# Evidence for a scalar state I = 0 0<sup>++</sup>(750) from measurements of $\pi N_{\uparrow} \rightarrow \pi^+ \pi^- N$ on a polarized target at 5.98, 11.85, and 17.2 GeV/c

M. Svec

Physics Department, Dawson College, Montreal, Quebec, Canada H3Z 1A4 and McGill University, Montreal, Quebec, Canada H3A 2T8

A. de Lesquen and L. van Rossum Département de Physique des Particules Élementaires, Centre d'Études Nucléaires, Saclay, 91191 Gif-sur-Yvette, France (Received 3 February 1992)

Measurements of the reactions  $\pi^+ n_{\uparrow} \rightarrow \pi^+ \pi^- p$  at 5.98 and 11.85 GeV/c and  $\pi^- p_{\uparrow} \rightarrow \pi^- \pi^+ n$  at 17.2 GeV/c enable model-independent amplitude analyses which yield two solutions for moduli and cosines of certain relative phases of the two S-wave and six P-wave production amplitudes describing the pion production process below the dipion mass of 1000 MeV. We obtain four solutions for the S- and P-wave intensities in the physical region of the  $\pi N \rightarrow \pi^+ \pi^- N$  reaction. The results for the S-wave intensity provide solution-independent evidence for a new resonant state  $I=0.0^{++}(750)$ . Its  $\pi^+\pi^-$  decay width depends on the solution and is estimated to be in the range of 100–250 MeV. The  $I=0.0^{++}(750)$  meson is best understood as the lowest-mass scalar gluonium  $0^{++}(gg)$ . Our results emphasize the need for a systematic study of pion production on the level of amplitudes in a new generation of dedicated experiments with spin at the recently proposed high-intensity hadron facilities.

PACS number(s): 14.40.Cs, 13.75.Gx, 13.85.Hd, 13.88.+e

#### I. INTRODUCTION

The isoscalar meson  $\sigma$  with a mass near  $\rho^0$  mass has a long and still controversial history. Following the discovery in 1961 of the  $\rho$  meson in  $\pi N \rightarrow \pi \pi N$  reactions [1], large and energy-independent differences were observed [2-4] in the  $\pi^-\pi^0$  and  $\pi^-\pi^+$  angular distributions over a broad region [5-7] of the dipion mass m. The  $\pi^{-}\pi^{0}$  forward-backward asymmetry changes sign [7] as it goes through the  $\rho^-$  mass while the  $\pi^-\pi^+$  distribution shows a maximum [7] at the  $\rho^0$  mass in its large forwardbackward asymmetry. The forward-backward asymmetry  $A \sim \text{Re}\rho_{0s}$  measures S- and P-wave interference. The behavior of  $A(\pi^{-}\pi^{0})$  corresponds to a large and resonating P wave interfering with a nonresonant and essentially real S wave. Two possible explanations were proposed [4] to account for the behavior of the asymmetry  $A(\pi^{-}\pi^{+})$ : (1) a strong isoscalar S wave or (2) the existence of an isoscalar resonance  $0^{++}$  near  $\rho^{0}$  mass. Early models of pion production invoking both alternatives described well the  $\pi^-\pi^0$  and  $\pi^-\pi^+$  unpolarized data [8] and later [9] the data on a polarized target in  $\pi^- p_{\uparrow} \rightarrow \pi^- \pi^+ n$ . Experiments on unpolarized targets do not measure enough observables to isolate the two contributing S-wave amplitudes, and thus are not selective.

An alternative is to search for resonances in  $\pi\pi \rightarrow \pi\pi$ reactions. In the absence of colliding pion beams, model-dependent extrapolations of the unpolarized data on  $\pi N \rightarrow \pi\pi N$  into the unphysical region of -t were used to obtain estimates of the phase shifts in  $\pi\pi \rightarrow \pi\pi$  scattering. Two solutions [10–13] similar to the alternatives (1) and (2) were invariably produced for the I=0 S-wave phase shift  $\delta_0^0$  in  $\pi^-\pi^+ \rightarrow \pi^-\pi^+$ . In principle, a solution can be selected from the study of  $\pi^-\pi^+ \rightarrow \pi^0\pi^0$  where the S wave can be determined without P-wave interference. Originally, the nonresonating solution was selected [12] by a comparison with  $\pi^0\pi^0$  mass spectrum from  $\pi^- p \rightarrow \pi^0 \pi^0 n$  at 8 GeV/c (Ref. [14]). A more recent study of  $\pi^-\pi^+ \rightarrow \pi^0\pi^0$  based on measurement of  $\pi^+ p \rightarrow \pi^0 \pi^0 \Delta^{++}$  at 8 GeV/c (Ref. [15]) prefers the resonating solution of Ref. [12]. The  $\pi^0\pi^0$  mass spectrum from  $\pi^+ p \rightarrow \pi^0 \pi^0 \Delta^{++}$  at 16 GeV/c has not resolved this issue [16]. One difficulty with such comparisons is that the  $\pi^0 \pi^0$  is integrated over various intervals of momentum transfer -t, and, while there are two S-wave amplitudes in  $\pi N \rightarrow \pi \pi N$  reactions, there are five S-wave amplitudes in  $\pi N \rightarrow \pi \pi \Delta$  reactions. Moreover, the determination of the  $\pi\pi \rightarrow \pi\pi$  phase shifts from unpolarized on several enabling assumptions data relies [11-13,17-19]. As we will discuss later, these assumptions are not well supported by the results of the experiments with spin.

The truly model-independent approach to the study of the structure of S-wave pion production in  $\pi N \rightarrow \pi \pi N$  reactions is amplitude analysis, i.e., the construction of production amplitudes directly from the experimental data. The first step in this direction was taken in 1973 by Estabrooks and Martin [20]. They showed that spin-densitymatrix (SDM) elements measured in  $\pi N \rightarrow \pi \pi N$  production on an unpolarized target for dipion masses below 1000 MeV enable an amplitude analysis provided that the amplitudes with exchanges of  $A_1$  quantum number can be neglected. The next step was taken in 1978 by Lutz and Rybicky [21] who showed that measurements of the pion production on a transversely polarized target determine enough observables that a nearly complete ampli-

46 949

tude analysis is possible, including the  $A_1$ -exchange amplitudes neglected by Estabrooks and Martin. However, the solution for amplitudes has a twofold ambiguity. It thus appeared possible that the old problem of two solutions for S-wave amplitudes, one nonresonating and another showing a possible scalar state near  $\rho^0$ , may not be decided even by experiments on polarized targets. Moreover, with the  $A_1$ -exchange amplitudes included, there now were four solutions for the S-wave intensity  $I_S(m,t)$ . The selection of the solution requires difficult measurements of the recoil nucleon polarization or S-wave intensity from the  $\pi N \rightarrow \pi^0 \pi^0 N$  reaction. However for the decision concerning the existence of the  $\sigma$  meson it would be sufficient to show that all solutions show resonant or nonresonant structure.

