)}(\tau) , \tag{11}$$

where $j_L^{(3)}$ and $j^{\rm em}$ are the usual electroweak neutral currents summing over all generations but $j_L^{(3)}(\tau)$ contains only those fields which transform under $SU(2)_3$. The couplings g_{12} and g_{123} are defined by $g_{12}^{-2}=g_1^{-2}+g_2^{-2}$ and $g_{123}^{-2}=g_{12}^{-2}+g_3^{-2}$, and $e^2/g_{123}^2=s^2+C^{1/2}(\xi-1)^{1/2}$, with $s^2=\sin^2\theta_W$. Now $Z=Z_1\cos\Phi+Z_2\sin\Phi$, where [8] $\tan\Phi\simeq-g_{123}^4g_{12}^{-3}g_3^{-1}(\xi-1)/\cos\theta_W$. Hence the interaction of Z is given by

$$H_{\rm int}=-g_{\rm eff}Z\left[j_L^{(3)}-\frac{(g_0^2+g_{123}^2)^{1/2}}{g_{\rm eff}}\frac{e^2}{g_{123}^2}j^{\rm em}\right] \tag{12}$$

for the first two generations, where $g_{\rm eff}\simeq(g_0^2+g_{123}^2)^{1/2}\{1+C^{1/2}[s^2(\xi-1)^{1/2}-C^{1/2}]/s^4\}$. Experimentally, the Z leptonic widths are measured and compared to the formula

$$\Gamma=\frac{G_FM_Z^3\rho}{24\sqrt{2}\pi}[1+(1-4\sin^2\theta_{\rm eff})^2] \tag{13}$$

to extract ρ. In our model, this width is proportional to $g_{\rm eff}^2M_Z$ instead; hence,

$$\rho=g_{\rm eff}^2/(4\sqrt{2}G_FM_Z^2) . \tag{14}$$

Using [8]

$$\frac{(g_0^2+g_{123}^2)s^2(1-s^2)}{e^2}\simeq1-\frac{\sqrt{C(\xi-1)}(1-2s^2)}{s^2(1-s^2)} \tag{15}$$

and

$$\frac{M_Z^2}{\mu_Z^2}\simeq1+\frac{\sqrt{C}\,[s^2\sqrt{\xi-1}-(1-s^2)\sqrt{C}\,]}{s^4(1-s^2)} , \tag{16}$$

where μ_Z is the standard-model Z mass, we find

$$\rho\approx1+\rho_{\rm rad}-C/s^4 , \tag{17}$$

where $\rho_{\rm rad}$ is the standard-model radiative correction, which is dominated by the m_t^2 term in Eq. (1). Since C is necessarily positive, this means that our model predicts a negative contribution to ρ, which is exactly what current experimental data favor, namely [6]

$$\rho=0.9968\pm0.0050 . \tag{18}$$

Strictly speaking, we should assume only e-μ universality in extracting the above ρ. However, it makes very little difference numerically. Using $m_t>91$ GeV, and neglecting the Higgs-boson contributions, we find $\rho_{\rm rad}>0.0026$; hence,

$$C/s^4>0.0058\pm0.0050 . \tag{19}$$

If we assume instead that $m_t=200$ GeV, then

$$C/s^4=0.0157\pm0.0050 . \tag{20}$$

This is the first indication that in addition to $\xi>1$ being confirmed, the other necessary feature of our model that $C>0$ may be correct as well.

Our third original prediction was that the observed W and Z masses should be greater than would be expected from low-energy data on the basis of the standard model. In addition to Eq. (16), we have

$$\frac{M_W^2}{\mu_W^2}\simeq1+\frac{\sqrt{C}\,[s^2\sqrt{\xi-1}-\sqrt{C}\,]}{s^4} . \tag{21}$$