The central claim of this work is that the measurements of  $\pi N \rightarrow \pi^+ \pi^- N$  on a polarized target provide evidence for an I = 0 0<sup>++</sup>(750) state which is solution independent.

This conclusion is based on an amplitude analysis of data from measurements of the reaction  $\pi^- p_{\uparrow} \rightarrow \pi^- \pi^+ n$  at 17.2 GeV/c in the kinematic region of small t [22,23],

$$600 \le m \le 900 \text{ MeV}$$
 with  $-t = 0.005 - 0.2 (GeV/c)^2$ 
(1.1)

and the reaction  $\pi^+ n_{\uparrow} \rightarrow \pi^+ \pi^- p$  at 5.98 and 11.85 GeV/c at larger t [24,25],

$$360 \le m \le 1040 \text{ MeV}$$
 with  $-t = 0.2 - 0.4 (\text{GeV}/c)^2$ .  
(1.2)

The method of amplitude analysis and the results for normalized nucleon transversity amplitudes in the reaction  $\pi^+ n_1 \rightarrow \pi^+ \pi^- p$  appeared earlier [24,26,27]. In this article we present our results for the moduli squared of unnormalized amplitudes for both reactions using the same method of analysis in the kinematic regions (1.1) and (1.2). We then study the mass distributions on the level of amplitudes and partial-wave cross sections. All mass distributions are normalized to 1 at their maximum value. The normalized S-wave intensities show resonantlike shapes in all solutions in both reactions at a mass around 750 MeV and with a width of 100-250 MeV. In addition, the S-wave amplitudes are approximately in phase with the dominating P-wave amplitudes in this mass region. These results provide model-independent and solution-independent evidence for the existence of an  $I = 0 0^{++}(750)$  state. The solution-independent nature of the evidence for this new scalar state has been pointed out already in the earlier presentations of the preliminary results [28-31]. This solution independence suggests that the only ambiguity in the amplitude analyses regarding the S-wave structure is the relative contribution of the S wave and P wave to the pion production process. This is a new and crucial contribution of the model-independent amplitude analysis of pion production processes to the experimental and theoretical hadron spectroscopy.

This paper is organized as follows. In Sec. II we briefly review the basic notation and describe our results for the mass dependence of unnormalized amplitudes at small momentum transfers in  $\pi^- p_1 \rightarrow \pi^- \pi^+ n$  and larger momentum transfers in  $\pi^+ n_1 \rightarrow \pi^+ \pi^- p$  corresponding to the binnings (1.1) and (1.2), respectively. In Sec. III we discuss the solution-independent evidence for an I=0 $0^{++}(750)$  state from the partial-wave intensities. Section IV is a brief note on the tests of assumptions used in phase-shift analyses of  $\pi\pi$  and  $K\pi$  scattering. In this section we also compare our results with the phase-shift analyses. In Sec. V we briefly discuss the possible constituent nature of the I=0  $0^{++}(750)$  resonance and conclude that it is best understood as the lowest mass gluonium state  $0^{++}(gg)$ . The paper closes with a summary in Sec. VI.

#### **II. AMPLITUDE ANALYSIS**

For invariant masses below 1000 MeV, the dipion system in  $\pi^+ N \rightarrow \pi^+ \pi^- N$  reactions is produced predominantly in spin states J = 0 (S wave) and J = 1 (P wave). The experiments on transversely polarized targets then yield 15 spin-density-matrix (SDM) elements describing the dimeson angular distribution. The measured SDM elements are [24,25]

$$\rho_{ss} + \rho_{00} + 2\rho_{11}, \quad \rho_{00} - \rho_{11}, \quad \rho_{1-1}, \quad (2.1a)$$

 $\operatorname{Re}\rho_{10}$ ,  $\operatorname{Re}\rho_{1s}$ ,  $\operatorname{Re}\rho_{0s}$ ,

$$\rho_{ss}^{\nu} + \rho_{00}^{\nu} + 2\rho_{11}^{\nu}, \ \rho_{00}^{\nu} - \rho_{11}^{\nu}, \ \rho_{1-1}^{\nu},$$
 (2.1b)

 $\operatorname{Re}\rho_{10}^{y}$ ,  $\operatorname{Re}\rho_{1s}^{y}$ ,  $\operatorname{Re}\rho_{0s}^{y}$ ,

 $\text{Im}\rho_{1-1}^{x}$ ,  $\text{Im}\rho_{10}^{x}$ ,  $\text{Im}\rho_{1s}^{x}$ . (2.1c)

The SDM elements (2.1a) are also measured in experiments on an unpolarized target. The observables (2.1b) and (2.1c) are determined by the transverse component of the target polarization perpendicular and parallel to the scattering plane  $\pi N \rightarrow (\pi^+ \pi^-)N$ , respectively.

For the purposes of amplitude analysis, the observables (2.1) and  $d^2\sigma/dm dt \equiv \Sigma$  can be expressed [21,26] in terms of two S-wave and six P-wave nucleon transversity amplitudes (NTA's). In our normalization

$$|S|^{2} + |\overline{S}|^{2} + |L|^{2} + |\overline{L}|^{2} + |U|^{2} + |\overline{U}|^{2} + |N|^{2} + |\overline{N}|^{2} = 1 ,$$
(2.2)

where A = S, L, U, N and  $\overline{A} = \overline{S}, \overline{L}, \overline{U}, \overline{N}$  are normalized nucleon transversity amplitudes with recoil nucleon transversity "down" and "up" relative to the scattering plane. The S-wave amplitudes are S and  $\overline{S}$ . The P-wave amplitudes  $L, \overline{L}$  have a dimeson helicity  $\lambda=0$  while the pairs  $U, \overline{U}$  and  $N, \overline{N}$  are combinations of amplitudes with helicities  $\lambda = \pm 1$  and have opposite t-channel-exchange naturality [24-27]. In  $\pi N \to \pi^+ \pi^- N$  reactions the unnatural exchange amplitudes  $S, \overline{S}, L, \overline{L}$  and  $U, \overline{U}$  receive contributions from " $\pi$ " and " $A_1$ " exchanges. The natural exchange amplitudes  $N, \overline{N}$  are dominated by " $A_2$ " exchange.

The SDM elements (2.1a) and (2.1b) organize themselves into two independent groups corresponding to their sum and differences [26]. The first group (sum) involves the following moduli of normalized NTA's and cosines of their relative phases:

$$|S|^{2}, |L|^{2}, |U|^{2}, |\bar{N}|^{2}, \cos(\gamma_{SL}), \cos(\gamma_{SU}), \cos(\gamma_{LU}).$$
(2.3)

The second group (differences) involves amplitudes of opposite transversity:

$$\begin{aligned} |\overline{S}|^2, \quad |\overline{L}|^2, \quad |\overline{U}|^2, \quad |N|^2, \\ \cos(\overline{\gamma}_{SL}), \quad \cos(\overline{\gamma}_{SU}), \quad \cos(\overline{\gamma}_{LU}). \end{aligned}$$
(2.4)

In each (m,t) bin amplitude analysis yields two analytical solutions for amplitudes (2.3) and (2.4). The technical details are given in Refs. [21,22,26].

One of the solutions provides *P*-wave moduli which are always larger than the moduli in the second solution. We label the larger and smaller solutions as solution 1 and solution 2, respectively. In some (m,t) bins we obtain two complex conjugate solutions for the moduli. In such a case we accept their real part as an approximate double solution which we label solution 0. In the following figures the results for solutions 1 and 2 are represented by the symbols  $\dagger$  and  $\phi$ , respectively. Solution 0 is represented by the symbol  $\bigcirc$  without error bars. The errors on solution 0 are comparable to nearby real solutions.

The results for normalized NTA in the reaction  $\pi^+ n_{\uparrow} \rightarrow \pi^+ \pi^- p$  at 5.98 and 11.85 GeV/c were reported in Ref. [26]. In this section we present the mass dependence of unnormalized amplitudes  $|A|^2 \Sigma$  and  $|\overline{A}|^2 \Sigma$ , A = S, L, U, N. For comparison we present also the results  $\pi^- p_{\uparrow} \rightarrow \pi^- \pi^+ n$  at 17.2 GeV/c. For  $\Sigma$  in  $\pi^+ n \rightarrow \pi^+ \pi^- p$  we used the estimate of  $d^2\sigma / dm dt$  given in Ref. [25]. For  $\Sigma$  in the reaction  $\pi^- p \rightarrow \pi^- \pi^+ n$  we used  $d^2\sigma / dm dt$  given in Ref. [32].

The results at small t in the kinematic region  $0.005 \le -t \le 0.2 \, (\text{GeV}/c)^2$  from reaction  $\pi^- p_{\uparrow} \rightarrow \pi^- \pi^+ n$  are given in Fig. 1. The results at larger t in the region  $0.2 \le -t \le 0.4 \, (\text{GeV}/c)^2$  for the reaction  $\pi^+ n_{\uparrow} \rightarrow \pi^+ \pi^- p$  at 5.98 GeV/c are presented in Fig. 2. The results at 11.85 GeV/c are similar. All presented results are in the t-channel helicity frame of the dipion system.

We first notice that the two solutions for the *P*-wave moduli are very similar. The principal difference between the two solutions are the *S*-wave moduli and the cosines of relative phases, and even there the differences are not pronounced.

The pion production process is clearly dominated by the amplitude  $|\bar{L}|^2 \Sigma$  at both momentum transfers. While the amplitudes  $|\bar{L}|^2 \Sigma$  and  $|L|^2 \Sigma$  show the expected resonance structure due to  $\rho^0$  resonance, the amplitudes  $|\bar{N}|^2 \Sigma$  and  $|U|^2 \Sigma$  appear broad and structureless, even though they resonate. At the lowest energy of 5.98 GeV/c, also the amplitude  $|\bar{U}|^2 \Sigma$  appears broad structured. Naively, one would expect all *P*-wave moduli to show a clear  $\rho^0$  peak.

At small t, solution 1 for amplitudes  $|\bar{S}|^2 \Sigma$  and  $|S|^2 \Sigma$ has structure suggestive of a resonance while solution 2 is broad structured. At the larger momentum transfers  $0.2 \le -t \le 0.4$  (GeV/c)<sup>2</sup>, solution 2 is appreciably larger and more suggestive of a resonance contribution in the S wave. This change is indicative for a significant t dependence of mass distributions, and was confirmed by a recent study of the t evolution of m dependence of moduli squared of NTA's over a broader interval of momentum transfers [33].

From the point of view of evidence for a scalar state near the  $\rho^0$  mass, important information is provided by the relative phases  $\overline{\gamma}_{SL}$  and  $\gamma_{SL}$  between the pairs of amplitudes  $\overline{S}, \overline{L}$  and S, L, respectively. At both momentum transfers the amplitudes  $\overline{L}$  and L dominate and show a clear resonant behavior. At small t, solution 1 gives amplitudes  $\overline{S}$  and S in phase with  $\overline{L}$  and L, respectively. In solution 2, there is a phase difference with  $\overline{\gamma}_{SL}$  and  $\gamma_{SL}$ ranging from 0° to about 50° in the mass region from 600 to 900 MeV. At the larger t, both solutions show some small phase difference which is constant or varying slowly in the mass region of  $\rho^0$ . If the S wave were entirely a structureless background, then one would expect a large and rapid variation of these relative phases. The observed relative phases are consistent with a constant or slowly varying phase difference in both solutions over the  $\rho^0$  mass region. Such behavior of relative phases strongly supports the view that the resonant structure in the Swave near the  $\rho^0$  mass is solution independent.

#### **III. PARTIAL-WAVE INTENSITIES**

The solutions for amplitudes with opposite transversities (2.3) and (2.4) are entirely independent. We can denote the two solutions for normalized nucleon transversity amplitudes as A(i) and  $\overline{A}(j)$  with i=1,2and j=1,2. Because the moduli squared in (2.3) and (2.4) are independent, there is a fourfold ambiguity in the partial-wave intensities. Using the indices *i* and *j* to label the four solutions, we get

$$I_{A}(i,j) = [|A(i)|^{2} + |\overline{A}(j)|^{2}]\Sigma, \qquad (3.1)$$

where A = S, L, U, N and  $\Sigma$  is the reaction cross section. The partial-wave intensities (3.1) are defined in the physical region and depend on the dipion mass *m* and momentum transfer *t*, i.e.,  $I_A = I_A(m, t)$ .

To study the similarities in shapes of partial-wave intensities in all solutions we work with partial-wave intensities  $R_A$  normalized to 1 at their maximum value at a given t. We define

$$R_{A}(i,j) = I_{A}(i,j) / I_{A}(i,j)_{\max}$$
, (3.2)

where

$$I_A(i,j)_{\max} = \max_{m} I_A(i,j) \tag{3.3}$$

is the maximum value over *m* at fixed *t*. For each pair of amplitudes *A* and  $\overline{A}$ , A = S, L, U, N there are four normalized resonant shapes  $R_A(i, j)$  which depend on *m* and *t*. Because of relatively large errors on  $R_S$  we have not performed fits with the Breit-Wigner formula. We estimate the width of each shape by its width at one-half the height of the normalized intensity.