To test these predictions, we determine μ_Z and μ_W from the value of $\sin^2\theta_W$ measured in νN deep-inelastic scattering as follows. Since we are now working entirely within the standard model, we will use the on-shell definition $\sin^2\theta_W=1-\mu_W{}^2/\mu_Z{}^2$, so that

$$\mu_Z = \frac{37.281 \text{ GeV}}{\sin\theta_W \cos\theta_W \sqrt{1-\Delta r}} , \tag{22}$$

where Δr represents all the relevant radiative corrections. Using [9] $\sin^2\theta_W = 0.231 \pm 0.006$ and the values $\Delta r = 0.057$ for $m_t = 100$ GeV and $\Delta r = 0.018$ for $m_t = 200$ GeV, with $m_H = 100$ GeV in both cases, we find, respectively,

$$\mu_Z = \begin{cases} 91.1 \pm 0.8 \text{ GeV} , \\ 89.3 \pm 0.8 \text{ GeV} , \end{cases} \tag{23}$$

and

$$\mu_W = \begin{cases} 79.9 \pm 1.0 \text{ GeV} , \\ 78.3 \pm 1.0 \text{ GeV} , \end{cases} \tag{24}$$

compared to the observed masses

$$M_Z = 91.175 \pm 0.021 \text{ GeV} , \tag{25}$$

and

$$M_W = 80.13 \pm 0.27 \text{ GeV} . \tag{26}$$

We see that our prediction that $M_Z > \mu_Z$ and $M_W > \mu_W$ is indeed consistent with data, especially if m_t is more near 200 GeV than 100 GeV. However, the ratios M_Z/μ_Z and M_W/μ_W are also bounded from above because of Eq. (9). Using Eqs. (10), (17), and (18), we find that these mass ratios should not exceed 1 by more than a percent or so. This means that m_t is still bounded from above. It is hard to make the above statement more precise at this time, but a consistent picture is certainly possible if we take $m_t = 150$ GeV and $m_H = 100$ GeV. In that case, $\Delta r = 0.041$ and $\mu_Z = 90.3 \pm 0.8$ GeV, $\mu_W = 79.2 \pm 1.0$ GeV. We also have $C/s^4 = 0.0101 \pm 0.0050$. Using $0.015 < \xi - 1 < 0.039$ and $2.7 \times 10^4 < C < 8.0 \times 10^4$, we then find $1.006 < M_Z{}^2/\mu_Z^2 < 1.016$ and $1.004 < M_W{}^2/\mu_W{}^2 < 1.009$, which are certainly consistent with the derived values $M_Z{}^2/\mu_Z^2 = 1.019 \pm 0.018$ and $M_W{}^2/\mu_W{}^2 = 1.024 \pm 0.026$ from νN data.

The second set of W and Z bosons have nearly equal masses and they are approximately given by [8]

$$M_{W'}^2 \simeq M_{Z'}^2 \simeq \frac{s^4(1-s^2)M_Z^2}{\sqrt{C}\,[s^2\sqrt{\xi-1} - \sqrt{C}\,]} , \tag{27}$$

which is bounded from below by $4(1-s^2)M_Z^2/(\xi-1)$. So, even without knowing the value of C, we find

$$M_{W'} \simeq M_{Z'} \geq 808 \text{ GeV} . \tag{28}$$

If C is known to have a lower bound greater than zero and an upper bound smaller than $s^4(\xi-1)$, then $M_{W'}$ and $M_{Z'}$ are also bounded from above. As discussed earlier, these bounds on C depend crucially on the value of m_t, which is of course yet to be determined.

We now consider the parameter $\sin^2\theta_{\text{eff}}$ in Eq. (13). In our model, the observed $\sin^2\theta_{\text{eff}}$ should be larger by the factor

$$\frac{(g_0^2 + g_{123}^2)^{1/2}}{g_{\text{eff}}} \frac{e^2}{s^2 g_{123}^2} \simeq 1 + \frac{C}{s^4} . \tag{29}$$

There is also the parameter s_0^2 which is defined as [6]

$$s_0^2(1-s_0^2) = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}G_F M_Z^2} . \tag{30}$$

In our model, because of Eq. (16), the observed s_0^2 is smaller that what the standard model would give by the factor

$$1 - \frac{\sqrt{C}\,[s^2\sqrt{\xi-1} - (1-s^2)\sqrt{C}\,]}{s^4(1-2s^2)} . \tag{31}$$

Using the relationship [6,10]

$$\sin^2\theta_{\text{eff}} = s_0^2 - \frac{s_0^2(1-s_0^2)}{1-2s_0^2}\Delta\rho + \frac{\alpha S}{4(1-2s_0^2)} , \tag{32}$$

where S contains only radiative corrections (which are calculable in terms of m_t and m_H) if we are dealing with just the standard model, we find that the tree-level contribution to S from our model in the case of e and μ is given by

$$\Delta S = \frac{4}{\alpha}[\sqrt{C(\xi-1)} - C/s^2] , \tag{33}$$

which cannot be negative, but does become zero in the limits $C = 0$ or $C = C_{\max}$ as given in Eq. (9). The experimentally derived value [10] of ΔS with reference to $m_t = 140$ GeV is -0.76 ± 0.71. Thus, our model cannot explain this shift by its tree-level contribution alone.