The ratios  $R_s$  and  $R_L$  for dimeson helicity states  $\lambda = 0$ are shown in Figs. 3 and 4 at 17.2 and 5.98 GeV/c, respectively. In Fig. 3 we notice the similarity of resonant



FIG. 1. The mass dependence of moduli squared of S-wave and P-wave nucleon transversity amplitudes and cosines of their relative phases at momentum transfers -t = 0.005 - 0.2 (GeV/c)<sup>2</sup> in reaction  $\pi^- p \rightarrow \pi^- \pi^+ n$  at 17.2 GeV/c. The symbols  $\ddagger$ ,  $\blacklozenge$ , and  $\bigcirc$  denote solution 1, solution 2, and solution 0 (real part of complex solution), respectively.



FIG. 2. The mass dependence of moduli squared of S-wave and P-wave nucleon transversity amplitudes and cosines of their relative phases at momentum transfers -t = 0.2 - 0.4 (GeV/c)<sup>2</sup> in reaction  $\pi^+ n \rightarrow \pi^+ \pi^- p$  at 5.98 GeV/c. The symbols are as in Fig. 1.

shapes in the intensities  $I_L$  corresponding to the  $\rho^0$  resonance with a width of 160 MeV. The resonant shapes  $R_L$  are smooth despite some points coming from the complex solutions. This suggests that we can consider with the same confidence also the structures in the S-wave ratios  $R_S$ . All ratios  $R_S$  exhibit a resonantlike shape at 750 or 730 MeV but there are differences in their widths. The solutions  $R_S(1,1)$  and  $R_S(2,1)$  have a clear resonant structure with an estimated width about 100–130 MeV and 160–180 MeV, respectively. The solutions  $R_S(1,2)$  and  $R_S(2,2)$  show broader structures but they are still compatible with a resonance with a width of about 200 and 250 MeV, respectively.

In Fig. 4, the results at 5.98 GeV/c show again nearly identical and smooth ratios  $R_L$  which at these larger momentum transfers show a width of 200 MeV. The  $\rho^0$  resonance in  $R_L$  has an asymmetric shape with a maximum value at 807 MeV. The S-wave ratios  $R_S$  show distinct structures at 750 or 730 MeV in all solutions and their shapes indicate the presence of an S-wave resonance at this mass. Because of large errors it is more difficult to estimate the width. The solutions  $R_S(1,1)$  and  $R_S(1,2)$  have a narrow width of about 70–160 MeV. The width of  $R_S(2,1)$  and  $R_S(2,2)$  is about 150–230 MeV. We note that the solution  $R_S(2,2)$  at these larger momentum transfers shows a more distinct and narrower resonant

shape than  $R_S(2,2)$  at the small momentum transfer shown in Fig. 3. The comparison of the ratios  $R_S$  and  $R_L$  in Figs. 3 and 4 suggests some t dependence of the widths of resonant shapes.

It is interesting to compare our results for  $R_S$  with the ratio of S-wave intensities in  $\pi^+\pi^- \rightarrow \pi^0\pi^0$  estimated recently [15] by extrapolation in the reaction  $\pi^+p \rightarrow \pi^0\pi^0\Delta^{++}$  at 8.0 GeV. The comparison is shown in Fig. 5 at 17.2, 5.98, and 11.85 GeV/c. The ratio  $R_S(\pi^+\pi^- \rightarrow \pi^0\pi^0)$  exhibits a broad structure. At all energies the narrower resonant shapes  $R_S(1,1)$  are within the structure in  $R_S(\pi^+\pi^- \rightarrow \pi^0\pi^0)$ . The solution  $R_S(2,2)$  agrees well with  $R_S(\pi^+\pi^- \rightarrow \pi^0\pi^0)$  only at small momentum transfers (at 17.2 GeV/c). At larger momentum transfers the solution  $R_S(2,2)$  is narrower. Since the solutions  $R_S(i,j)$  are in the physical region of momentum transfers t, we cannot use their comparison with  $R_S(\pi^+\pi^- \rightarrow \pi^0\pi^0)$  to select a unique solution. Moreover, there are five S-wave amplitudes in the reaction  $\pi^+p \rightarrow \pi^0\pi^0\Delta^{++}$  in contrast with two S-wave amplitudes in  $\pi N \rightarrow \pi^+\pi^- N$  reactions.

For completeness we show in Fig. 6 the ratios  $R_U$  and  $R_N$  for the amplitudes U and N corresponding to dipion helicities  $\lambda = \pm 1$ . At 17.2 GeV/c, the  $\rho^0$  resonance shape in  $R_U(2,2)$  and  $R_N(2,2)$  peaks at 750 MeV while in  $R_L$  with  $\lambda = 0$  it peaks at 770 MeV. We conclude that the ap-



FIG. 3. The mass dependence of S-wave and P-wave partial-wave intensities scaled to 1 at maximum value showing the spectral shapes for dimeson helicity  $\lambda = 0$  at momentum transfers -t = 0.005 - 0.2 (GeV/c)<sup>2</sup> in reaction  $\pi^- p \rightarrow \pi^- \pi^+ n$  at 17.2 GeV/c. The symbols are as in Fig. 1. The line is to guide the eye.



FIG. 4. The mass dependence of S-wave and P-wave partial-wave intensities scaled to 1 at maximum value showing the spectral shapes for dimeson helicity  $\lambda = 0$  at momentum transfers  $-t = 0.2 - 0.4 (\text{GeV}/c)^2$  in reaction  $\pi^+ n \rightarrow \pi^+ \pi^- p$  at 5.98 GeV/c. The symbols are as in Fig. 1. The line is to guide the eye.

parent position of the  $\rho^0$  resonance depends on the dimeson helicity in this solution. There is some dependence of the width of resonant shapes in  $R_U$  and  $R_N$  on the solution. The width of  $R_U$  is ~150 MeV for the solution  $R_U(1,1)$  and about 125 MeV for  $R_U(2,2)$ . Similarly, the apparent width of  $R_N$  is about 200 MeV for  $R_N(1,1)$  and 160 MeV for  $R_N(2,2)$ . This decrease of the width in  $R_U(2,2)$  and  $R_N(2,2)$  correlates with the increase of the apparent width in  $R_S(2,2)$ .

The results at larger momentum transfer at 5.98 GeV/c show a somewhat asymmetric shape of  $R_U$ . The ratios  $R_N$  for natural exchange combinations show smooth resonant shapes. For the solution  $R_N(1,1)$  the apparent position of the  $\rho^0$  peak is at 750 MeV and its width is about 150 MeV. The solution  $R_N(2,2)$  peaks at 770 MeV and its width is also about 150 MeV. With the  $\rho^0$  peak at 807 MeV in the ratio  $R_L$  we again find that the apparent position of the  $\rho^0$  resonance depends on its helicity state.

# IV. A NOTE ON PHASE-SHIFT ANALYSES OF $\pi\pi \rightarrow \pi\pi$ REACTIONS

It is the similarity of the two resonating S-wave solutions in which our amplitude analysis in the physical region of the process  $\pi N \rightarrow \pi^+ \pi^- N$  differs from the conclusions of the phase-shift analyses of pion-pion scattering. We recall that those analyses produced two solutions for the S-wave phase shifts: one which resonates at 750 MeV and another one with a broad nonresonating structure [12].

Of course, there are no actual measurements of pionpion scattering. The phase-shift analyses used  $\pi\pi$ scattering "data" obtained from measurements of  $\pi N \rightarrow \pi^+ \pi^- N$  on unpolarized targets together with several necessary enabling assumptions: (i) the factorization of the dependence of production amplitudes on the dipion mass and momentum transfer t (Refs. [17,18]); (ii) the vanishing of nucleon helicity nonflip amplitudes with unnatural " $A_1$ " exchange [11-13,19]; (iii) the proportionality of the S-wave and P-wave amplitudes with dimeson helicity  $\lambda = 0$  (Refs. [11,12]). These assumptions lead to predictions which are testable in experiments with polarized targets. Assumption (i) leads to the prediction that the moduli  $|A|^2$  and  $|\overline{A}|^2$  of nucleon transversity amplitudes with opposite transversities have the same resonant shape independent of t. This is not observed in the amplitude analyses based on polarized data [26]. Assumption (ii) implies

$$\rho_{ss}^{y} + \rho_{00}^{y} + 2\rho_{11}^{y} = -2(\rho_{00}^{y} - \rho_{11}^{y}) = +2\rho_{1-1}^{y} ,$$
  

$$\mathbf{R}\rho_{10}^{y} = \mathbf{R}\rho_{1s}^{y} = \mathbf{R}\rho_{0s}^{y} = 0 .$$
(4.1)

The data for polarized SDM elements clearly rule out these predictions [25]. Assumption (iii) is also ruled out by the explicit results of the amplitude analyses [26].

Another consequence of assumption (ii) is that  $A = -\overline{A}$  for nucleon transversity amplitudes with unnatural exchange [26]. This means  $|A|^2 \Sigma = |\overline{A}|^2 \Sigma$  for A = S, L, U. Figures 1 and 2 show clearly that this is not the case and that  $A_1$ -exchange contributions to the nucleon transversity amplitudes are large and nontrivial, in

particular to  $|\bar{L}|^2 \Sigma$  and  $|L|^2 \Sigma$ . In phase-shift analyses of  $\pi\pi \rightarrow \pi\pi$  scattering assumption (ii) was made [12,13] to enable the extrapolation of  $\pi N \rightarrow \pi^+ \pi^- N$  data to the pion pole. The presence of large  $A_1$  exchange means that these extrapolations, and consequently also the  $\pi\pi$  phase shifts, are only approximate.

A more general form of assumption (ii) was suggested by Ochs in Ref. [19]. Consider nucleon helicity amplitudes  $A_n$  where A = S, L, U and n = 0, 1 is the nucleon



FIG. 5. The normalized S-wave partial-wave intensities at 17.2, 5.98, and 11.85 GeV/c compared to the normalized S-wave intensity  $R_S$  ( $\pi^+\pi^- \rightarrow \pi^0\pi^0$ ) from Ref. [15]. The S-wave partial-wave intensities are normalized to 1 at their maximum value. The lines are to guide the eye.

helicity flip. Instead of  $A_0=0$  required by assumption (ii), Ochs proposed a proportionality  $A_0=cA_1$  where c is a factor common to A=S,L,U amplitudes. A phaseshift analysis based on Ochs' assumption differs from the analysis using  $A_0\equiv 0$  by a normalization factor  $\sqrt{1+|c|^2}$ for the magnitudes of the amplitudes. The behavior of phase shifts is the same in both analyses. Ochs' assumption leads to testable predictions for polarized SDM elements. One can show that it implies relations

$$0 < \frac{\mathbf{Re}\rho_{10} - \mathbf{Re}\rho_{10}^{y}}{\mathbf{Re}\rho_{10} + \mathbf{Re}\rho_{10}^{y}} = \frac{\mathbf{Re}\rho_{1s} - \mathbf{Re}\rho_{1s}^{y}}{\mathbf{Re}\rho_{1s} + \mathbf{Re}\rho_{1s}^{y}} = \frac{\mathbf{Re}\rho_{0s} - \mathbf{Re}\rho_{0s}^{y}}{\mathbf{Re}\rho_{0s} + \mathbf{Re}\rho_{0s}^{y}} = r$$
(4.2)

where

$$r = \frac{|c-i|^2}{|c+i|^2}$$

and

$$q = \frac{A - 2\rho_{1-1}^{y}}{1 - 2\rho_{1-1}} = \frac{A + 2(\rho_{00}^{y} - \rho_{11}^{y})}{1 + 2(\rho_{00} - \rho_{11})} = \frac{(\rho_{00}^{y} - \rho_{11}^{y}) + \rho_{1-1}^{y}}{(\rho_{00} - \rho_{11}) + \rho_{1-1}}$$
$$= \frac{\operatorname{Re}\rho_{10}^{y}}{\operatorname{Re}\rho_{10}} = \frac{\operatorname{Re}\rho_{1s}^{y}}{\operatorname{Re}\rho_{1s}} = \frac{\operatorname{Re}\rho_{0s}^{y}}{\operatorname{Re}\rho_{0s}}$$
(4.3)

where



Relations (4.2) and (4.3) are not well supported by the data on polarized targets in particular at larger momentum transfers [27].

For a comparison of our S-wave intensity  $R_S$  with the results from conventional phase-shift analyses of  $\pi\pi$ scattering, we refer the reader to examine Fig. 19 in Ref. [15]. This figure shows S-wave intensities (normalized to 1 at maximum value) from several conventional phaseshift analyses of the  $\pi^+\pi^- \rightarrow \pi^+\pi^-$  reaction. There are three broad and structureless solutions (labeled A, B, C) and one resonant solution (labeled D) which resembles strongly our results shown in Fig. 5. The curves A and Dcorrespond to solutions 1 and 2 in the analysis of Estabrooks and Martin in Ref. [12]. As discussed in the Introduction, the resonant solution 2 (curve D in Fig. 19 of Ref. [15]) has been conventionally rejected.