We now consider the radiative corrections to S and $T = \alpha^{-1}\Delta\rho$ in our model. Because there are now many physical scalar bosons, we would expect in principle significant radiative contributions which could change our results for S and T. However, as it turns out, there is an automatic custodial SU(2) symmetry in the Higgs sector [8], which screens out the effects of all the scalar masses of T except small logarithmic corrections. In other words, m_t remains the dominant radiative contribution. On the other hand, this argument does not apply to S, so its true value is still unknown in this model.

As pointed out already in Ref. [8], the $Z \rightarrow \tau^+\tau^-$ partial width should also differ from that of $Z \rightarrow e^+e^-$ and $Z \rightarrow \mu^+\mu^-$ by the factor

$$1 - \frac{\xi-1}{s^2}[s^2 - \sqrt{C/(\xi-1)}]\frac{2-4s^2}{1-4s^2+8s^4} . \tag{34}$$

For $m_t = 150$ GeV, this amounts to a reduction of 2.3% if the central values of ξ and C are used. Experimentally, the measured partial widths are [11] $\Gamma_e = 83.0 \pm 0.5$ MeV, $\Gamma_\mu = 83.8 \pm 0.8$ MeV, and $\Gamma_\tau = 83.3 \pm 1.0$ MeV. Hence the possible deviation of Γ_τ from $(\Gamma_e + \Gamma_\mu)/2$ is only about 1.3%, and this prediction of our model appears to be only marginally satisfied by the present data. However, the situation here is not as clearcut as in the τ lifetime which depends only on $\xi > 1$, because in the limit $C = C_{\max} = s^4(\xi-1)$, Γ_τ becomes equal to Γ_μ and Γ_e, and that is possible here if we take $\xi - 1$ near its minimum value of 0.015. As for the number of neutrinos corresponding to the invisible width of the Z, our model now gives 2.98 ± 0.02 (for $m_t = 150$ GeV) which is certainly consistent with the present experimental value [7] of

2.99 ± 0.05.

In conclusion, we have shown that our gauge model of generation nonuniversality proposed 10 years ago is now gaining support from experimental data on many of its predictions. The effects are all small, and each by itself does not have great significance. However, our model predicts correctly how the data may deviate from the standard model.

The strongest evidence at present is the τ lifetime which is predicted by our model to be longer, and that has been confirmed by LEP data [7]. Our model also gives a negative contribution of order -1 to the parameter T, which is supported by present analyses [6,10,12], whereas the parameter S has a non-negative tree-level contribution. Radiative corrections from the Higgs sector will change S, but not T to first approximation. Our model predicts that the observed W and Z masses should be greater than would be obtained from νN data, and that is indeed the tendency as we have shown here. Our model does less well in the prediction that $\Gamma_\tau < \Gamma_\mu = \Gamma_e$, but that is still allowed within the experimental errors. An unambiguous test of our model in the future would be the discovery of a second set of W and Z bosons of the same mass and with properties as already specified in our model. Meanwhile we can tighten up our comparison with data once m_t is measured.

Note added. Since this manuscript was submitted for publication, new preliminary data have been announced. In particular, m_τ has been measured by the BES Collaboration to have the value $1776.9 \pm 0.4 \pm 0.3$ MeV [13]. Combined with new LEP values [14] for the τ lifetime and leptonic branching fractions, the updated value of $\xi - 1$ is now 0.015 ± 0.008, whereas we have used 0.027 ± 0.012. The discrepancy is now reduced from 2.3σ to 1.8σ.

This work was supported in part by the U.S. Department of Energy under Contract No. DE-AT03-87ER40327.

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