In Fig. 7 we show the cosine of the relative phase between the I = 0 S wave and the dominant P wave for the two solutions for partial waves in the phase-shift analysis of Estabrooks and Martin taken from Ref. [12]. Solution 1 shows a large change of the cosine in the  $\rho^0$  mass range as expected from the interference of a structureless and broad S wave with a resonating P wave. Solution 2 shows essentially a constant cosine of the relative phase in the



FIG. 6. The mass dependence of *P*-wave unnatural and natural exchange partial-wave intensities scaled to 1 at maximum value showing the  $\rho^0$  spectral shapes for dimeson helicities  $\lambda = \pm 1$ . The solutions (1,2) and (2,1) are similar.



FIG. 7. Two solutions for the cosine of relative phase between S- and P-wave partial-wave amplitudes from phase-shift analysis of  $\pi^+\pi^- \rightarrow \pi^+\pi^-$  by Estabrooks and Martin [12].

whole mass region considered, and corresponds to a resonant S-wave intensity (curve D in Fig. 19 of Ref. [15]). When we compare the cosines in Fig. 7 with  $\cos\overline{\gamma}_{SL}$  and  $\cos\gamma_{SL}$  in Fig. 1, we notice that none of our solutions behaves like the nonresonant solution 1 in Fig. 7. Our solution 1 for  $\cos\overline{\gamma}_{SL}$  and  $\cos\gamma_{SL}$  at 17.2 GeV/c is very similar to solution 2 of Estabrooks and Martin. Our solution 2 for  $\cos\overline{\gamma}_{SL}$  and  $\cos\gamma_{SL}$  in Fig. 1 changes slowly in the  $\rho^0$  resonance region and is thus compatible with a resonant S wave.

At this point we remind the reader that our results are in the physical region of  $\pi^- p \rightarrow \pi^- \pi^+ n$  and  $\pi^+ n \rightarrow \pi^+ \pi^- p$  reactions. In making the above comparisons we have neglected the off-mass-shell dependence of  $\pi \pi$  scattering amplitudes. Also, our results contain  $A_1$ exchange contributions which were neglected in all phase-shift analyses. Finally, our S wave includes a small isospin I = 2 contribution which we cannot isolate.

The principal reason for the discrepancy between our results and the conventional results for  $R_S$  (the curves A, B, C in Fig. 19 of Ref. [15]) is the existence of large and nontrivial  $A_1$ -exchange contribution to the nucleon helicity nonflip amplitudes  $S_0$ ,  $L_0$ , and  $U_0$ . The  $A_1$ exchange contribution is particularly large in the P-wave amplitude  $L_0$  which is responsible for the large differences between  $|\overline{L}|^2 \Sigma$  and  $|L|^2 \Sigma$ . The large amplitude  $L_0$  is also responsible for distortions in the S-wave amplitude in phase-shift analyses of  $\pi\pi$  scattering, leading to the observed discrepancies between our results and the conventional results for the S-wave intensity  $R_S$ . Since there is no P wave in  $\pi^+\pi^- \rightarrow \pi^0\pi^0$  reaction to distort the S-wave amplitude in phase-shift analysis, our results for the S-wave intensity  $R_S$  are in a reasonable agreement with the results of Cason et al. (Fig. 5).

We conclude that the existing phase-shift analyses of  $\pi\pi \rightarrow \pi\pi$  and  $K\pi \rightarrow K\pi$  reactions do not contain the information provided by the measurements of  $\pi N \rightarrow \pi^+\pi^- N$  and  $KN \rightarrow K\pi N$  reactions on polarized targets. We may expect that phase-shift analyses which do take into account the information from the experiments with polarized targets would yield results for the S-wave structure in better agreement with the results from amplitude analyses in the physical region of t.

### V. CONSTITUENT STRUCTURE OF THE $I = 0.0^{++}(750)$ STATE

In the usual quark model meson resonances are  $q\bar{q}$  states. The mass of  $0^{++}(750)$  is too low for it to be a  $q\bar{q}$  state. The mass M of the  $q\bar{q}$  state increases with its angular momentum L as  $M = M_0(2n + L)$  where n is the degree of radial excitation. The lowest mass scalar mesons are  ${}^{3}P_0$  states with masses expected to be around 1000 MeV or higher. The established scalar mesons seem to populate this  $0^{++}(q\bar{q})$  nonet.

It was suggested that  $0^{++}(700)$  could be a four-quark  $q\bar{q}q\bar{q}$  scalar state in the MIT bag model [34]. However, more detailed studies of  $q\bar{q}q\bar{q}$  systems conclude that pure multiquark hadrons do not exist [35,36] with  $\pi^+\pi^-$  decay [37]. We can also exclude the possibility that  $0^{++}(750)$  is a hybrid state  $q\bar{q}g$ . The lowest mass hybrid state must be a  $0^{-+}$  or  $1^{-+}$  state. Calculations based on bag models, QCD sum rules, lattice QCD, and a string model all estimate [38] the masses of  $0^{++}(q\bar{q}g)$  states to be above 1500 MeV.

Lattice QCD calculations by several groups [39-42] concluded that the gluonium ground state  $0^{++}(gg)$  has a mass near the  $\rho^0$  meson:  $740\pm40$  MeV. More recent calculations [43] suggest that the mass of  $0^{++}$  gluonium is above 1000 MeV. However, the inclusion of quark flavors in the calculations could lower the gluonium mass substantially [44]. Recent work [45] using a version of QCD with two colors and four light flavors finds that the  $0^{++}(gg)$  gluonium mass is 850 MeV. This is a mass region of great phenomenological interest as it is accessible by amplitude analyses of  $\pi N \rightarrow \pi^+\pi^-N$  reactions on polarized targets.

On the basis of the above discussion we propose to identify the I = 0 0<sup>++</sup>(750) meson with the 0<sup>++</sup>(gg) gluonium ground state. The 100-250 MeV width of the 0<sup>++</sup>(750) state is consistent with glueball phenomenology [46].

Since the observed and calculated masses of the 0<sup>++</sup> state are so close we also conclude that the 0<sup>++</sup>(750) state has no  $q\bar{q}$  content or only a weak  $q\bar{q}$  mixing. Since gluons do not couple directly to photons we expect 0<sup>++</sup>(750) not to appear in reaction  $\gamma\gamma \rightarrow \pi^+\pi^-$ . This conclusion is supported by the PLUTO and DELCO data [47,48]. However, the more recent DM1/2 data [49,50] show an excess over the Born term expectation that is attributed to the formation of a broad scalar resonance 0<sup>++</sup>(700) with a two-photon width of (10±6) MeV. This would suggest some  $q\bar{q}$  component in the 0<sup>++</sup>(750) state. The most recent results [51] are on  $\gamma\gamma \rightarrow \pi^0\pi^0$  which show no evidence for a scalar state near 750 MeV. The reaction  $pp \rightarrow pp \pi^+ \pi^-$  was measured [52] at the CERN Intersecting Storage Rings (ISR) in a search for scalar gluonium. The structures at  $m(\pi^+\pi^-)$  near 750 MeV reported in the moments H(11) and H(31) are consistent with  $0^{++}(750)$  and  $\rho^0(770)$  interference. The experiment does not separate the S- and P-wave amplitudes and thus it is not conclusive.

The anomalous energy dependence of pp and np elastic polarizations and the departure from mirror symmetry in  $\pi N$  elastic polarizations at intermediate energies require a low-lying Regge trajectory [53,54] corresponding to  $0^{++}(750)$ . These anomalous structures in the polarization data may have been the first evidence for a gluonium exchange in two-body reactions.

# VI. SUMMARY

The measurements of reactions  $\pi^- p_{\uparrow} \rightarrow \pi^- \pi^+ n$  at 17.2 GeV/c and  $\pi^+ n_{\uparrow} \rightarrow \pi^+ \pi^- p$  at 5.98 and 11.85 GeV/c on polarized targets enable a model-independent amplitude analysis for dimeson masses below 1000 MeV where S-wave and P-wave contributions dominate the pion production process. We studied the dependence on the dipion mass of moduli squared of S- and P-wave unnormalized nucleon transversity amplitudes and cosines of certain relative phases at small momentum transfer -t=0.005-0.2 (GeV/c)<sup>2</sup> in the reaction  $\pi^- p_{\uparrow} \rightarrow \pi^- \pi^+ n$  and at a larger momentum transfer -t=0.2-0.4 (GeV/c)<sup>2</sup> in the reaction  $\pi^+ n_{\uparrow} \rightarrow \pi^+ \pi^- p$ . The amplitude analysis yields two similar solutions for the moduli and cosines, and partial-wave intensities with a fourfold ambiguity.

The principal finding claimed in this report is solution-independent evidence for a new scalar state I = 0 $0^{++}(750)$ . At both momentum transfers, both solutions yield small and nearly constant relative phases between the S-wave amplitudes S and  $\overline{S}$  and the dominant resonating amplitudes L and  $\overline{L}$ . This suggestion of resonant Swave behavior is confirmed by the behavior of the S-wave partial-wave intensities  $R_S$  normalized to 1 at their maximum values. For all four solutions, the normalized intensities  $R_S$  show resonant structures at masses 730 or 750 MeV. The width of this resonant structure depends somewhat on the solution and varies within a range of 100-250 MeV. Only the width of the solution  $R_s(2,2)$  at lower momentum transfers may be broader than 150 MeV.

We have proposed to identify this new scalar state with the lowest mass scalar gluonium  $0^{++}(gg)$  predicted by QCD. This proposal is supported by cross-section data on reactions  $\gamma\gamma \rightarrow \pi^+\pi^-$  and  $\gamma\gamma \rightarrow \pi^0\pi^0$ . The apparent position of the  $\rho^0$  resonance in the nor-

The apparent position of the  $\rho^0$  resonance in the normalized partial-wave intensity  $R_L$  corresponding to dipion helicity  $\lambda=0$  is 770 and 807 MeV at the small and larger momentum transfers, respectively. This contrasts with the lower apparent position of  $\rho^0$  resonance in the intensities  $R_U$  and  $R_N$  corresponding to dipion helicities  $\lambda=\pm 1$ . At 17.2 GeV/c, the solutions  $R_U(2,2)$  and  $R_N(2,2)$  peak at 750 MeV. At 5.98 GeV/c, the intensity  $R_N$  peaks at 750–770 MeV. These results are the first evidence for the possible dependence of the apparent position of a resonance on its helicity state. The commonly assumed degeneracy of resonance mass with respect to its helicity state appears to be broken in these solutions. If this effect is confirmed by further experimental studies, it will have a profound influence on hadron spectroscopy and on our understanding of what is a hadron resonance.

Experiments with polarized targets have opened a whole new approach to experimental hadron spectroscopy by making accessible the study of hadron production on the level of production amplitudes. We may expect that this new field of amplitude spectroscopy will be further developed at the new proposed advanced hadron facilities [55-66].

## ACKNOWLEDGMENTS

We thank J.G.H. de Groot for providing us with numerical results of Ref. [22]. One of us (M.S.) would like to thank N. M. Cason, S. U. Chung, R. Longacre, B. Margolis, R. Mendel, H. Navelet, and J. Soffer for useful discussions. This work was supported by Fonds pour la Formation de Chercheurs et l'Aide à la Recherche (FCAR), Ministère de l'Education du Québec, Canada, and by Commissariat à l'Energie Atomique, Saclay, France.

- [1] A. R. Erwin et al., Phys. Rev. Lett. 6, 628 (1961).
- [2] E. Pickup et al., Phys. Rev. Lett. 9, 170 (1962).
- [3] J. Alitti et al., Nuovo Cimento 29, 515 (1963).
- [4] V. Hagopian and W. Selove, Phys. Rev. Lett. 10, 533 (1963).
- [5] Z. G. T. Guiragossian, Phys. Rev. Lett. 11, 85 (1963).
- [6] L. Boudar et al., Phys. Lett. 5, 153 (1963).
- [7] J. P. Baton et al., Nuovo Cimento 35, 713 (1965).
- [8] M. M. Islam and R. Pinon, Phys. Rev. Lett. 12, 310 (1964);
   S. H. Patil, *ibid.* 13, 261 (1964); L. Durand III and Y. T. Chiu, *ibid.* 14, 329 (1965).
- [9] J. T. Donohue and Y. Leroyer, Nucl. Phys. B158, 123 (1979).

- [10] G. Wolf, Phys. Lett. 19, 328 (1965); W. D. Walker et al., Phys. Rev. Lett. 18, 630 (1967).
- [11] L. J. Gutay et al., Nucl. Phys. B12, 31 (1969); J. P. Baton et al., Phys. Lett. 33B, 528 (1970).
- [12] P. Estabrooks *et al.*, in  $\pi$ - $\pi$  Scattering—1973, Proceedings of the International Conference on Scattering and Associated Topics, Tallahassee, edited by D. K. Williams and V. Hagopian, AIP Conf. Proc. No. 13 (AIP, New York, 1973), p. 37.
- [13] B. Hyams et al., Nucl. Phys. B64, 134 (1973); P. Estabrooks and A. D. Martin, *ibid.* B79, 301 (1974).
- [14] W. D. Apel et al., Phys. Lett. 41B, 542 (1972).
- [15] N. M. Cason et al., Phys. Rev. D 28, 1586 (1983).

- [16] R. K. Clark et al., Phys. Rev. D 32, 1061 (1985).
- [17] P. E. Schlein, Phys. Rev. Lett. 19, 1052 (1967).
- [18] A. B. Wicklund et al., Phys. Rev. D 17, 1197 (1978).
- [19] W. Ochs, in Nucleon-Nucleon Interactions—1977 (Vancouver), Proceedings of the Second International Conference on Nucleon-Nucleon Interactions, edited by H. Fearing, D. Measday, and A. Strathdee, AIP Conf. Proc. No. 41 (AIP, New York, 1978), p. 326.
- [20] P. Estabrooks and A. D. Martin, Phys. Lett. 41B, 350 (1973).
- [21] G. Lutz and K. Rybicki, Max Planck Institute, Munich, Internal Report No. MPI-PAE/Exp. EI.75, 1978 (unpublished).
- [22] H. Becker et al., Nucl. Phys. B150, 301 (1979).
- [23] H. Becker et al., Nucl. Phys. B151, 46 (1979).
- [24] A. de Lesquen et al., CEN-Saclay Internal Report DPhPE No. 82-01, 1982 (unpublished).
- [25] A. de Lesquen et al., Phys. Rev. D 32, 21 (1985).
- [26] M. Svec, A de Lesquen, and L. van Rossum, Phys. Rev. D 45, 55 (1992).
- [27] A. de Lesquen et al., Phys. Rev. D 39, 21 (1989).
- [28] M. Svec, in Proceedings of the XXII International Conference on High Energy Physics, Leipzig, East Germany, 1984, edited by A. Meyer and E. Wieczorek (Akademie der Wissenschaften der DDR, Zeuthen, 1984).
- [29] M. Svec, J. Phys. (Paris) Colloq. 46, C2-281 (1985).
- [30] M. Svec, in *Hadron Spectroscopy*—1985, Proceedings of the International Conference on Hadron Spectroscopy, College Park, Maryland, edited by S. Oneda, AIP Conf. Proc. No. 132 (AIP, New York, 1985), p. 68.
- [31] In Refs. [28-30] the amplitude analysis of  $\pi^- p \rightarrow \pi^- \pi^+ n$  at 17.2 GeV/c is in error. A numerical factor  $\epsilon_M$  defined in Refs. [21-23] was mistakenly taken to be equal to 1 for all M. This error does not affect the results of analysis with the Saclay data at 5.98 and 11.85 GeV/c. One of us (M.S.) wishes to thank R. Longacre for pointing out this numerical error.
- [32] G. Grayer et al., Nucl. Phys. B75, 189 (1974).
- [33] M. Svec, A. de Lesquen, and L. van Rossum, Phys. Rev. D 42, 934 (1990).
- [34] R. J. Jaffe, Phys. Rev. D 15, 267 (1977).
- [35] R. P. Bickerstaff and B. H. McKellar, Z. Phys. C 16, 171 (1982).
- [36] J. Weinstein and N. Isgur, Phys. Rev. D 27, 588 (1983).
- [37] B. A. Liu and K. F. Liu, Phys. Rev. D 30, 613 (1984).
- [38] F. E. Close, Nucl. Phys. A416, 55c (1984).
- [39] K. Ishikawa et al., Phys. Lett. 116B, 429 (1982).
- [40] K. Ishikawa et al., Z. Phys. C 19, 327 (1983); 21, 167

(1983).

- [41] B. Berg and A. Billoire, Nucl. Phys. B221, 109 (1983).
- [42] W. H. Hamber and M. Urs Heller, Phys. Rev. D 29, 928 (1984).
- [43] R. Gupta et al., Phys. Rev. D 43, 2301 (1991).
- [44] R. Gupta (private communication).
- [45] J. B. Kogut, D. K. Sinclair, and M. Teper, Phys. Rev. D 44, 2869 (1991).
- [46] H. Fritzsch and O. Minkowski, Nuovo Cimento 30A, 409 (1975).
- [47] Ch. Berger et al., Z. Phys. C 26, 199 (1984).
- [48] H. Aihara et al., Phys. Rev. Lett. 57, 404 (1986).
- [49] A. Coureau et al., Nucl. Phys. B271, 1 (1986).
- [50] Z. Ajaltouni *et al.*, Phys. Lett. B **194**, 573 (1987); **197**, 565E (1987).
- [51] H. Marsiske et al., Phys. Rev. D 41, 3324 (1990).
- [52] T. Akesson et al., Phys. Lett. 133B, 268 (1983).
- [53] J. W. Dash and H. Navelet, Phys. Rev. D 13, 1940 (1976).
- [54] G. Girardi and H. Navelet, Phys. Rev. D 14, 280 (1976).
- [55] E. Vogt, Nucl. Phys. A450, 473c (1986); J. Domigo, *ibid*.
   A450, 473c (1986); G. T. Garvey, *ibid*. A450, 539c (1986).
- [56] Canadian KAON Factory (TRIUMF, Vancouver, 1985).
- [57] Physics and a Plan for a 45 GeV Facility (LANL Report No. LA-10720-MS, Los Alamos, 1986).
- [58] Proceedings of the International Conference on a European Hadron Facility, Mainz, Germany, 1986, edited by T. Walcher [Nucl. Phys. B279, 2 (1987)].
- [59] International Workshop on Hadron Facility Technology, Proceedings, Los Alamos, New Mexico, 1987, edited by H. A. Thiessen (LANL Report No. LA-11130-C, 1987).
- [60] Japanese Hadron Facility, in International Workshop on Hadron Facility Technology [59], p. 67.
- [61] Proceedings of Workshop on Hadron Spectroscopy at the Kaon Factory, Vancouver, Canada, 1989 (TRIUMF Report No. TRI-89-4, Vancouver, 1989).
- [62] Physics at Kaon, 1990, edited by D. Frekers, D. R. Gill, and J. Speth [Z. Phys. C 46, S1 (1990)].
- [63] Multi-user Hadron Spectrometer Workshop, edited by M. Comyn (TRIUMF, Vancouver, Canada, in press).
- [64] Workshop on Science at Kaon, edited by D. R. Gill (TRIUMF, Vancouver, Canada, in press).
- [65] G. Preparata, in *Intense Hadron Facilities and Antiproton Physics*, Conference Proceedings, Vol. 26, edited by T. Bressani, F. Iazzi, and G. Pauli (SIF, Bologna, 1990).
- [66] M. Svec, in High-Energy Spin Physics: Eighth International Symposium, Proceedings, Minneapolis, Minnesota, 1988, edited by Kenneth J. Heller, AIP Conf. Proc. No. 187 (AIP, New York, 1989), Vol. 2, p. 1181.